Next-to-leading power gluon TMDs from back-to-back DIS dijets at next-to-eikonal accuracy at low x

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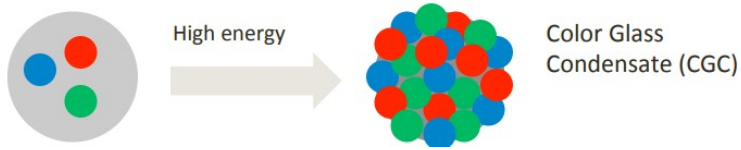
On the base of current work of **Tolga Altinoluk, Guillaume Beuf** and **Alina Czajka**

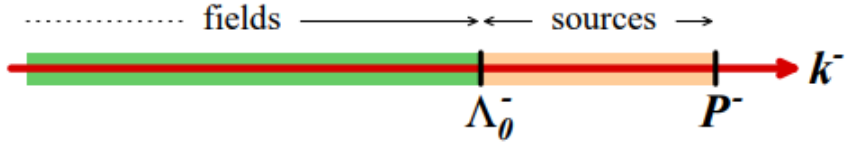
Setup

The most prominent effective theory regarding small-x regime is known as **Color Glass Condensate (CGC).**



Main idea behind CGC: Fast moving gluons and slow moving gluons ($x << 1$) are separated due to time dilation.

From the perspective of slow gluons, the fast moving ones are “frozen” in time – act like **semi-classical, background color sources.**



Most works are based on **eikonal approximation –** taking into account only terms with leading powers of energy.

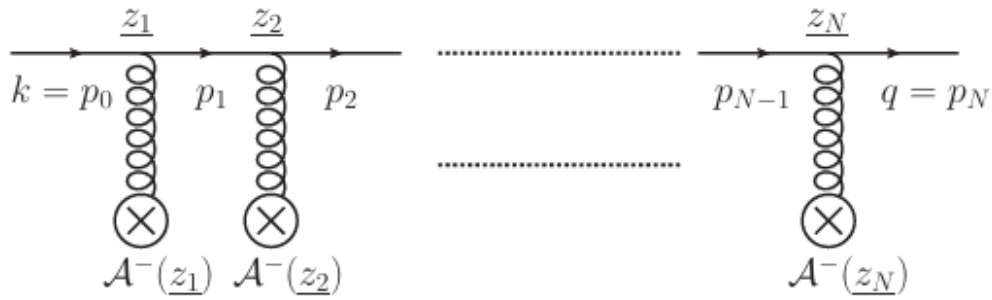
Setup

We examine dilute-dense system:

- The dense target as a background gluon field within CGC
- Projectile is dilute (quark or anti-quark)
- **Next-to-eikonal (Eik) approximation**

$$\mathcal{A}^- = O(\gamma) \gg \mathcal{A}_j = O(1) \gg \mathcal{A}^+ = O(1/\gamma)$$

Next-to-eikonal corrections to the background quark propagator have been calculated several years ago



T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska, Phys.Rev.D 104 (2021) 1, 014019

Setup

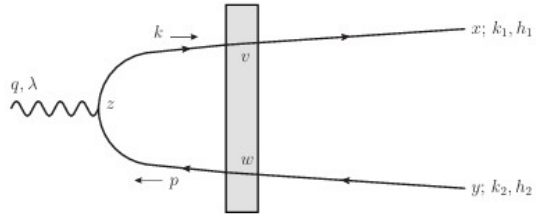
Considered process - Dijet production via Deep Inelastic Scattering (DIS):

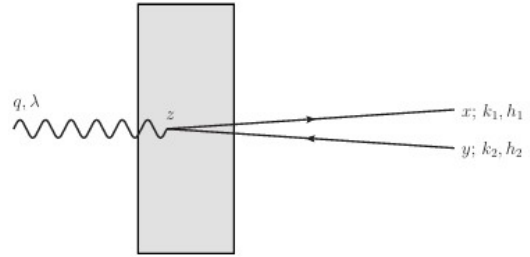
$$\frac{d\sigma^{l+\text{target}\to l'+\text{dijet}+X}}{dx_{Bj}\,dQ^2\,d\text{P.S.}} = \frac{\alpha_{\text{em}}}{\pi\,x_{Bj}\,Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\frac{d\sigma_{\gamma_T^*\to\text{dijet}}}{d\text{P.S.}}(x_{Bj},Q^2)+(1-y)\,\frac{d\sigma_{\gamma_L^*\to\text{dijet}}}{d\text{P.S.}}(x_{Bj},Q^2)\right]$$

- Exchange of a virtual photon
- The photon splits into quark-antiquark pair (dipole approximation)

Cross section for DIS dijet production at Neik can be calculated using the propagator mentioned in the previous site

Already known at Neik accuracy





T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska, Phys.Rev.D 107 (2023) 7, 074016

Setup

Current goal: We would like to find a connection between the TMD formalism and CGC computations

High energy limit of the TMD distributions may be achieved from CGC calculations within the **correlation limit** (or back-to-back).

Next-to-leading power (NLP) correction: $\frac{|\mathbf{k}|^2}{|\mathbf{P}|^2}$

Total transverse momentum of the dijet.

Relative transverse momentum between the two jets

Fourier conjugated variables:

- Dipole size **r** – related to relative momentum **P**
- Impact parameter **b** – related to total transverse momentum **k**

Hence $|\mathbf{k}| \ll |\mathbf{P}| \longrightarrow |\mathbf{r}| \ll |\mathbf{b}|$

Next-to-eikonal (Neik) correction: $\frac{1}{W^2}$

General Procedure

How to proceed: possible to utilize Eik + Neik operators by expanding Wilson line structure in powers of |**r**| up to the desired order.

Example: $\mathcal{O}^{eik.} \sim U_F(\mathbf{b}+z_2\mathbf{r},b^-)U_F(\mathbf{b}+z_1\mathbf{r},b^-)^\dagger - 1 = \left(1+i\frac{(z_2-z_1)}{2}\mathbf{r}\cdot\mathbf{k}\right)\mathbf{r}^i[\partial_i U_F(\mathbf{b},b^-)]U_F(\mathbf{b},b^-)^\dagger$

$$-\frac{1}{2}\mathbf{r}^i\mathbf{r}^j[\partial_i U_F(\mathbf{b},b^-)][\partial_j U_F(\mathbf{b},b^-)]^\dagger + \mathcal{O}\left(\frac{|\mathbf{r}|^3}{|\mathbf{b}|^3}\right)$$

Where $\partial_j \mathcal{U}_F(\mathbf{b},b^-) = -\int_{-\infty}^{\infty} dz^+ \,\mathcal{U}_F(\infty,z^+;\mathbf{b},b^-)\,(ig\,\mathcal{F}_j^-(z))\,\mathcal{U}_F(z^+,-\infty;\mathbf{b},b^-)$

At the end, we arrive at a sum of terms:

$$\left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\text{Eik+NEik}} = \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}} + \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}} + \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{+-}\mathcal{F}^{\perp-}}$$

Example: two-point gluon field insertion:

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^2 \int_{\mathbf{b}_1,\mathbf{b}_2} e^{i(\mathbf{b}_2-\mathbf{b}_1)\cdot\mathbf{k}} \int dz_1^+ dz_2^+ \left\langle F_j^-(z_2^+,\mathbf{b}_2)F_i^-(z_1^+,\mathbf{b}_1)\right\rangle \cdot \Pi^{ij}(\mathbf{P},\mathbf{k},\{z\})$$

General Procedure

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^2 \int_{\mathbf{b}_1,\mathbf{b}_2} e^{i(\mathbf{b}_2-\mathbf{b}_1)\cdot\mathbf{k}} \int dz_1^+ dz_2^+ \left\langle F_j^-(z_2^+,\mathbf{b}_2) F_i^-(z_1^+,\mathbf{b}_1)\right\rangle \cdot \Pi^{ij}(\mathbf{P},\mathbf{k},\{z\})$$

Eikonal + next-to-eikonal coefficients

The bridge between CGC and TMD formalism can be formalized via:

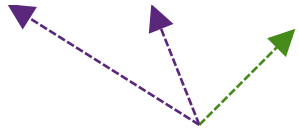
$$\langle \dots \rangle \longrightarrow \lim_{P'\to P} \frac{\langle P'|\dots|P\rangle}{\langle P'|P\rangle}$$

So that we can identify the color average of n-point gluon field insertion with n-point TMD correlator. Two-point functions already discussed:

T. Altinoluk, G. Beuf, A. Czajka, C. Marquet, Phys.Rev.D 111 (2025) 1, 014010

What about the three-point function?

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^3 \int_{\mathbf{b}_1,\mathbf{b}_2} e^{i(\mathbf{b}_2-\mathbf{b}_1)\cdot\mathbf{k}} \int dz_1^+ dz_2^+ dz_3^+ \left\langle F_k^-(z_3^+,\mathbf{b}_2) F_j^-(z_2^+,\mathbf{b}_2) F_i^-(z_1^+,\mathbf{b}_1)\right\rangle \cdot \Pi^{ijk}(\mathbf{P},\{z\})$$



Only two different transverse points

Issues and caveats:

- Does time ordering matter at next-to-eikonal level?
  - For two-point functions, time ordering is trivial. Is it so in the three-point case?
- What is the appropriate definition of the 3-point gluon TMD that would match our CGC calculations?

What about the three-point function? – Time order

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim \left\langle \mathcal{O}_k^c(z_1^+, \mathbf{b}_\perp + \Delta\mathbf{b}_\perp)^\dagger \mathcal{O}_{ij}^{ab}(z_3^+, z_2^+, \mathbf{b}_\perp) \right\rangle \cdot \left\{ d^{abc}\Big[\dots\Big] + f^{abc} \cdot \frac{(z_1 - z_2)(b_2^+ - b_3^+)}{2q^+ z_1 z_2} \cdot \frac{4\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{[\mathbf{P}^2 + \bar{Q}^2]^4} \right\}$$

Time ordered gluonic operator

Cannot differentiate quark and antiquark jets – only this term realistically remains.
Next-to-eikonal term.

$$\mathcal{O}_a^i(z^+, \mathbf{z}) = T[U_A(\infty, z^+, \mathbf{z})_{aa'} F_{a'}^i(z^+, \mathbf{z})]$$

$$\mathcal{O}_{ab}^{ij}(z_1^+, z_2^+, \mathbf{z}) = T[U_A(\infty, z_1^+, \mathbf{z})_{aa'} F_{a'}^i(z_1^+, \mathbf{z}) U_A(\infty, z_2^+, \mathbf{z})_{bb'} F_{b'}^j(z_2^+, \mathbf{z})]$$

The formulas are explicitly time-ordered at the amplitude level

- Required splitting of the phase space in the integration + symmetrization:

$$\mathcal{O}_{ij}^{ab}(z_3^+, z_2^+, \mathbf{0}_\perp) = \mathcal{O}_{ji}^{ba}(z_2^+, z_3^+, \mathbf{0}_\perp)$$

What about the three-point function? – the operator

TMD Definition: $\Phi^{g;cab}_{12;kij}(z_1^+, z_2^+, z_3^+, \Delta\mathbf{b}_\perp) = \langle P|\, g\mathcal{O}^c_k(z_1^+, \Delta\mathbf{b}_\perp)^\dagger \mathcal{O}^{ab}_{ij}(z_3^+, z_2^+, \mathbf{0}_\perp)\,|P\rangle$

Based on:
S. Rodini, A. Vladimirov, arXiv:2204.03856 (2022)

Can be shifted thanks to translational invariance

$$\Phi^{g;cab}_{12;kij}(z_1^+, z_2^+, z_3^+, \Delta\mathbf{b}_\perp) = \Phi^{g;cab}_{12;kij}(0, z_2^+ - z_1^+, z_3^+ - z_1^+, \Delta\mathbf{b}_\perp)$$

With our prescription, the operator part takes the form:

$$\int dz_1^+ \int d^2\mathbf{b} \cdot \left\langle \mathcal{O}^c_k(z_1^+, \mathbf{b}_\perp + \Delta\mathbf{b}_\perp)^\dagger \mathcal{O}^{ab}_{ij}(z_3^+, z_2^+, \mathbf{b}_\perp) \right\rangle = \frac{1}{2P^-} \Phi^{g;cab}_{12;kij}(0^+, \Delta z_2^+, \Delta z_3^+, \Delta\mathbf{b}_\perp)$$

What about the three-point function? – the operator

$$\int dz_1^+ \int d^2\mathbf{b} \cdot \langle \mathcal{O}_k^c(z_1^+, \mathbf{b}_\perp + \Delta\mathbf{b}_\perp)^\dagger \mathcal{O}_{ij}^{ab}(z_3^+, z_2^+, \mathbf{b}_\perp)\rangle = \frac{1}{2P^-}\Phi_{12;kij}^{g;cab}(0^+, \Delta z_2^+, \Delta z_3^+, \Delta\mathbf{b}_\perp)$$

After Fourier transformation to momentum fraction space:

$$\int dz_1^+ \int d^2\mathbf{b} \cdot \langle \mathcal{O}_k^c(z_1^+, \mathbf{b}_\perp + \Delta\mathbf{b}_\perp)^\dagger \mathcal{O}_{ij}^{ab}(z_3^+, z_2^+, \mathbf{b}_\perp)\rangle = \frac{(P^-)^3}{2P^-}\int [dx] e^{-i(x_2^+\Delta z_2^+ + x_3^+\Delta z_3^+)P^-}\Phi_{12;kij}^{g;cab}(x_1^+, x_2^+, x_3^+, \Delta\mathbf{b}_\perp)$$

$$\int [dx] = \int_{-1}^{1} dx_1^+ dx_2^+ dx_3^+ \delta(x_1^+ + x_2^+ + x_3^+)$$

What about the three-point function?

What we have so far:

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^3 \int_{\mathbf{b}_1,\mathbf{b}_2} e^{i(\mathbf{b}_2-\mathbf{b}_1)\cdot\mathbf{k}} \int dz_1^+ dz_2^+ dz_3^+ \left\langle F_k^-(z_3^+,\mathbf{b}_2) F_j^-(z_2^+,\mathbf{b}_2) F_i^-(z_1^+,\mathbf{b}_1)\right\rangle \cdot \Pi^{ijk}(\mathbf{P},\{z\})$$

$$\left.\frac{d\sigma}{dP.S.}\right|_{3\times\mathcal{F}^{\perp+}} \sim g^2 \frac{(P^-)^3}{2P^-} \int_{\Delta z_2^+,\Delta z_3^+,\Delta\mathbf{b}} e^{-i\mathbf{k}\cdot\Delta\mathbf{b}} \int [dx] e^{-i(x_2^+\Delta z_2^+ + x_3^+\Delta z_3^+)P^-} \Phi^{g;cab}_{12;kij}(x_1^+, x_2^+, x_3^+, \Delta\mathbf{b}_\perp)$$
$$\cdot\left\{d^{abc}\mathcal{A}^{ijk}(\mathbf{P},\{z\}) + f^{abc}\mathcal{B}^{ijk}(\mathbf{P},\{z\})\right\}$$

(full expression in the backup slide)

$$\mathcal{B}^{ijk} = \frac{(z_1-z_2)(\Delta z_2^+ - \Delta z_3^+)}{2q^+ z_1 z_2} \cdot \frac{4\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{[\mathbf{P}^2+\bar{Q}^2]^4}$$

What about the three-point function?

Conclusion: We can write the operator from CGC as a (Fourier-transformed) 3-point TMD correlator with some momentum-dependent eikonal + next-to-eikonal coefficients in a time-ordered way.

Some algebra later, the 3-point part of the cross section is, schematically:

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim (\dots)\cdot\Phi(0,0,0,\Delta\mathbf{b}) + (\dots)\cdot\int_{-1}^{1} dx\delta'(x)\Phi(x,-\frac{x}{2},-\frac{x}{2},\Delta\mathbf{b}) + f^{abc}(\dots)\cdot\int_{-1}^{1} dx\delta'(x)\Phi(0,x,-x,\Delta\mathbf{b})$$
$$+(\dots)\cdot\int_{-1}^{1} dx\left[\frac{1}{(x+i\epsilon)^2}+\frac{1}{(x-i\epsilon)^2}\right]\Phi(x,-\frac{x}{2},-\frac{x}{2},\Delta\mathbf{b})$$

(full expression in the backup slide)

Quick Summary

- Dijet production via Deep Inelastic Scattering provides a reliable tool to study bridging between CGC and TMD formalisms within the correlation limit (+ Neik, + NLP order).
- At Neik + NLP order, the cross section (for both longitudinal and transverse photon) may be split into terms with 2-point and 3-point gluon field insertions.
- The 3-point term and its connection to 3-point TMDs have been recently studied, including potential issues like time-ordering and proper 3-point gluon TMD definition.
- Only one of the coefficient is relevant from the experimental standpoint.

[Draft already in preparation]

Full three-point contribution

$$(2q^+)2\pi\delta(p^+-k^+)4Q^2e^2e_f^2z^3(1-z)^3\int d^2\Delta\mathbf{b}\,e^{i\Delta\mathbf{b}\cdot\mathbf{k}}\frac{(P^-)^3}{2P^-}\int[dx]\Phi^{g;cab}_{12;ijk}(x_1,x_2,x_3,\Delta\mathbf{b})\int d\Delta z_2^+\int d\Delta_3^+\,e^{-i(x_2\Delta z_2^+ + x_3\Delta z_3^+)P^-}$$
$$\left\{f^{abc}\frac{(1-2z)\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{z(1-z)q^+[\mathbf{P}^2+\bar{Q}^2]^4}[\Delta z_2^+-\Delta z_3^+]+d^{abc}\left[\frac{\delta^{ij}}{[\mathbf{P}^2+\bar{Q}^2]^4}-\frac{4\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{[\mathbf{P}^2+\bar{Q}^2]^5}\right]\left[1+\frac{i(\mathbf{P}^2+\bar{Q}^2)}{2z(1-z)q^+}\left(\frac{\Delta z_2^++\Delta z_3^+}{2}\right)\right]\right.$$
$$\left.-\,d^{abc}\frac{i\delta^{ij}\mathbf{P}^k}{4z(1-z)q^+[\mathbf{P}^2+\bar{Q}^2]^3}|\Delta z_2^+-\Delta z_3^+|\right\}$$

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^\perp+\mathcal{F}^\perp+\mathcal{F}^\perp+}\sim d^{abc}\left[\frac{\delta^{ij}}{[\mathbf{P}^2+\bar{Q}^2]^4}-\frac{4\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{[\mathbf{P}^2+\bar{Q}^2]^5}\right]\left[\Phi(0,0,0,\Delta\mathbf{b})+\frac{\mathbf{P}^2+\bar{Q}^2}{2q^+P^-z(1-z)}\cdot\int_{-1}^1dx\delta'(x)\Phi(x,-\frac{x}{2},-\frac{x}{2},\Delta\mathbf{b})\right]$$
$$+\,f^{abc}\frac{2i(1-2z)\mathbf{P}^i\mathbf{P}^j\mathbf{P}^k}{z(1-z)2q^+P^-[\mathbf{P}^2+\bar{Q}^2]^4}[\Delta z_2^+-\Delta z_3^+]\cdot\int_{-1}^1dx\delta'(x)\Phi(0,x,-x,\Delta\mathbf{b})+$$
$$d^{abc}\frac{i\delta^{ij}\mathbf{P}^k}{4\pi z(1-z)2q^+P^-[\mathbf{P}^2+\bar{Q}^2]^3}\cdot\int_{-1}^1dx\left[\frac{1}{(x+i\epsilon)^2}+\frac{1}{(x-i\epsilon)^2}\right]\Phi(x,-\frac{x}{2},-\frac{x}{2},\Delta\mathbf{b})$$