EXTENSIONS OF MEAN-FIELD DESCRIPTIONS IN DYNAMICAL PROCESSES OF NUCLEONIC DEGREES OF FREEDOM

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motivation

- extended time-dependent mean-field theories (e-tdmf)

  dissipative and irreversible behaviour

  toward equilibrium

nuclear Boltzmann transport equations

- stochastic time dependent mean-field theories (s-tdmf)

  observable dispersions

  multi-fragment formation, vaporization, …

  fluctuations, bifurcations

Boltzmann – Langevin approaches

e-tdmf description

$$i\hbar\dot{\rho} = [\mathbf{h}, \rho] + i\mathcal{K}(\rho)$$

$$\mathbf{h} \;= \frac{\mathbf{p^2}}{2m} + V^{HF}(\rho(t))$$

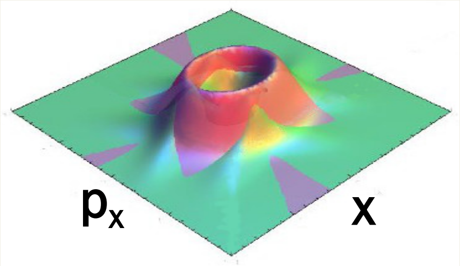
V^{HF} *:* Hartree-Fock potential, Skyme-type effective force, density and isospin dependent and with surface term (Skt5)

$$\mathcal{K}_\alpha(\rho) = \sum_{\beta,\gamma,\delta} W_{\alpha\beta\gamma\delta}\, [\rho_\gamma\rho_\delta(1-\rho_\alpha)(1-\rho_\beta) - \rho_\alpha\rho_\beta(1-\rho_\gamma)(1-\rho_\delta)]$$

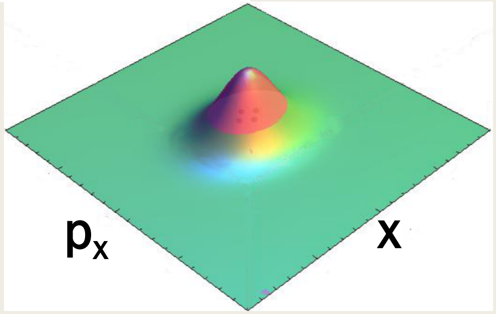
coherent states expansion:

$$|\varphi_\lambda\rangle(t) = \sum_i^{m_\lambda} c_i^\lambda |\alpha_i^\lambda\rangle(t)$$ sp wave functions

$$\rho = \sum_{\lambda=0}^{N} \sum_i n_i^\lambda(t) |\alpha_i^\lambda >< \alpha_i^\lambda|$$ 1-b density matrix



α Wigner transforms



coherent states (cs)

$$\begin{cases} \alpha_x(x) = \mathcal{N} \exp\{-a(\chi,\phi)(x-\langle x\rangle)^2 + i\dfrac{\langle p_x\rangle}{\hbar}(x-\langle x\rangle)\} \\ \alpha(\vec{r}) = \alpha_x(x)\alpha_y(y)\alpha_z(z) \end{cases}$$

e-tdhf ⇒

$$\left[\begin{array}{l} i\hbar \dfrac{\partial |\alpha_k^\lambda(t)\rangle}{\partial t} = \mathbf{h} |\alpha_k^\lambda(t)\rangle \\ \dot{n}_i^\alpha = \sum_{\beta,\gamma,\delta} \sum_{j,k,l} W_{\alpha\beta\gamma\delta} \, [n_j^\gamma n_k^\delta (|c_i^\alpha|^2 - n_i^\alpha)(|c_l^\beta|^2 - n_l^\beta) - n_i^\alpha n_l^\beta (|c_j^\gamma|^2 - n_\gamma)(|c_k^\delta|^2 - \rho_\delta)] \end{array} \right.$$

providing all the information about the mean behaviour of the system

untangling fluctuations

observable dispersions

fluctuations around the average density

pure m.f. evolution: $|\boldsymbol{\Psi}^{SD}\rangle(0)=\mathcal{A}\prod\boldsymbol{\varphi}_i(0) \Rightarrow |\boldsymbol{\Psi}^{SD}\rangle(t)=\mathcal{A}\prod\boldsymbol{\varphi}_i(t)$

$|\boldsymbol{\Psi}^{SD}\rangle \Leftrightarrow \rho(1) = \rho^2(1)$

m.f. + residual interactions: $\rho \neq [\rho]^2$

the correspondence between $|\boldsymbol{\Psi}^{SD}\rangle$ and ρ is lost residual interactions

- a single $|\Psi^{SD}\rangle$ cannot be a solution of the many-body problem
- but a manifold of microscopic N-body configurations:

  $\{ |\Psi^{SD}\rangle \} \equiv \{ \psi_1, \psi_2, \psi_3, \ldots, \psi_k, \ldots \}$

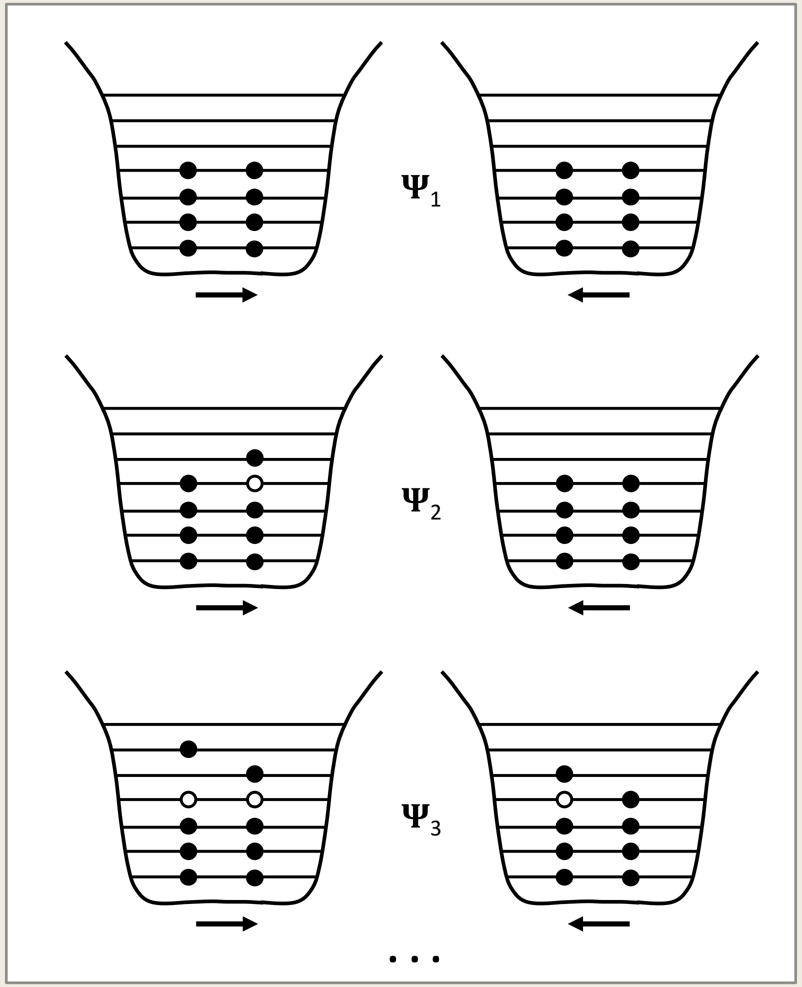
- allowing transitions between $|\Psi^{SD}\rangle$ (as a result of n-n interactions)

$$|\Psi\rangle = \sum_k a_k(t)|\Psi_k\rangle$$

$$|\Psi_k\rangle = \mathcal{A}|\varphi_{\lambda_1}\rangle|\varphi_{\lambda_2}\rangle...|\varphi_{\lambda_N}\rangle$$

C.-Y. Wong and J.A. McDonald, PRC 16 (1977) 1196

t=0



adopting a statistical point of view:

the least-biased N-body state compatible with our 1-and-2-body description is a statistical mixture of Slater Determinants $\boldsymbol{\psi}_k$:

$$\rho^N = \sum_{k=1}^{M} p_k |\Psi_k\rangle\langle\Psi_k| \qquad \sum_{k}^{M} p_{k=1} = 1$$

{ | $\boldsymbol{\psi}_k$ ⟩(t)} evolving in a "**modified**" mean-field: $\boldsymbol{V}^{HF}(\rho[n_i])$ { n_i } ☜ master equation

since $$|\varphi_\lambda\rangle(t) = \sum_{i}^{m_\lambda} c_i^\lambda |\alpha_i^\lambda\rangle(t) \qquad |c_i^\lambda|^2 = \frac{1}{m_\lambda}$$ $c_i^{\boldsymbol{\lambda}}$ time-independent

$$\Rightarrow \qquad \rho^N = \sum_{k=1}^{M} \sum_{q=1}^{\mathcal{N}_q^{(k)}} |c_q^{(k)}|^2 |\Pi_q^{(k)}\rangle\langle\Pi_q^{(k)}| \qquad |\Pi_q^{(k)}\rangle = \mathcal{A}|\alpha_{i_1}\alpha_{i_2}...\alpha_{i_N}\rangle$$

coherent states Slater Determinants (CSSD)

(decoherence assumption)

m^{λ} : number of coherent states per single particle level

M : the total number of SD $|\psi_k\rangle$

$\mathcal{N}_q^{(k)}$: the total number of CSSD for a given $|\psi_k\rangle$ n-body state

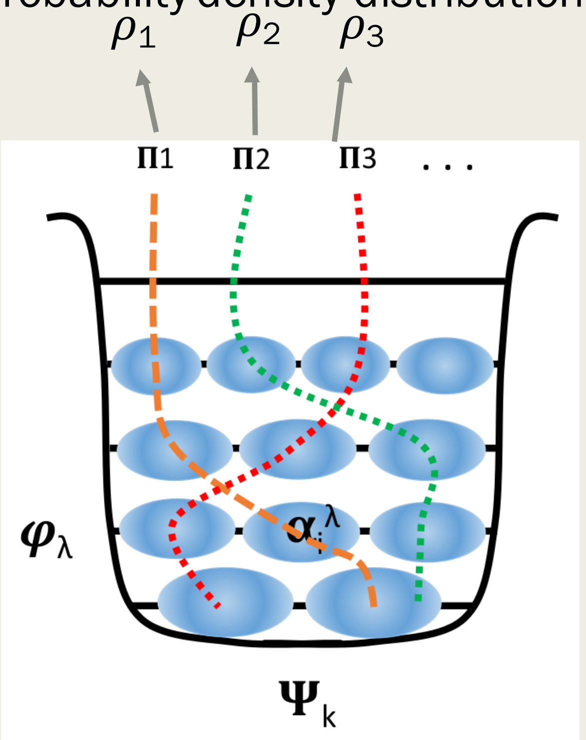
$$\mathcal{N}_q^{(k)} = \prod_{j=1}^{N} m_{\lambda j}$$

total number of CSSD to ρ^N is high $\Rightarrow$ simple sampling with uniform probability density distribution

$$\rho^N \sim \sum_I \omega_I |\Pi_I\rangle\langle\Pi_I|$$

with $\boldsymbol{\omega}_I = \dfrac{1}{\boldsymbol{\Omega}(I)}$

$\boldsymbol{\Omega}$: sample dimension



reduced distribution functions:

$$\Rightarrow \quad \rho^{(s)} \sim \sum_I \omega_I \rho_I^{(s)} \qquad s=1, 2, \ldots$$

with $$\rho_I^{(s)} = Tr_{s+1\ldots N}\{|\Pi_I\rangle\langle\Pi_I|\}$$

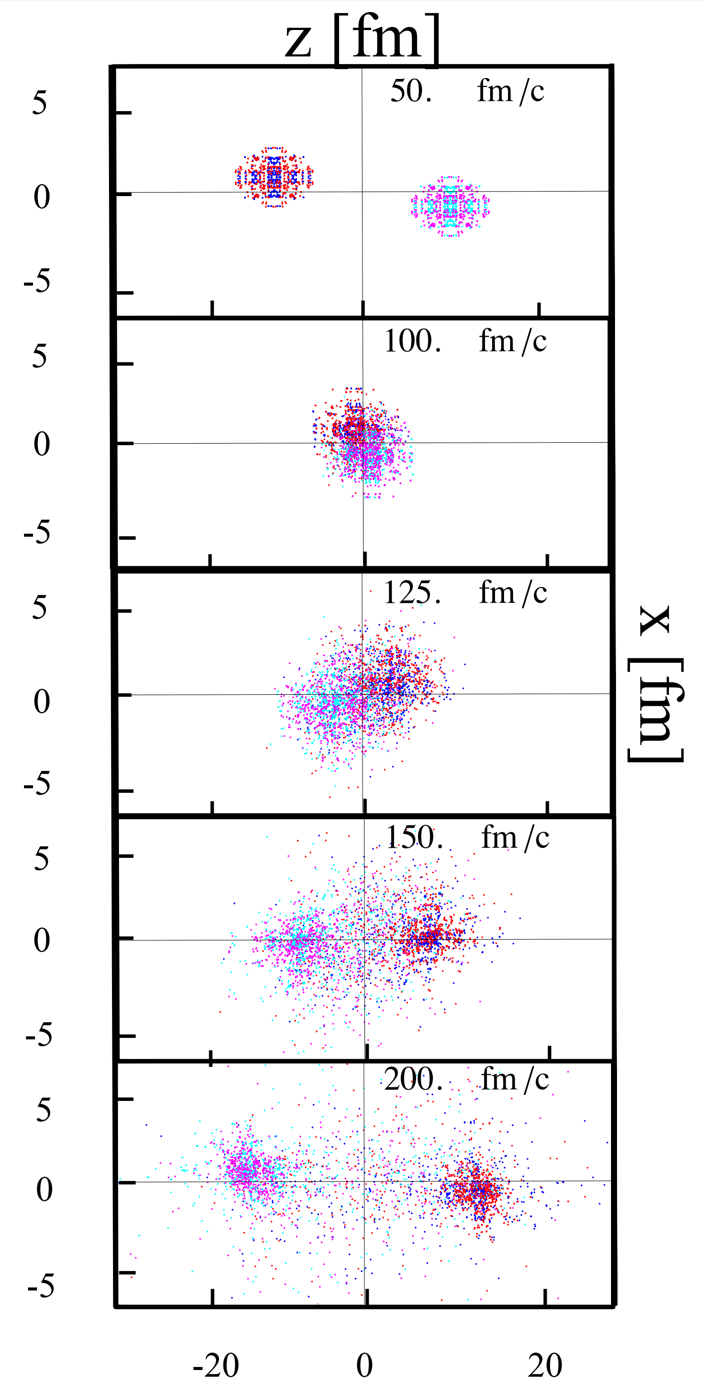
mean values of s- body observables: $$\langle \hat{O}^{(s)} \rangle = \frac{1}{s!} \sum_I \omega_I \langle \hat{O}^{(s)} \rangle_I$$

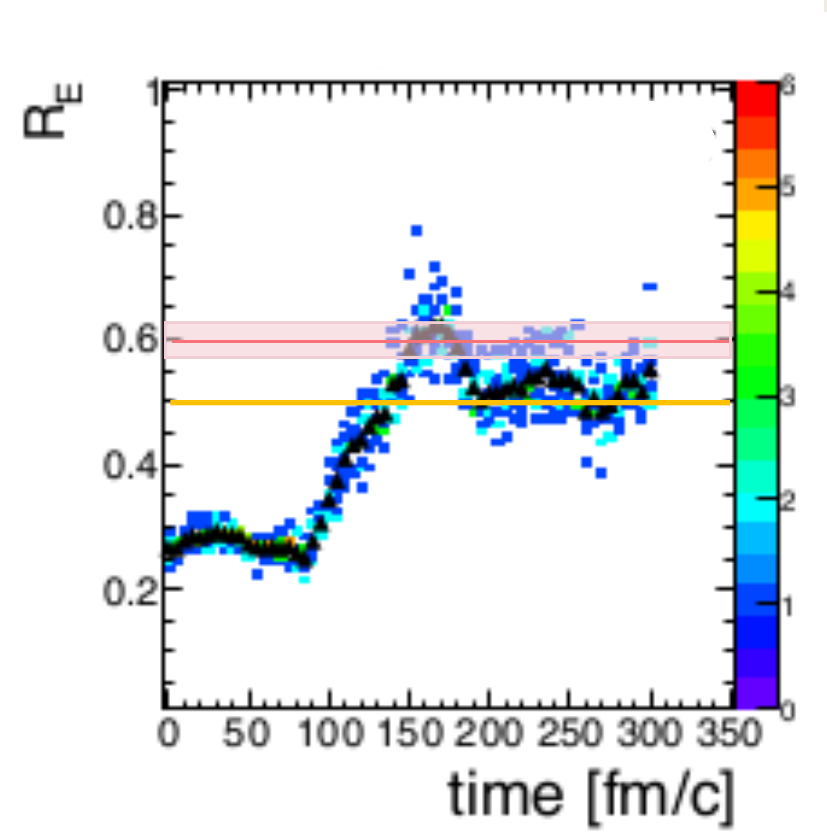
$^{58}Ni + ^{58}Ni$ b=3 fm E_{inc}= 50 A MeV

kinetic energy transfers
stopping power

$$R_E = \frac{E_\perp}{2E_\parallel}$$

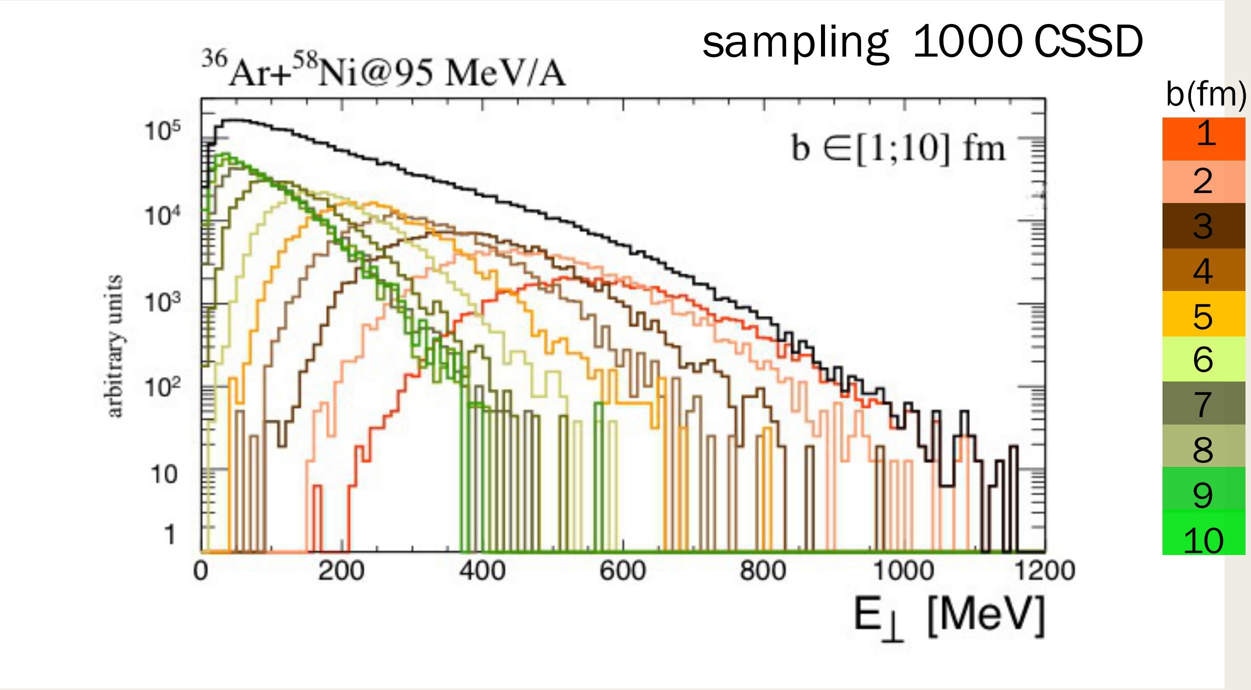
isotropy ratio



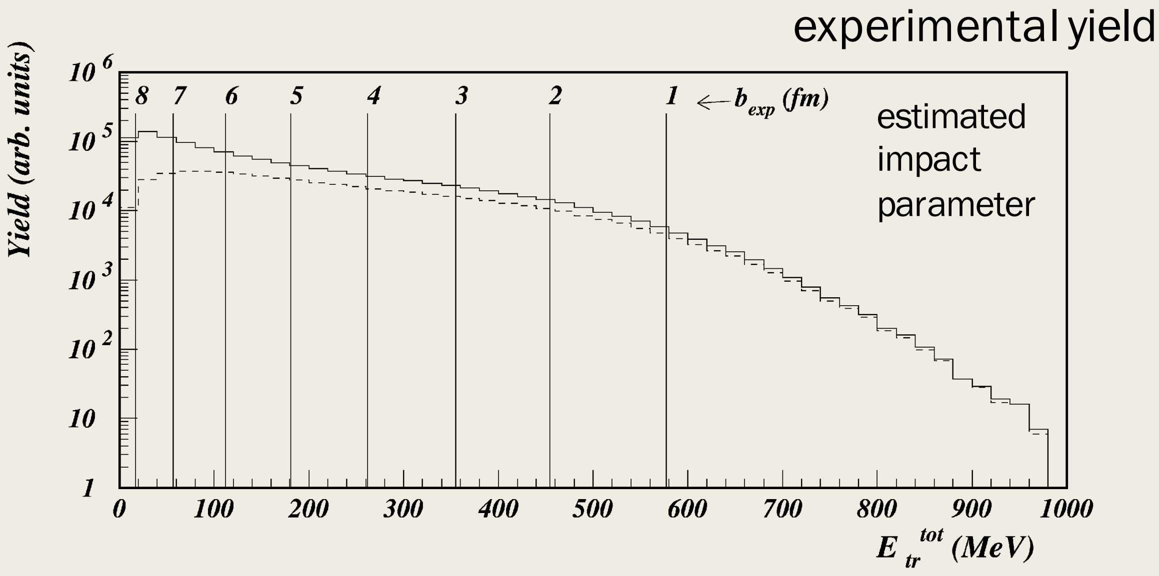


— O. Lopez et al. PRC 90 064602 (2014)

— G. Lehaut et al. PRL 104 232701 (2010)



D. Doré et al. / Physics Letters B 491 (2000) 15–22



the transverse energy
characterizing the violence of the collision

$$E_{\perp} = \frac{p_x^2 + p_y^2}{2m}$$

calculated $E_{\perp}$ maxima ≈ experimental value
for each estimated impact parameter **b**

the correlation between $E_{\perp}$ and **b**
in consistence with the data

beyond e-tdhf

the Boltzmann-Langevin (BL) equation:

$$\frac{\partial f}{\partial \mathrm{t}} = \{h[f], f\} + \bar{\mathcal{K}}[f] + \delta\mathcal{K}[f]$$

describes the time evolution of the semi-classical one-body distribution function $f(\vec{r}, \vec{p})$ through a kinetic Boltzmann-like equation supplemented by fluctuating collisional term.

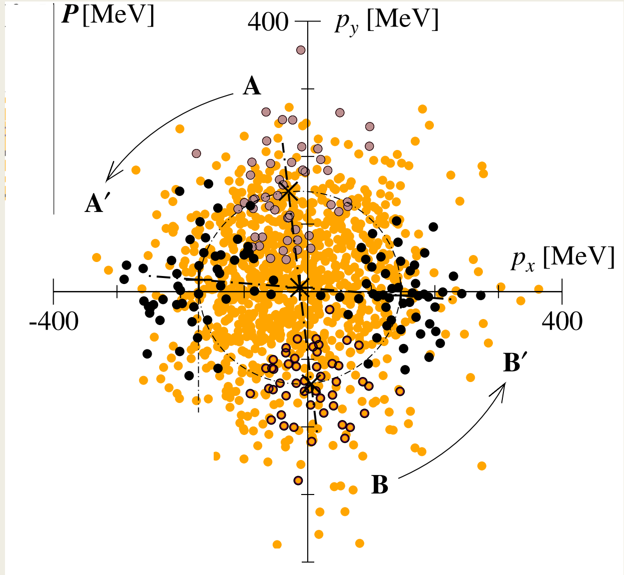
large density fluctuations generated by multi-particle correlations

The Bolzmann-Langevin One-Body (BLOB) model is a particular realization of the BL equation

numerical implementation in semi-classical description: test-particle method solve BL equation:

test-particles organized in agglomerates
representing nucleonic wave packets
locally defined at each time step

true nucleon-nucleon collisions with an
accurate treatment of Pauli principle



P. Napolitani EPJ W. Conf. (2012)

in HIC E_{inc} around E_F

wide variety of dissipative phenomena:
global phenomena from fusion to fragmentation in central collisions,
the occurrence of bifurcations and bimodal behaviour in dynamical trajectories,
links to fragment-formation mechanisms identified [Napolitani plb726]
the validity of the model is now well established.

Stochastic **e-tdmf** approach in the spirit of BLOB model ⟵ available microscopic information in CSSD at all time

cornerstones:

at each time step:

. sample a given CSSD

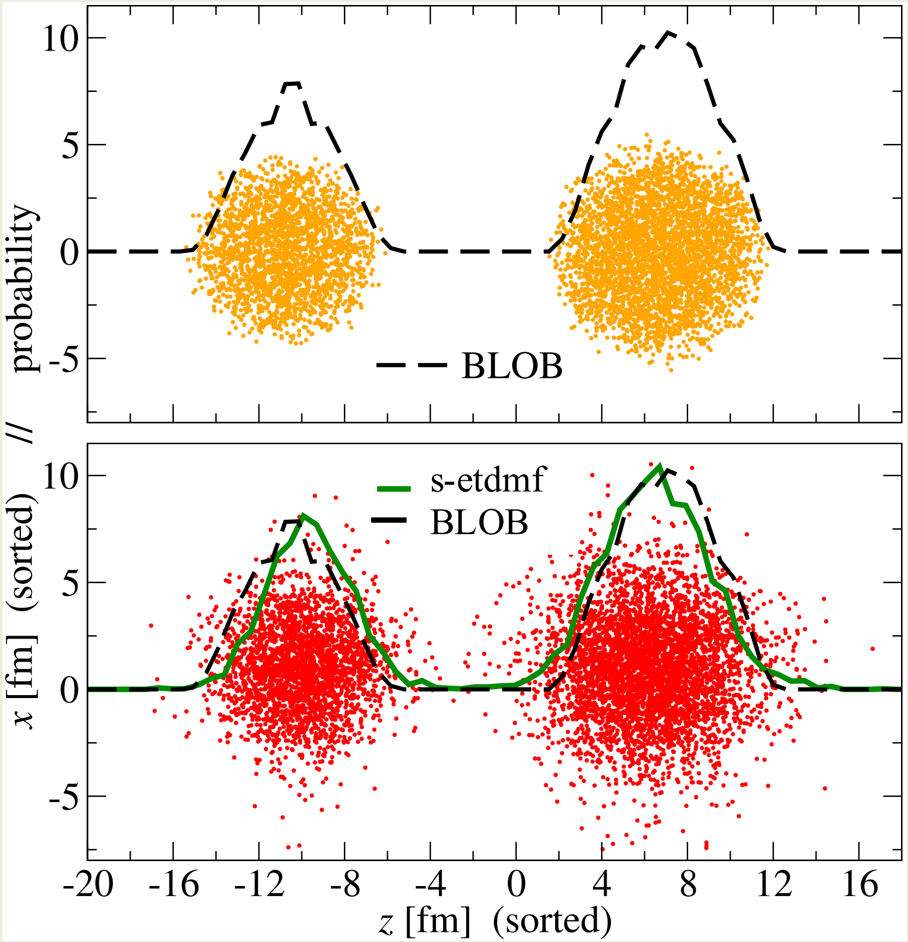
. define possible locations of nucleonic wave-packets (NWP)
   - . built around a reference CS
   - . include the closest CS in phase-space

. "collisions" between NWP
   - . scattered NWP conform to Pauli principle

preliminary results

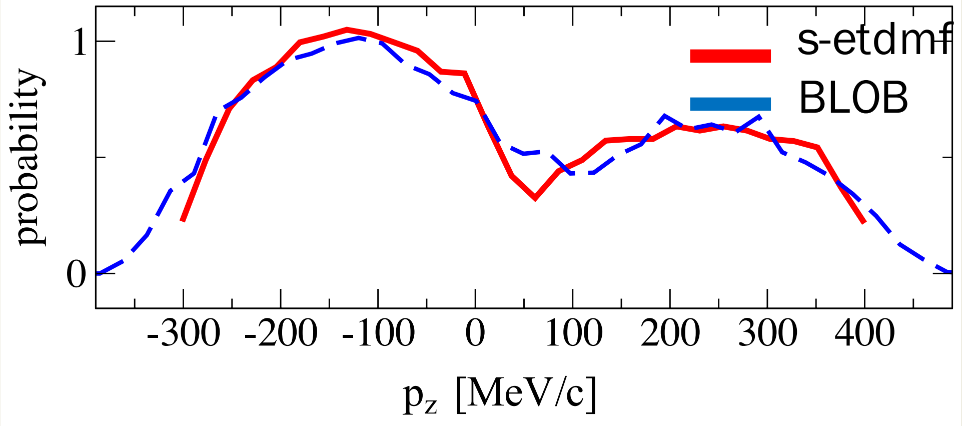
density profiles
central $^{36}Ar+^{58}Ni$ collision @ E_{inc}= 74 A MeV

density distribution along the beam axis



initial stage

longitudinal momentum distribution



TP number/nuc ≈ mean CS number/nuc

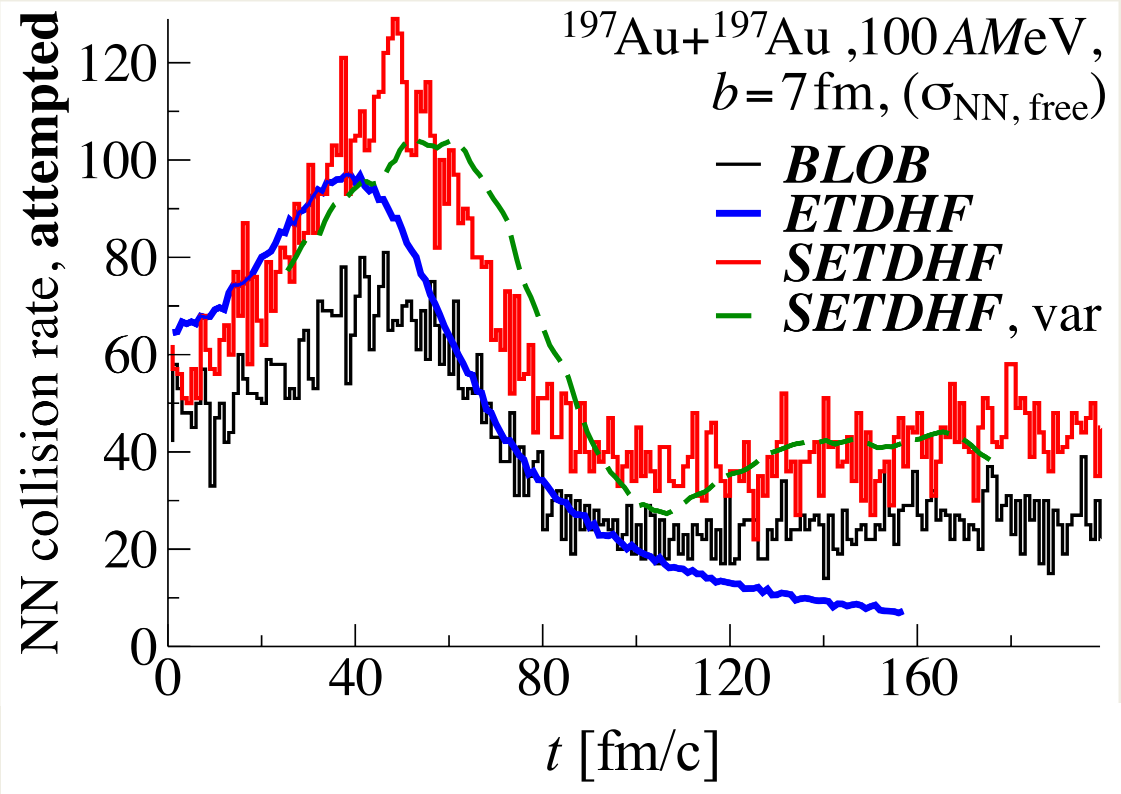
effective force of the Skyrme-type

free isospin- and E-dependent n-n cross-section with a 10 fm^2 (100 mb) threshold value analysis of collision rates and variances

^{197}Au+ ^{197}Au @ E_{inc}=100 AMeV b=7fm

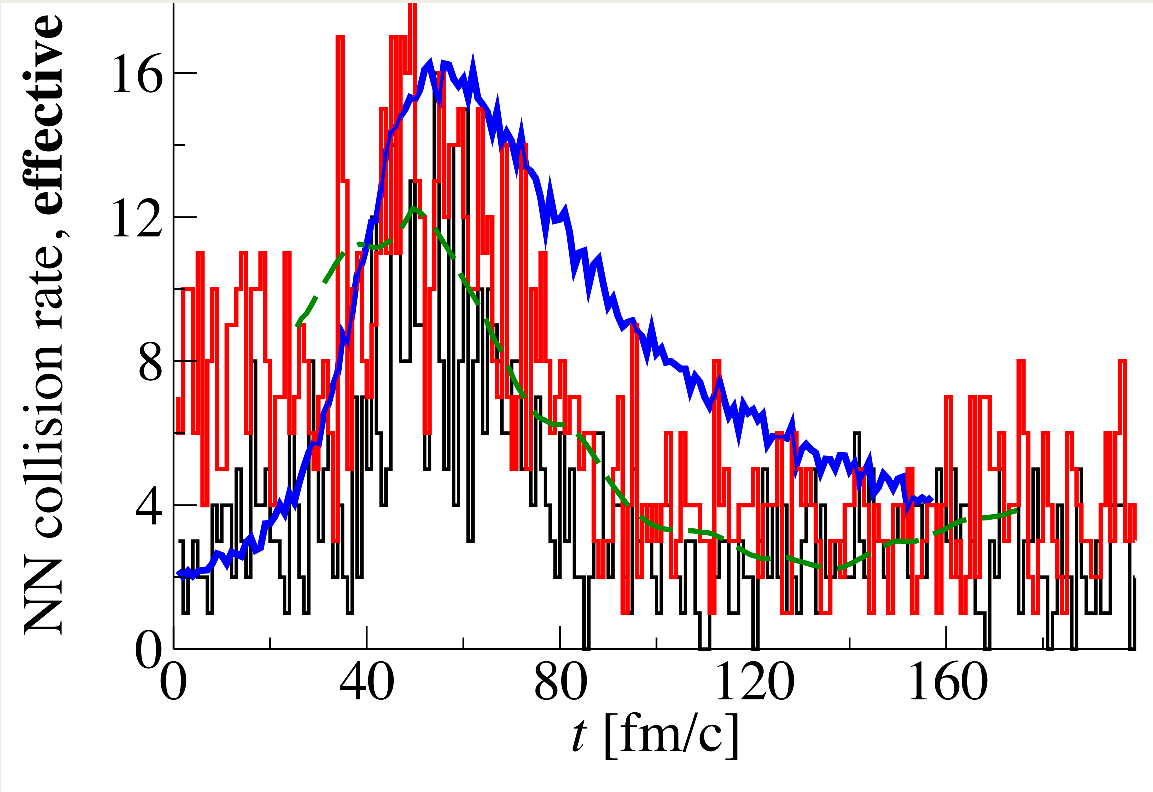
var=⟨(N-⟨N⟩)^2⟩

N: number of collisions

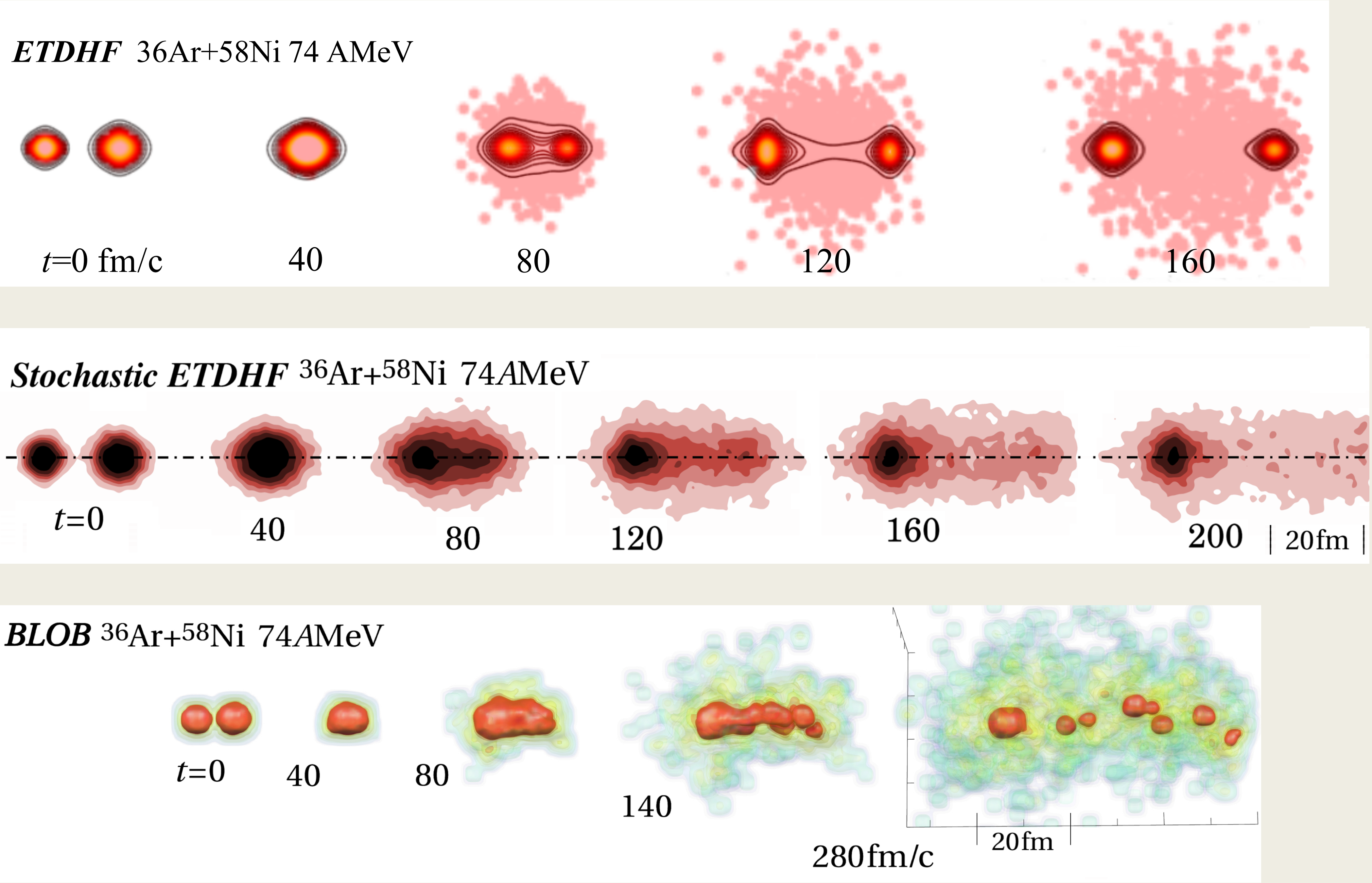


compression

dilution



after elimination of those collisions violating Pauli principle


ETDHF 36Ar+58Ni 74 AMeV
t=0 fm/c
40
80
120
160
Stochastic ETDHF 36Ar+58Ni 74AMeV
t=0
40
80
120
160
200
20fm
BLOB 36Ar+58Ni 74AMeV
t=0
40
80
140
280fm/c
20fm

conclusions

- the occurrence of a variety of mechanisms in the exit channel in hic and the knowledge of their relative probabilities constitute a challenge for transport models
- small amplitude (statistical) fluctuations from e-tdmf description reliable in describing observable dispersions
- large amplitude fluctuations in the spirit of Boltzmann-Langevin approach through the BLOB prescription
- the BL density fluctuations and variances present the correct behaviour
- clusters recognition is in progress
- next step is to understand what are those ingredients inherent to the model by which the system fragments.

🌸 thank you for your attention !

$$V^{HF}(\rho,\xi) = \frac{3}{4}t_0\rho + \frac{(\sigma+2)}{16}t_3\rho^{\sigma+1} + \tau_{(p,n)}\frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right)+t_2\left(x_2+\frac{1}{2}\right)\right]\Delta\xi - \frac{\sigma t_3}{24}\left(x_3+\frac{1}{2}\right)\rho^{\sigma-1}\xi^2$$

$$-\tau_{(p,n)}\frac{t_3}{12}\left(x_3+\frac{1}{2}\right)\rho^{\sigma}\xi - \tau_{(p,n)}\frac{1}{2}t_0\left(x_0+\frac{1}{2}\right)\xi - \frac{1}{8}\left[\frac{9}{4}t_1 - t_2\left(x_2+\frac{5}{4}\right)\right]\Delta\rho$$

$$\rho = \rho_n + \rho_p \qquad \tau_n = \frac{\partial\xi}{\partial\rho_n} = +1 \quad \text{neutrons}$$

$$\xi = \rho_n - \rho_p \qquad \tau_p = \frac{\partial\xi}{\partial\rho_p} = -1 \quad \text{protons}$$

SkT5

t_0[MeV.fm^3]	t_1[MeV.fm^5]	t_2[MeV.fm^5]	t_3[MeV.fm$^{\alpha+1}$]	x_0	x_1	x_2	x_3	σ
-2917.1	328.2	-328.2	18584	-0.295	-0.5	-0.5	-0.5	1/6