MSHT PDFs - High x and Uncertainty related issues.

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MSHT PDFs [1] - High x and uncertainty related issues.

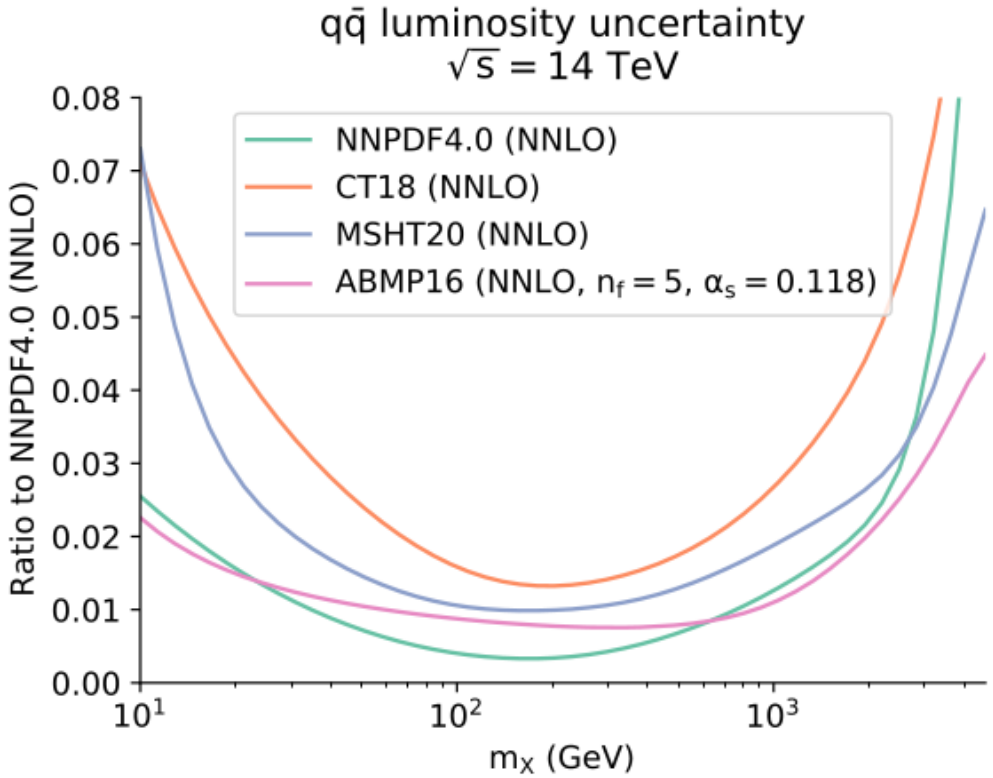
Partial update of MSHT20 PDFs focussing on high-x including re-analysis of fixed-target data, investigation of target-mass corrections and higher twist and inclusion of some new data sets [2].

Investigation of alternative treatments of errors and correlations in data. Implication for 13 TeV LHC jet data.

High-x PDFs.

High x quarks/antiquarks vital for BSM searches, but uncertainties high, both experimental and due to methodology.

Constraints from high-mass Drell-Yan, jets, data, but possible interplay with BSM physics.



"Cleaner" constraint from DIS data, but currently mainly from older fixed-target data with less sophisticated error treatments and potential non-perturbative (target-mass, higher twist) sensitivities.

Revist these data and issues.

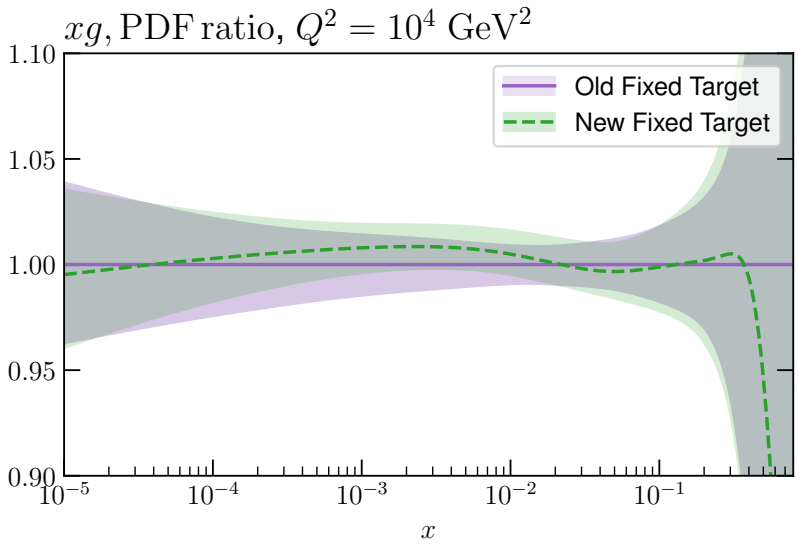
Fixed target DIS data updates.

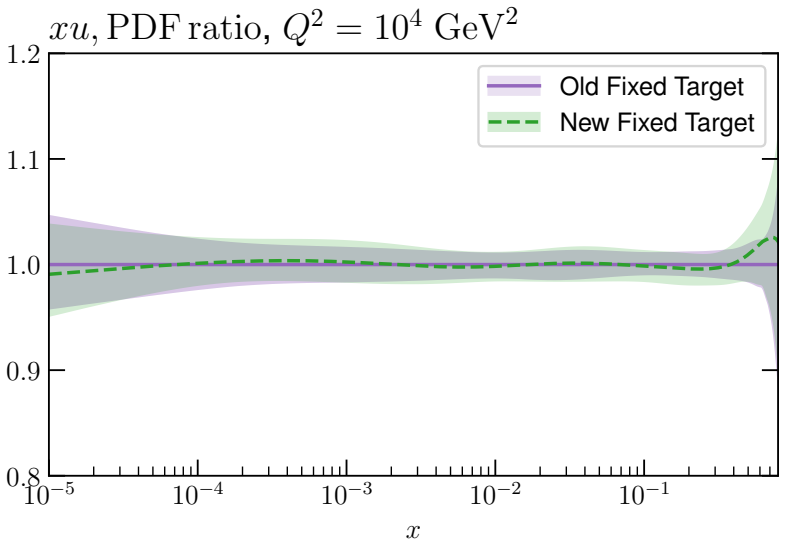
	Old FT (aN^{3}LO)	New FT (aN^{3}LO)
Fixed Target	1424.8 (0.96)	2201.1 (1.11)
HERA	1593.3 (1.26)	1626.6 (1.29)
Hadron Collider	2391.9 (1.34)	2387.1 (1.33)
Global	5410.0 (1.19)	6214.7 (1.23)

- (Similar story at NNLO)

Previous MSHT/MMHT/.... fits tended to fit beam energy averaged structure function data for e.g. BCDMS/NMC/E665 losing information on correlated systematics.

Now updated. Increase of $\sim$ 500 data point. Some fit quality deterioration. PDF impact moderate, mainly in gluon and up quark.





Target Mass and Higher-Twist Corrections.

Target Mass Correction - well determined $\mathcal{O}(m_p^2/Q^2)$ corrections [3] to leading twist structure functions.

$$F_2^{\mathrm{TMC}}(x,Q^2) \approx \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi,Q^2)\left[1+\frac{6x\mu\xi}{r}(1-\xi)^2\right],$$

$$\xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}.$$

Higher Twist - less well-determined, parameterised in this study.

$$F_2^{\mathrm{HT}}(x,Q^2) = F_2^{\mathrm{LT}}(x,Q^2)\left(1+\frac{D_i(x)}{Q^2}\right),$$

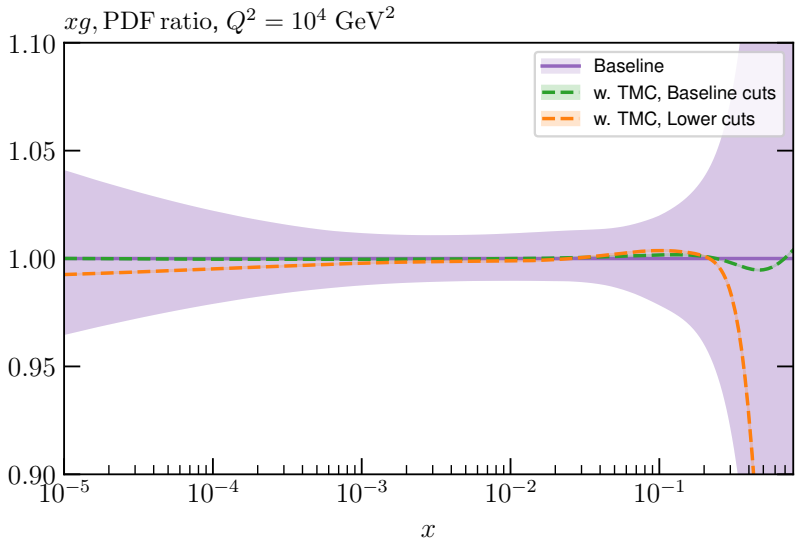
In standard MSHT fits cuts have been imposed with values chosen to minimise impact of these terms, i.e. $Q^2 > 2\mathrm{GeV}^2, W^2 > 15\mathrm{GeV}^2$. But when including these corrections can investigate lowering these.

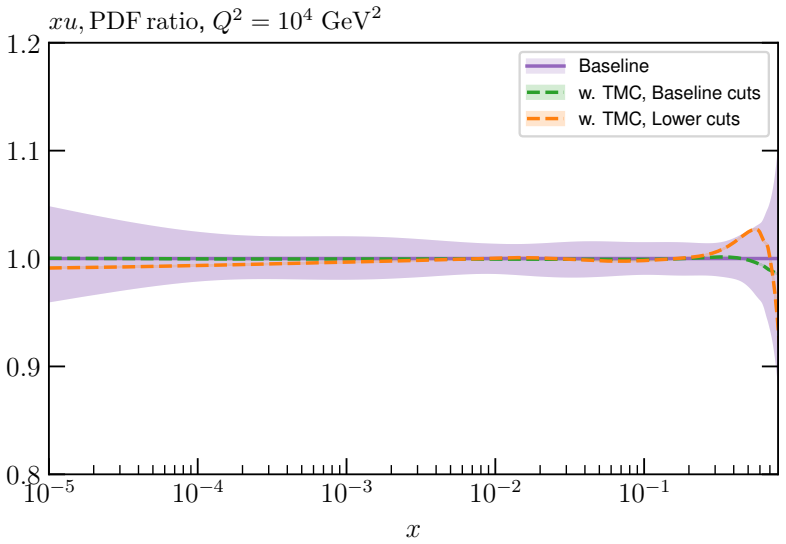
Target Mass Corrections.

Not significant with default cuts, but important if lowering W^2.

	$W^2 > 15\,\mathrm{GeV}^2$	$W^2 > 5\,\mathrm{GeV}^2$	
	Baseline cuts, no TMC	no TMC, lower W^2 cut	w. TMC, lower W^2 cut
Fixed Target	**2201.1 (1.11)**	**2590.7 (1.15)**	**2511.1 (1.12)**
Global	**6214.7 (1.23)**	**6646.4 (1.26)**	**6526.6.4 (1.23)**

PDF impact not very significant, except at very high x and only with lowered cuts.





Higher Twist Corrections.

Including these alongside TMCs and fiting parameters gives improvement in fit quality for lower W^2 cuts.

$W^2 > 5\ \mathrm{GeV}^2$ – TMC $\chi^2 = 6527 \rightarrow$ TMC+HT $\chi^2 = 6470$.

However, also improvement for baseline case

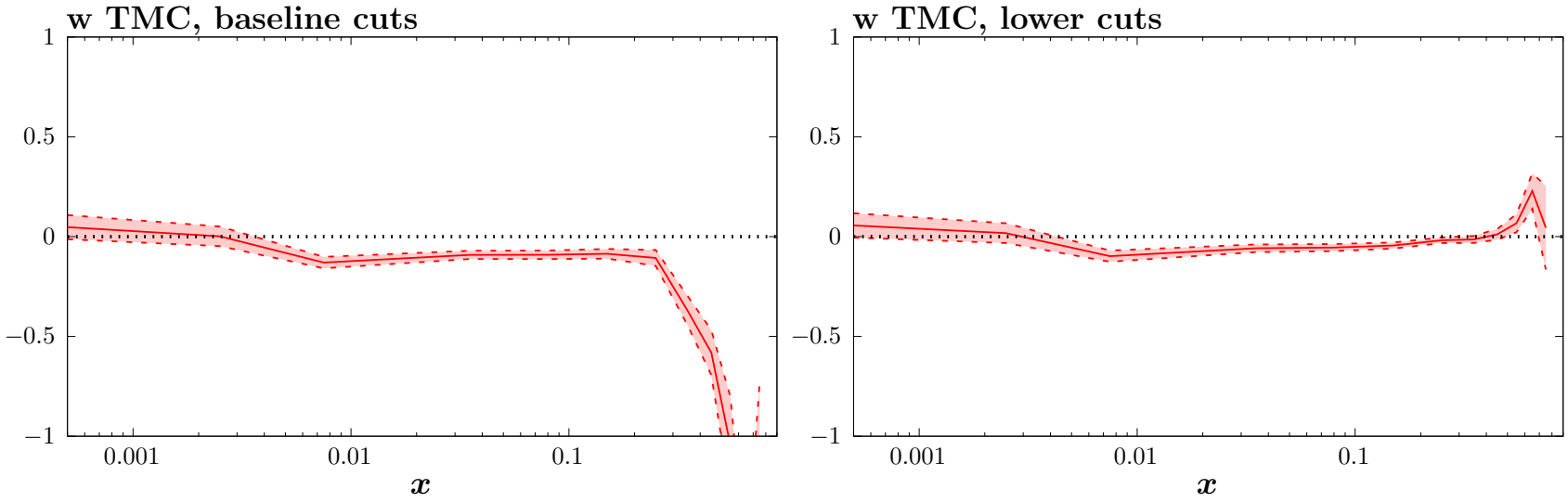
$W^2 > 15\ \mathrm{GeV}^2$ – TMC $\chi^2 = 6215 \rightarrow$ TMC+HT $\chi^2 = 6122$.

However, fixing HT terms to lower cuts value gives less improvement

$W^2 > 15\ \mathrm{GeV}^2$ – TMC $\chi^2 = 6215 \rightarrow$ TMC+HTfix $\chi^2 = 6176$.

and using baseline cut HT at low W^2 is a disaster.

Completely different fitted Higher Twist terms in the two cases

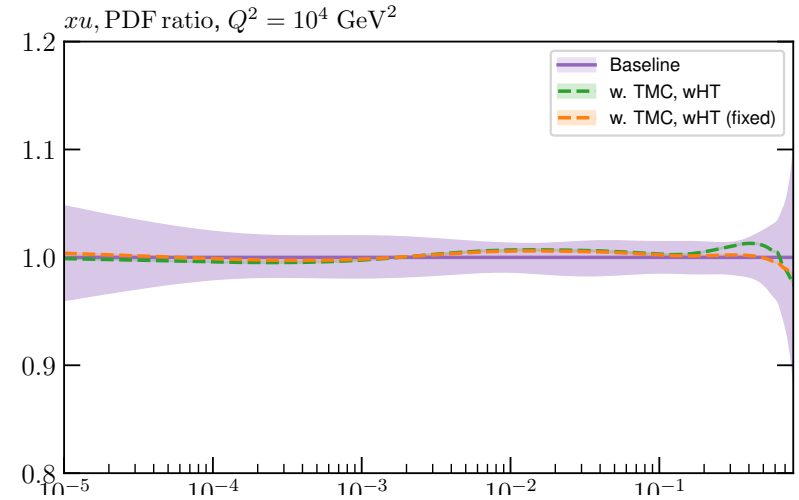


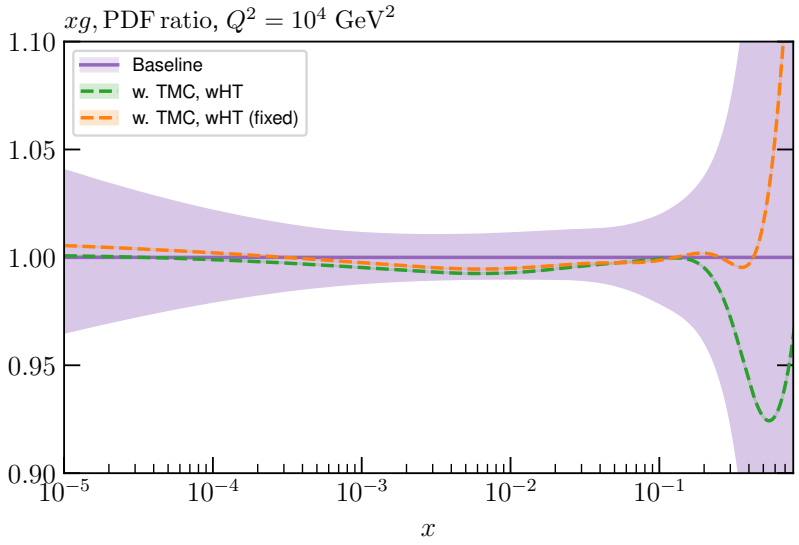
Suggests “higher twist” in baseline cuts simply accounting for higher order corrections not genuine higher twist.

For lower cut wider kinematic range and true higher twist dominates.

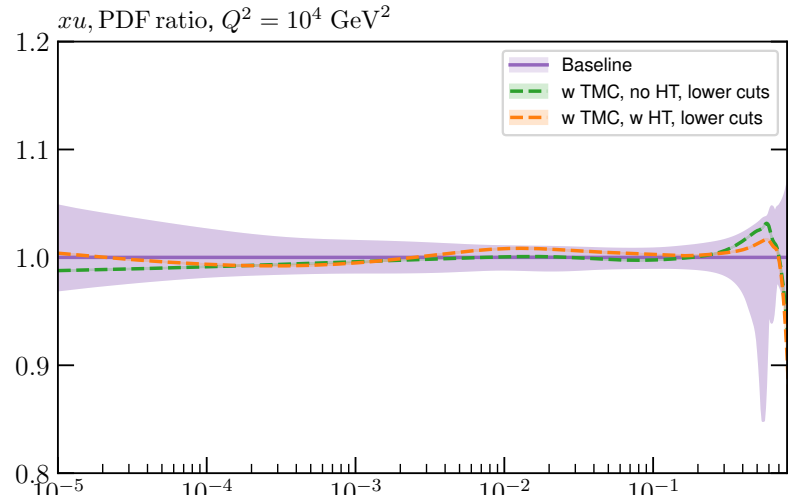
Changes in PDFs.

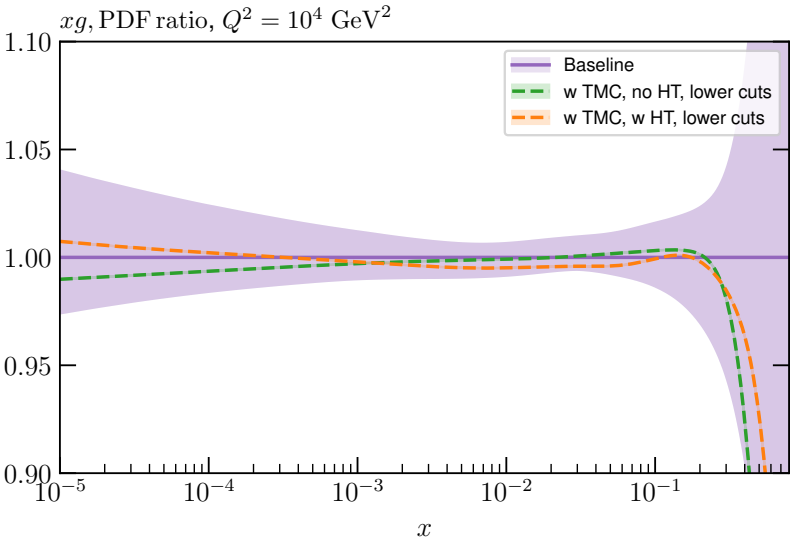
$W^2 > 15GeV^2$





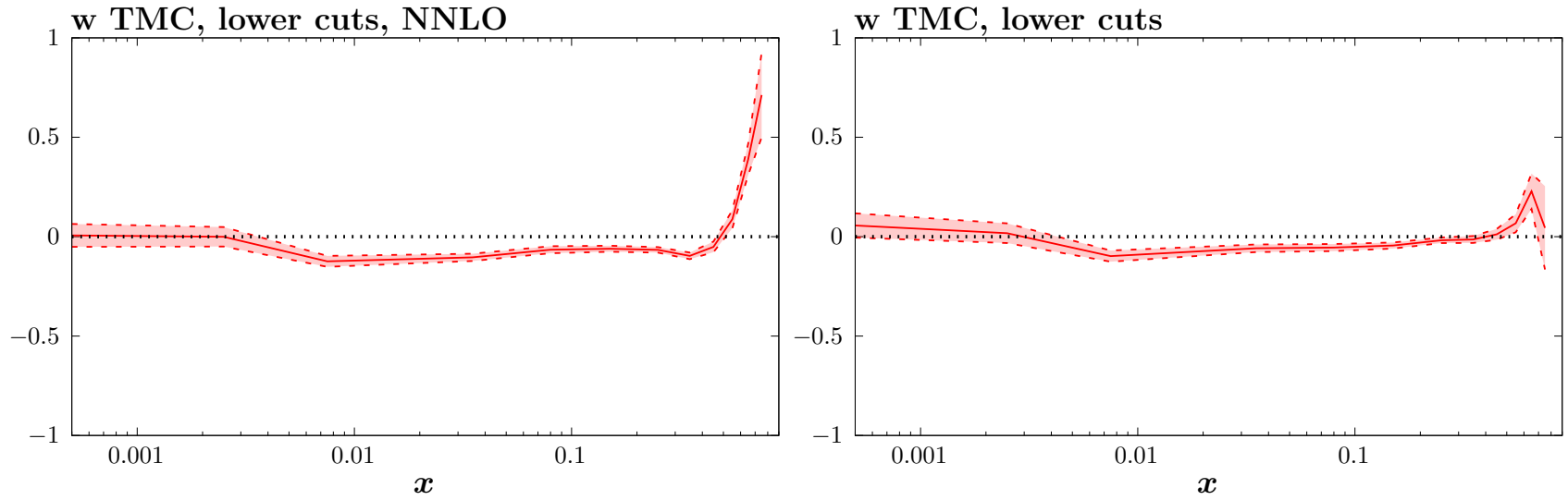
$W^2 > 5GeV^2$





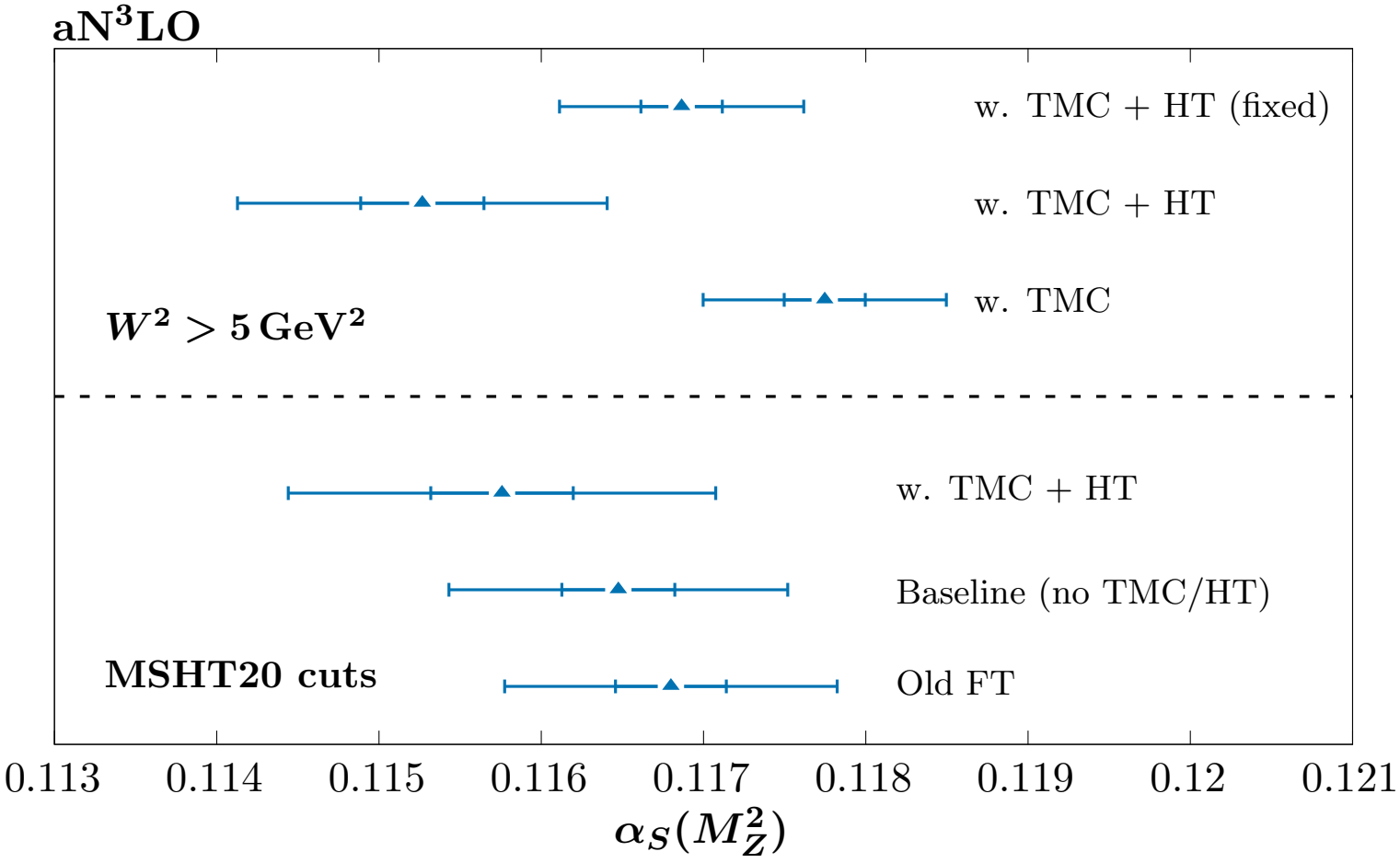
PDF impact limited, but some reduction in moderately small x with TMC+HT and lower cuts.

The size of higher twist corrections decreases at aN^{3}LO (right) compared to NNLO, suggesting that some of fit values mimicking higher orders.



Beyond N^{3}LO higher twist and higher orders difficult to distinguish at high x due to renormalon effects, i.e. perturbation theory series convergence.

Impact on $\alpha_S(M_Z^2)$ determination.



Including the higher twist contributions reduces the best fit value of $\alpha_S(M_Z^2)$, particularly with the lower W^2 cut.

However, within original MSHT uncertainties.

Investigation of special high-x ZEUS data [4].

ZEUS re-analysis of DIS data with finer bining and $x \to 1$.

Event counts small, apply Poissonian probability

$$\chi^2_{\rm ZEUS} = -2 \sum_{j=1}^{N_{data}} \ln(e^{-\nu_j} \nu_j^{n_j}/n_j)$$

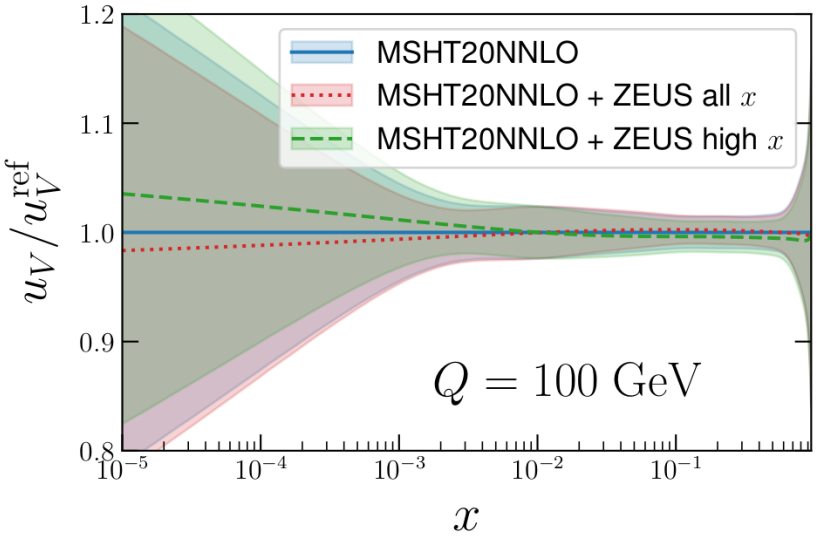
($\nu_j =$ expected number, $n_j =$ actual number).

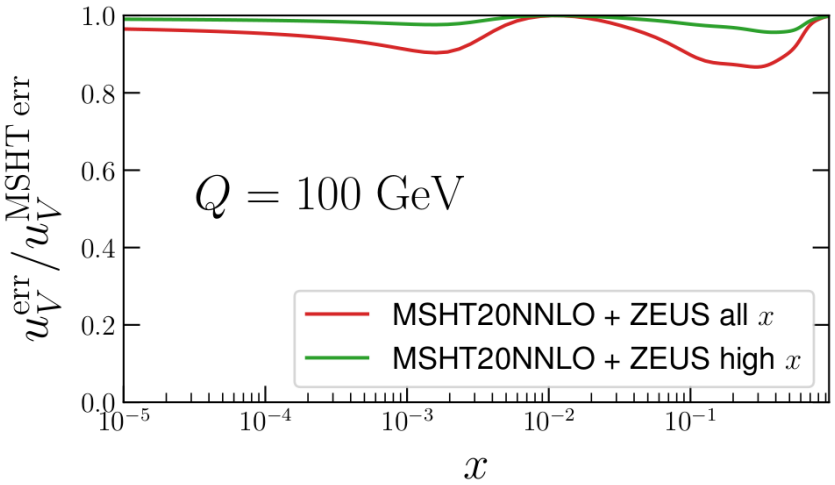
Assess impact on MSHT20 PDFs via profiling:

$$\chi^2(w) = \chi^2_{\rm MSHT20} + \chi^2_{\rm ZEUS}(w_k) + \sum_k T_k^2 w_k^2,$$

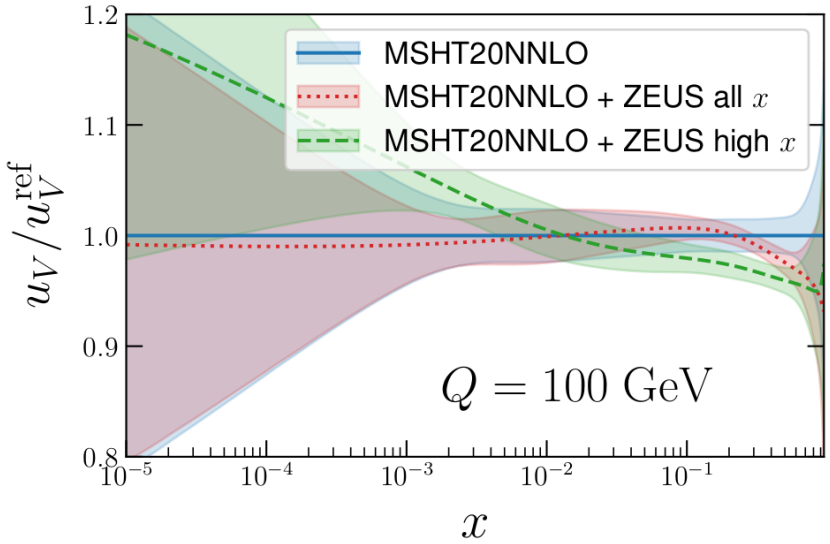
where k is the number of eigenvector pairs, w_k is the fractional shift along an eigenvector direction and T^2 is the tolerance. For MSHT T_k is dynamical but $T^2 = 9$ is a very good approx.

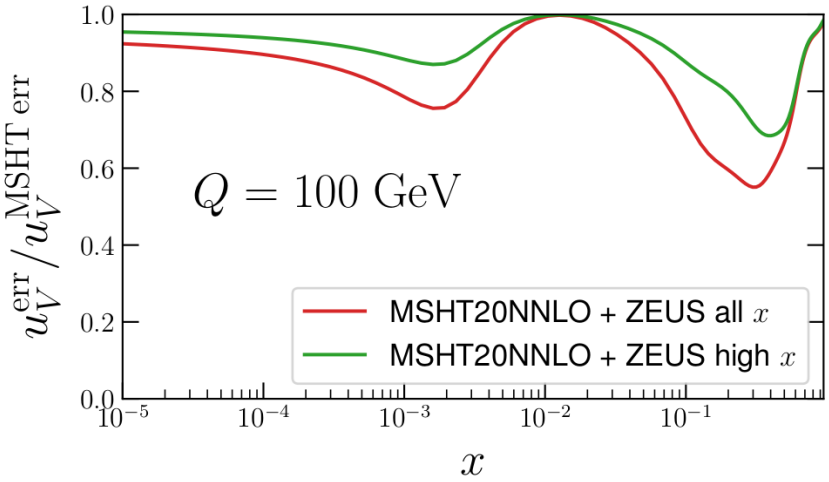
Impact small, especially if low-x data omitted to avoid double counting.





Much more significant if tolerance incorrectly excluded from profiling

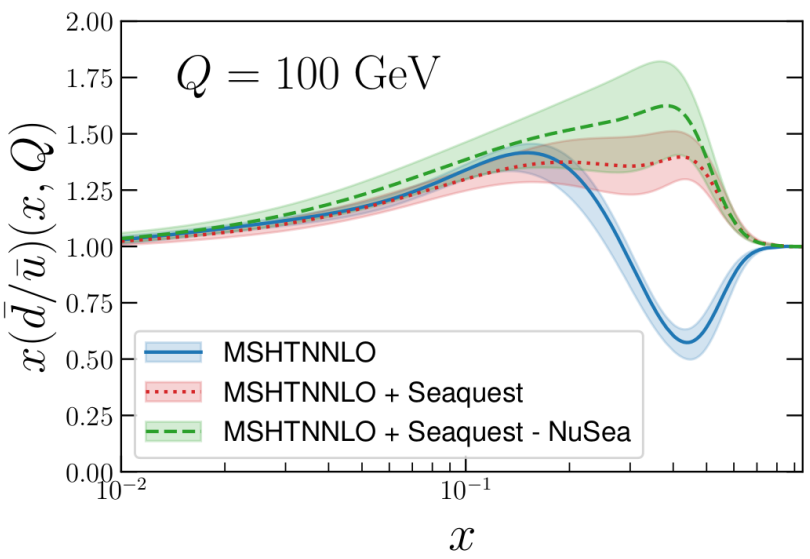


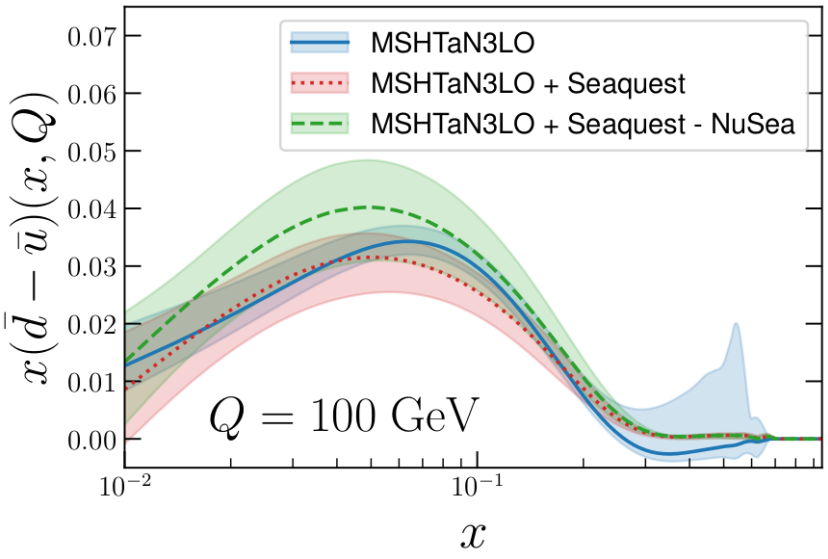


SeaQuest data on Drell-Yan ratio.

SeaQuest (E906) fixed target DY data [5] raises $\bar{d}/\bar{u}$. In tension with NuSea (E866).

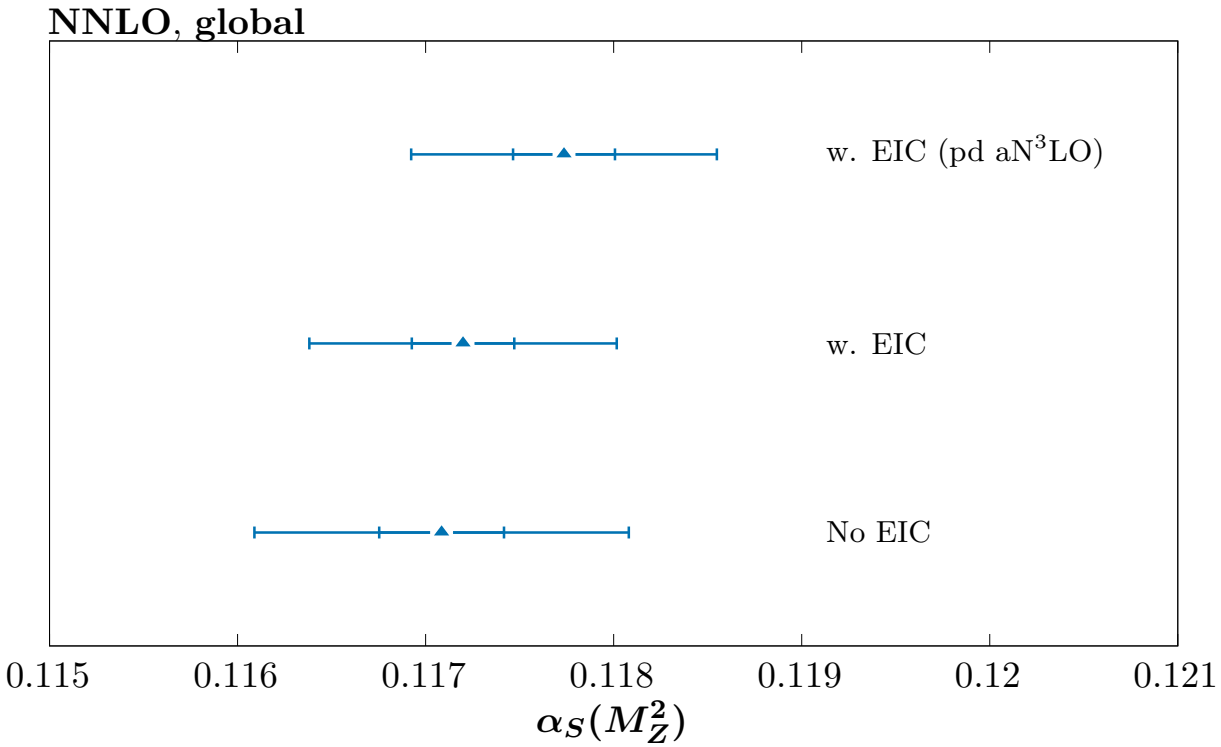
Strong constraint on MSHT pdfs.





χ^2 for NuSea (E866) doubles with inclusion. Some tension with other data, mainly fixed-target DIS.

$\alpha_S(M_Z^2)$ determination with inclusion of EIC pseudodata [6].



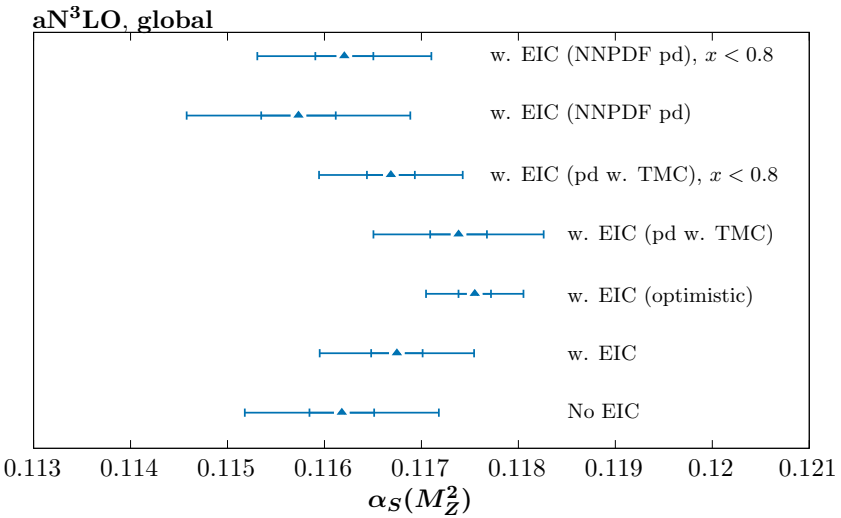
Main constraint from high-x due to the evolution of the non-singlet compent of the structure function - clean probe.

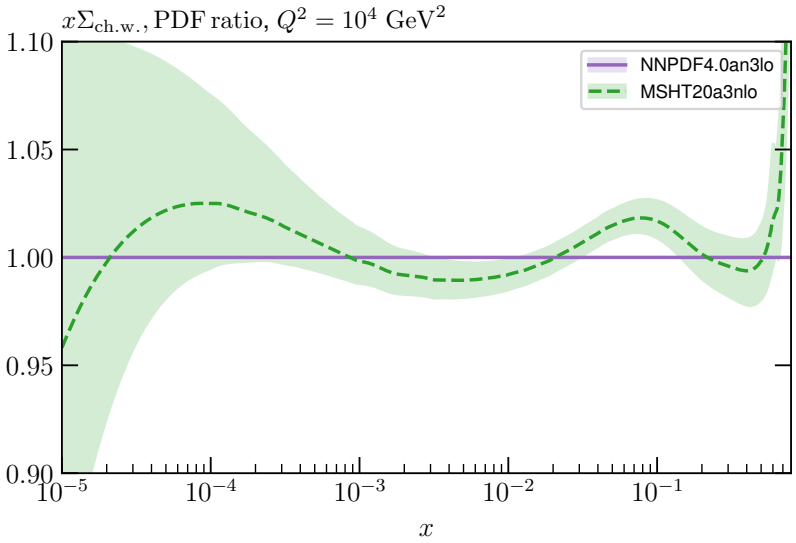
Introduce EIC pseudodata generated from theory with $\alpha_S(M_Z^2) = 0.118$.

Significant constraint, but weaker in full global fit than DIS-only fit.

Impact different if pseudodat from N^{3}LO theory - example of theoretical uncertainty.

Also consider N^{3}LO, again EIC pseudodata generated from theory with $\alpha_S(M_Z^2) = 0.118$





"Optimistic" pseudodata with 0.5% uncorrelated and correlated uncertainties has much more effect.

Generate possible "theory" uncertainties by generating pseudodata with TMC corrections (not in default fit here) or alternative PDF set (NNPDF4.0 [7]) where high-x structure function prediction different.

Significant effect, but less so if very limited data for $x \geq 0.8$ omitted.

PDF sensitivity to treatment of (correlated) systematic errors.

Potential issues due to lack of complete knowledge of what the error is. Need to consider the “error on the error”. Work by M. Reader.

- Model the estimated variances, v_i, of $\sigma^2_{u_i}$, as Gamma distributed gives:

$$L(\mu, \theta, \sigma^2_{u_i}) = P(y|\mu, \theta) \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_{u_i}} e^{-(u_i-\theta_i)^2/2\sigma^2_{u_i}} \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} v_i^{\alpha_i-1} e^{-\beta_i v_i}$$

$$\alpha_i = \frac{1}{4r_i^2} \qquad \beta_i = \frac{1}{4r_i^2\sigma^2_{u_i}}$$

- r_i is defined as the relative uncertainty in the estimate of the systematic error. The parameters r_i can therefore be referred to as the ”error on errors”.

[2]Cowan arXiv:1809.05778v3

Gaussian plus Gamma = t-distribution

- This model can be identically reinterpreted as a Student’s t-distribution

$$L(\mu, \theta, \sigma^2_{u_i}) = P(y|\mu, \theta) \prod_{i=1}^{N} \frac{\Gamma(\frac{\nu_i+1}{2})}{\sqrt{\nu_i\pi}\Gamma(\nu_i/2)} \left(1 + \frac{t_i^2}{\nu_i}\right)^{-\frac{\nu_i+1}{2}}$$

where $t_i = \frac{u_i-\theta_i}{\sqrt{v_i}}$ and $\nu_i = \frac{1}{2r_i^2}$.

- So we can treat our nuisance parameters as t-distributed!

- Consider the case:

$$y_i = d_i + errors = d_i + \sigma_i z_i + \sigma_{u_i} t_{u_i} + \sum_{j=1}^{M} \beta_{ij} t'_j \qquad (3)$$

where for each observable y_i we have

- One statistical error σ_i, with a z_i that is a Normally distributed fluctuating variable.
- One uncorrelated systematic error σ_{u_i} with a t_{u_i} that is a t-distributed fluctuating variable with dof of $\nu = 1/2r^2_{\chi^2}$
- M correlated systematic errors, β_{ij}, each with a fluctuation t'_j that are t-distributed with dof of $\nu = 1/2r^2_{\chi^2}$. These fluctuations are the same for all i.
- The likelihood function can be written up to some constants as:

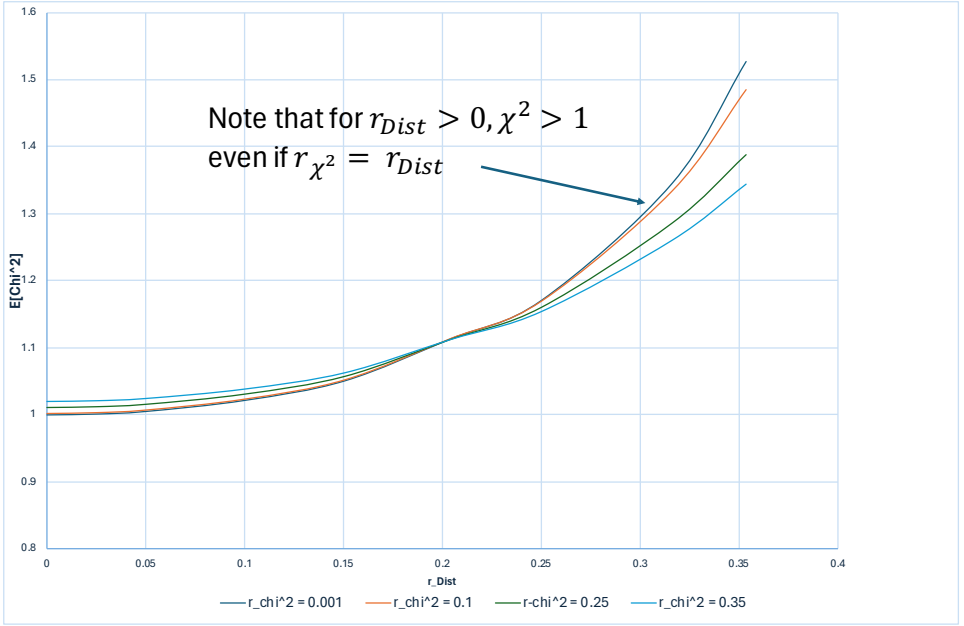
$$L \propto \prod_{i=1}^{N} \exp\left[-\frac{1}{2}\frac{(y_i - d_i - t_{u_i}\sigma_{u_i} - \sum_{j=1}^{M}\beta_{ij}t'_j)}{\sigma_i^2}\right]\left(1 + \frac{{t_{u_i}}^2}{\nu}\right)^{-\frac{\nu+1}{2}} \times \prod_{j=1}^{M}\left(1 + \frac{{t'_j}^2}{\nu}\right)^{-\frac{\nu+1}{2}} \qquad (4)$$

- Naturally leading to the Loglikehihood equation:

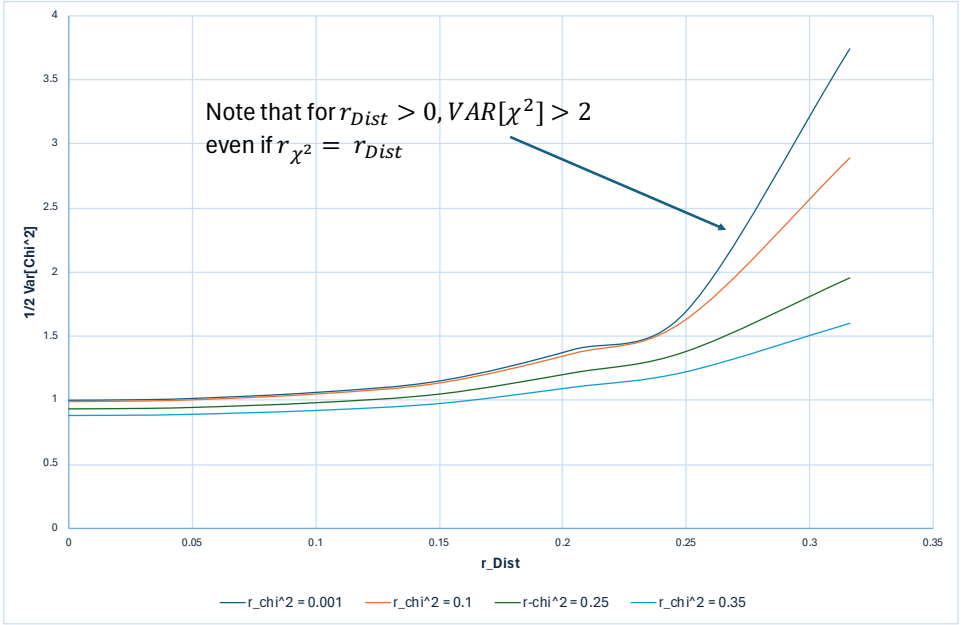
$$\chi^2 \equiv -2LnL = \sum_{i=1}^{N}\left(\frac{m_i - d_i - \sigma_{u_i}t_{u_i} - \sum_j \beta_{ij}t'_j}{\sigma_i}\right)^2 + (\nu+1)\sum_{i=1}^{N} Ln\left(1 + \frac{t_{u_i}^2}{\nu}\right) + (\nu+1)\sum_{j=1}^{M} Ln\left(1 + \frac{{t'}_j^2}{\nu}\right) \qquad (5)$$

Expectation and Variance of χ^2 as a function of r [8].

χ^2 increases if $r_i > 0$ especially (but not only) if χ^2 calculation not modified to acknowledge this.



Similarly variance increases if $r_i > 0$ particularly if χ^2 calculation not modified.



Decorrelation of Systematic Uncertainties.

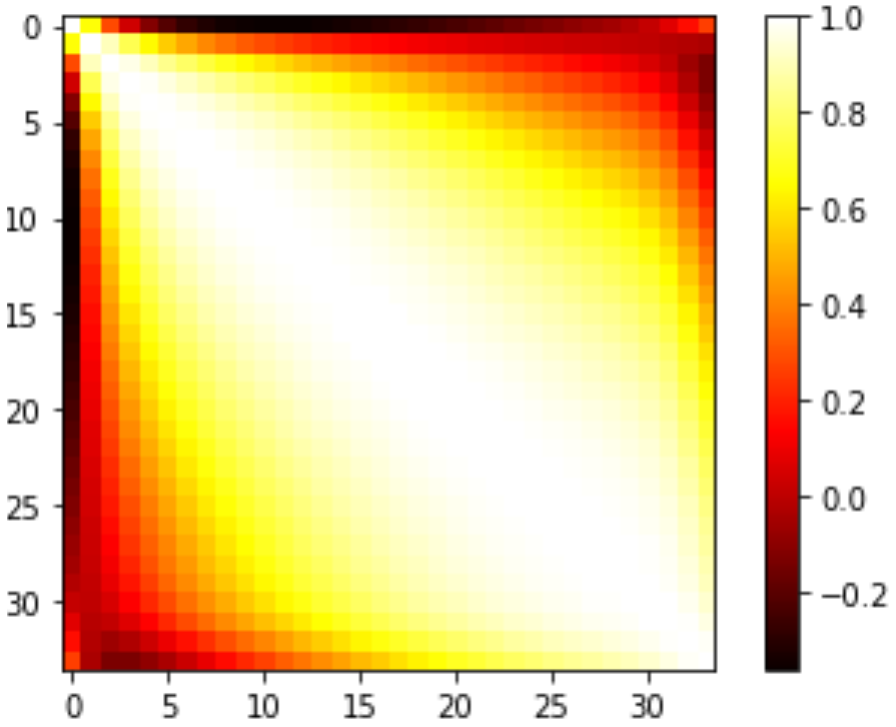
Sometimes necessary to use in fits to give reasonable χ^2.

Can be justified/explained by the fact that systematic uncertainties often depend on e.g. Monte Carlo variation where a 100% point-to-point correlation would not be expected.

Example, ATLAS 8 TeV inclusive jet data [9].

4 recommended candidate systematic errors for decorrelation.

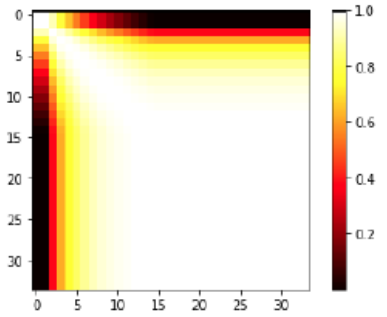
MSHT20 procedure turns each into combination of 4 correlated errors $\rightarrow \chi^2 = 345/171$.



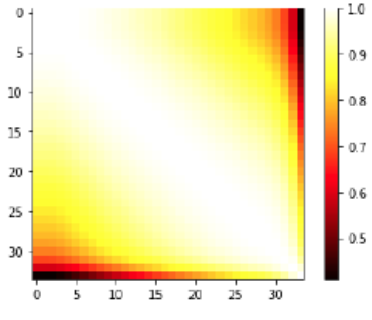
Decorrelation models proposed for ATLAS 8 TeV Jet data.

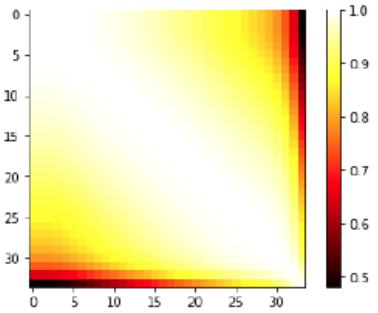
- Below are correlation matrices described by 4 different de-correlation methods described in the ATLAS paper.[8]

Splitting Option 5 $\chi^2 = 383$ $(1 \rightarrow 2)$

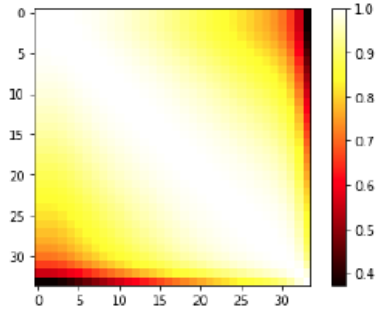


Splitting Option 16 $\chi^2 = 349$ $(1 \rightarrow 3)$





Splitting Option 17 $\chi^2 = 346$ $(1 \rightarrow 3)$



Splitting Option 18 $\chi^2 = 358$ $(1 \rightarrow 3)$

[8] For ease of presentation, heat maps only show the correlation structure due to the de-correlation of P_T in the lowest y bin. χ^2 numbers refer to full de-corelation including y space. $1 \rightarrow n$ refers to 1 correlated systematic error converted to n correlated errors.

Correlation high except for point at the edge of kinematics. Difficult to avoid using this approach.

Alternative Approach.

Aim for something where the correlation decreases in a smooth fashion between data points.

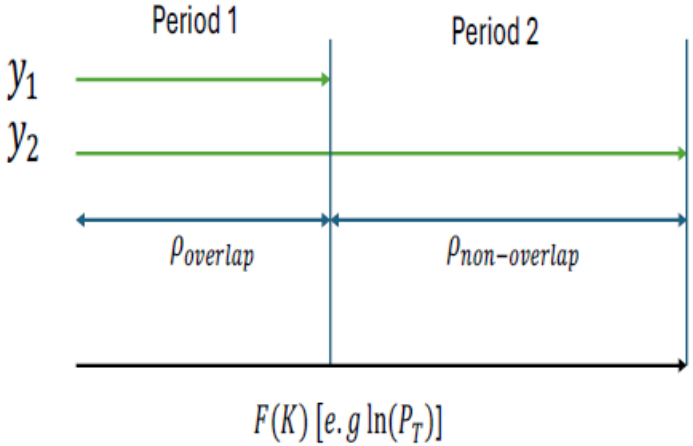
- For a given kinematic variable , K, we may want the following behaviour for a 2-point error [$z_i \sim N(0,1)$]:

$$y_1 \sim \sqrt{F(K_1)}z_1$$

$$y_2 \sim \sqrt{F(K_1)}z_2 + \sqrt{F(K_2) - F(K_1)}z_3$$

with correlations over the 2 periods:

$$\rho_{z_1,z_2} = \rho_{overlap} \quad \rho_{z_2,z_3} = \rho_{non-overlap}$$



- We can simply write down the correlation between y_1 and y_2 as:

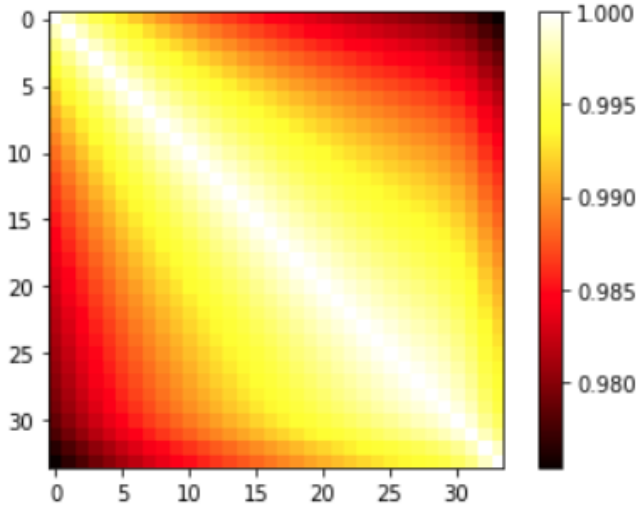
$$\rho_{1,2}(F(K)) = \frac{\rho_{overlap}(\sqrt{F(K_1)} + \rho_{non-overlap}\sqrt{F(K_2) - F(K_1)})}{\sqrt{F(K_2) + 2\rho_{non-overlap}\sqrt{F(K_1)}\sqrt{F(K_2) - F(K_1)}}}$$

- If we now assume for simplicity that $\rho_{1,2} = \rho_{1,2}(F(P_T)) \times \rho_{1,2}(F(y))$
- and using the simple functions:

$$F(P_T) = Ln(P_T) \qquad F(y) = y$$

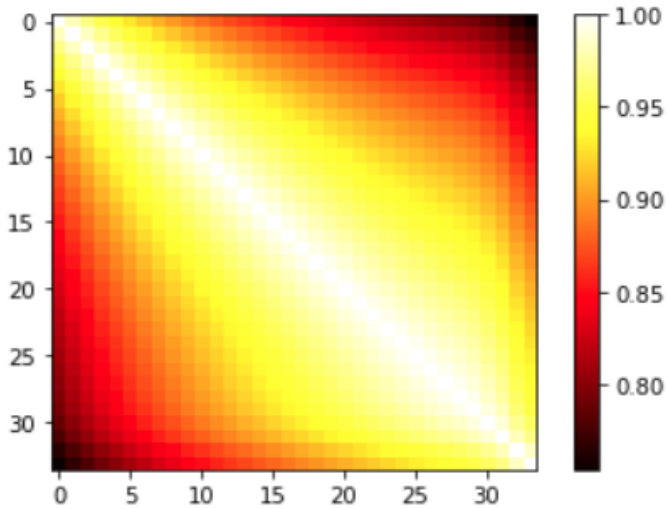
- We obtain the following correlation matrices in the case $\rho_{overlap} = 1$

$\rho_{non-overlap} = 0.90$



$\chi^2 = 329$

$\rho_{non-overlap} = 0$



$\chi^2 = 233$

- To match the MSHT20 de-correlated $\chi^2 = 345$, $\rho_{non-overlap} = 0.94$

Can decompose matrix into orthogonal eigenvectors. In principle all correlated if number of eigenvectors $m_\lambda = N_{data}$

M_λ	10	9	8	7	6
Eigen Values	31.38354882	31.38289389	31.38205569	31.38091469	31.37931286
	1.420410959	1.419709761	1.418840345	1.417594975	1.415728003
	0.411270394	0.410624252	0.409794869	0.40860557	0.406808537
	0.196099805	0.195495108	0.194702518	0.193557435	0.19178369
	0.114456638	0.113892137	0.113117527	0.111999399	0.110262776
	0.071790292	0.071221206	0.070443423	0.069314649	0.067622144
	0.047140754	0.046536308	0.045745646	0.04458896	0
	0.032334442	0.031680255	0.03086293	0	0
	0.022766932	0.022037056	0	0	0
	0.01658088	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0
Error	0.019221612	0.024957612	0.032812948	0.044392454	0.061981092

M_λ	10	9	8	7	6
Diagonal	0.035582031	0.035929879	0.036538739	0.037945115	0.041001278
	0.000408356	0.000832162	0.00153446	0.003095487	0.006383048
	0.015090702	0.015600952	0.01599194	0.016044019	0.015366201
	0.009435912	0.0120917	0.014482	0.017118397	0.019426471
	0.009396629	0.008651851	0.009545099	0.0129726	0.017872201
	0.011215169	0.011750679	0.011356818	0.010936442	0.014120954
	0.008234458	0.010619169	0.012359776	0.012485513	0.012035364
	0.008188891	0.008151197	0.010510364	0.012970766	0.01281777
	0.008896867	0.008769956	0.00856325	0.011504974	0.013874978
	0.007443871	0.009213886	0.009097752	0.00967664	0.01364969
	0.006991453	0.00823039	0.009930602	0.00938482	0.012303194
	0.007639395	0.007123286	0.009133473	0.01009568	0.010574353
	0.007137234	0.007528077	0.007733555	0.010369945	0.009822683
	0.006210641	0.007900123	0.007521174	0.009801989	0.010300209
	0.006453615	0.007213061	0.008274772	0.008710751	0.011133381
	0.006896389	0.006454176	0.008526198	0.008076766	0.011445471
	0.006362919	0.006764188	0.007823212	0.008407046	0.0109289
	0.005647598	0.00718155	0.006890894	0.009043241	0.009837387
	0.005940989	0.006775942	0.006810149	0.009148273	0.008806768
	0.00626523	0.005962435	0.007403025	0.008491055	0.008466895
	0.005772569	0.005854674	0.007633826	0.007482802	0.008908548
	0.005099902	0.006356001	0.007076803	0.006913211	0.009625934
	0.005292178	0.006422932	0.006163354	0.007224009	0.010017279
	0.005710275	0.005791662	0.005846973	0.007952638	0.009760035
	0.005483942	0.005190794	0.006440922	0.008335222	0.008886912
	0.004759427	0.005421187	0.007051946	0.007912831	0.007667311
	0.004623886	0.005970159	0.006859657	0.00679318	0.006767717
	0.005253385	0.006002597	0.005965431	0.005786801	0.006938772
	0.005751624	0.005572998	0.0053464	0.00615733	0.008520053
	0.005780677	0.005699734	0.006882869	0.008965817	0.011544099
	0.007232282	0.0097232	0.011877948	0.013651902	0.015033126
	0.013744934	0.014698391	0.015454596	0.015807518	0.015578056
	0.000372964	0.00081183	0.001526892	0.00275724	0.005294355
	0.029283696	0.029649209	0.030282189	0.031404312	0.033772608

However, using technique in [10] can accurately approximate by interpreting as smaller M_λ and some uncorrelated component.

Works well for $M_\lambda = 6 - 10$ with uncorrelated contribution 1% of total.

Investigations with 13 TeV LHC inclusive jet/dijet data.

Studied the impact using various frameworks for addressing systematic uncertainties for 13 TeV ATLAS inclusive jet/dijet data [11] and 13 TeV CMS inclusive jet data [12] and di-jet data [13].

In practice the biggest impact is from ATLAS inclusive jet data ($R = 0.4$) and CMS di-jet data ($R = 0.8$). So only present results for this.

Investigate impact of data using the profiling procedure

Additional χ^2 due to new data set in global fit

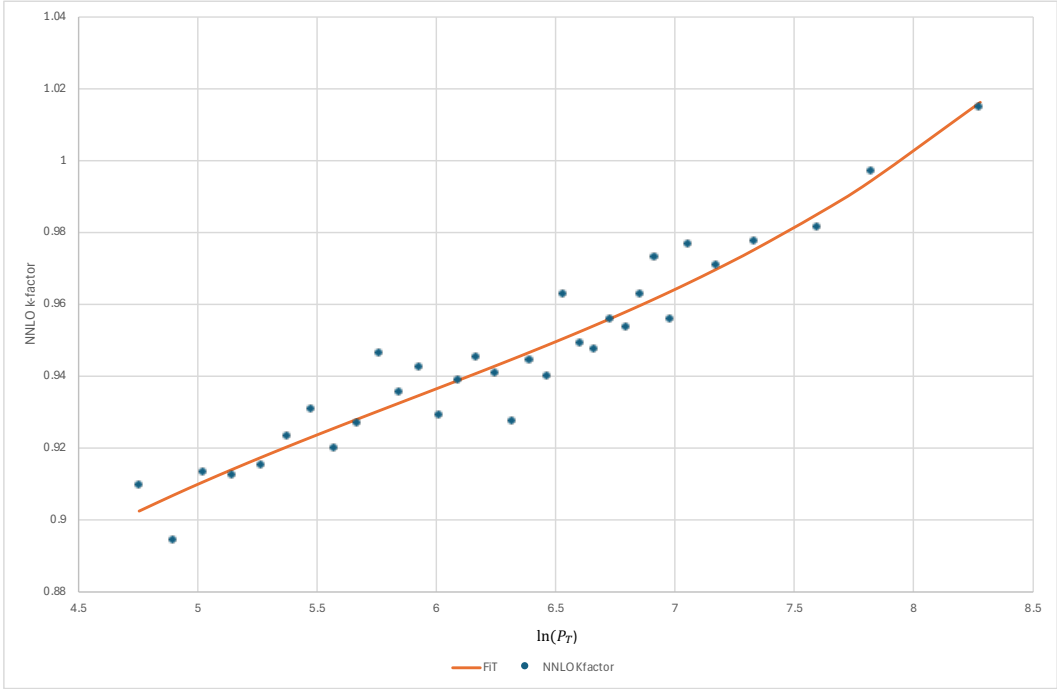
$$\chi^2(\beta_{exp}, w) = \sum_{i=1}^{N_{dat}} \frac{(D_i + \sum_j \lambda_{ij}^{exp} \beta_{j,exp} - T_i - \sum_k \lambda_{ik}^{th} w_k)^2}{\sigma_{i,uncor}^2} + \sum_j \beta_{j,exp}^2 + \sum_k T_k^2 w_k^2,$$

where k is the number of eigenvector pairs and T^2 is the tolerance. For MSHT T_k is dynamical but $T^2 = 9$ is a very good approx.).

Allows for a study in variations of "error on error" and correlation procedures.

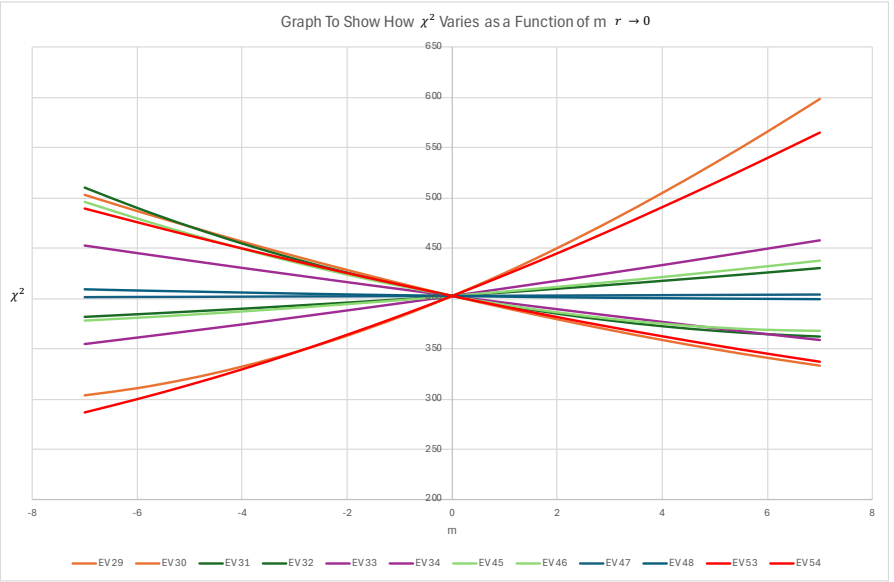
Need to smooth the K-factors obtained from the grids (alternative, provide additional uncorrelated uncertainty source).

Use polynomial in $\ln(p_T)$. In principle uncertainty associatted with freedom in coefficients.

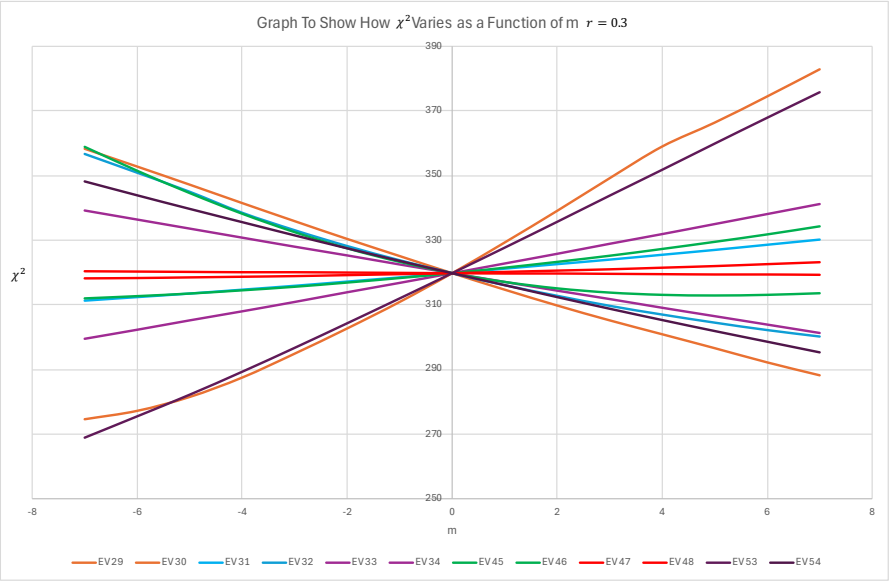


In fit we decorrelate a small number of Monte Carlo dependent systematic errors using $\rho_{overlap} = 1, \rho_{non-overlap} = 0$.

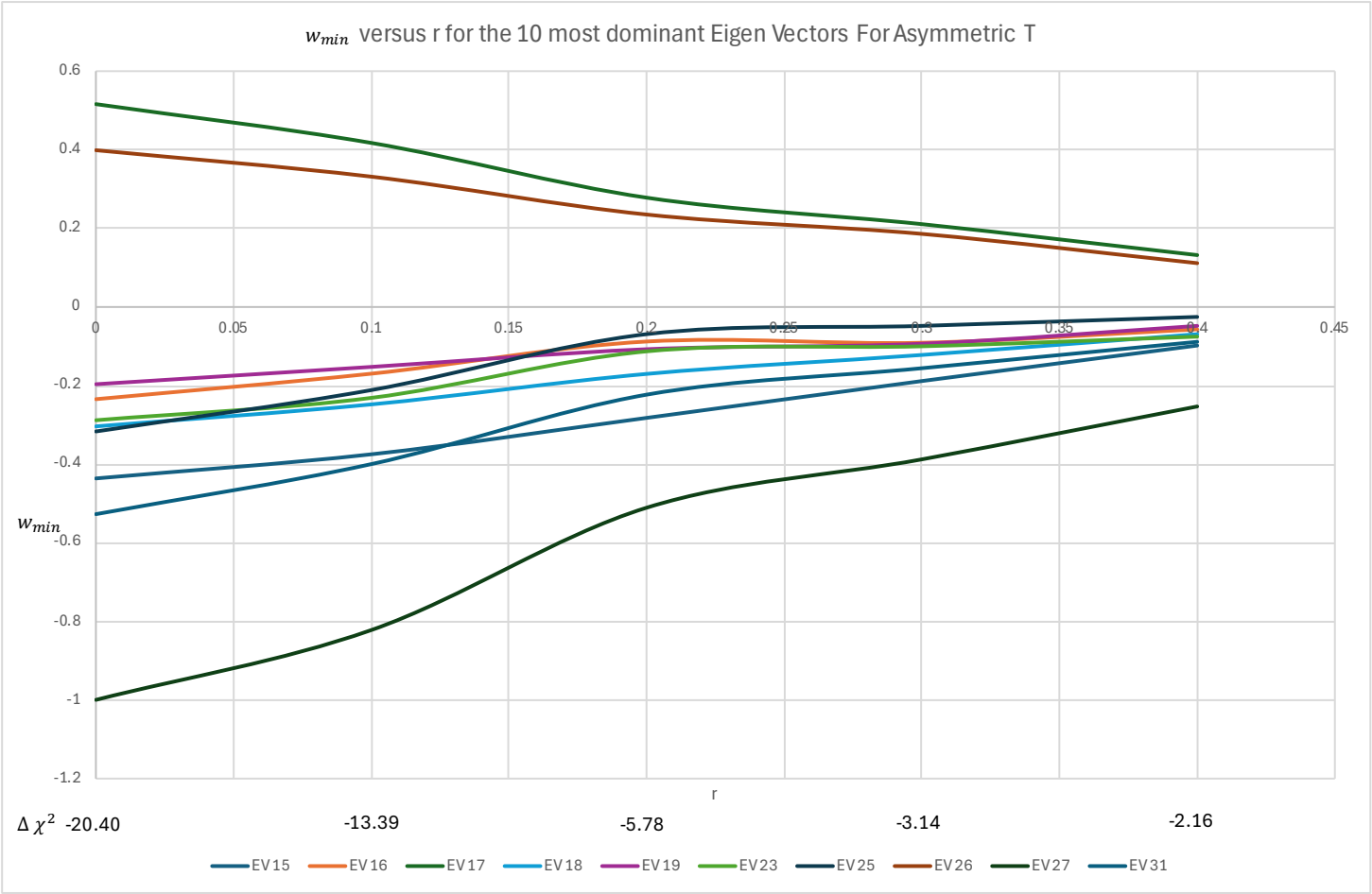
Fit quality highly dependent on PDF variation along some of the eigenvector directions, e.g ATLAS inclusive jets.



Both best fit and χ^2 variation modified significant by assumed "error on error" on systematic uncertainties, e.g. $r_i = 0.3$ (less so for CMS dijets).

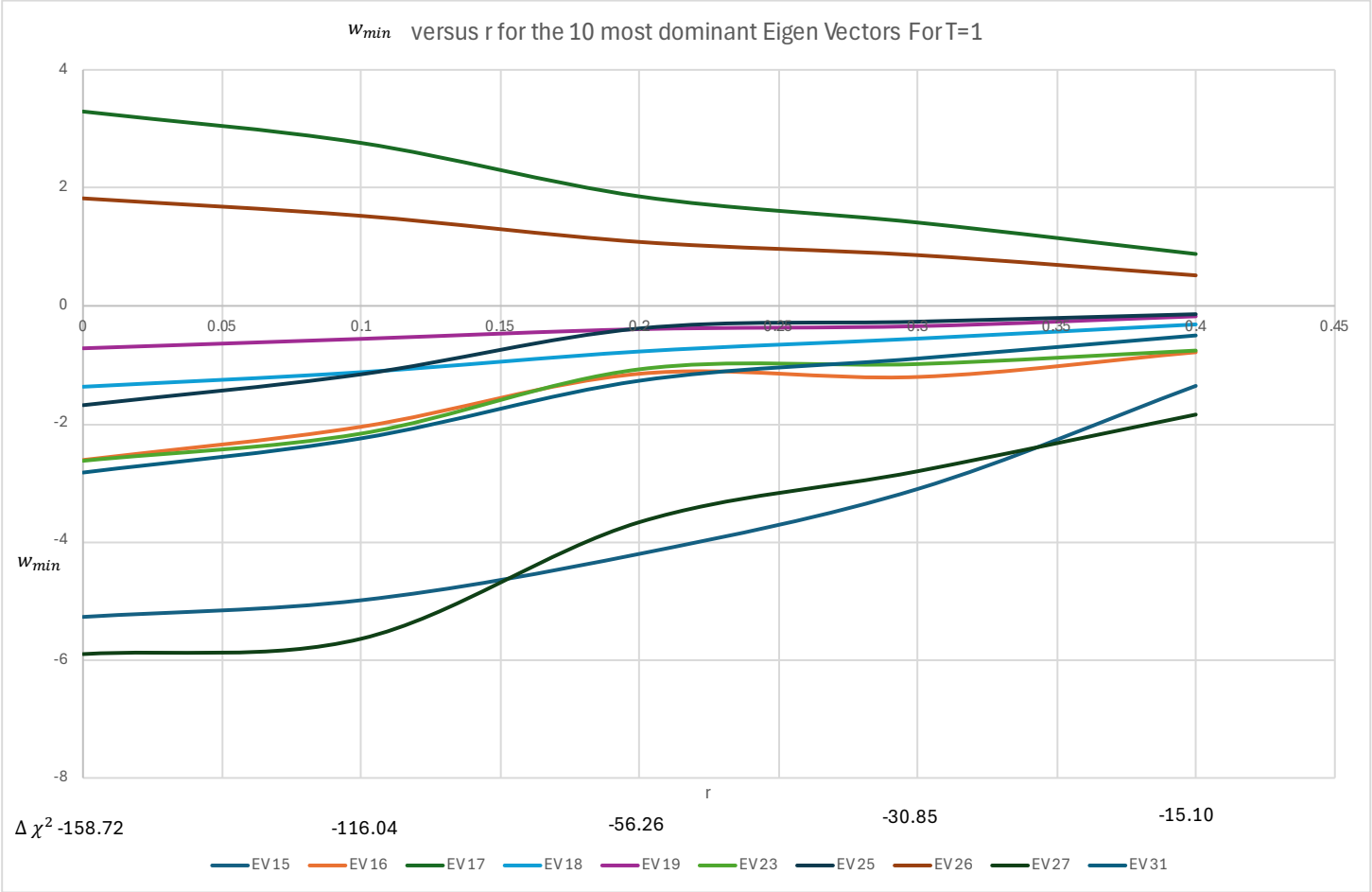


Impact clear on obtained values of $w_{i,min}$ for ATLAS inclusive jets as a function of r_i for default tolerance definition.



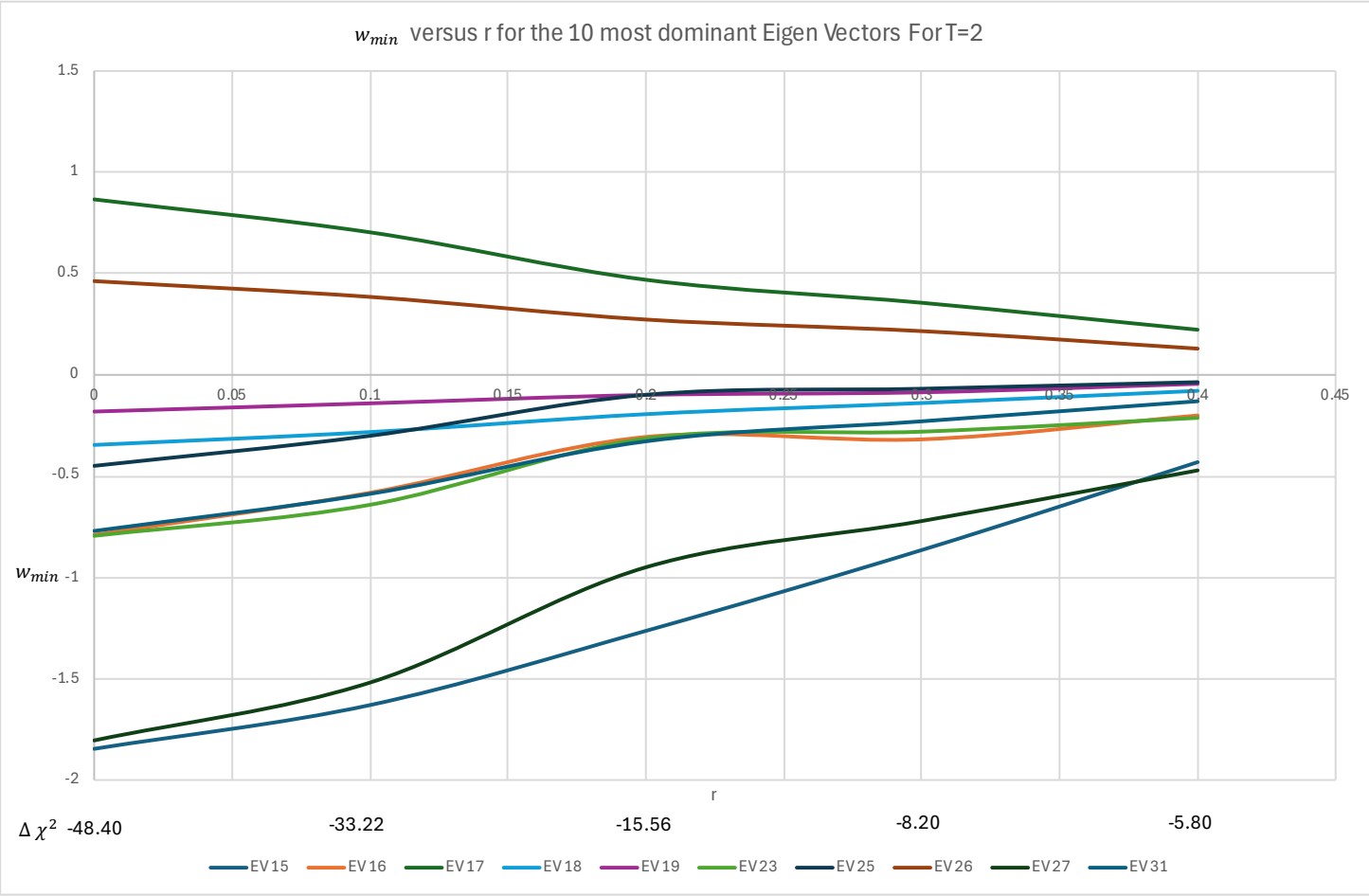
$\Delta\chi^2$ refers to jet data. Penalty from eigenvector shifts, i.e. deterioration in fit to global data $\Delta\chi^2_{MSHT} \approx -0.5\Delta\chi^2_{jet}$.

If instead using naive $T^2 = 1$, $w_{i,min}$ values much greater, but again these, and χ^2 changes moderated significantly as r_i increase.

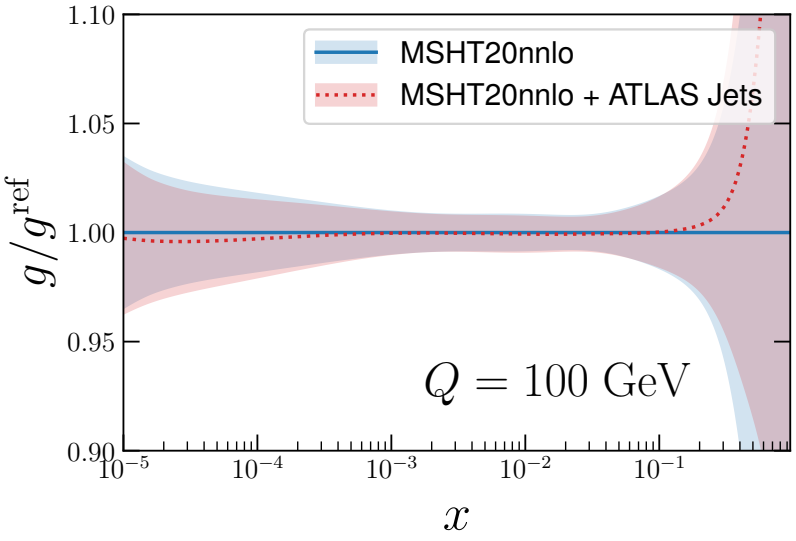


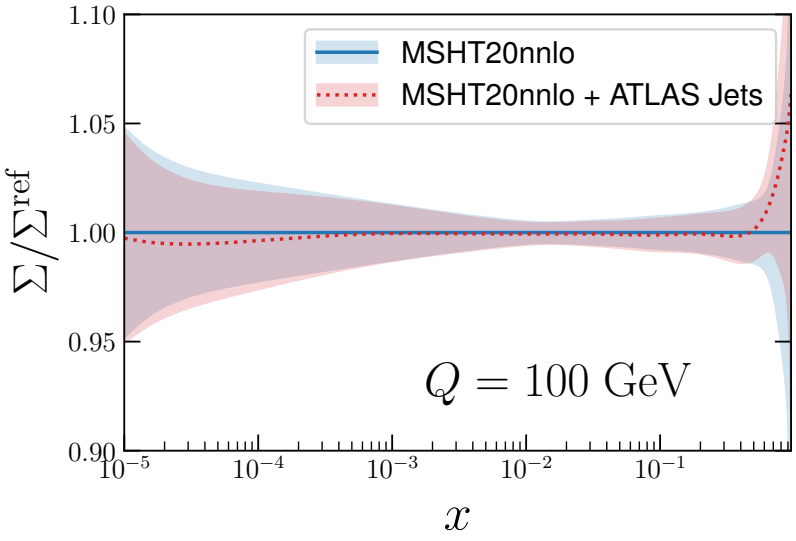
Now $\Delta\chi^2_{MSHT} \approx -0.5\Delta\chi^2_{jet}$ when global deterioration calculated using $T^2 = 1$. However, real deterioration in global fit $\Delta\chi^2_{MSHT} \approx -5\Delta\chi^2_{jet}$

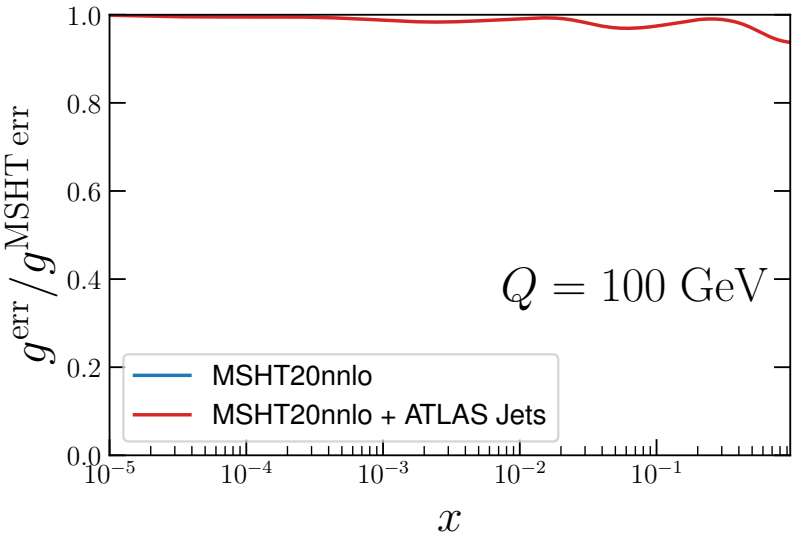
But for $r_i \neq 0$ then we have seen $T^2 \neq 1$. Try $T = 2$ as a conservative assumption for tolerance to assign to MSHT eigenvectors if data in global fit had $r_i \neq 0$.

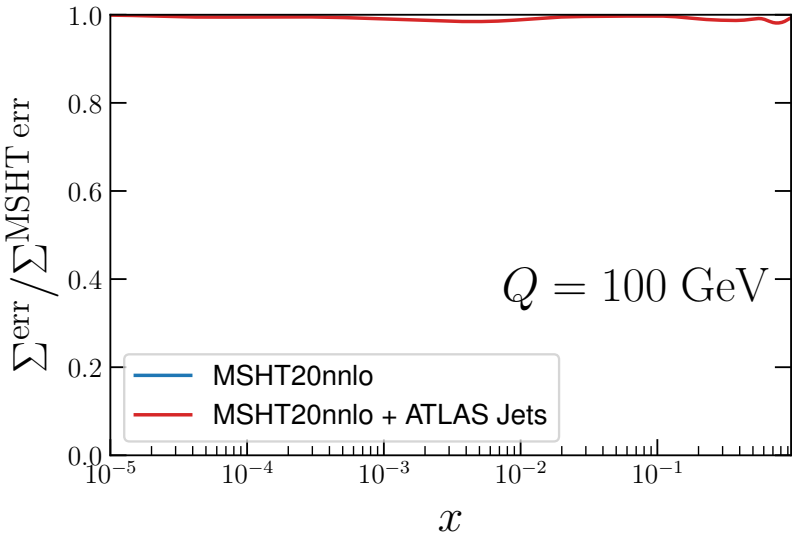


Effect of profiling using 13 TeV ATLAS inclusive jets [11]- default approach.

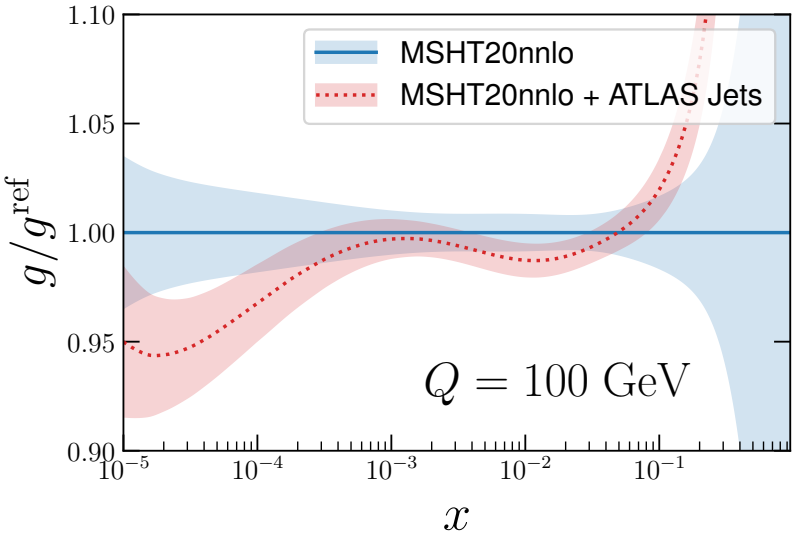


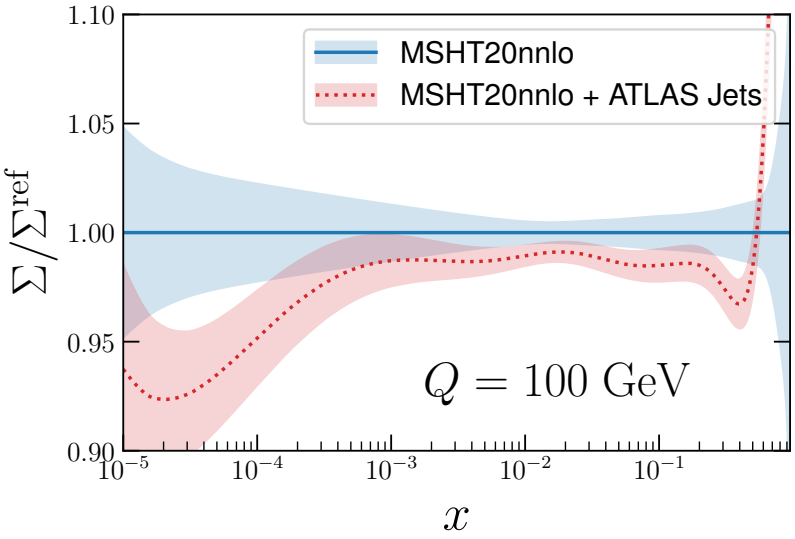


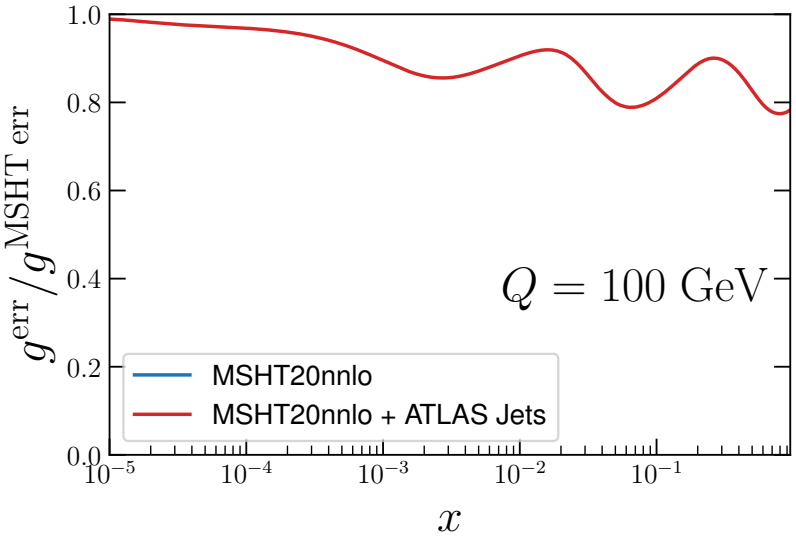


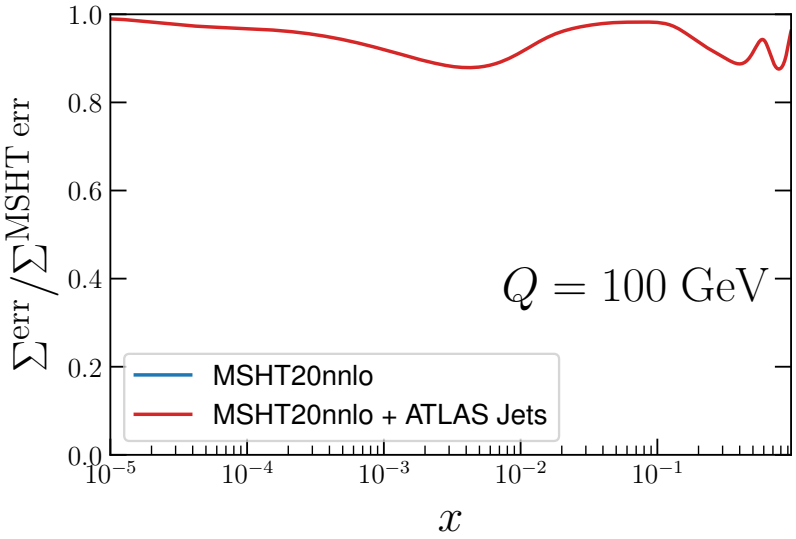


Effect of profiling using 13 TeV ATLAS inclusive jets [11] - $T = 1$.

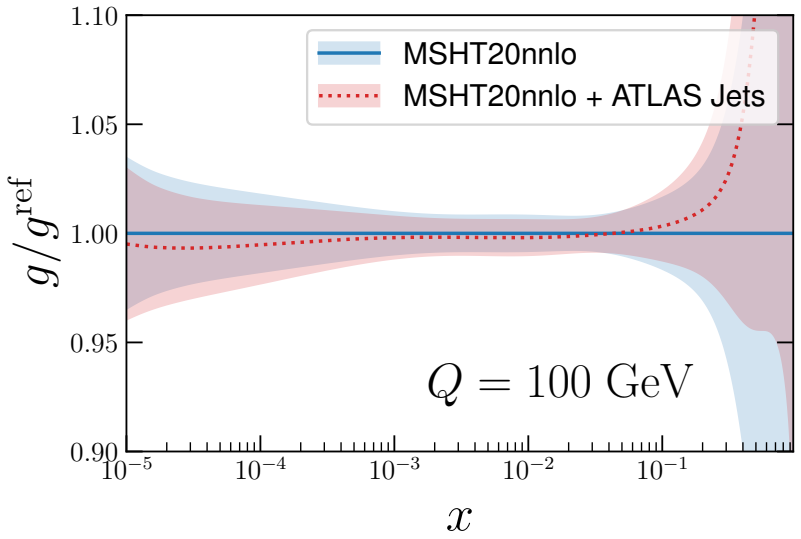


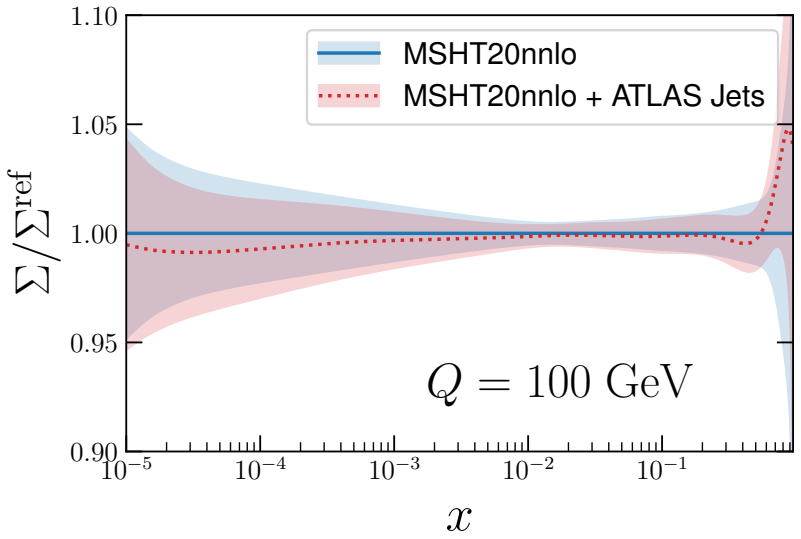


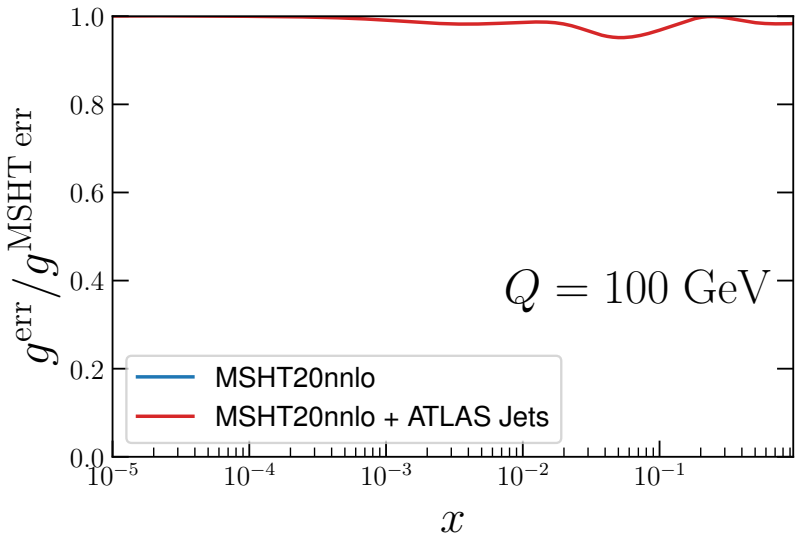


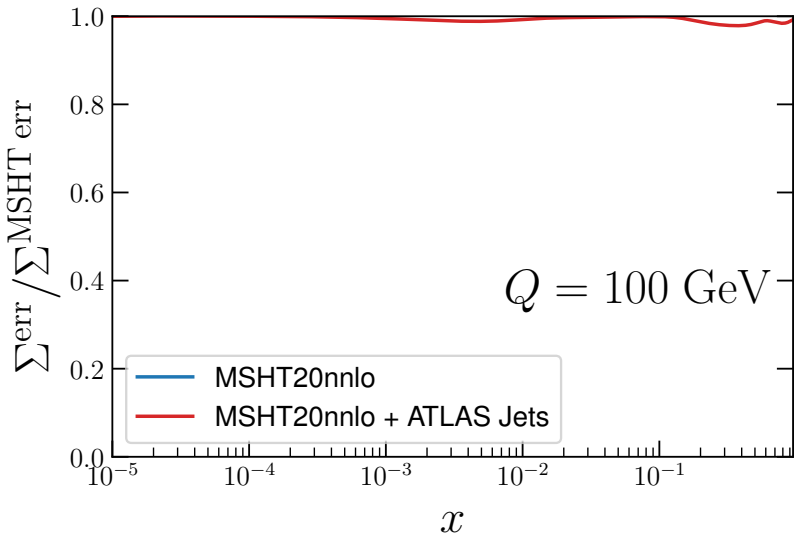


Effect of profiling using 13 TeV ATLAS inclusive jets [11] - $T = 2, r_i = 0.3$.

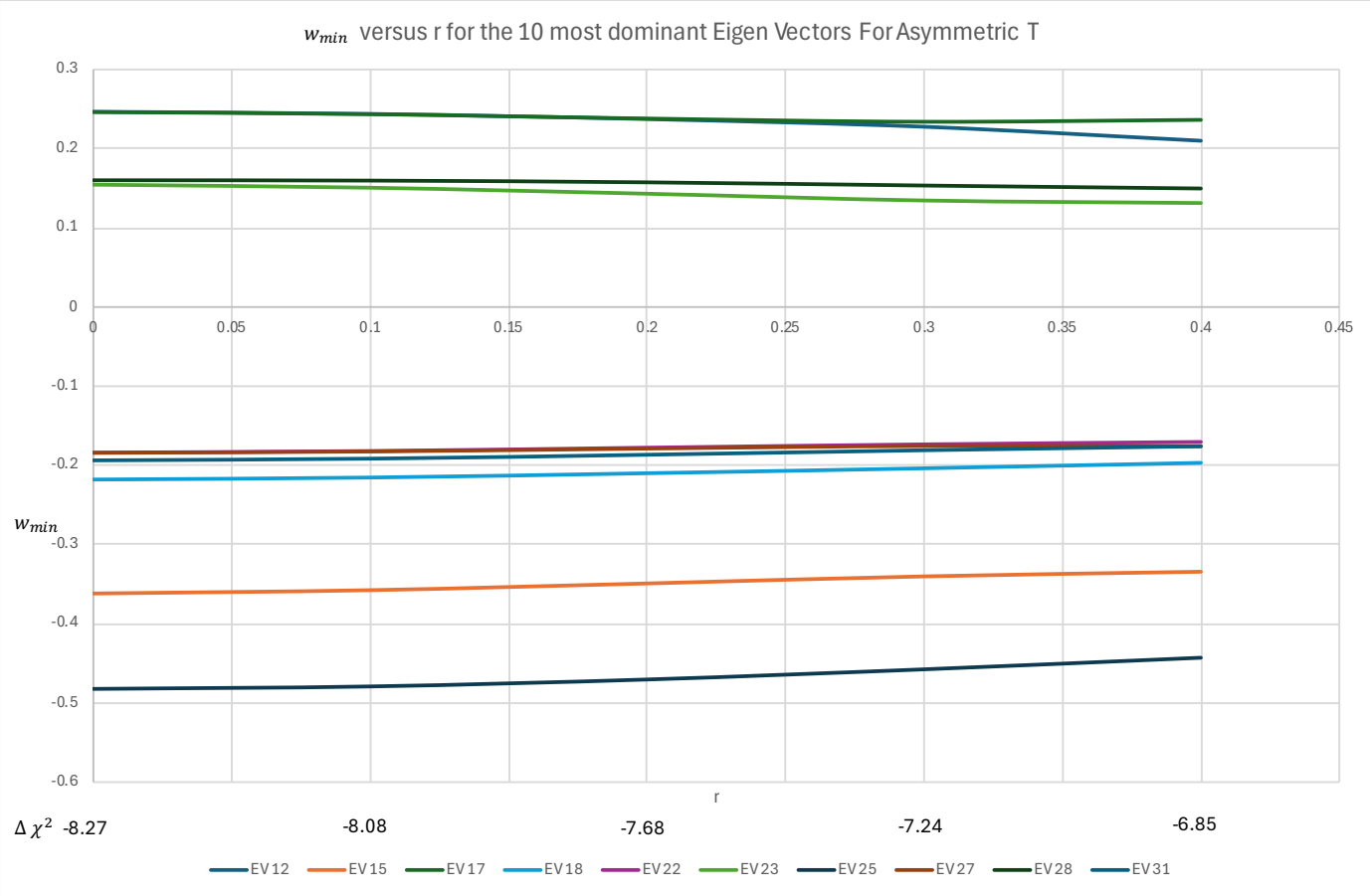






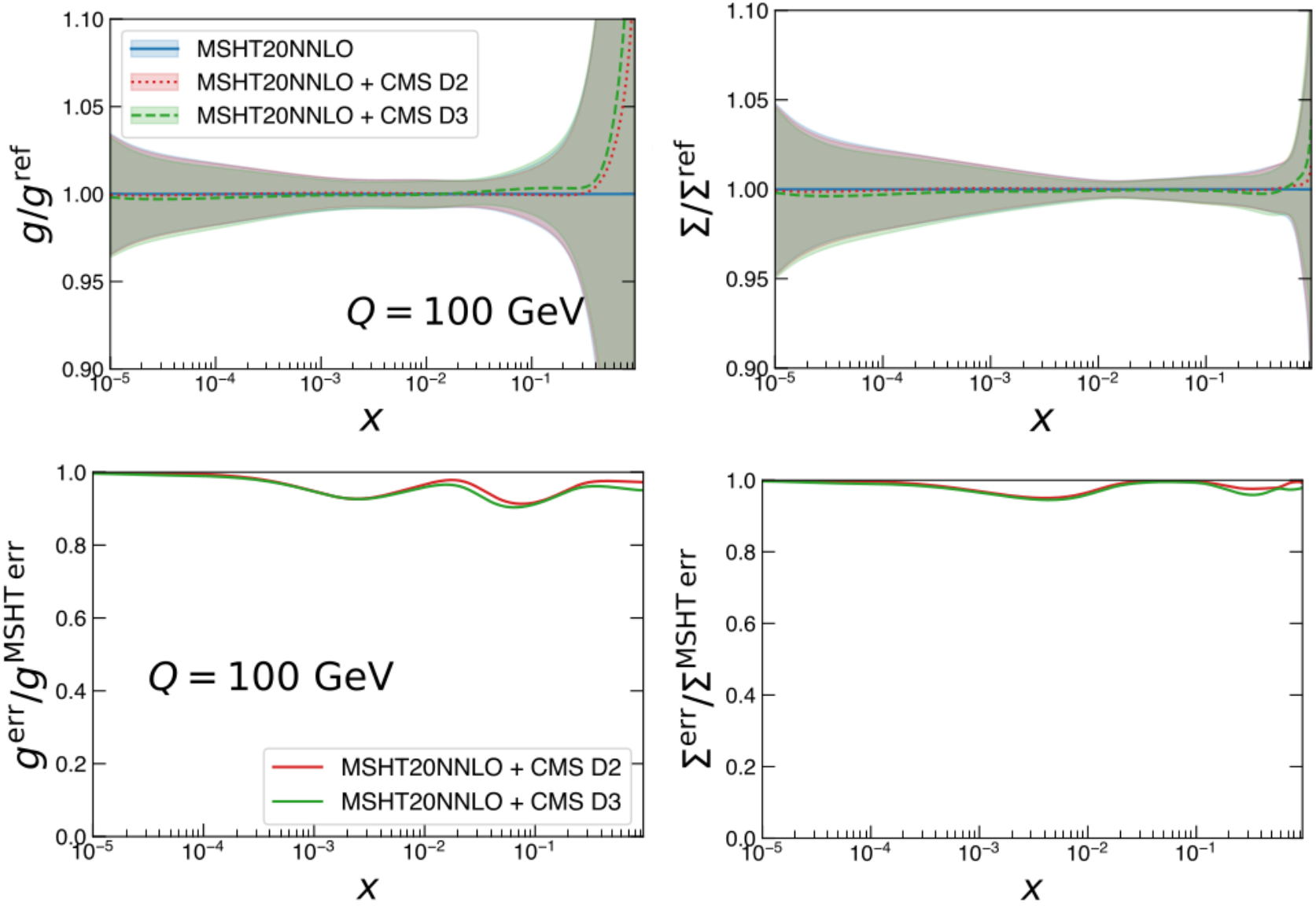


Impact less clear on obtained values of $w_{i,min}$ for CMS di-jets as a function of r_i for default tolerance definition (for triple-differential data, similar for double differential).

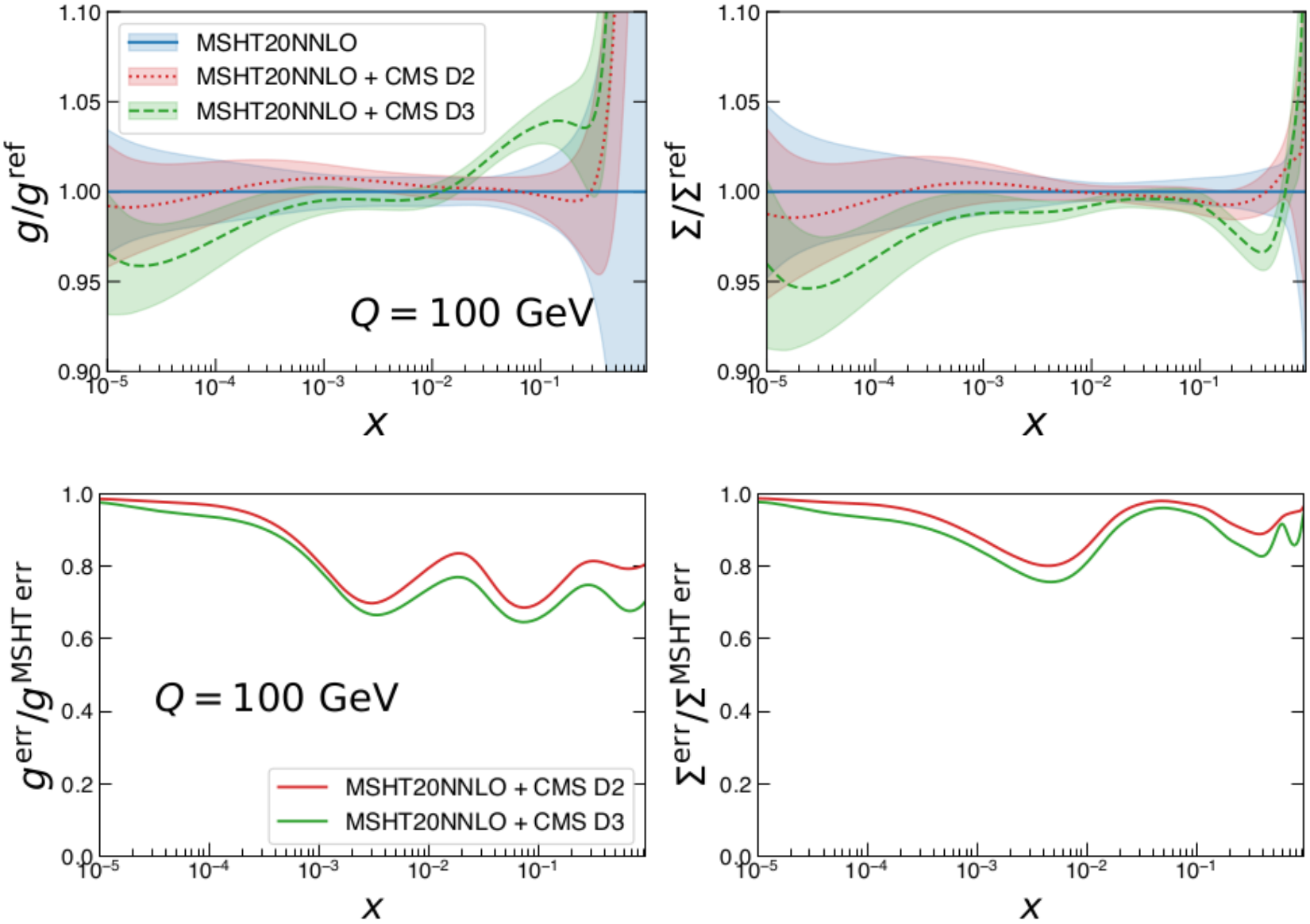


Similar r dependence if T reduced, though again absolute values of $w_{i,min}$ increase significantly.

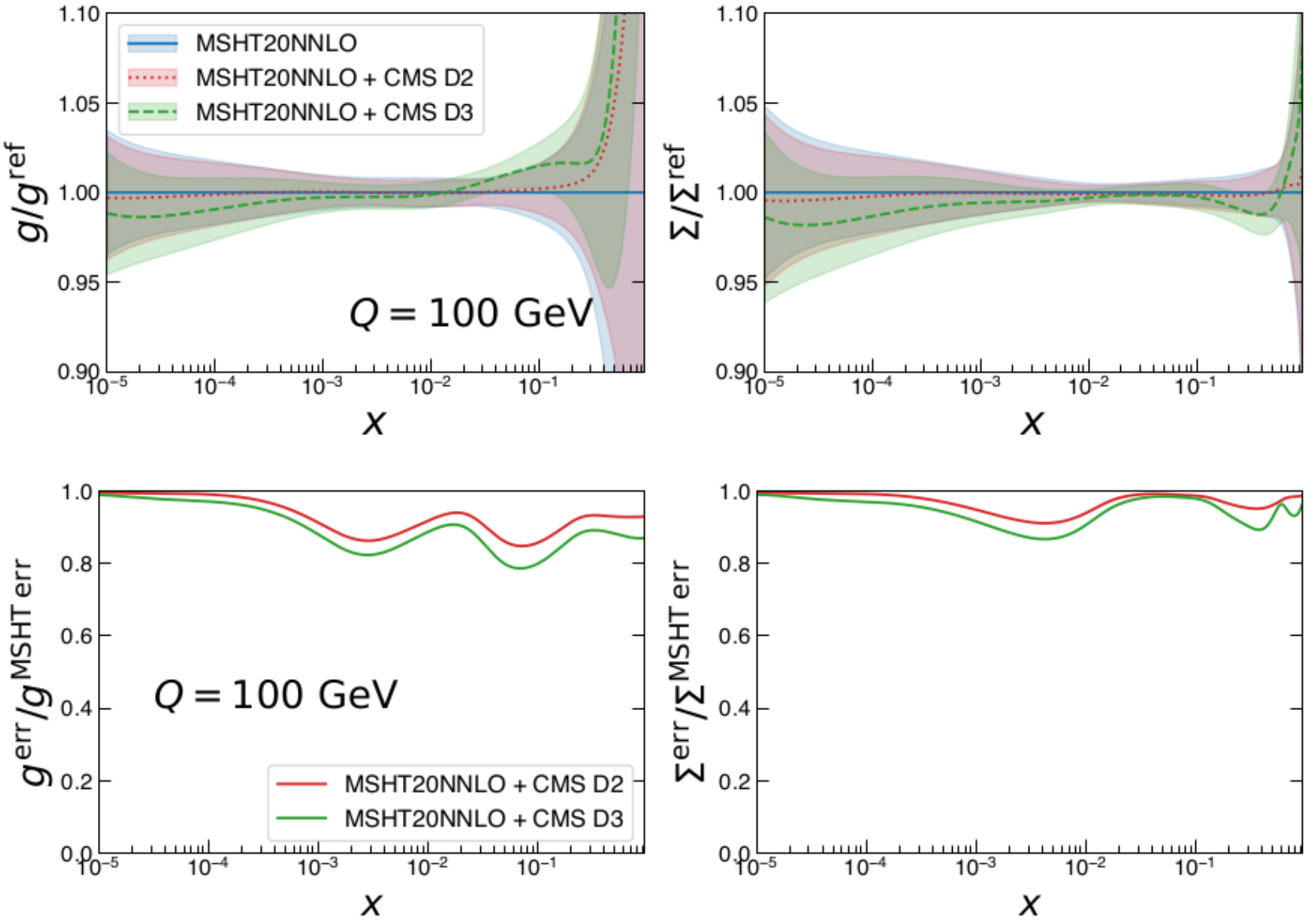
Effect of profiling using 13 TeV CMS di-jet data [13]- default approach.



Effect of profiling using 13 TeV CMS di-jet data [13]- $T = 1$.



Effect of profiling using 13 TeV CMS di-jet data [13]- $T = 2, r_i = 0.3$.



Conclusions

Investigate sensitivity of fit quality and PDFs to modifications in both data and inclusion of non-perturbative theoretical corrections at high x.

Target Mass and Higher twist corrections required for lower W^2 cuts, but with default cuts hard to distinguish from higher order corrections. Impact on PDFs mild.

Impact of improved treatment of fixed-target DIS data and special ZEUS data mild. Antiquark decompostion sensitive to SeaQuest data.

EIC pseudodata improves $\alpha_S(M_Z^2)$ determination, with degree related to "optimism" about data. Sensitivity to theoretical uncertainties.

Look at "errors" on errors - implies some increased tolerance. Also investigated the impact of new constraining jet data using this approach. Significant sensitivity when looking at inconsistent data.

Also investigated ways of incorporating lack of perfect correlations in errors. Sensitivity also to this choice.

Implications of 13 TeV LHC jet data due to both using profiling techniques.

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[10] - T.J. Hou et al. Phys. Rev. D 103 (2021) 014013.

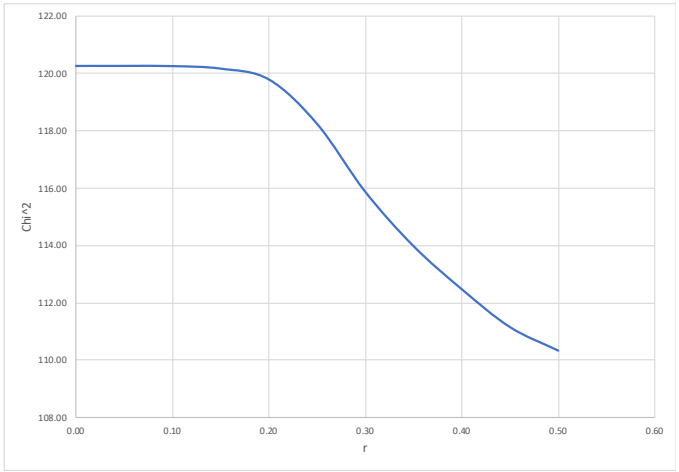
[11] - M. Aaboud et al. [ATLAS], JHEP 05 (2018), 195.

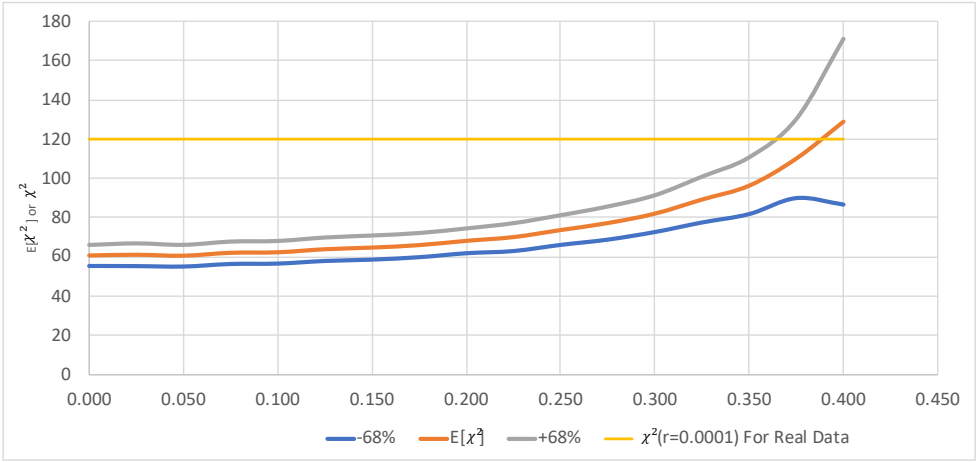
[12] - A. Tumasyan et al. [CMS], JHEP 02 (2022), 142.

[13] - A. Hayrapetyan *et al.* [CMS], Eur. Phys. J. C 85 (2025) no.1, 72.

Applications and Implications for LHC data.

ATLAS W, Z data [7] is very precise and gives poor χ^2 in fits (though depends on details of theory).





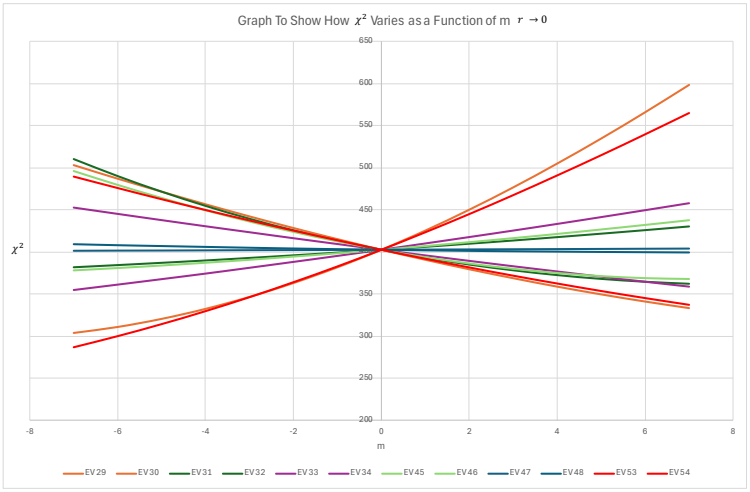
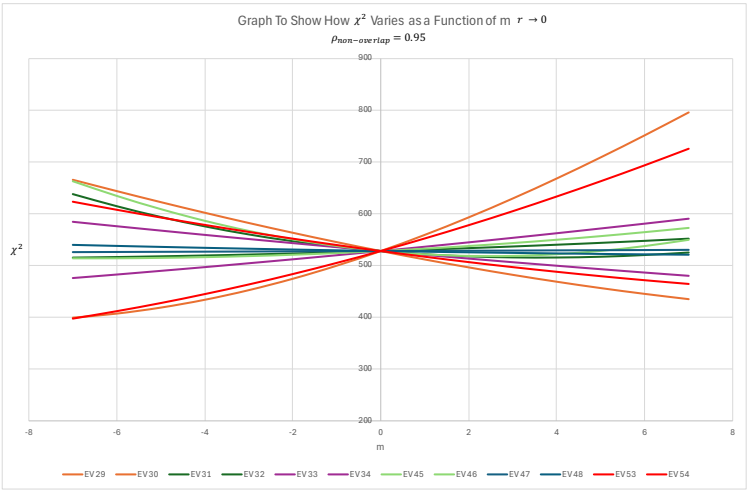
Left shows χ^2 decrease of real fit as r in χ^2 definition increases.

Right shows quality of simple χ^2 as r in pseudodata increases.

Effect of assumption about degree of correlation.

Change in χ^2 and w_{min} values as a function of $\rho_{non-overlap}$.

Absolute value of χ^2 particularly sensitive.

$\rho_{non-overlap}$	0	0.75	0.9	0.95	1
1	-0.001355	-0.001338	-0.001353	-0.001362	-0.001364
2	-0.000392	-0.000389	-0.000391	-0.000387	-0.000385
3	-0.003316	-0.00323	-0.003302	-0.003441	-0.003466
4	0.002895	0.002875	0.002891	0.003002	0.003028
5	-0.015409	-0.013523	-0.015993	-0.020943	-0.022659
6	-0.000701	-0.00071	-0.000702	-0.000683	-0.000673
7	-0.009261	-0.00949	-0.009239	-0.010088	-0.010647
8	-0.003841	-0.00371	-0.003865	-0.004272	-0.004377
9	-0.066073	-0.09156	-0.116706	-0.112925	-0.133325
10	0.093793	0.117815	0.163521	0.151092	0.175825
11	-0.006702	-0.006629	-0.006701	-0.007113	-0.007116
12	0.096437	0.096628	0.072026	0.085733	0.063296
13	0.047198	0.045331	0.026602	0.037282	0.013036
14	0.090813	0.096843	0.08892	0.093175	0.086704
15	-0.417583	-0.494784	-0.57524	-0.562808	-0.624583
16	-0.237356	-0.209886	-0.14216	-0.161487	-0.096531
17	0.486426	0.515661	0.515416	0.525367	0.523651
18	-0.282812	-0.317711	-0.349962	-0.352271	-0.373897
19	-0.174866	-0.174253	-0.13282	-0.1502	-0.123608
20	0.033378	0.033078	0.038415	0.035175	0.029984
21	0.117846	0.129293	0.143876	0.137593	0.142969
22	-0.174592	-0.207044	-0.233043	-0.230799	-0.2532
23	-0.225225	-0.215957	-0.175054	-0.206305	-0.187028
24	-0.028007	-0.030667	-0.036881	-0.041759	-0.048912
25	-0.326005	-0.358265	-0.333187	-0.353443	-0.331915
26	0.350559	0.380167	0.394625	0.402987	0.408386
27	-0.346515	-0.36818	-0.369655	-0.38015	-0.379398
28	0.042196	0.045256	0.038482	0.039447	0.031178
29	0.016207	0.021242	0.014842	0.011473	0.010638
30	-0.029148	-0.04451	-0.067257	-0.061387	-0.073992
31	-0.162451	-0.176395	-0.178108	-0.181563	-0.179772
32	-0.117634	-0.135755	-0.151849	-0.147744	-0.161184
$\chi^2(w=0)$	402.34	470.51	506.24	528.09	575.83
$\Delta\chi^2(w_{min})$	-13.19	-15.83	-16.89	-18.27	-19.88

Also compare change in χ^2 along eigenvector direction when $\rho_{non-overlap} = 0.95$.