The Pion Parton Distribution Puzzle in the High x Region

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U.S. DEPARTMENT OF ENERGY

33rd International Workshop on
Deep Inelastic Scattering
04-08-May 2026, Bologna, Italy

- **Parton Distribution Function**
- **Generalized Parton Distributions**
- **Transverse Momentum Dependent Part on Distributions**

Focus on Interacting with the probe parton

- **Residual Hadron Distributions**

Focus on non-interacting with the probe residual system

Outline

– New Approach in Calculation of Valence Quark Structure of Hadrons

– Unexpected Observation for Valence PDF's of Nucleons

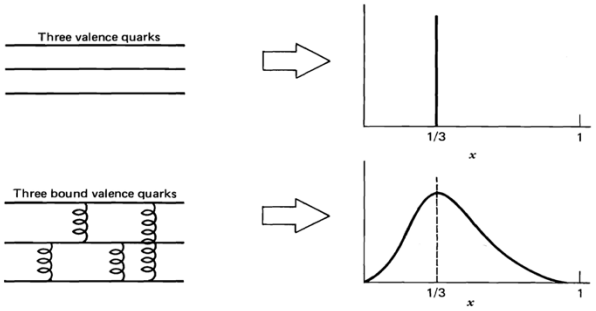
– Possible Resolution of Pion Valence PDF Puzzle at High x

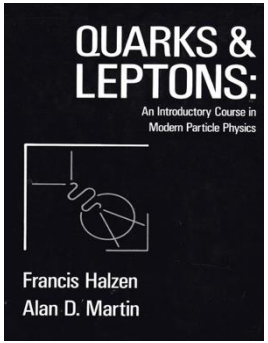
Nucleon:

- What is the average momentum fraction carried by a valence quark in the nucleon?

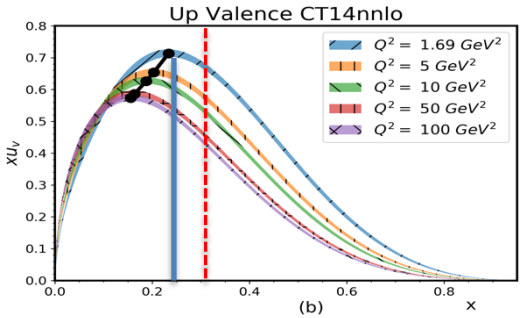
(at starting Q_0)

- Natural answer is 1/3, since there are three equal mass valence quarks in the nucleon





- Our observation is that it is $\leq 1/4$ supported by the data



Pion: Dual Nature

- Pion is a Goldstone particle of spontaneously broken chiral symmetry of QCD

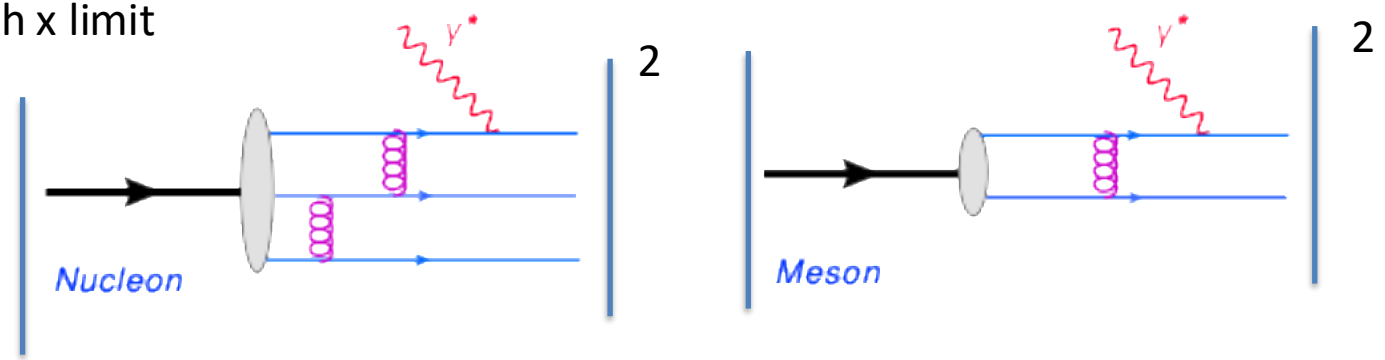
$$m_\pi^2 = \frac{(m_u + m_d)}{f_\pi^2} \langle q\bar{q} \rangle$$

- Pion contains valence quarks, see quarks and gluons which can be probed in Deep Inelastic Processes by extracting their distribution functions

$$f_V(x),\ f_s(x) \text{ and } f_G(x)$$

Pion Puzzle

At high x limit



$$f_V(x)_{x\to 1} \sim (1-x)^{\beta}$$

Farrar, Jackson PRL 1979

Berger Brodsky PRL 1979

$$\beta = 2N - 3 + 2\left|\lambda_h - \lambda_i\right|$$

For nucleon $\beta = 3$

For pion $\beta = 2$

Barry, Sato, Melnitchouk, Ji, PRL 2018

Extracted from the fit: $\beta \approx 1$ or $\beta \approx 1 - 1.2$

Barry, Ji, Sato, Melnitchouk, PRL 2021

Motivation: Valence quarks in the hadrons

Valence quarks play a unique role in QCD dynamics of hadrons

- They define baryonic number of hadrons:

- They represent "effective fermions" whose number is conserved with complex interaction among themselves and with hadronic environment

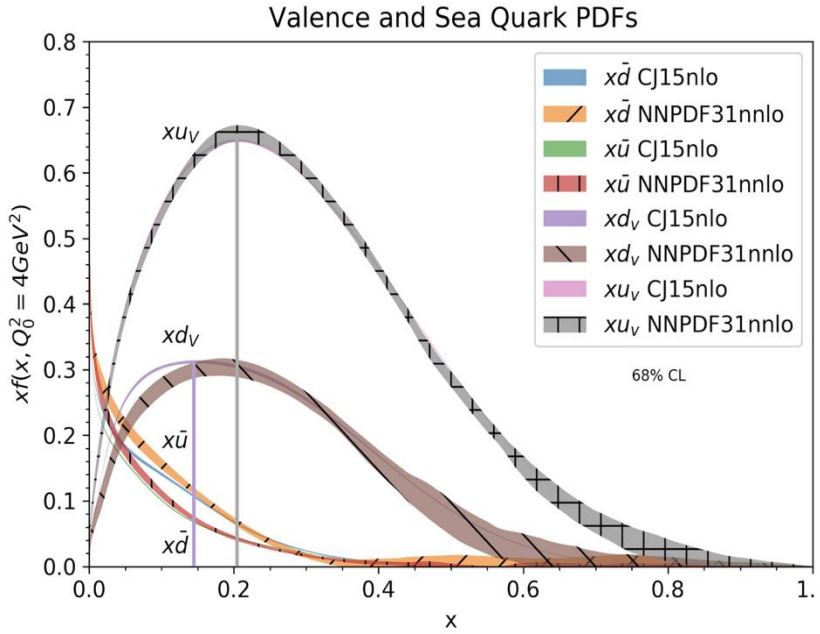
- Because of their conserved number "quantum mechanics can be applied" introducing concept of mean-field interaction with the hadronic environment

- **Quantum mechanically, this becomes a problem of fermions in the strong external field** – (see e g. Migdal Fermions in the Strong Field)

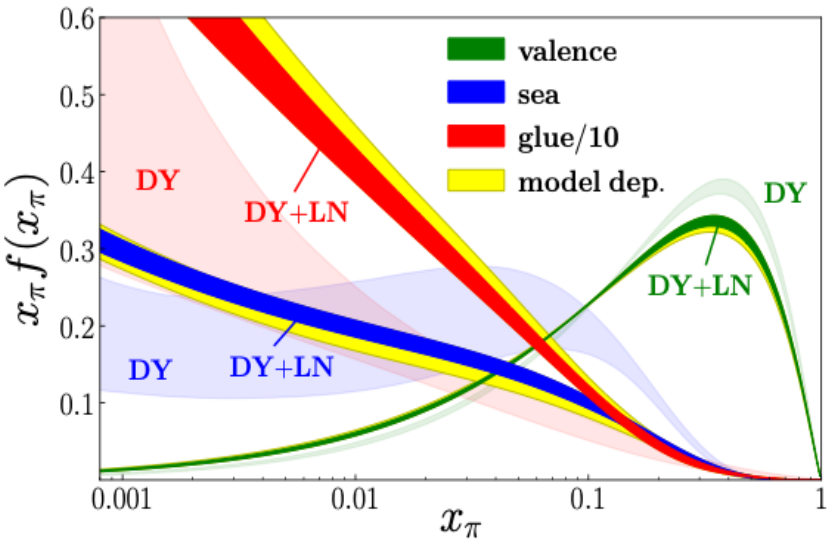
- Short range interaction among three valence quarks responsible to the generation of high x distribution of PDFs

Pion Valence Quark Distribution

Nucleon



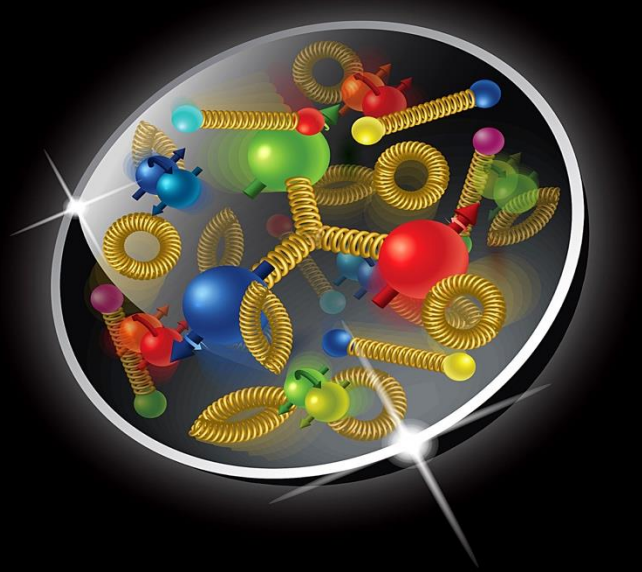
Pion

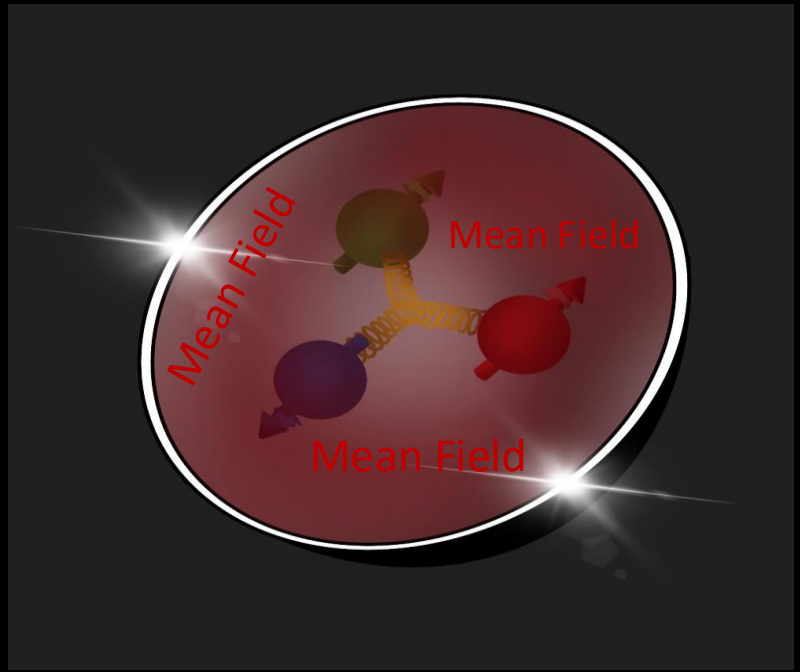


$xf(x)$- distributions

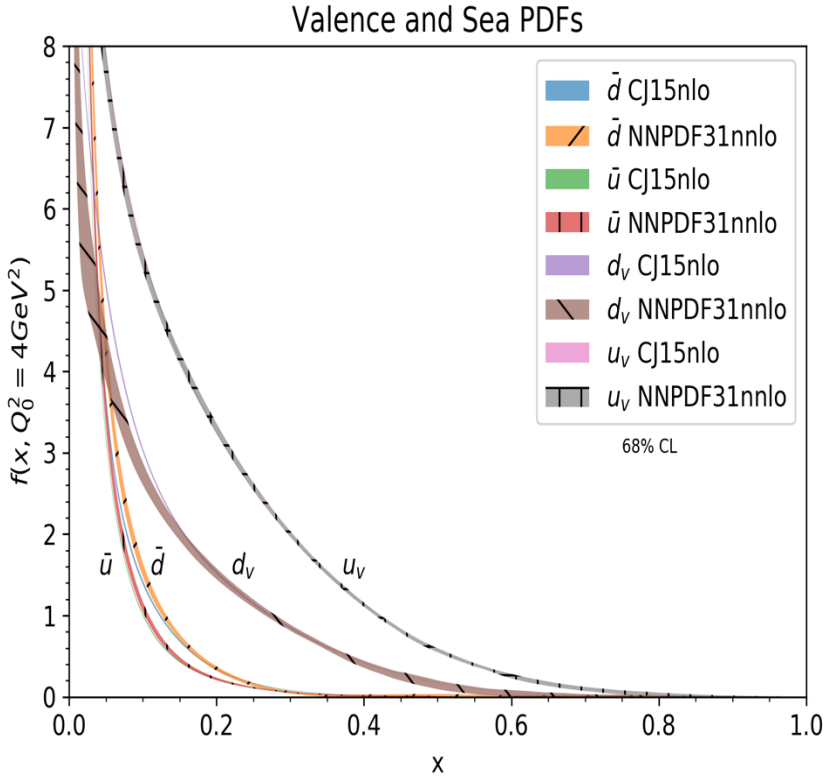
Valence quarks in the hadron

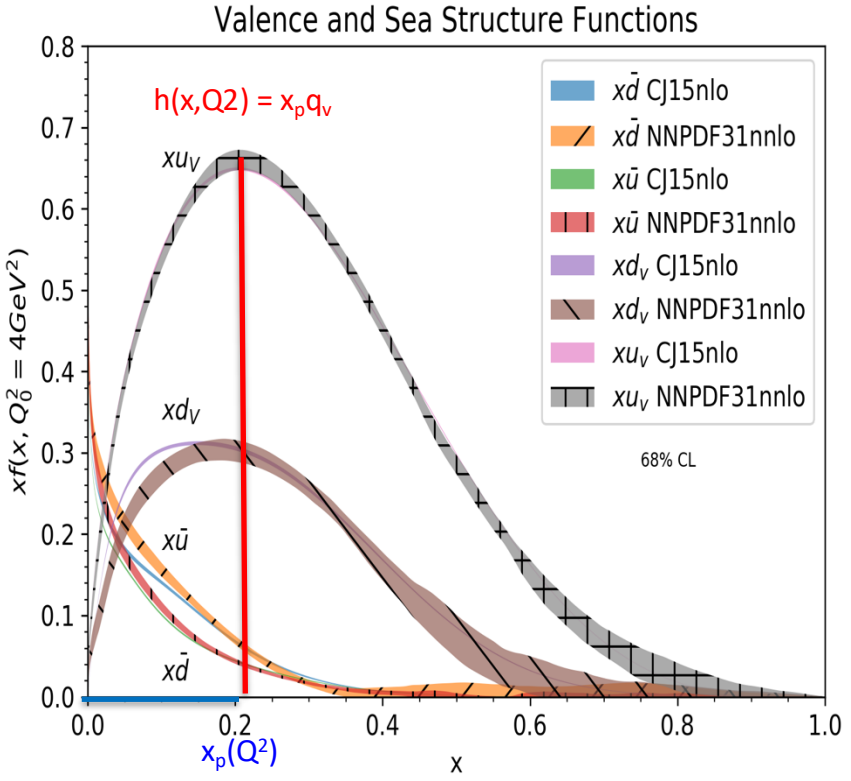
at medium to high x: 0.1 < x < 1





- **Treating the height of the peak $h(x_p,Q^2)$ and position of the peak x_p as "physical observables":**



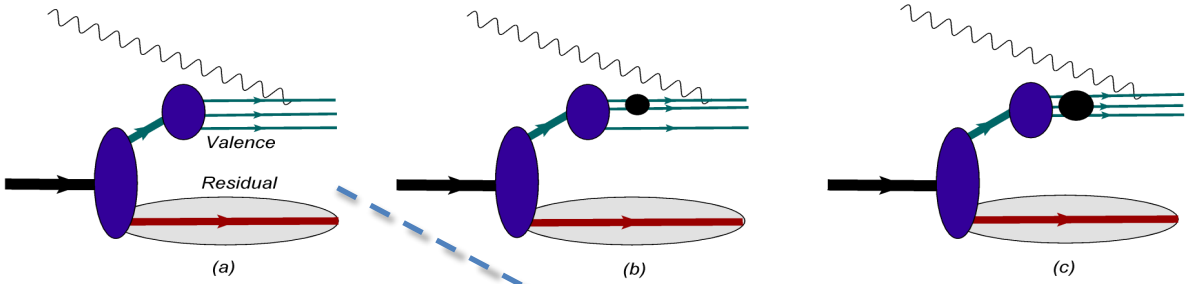


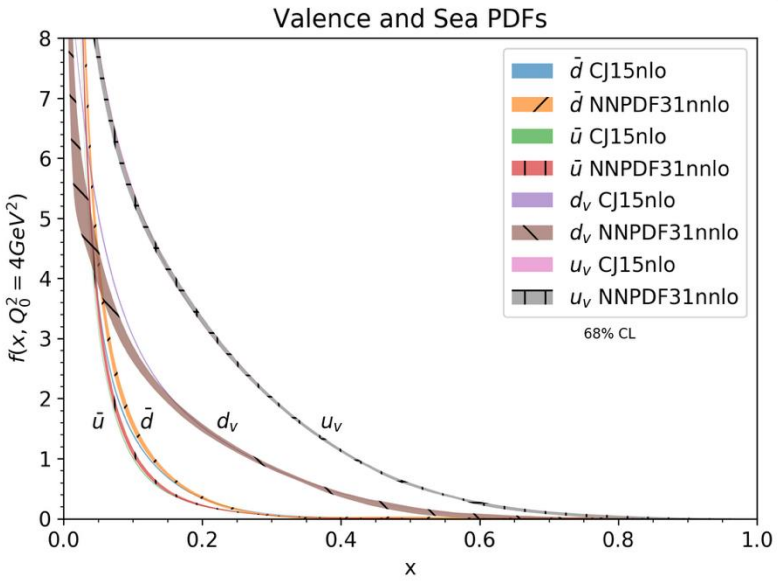
"New Approaches" in modeling valence quark dynamics

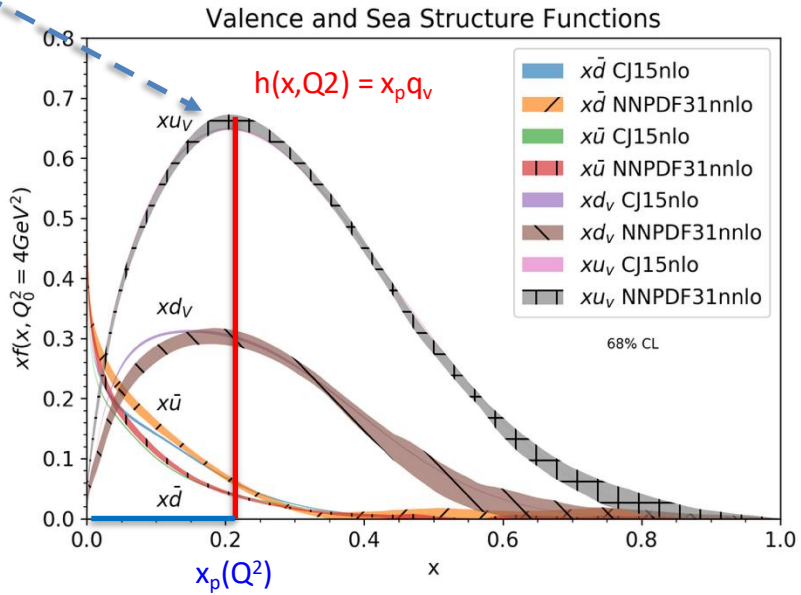
- In general the peaking property of bound Fermi-system is a hallmark for mean-field dynamics

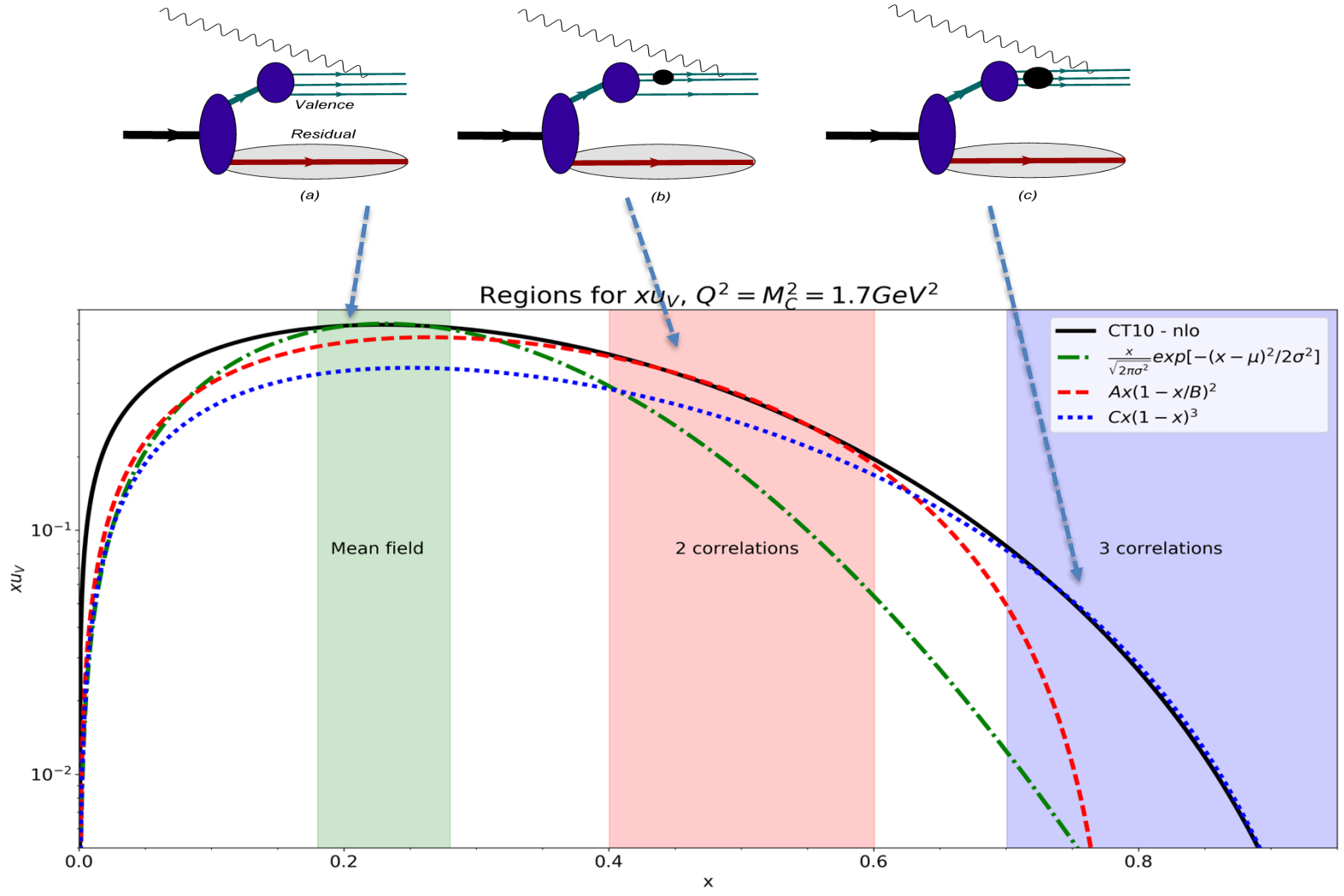
- Our assumption is that the peaking feature of valence quark distributions is due to interaction of valence quarks in the strong mean field generated by "residual hadronic system"

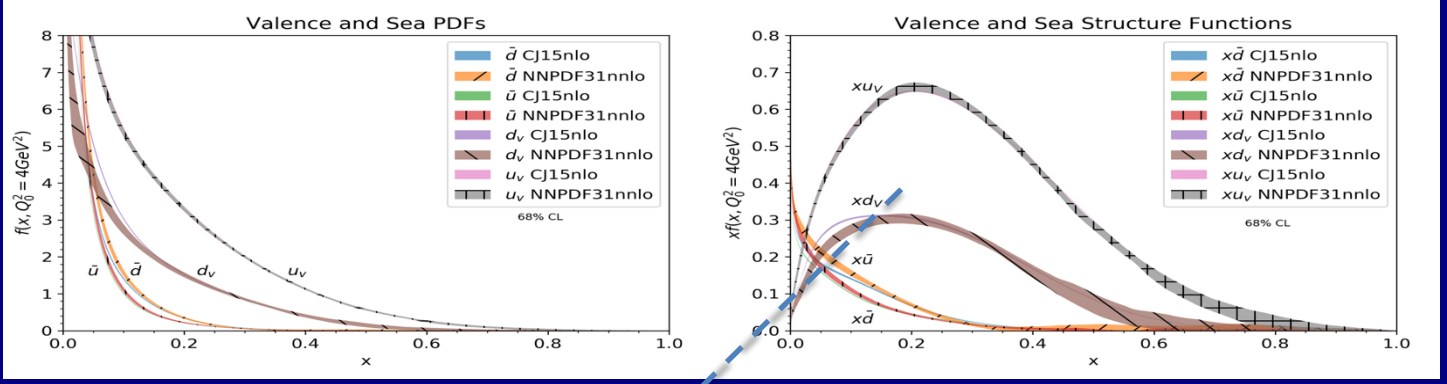
- We introduce concept of "residual hadronic system" as a composite part of the light-front wave function of valence quarks


Valence
Residual
(a)
(b)
(c)

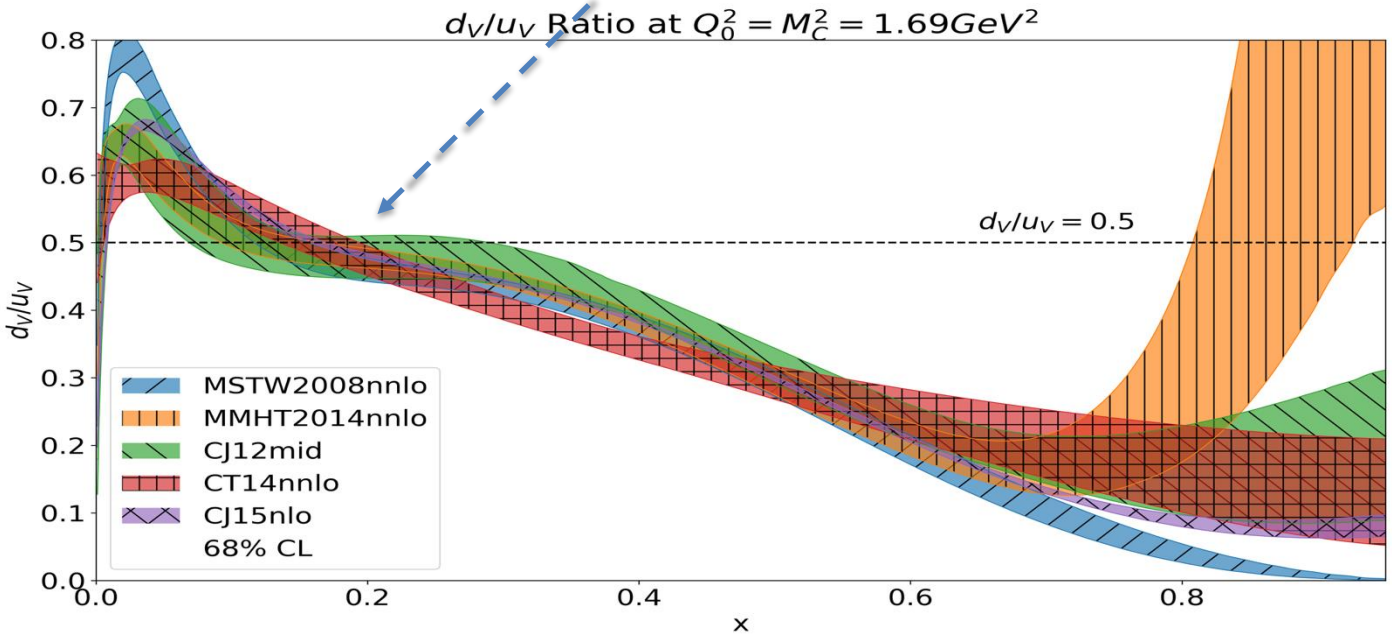

Valence and Sea PDFs
$\bar{d}$ CJ15nlo
$\bar{d}$ NNPDF31nnlo
$\bar{u}$ CJ15nlo
$\bar{u}$ NNPDF31nnlo
d_v CJ15nlo
d_v NNPDF31nnlo
u_v CJ15nlo
u_v NNPDF31nnlo
68% CL
$f(x, Q_0^2 = 4GeV^2)$
x
$\bar{u}$
$\bar{d}$
d_v
u_v


Valence and Sea Structure Functions
h(x,Q2) = $x_p q_v$
$x\bar{d}$ CJ15nlo
$x\bar{d}$ NNPDF31nnlo
$x\bar{u}$ CJ15nlo
$x\bar{u}$ NNPDF31nnlo
xd_v CJ15nlo
xd_v NNPDF31nnlo
xu_v CJ15nlo
xu_v NNPDF31nnlo
68% CL
$xf(x, Q_0^2 = 4GeV^2)$
x
xu_v
xd_v
$x\bar{u}$
$x\bar{d}$
$x_p(Q^2)$


Valence
Residual
(a)
(b)
(c)
Regions for xu_V, $Q^2 = M_C^2 = 1.7GeV^2$
CT10 - nlo
$\frac{x}{\sqrt{2\pi\sigma^2}} exp[-(x-\mu)^2/2\sigma^2]$
$Ax(1-x/B)^2$
$Cx(1-x)^3$
Mean field
2 correlations
3 correlations
xu_V
10^{-1}
10^{-2}
0.0
0.2
0.4
0.6
0.8
x



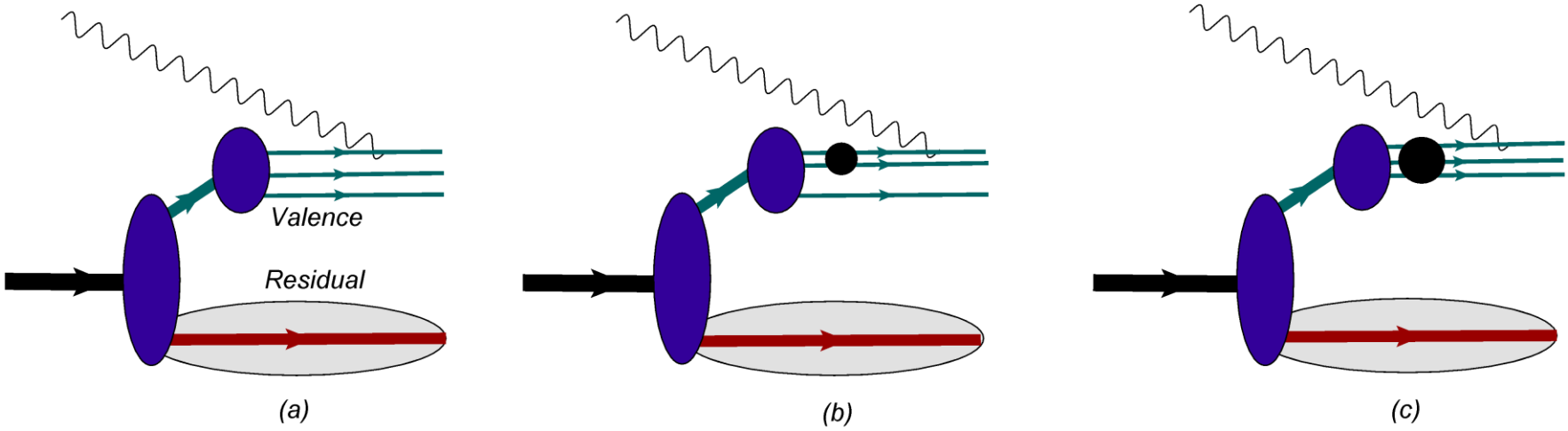
SU3 Symmetry



Main assumptions of the model

Dynamics: The main assumption is that the mean field, two- and three- quark short-range correlations define the dynamics of the valence quarks in the range of

$0.1 \leq x \leq 1$.



Valence Quarks in the Hadron:

- The model assumes an existence of almost *massless* valence quark cluster V in the nucleon.

- The cluster is compact with the transverse separation between any qq,

- Valence quark system defines the baryonic number but not necessarily the total isospin of the nucleon. It can have total total isospin, I_V = 1/2 or 3/2 each of them corresponding to the different excitations or masses of the residual nucleon system.

(For the lowest mass of the recoil system one expects the 3q system to have the same isospin and its projection that the considered nucleon has.)

Residual Structure:

- Introducing residual structure of the hadron with the *spectrum of mass*, m_R - spectral function formalism in the description of the hadron structure.

- The model assumes a certain universality of the *residual structure*, R, entering in all other mechanisms of generation of valence quark distribution.

- This universality is reflected in the fact that one can fix its main properties within mean field and apply it in the calculation of 2q- and 3q- correlation contributions.

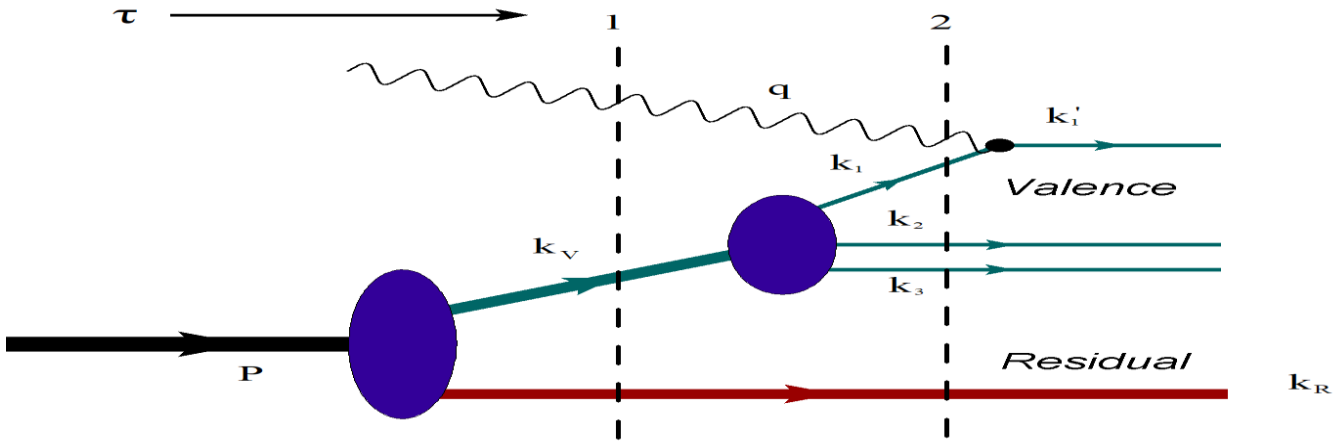
-The mass spectrum of the residual system is continuous and effectively *depends* on whether u- or d- valence quarks are probed in the nucleon.

$$m_R(u/d) = \alpha_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = \frac{1}{2}) + \beta_{u/d} \cdot m_R(I_V = \frac{1}{2}, I_V^3 = -\frac{1}{2}) + \gamma(u/d) \sum_{I_V^3 = -\frac{3}{2}}^{\frac{3}{2}} m_R(I_V = \frac{3}{2}, I_V^3) + \cdots$$

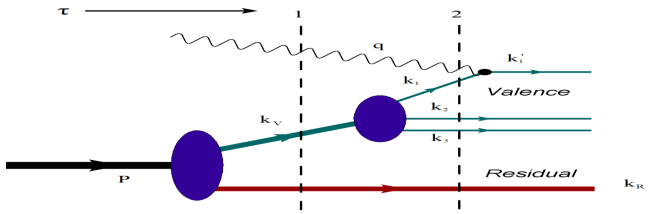
For proton: $m_R(u) < m_R(d)$

Mean-Field Model of Valence Quark Distributions

- The valence 3q system occupies a region of ~0.6Fm and is described by mutually coupled three-dimensional harmonic oscillators, thus satisfying confinement condition.
- The valence system does not define the total radius of the nucleon.
- Valence quarks are almost massless with the invariant energy characterizing its virtuality
- The residual system generates the mean field and occupies a volume less or equal to the hadronic volume.



Light-Front Wave Functions



$|Hadron\rangle$ described as Fock expansion in Light Front Wave Functions:

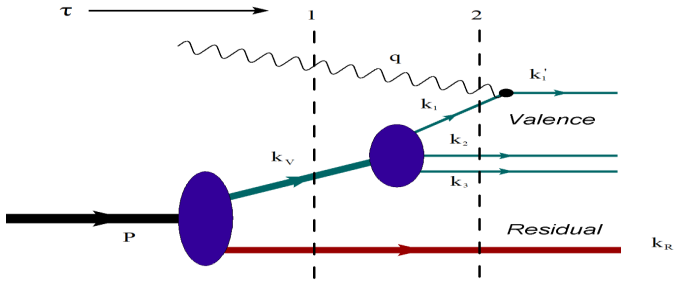
$$|Hadron\rangle = \sum_i \int [d\mu_i]\, \Psi_i \;\; |i\rangle$$

$$|N\rangle = \int [d\mu_{3q}]\, \psi_{3q} |qqq\rangle + \int [d\mu_{4q,\bar{q}}]\, \psi_{4q,\bar{q}} \, |qqq\,\bar{q}q\rangle + \int [d\mu_{3q,1g}]\, \psi_{3q,1g} |qqq\;\rangle + \cdots$$

Infinite set of coupled integral equations

$$\sum_{n'} \int [d\mu'_{n'}]\, \langle n : x_i, \vec{k}_{\perp i}, \lambda_i | H | n' : x'_i, \vec{k}'_{\perp i}, \lambda'_i \rangle\, \Psi_{n'/h}(x'_i, \vec{k}'_{\perp i}, \lambda'_i) = \frac{M^2 + \vec{P}_\perp^2}{2P^+} \Psi_{n/h}(x_i, \vec{k}_\perp, \lambda_i) \;,$$

Phenomenological Light-Front Wave Functions



$$\psi_{VR}(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp}) = \frac{\bar{\chi_V}\bar{\chi_R}\Gamma^{B\to VR}u(p_N, h_N)}{m_N^2 - \frac{k_{V,\perp}^2+m_V^2}{x_V} - \frac{k_{R,\perp}^2+m_R^2}{x_R}}$$

$$\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3) = \frac{\prod_{i=1}^3 \bar{u}(k_i, h_i)\Gamma^{V\to 3q}\chi_V}{m_V^2 - \sum_{i=1}^3 \frac{k_{i,\perp}^2+m_i^2}{\beta_i}}$$

where $\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^3$ denotes the LC momenta and helicities of the three valence quarks in the wave function.

Reference Frame, Kinematics and Structure Function

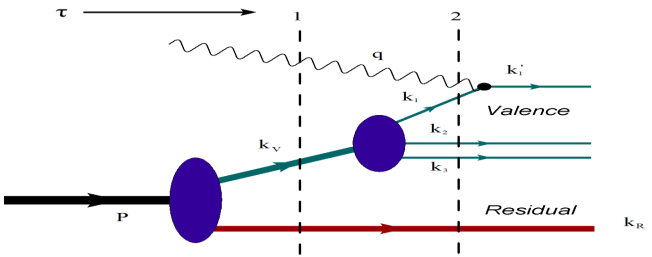
Drell-Yan-West Reference Frame

$$p_N^+ \gg m_N, k_i^-, k_{i,\perp},$$

$$p_N^\mu = (p_N^+, \frac{m_N^2}{p_N^+}, \mathbf{0}_\perp), \quad q^\mu = (0, \frac{2p\cdot q}{p_N^+}, \mathbf{q}_\perp), \quad Q^2 = -q^2 = |\mathbf{q}_\perp|^2,$$

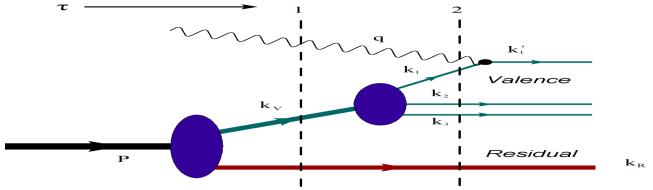
$$F_2(x,Q^2) \equiv \sum_i e_i^2 x f_i(x,Q^2), = \frac{MQ^2}{2x(p_N^+)^2)} W_N^{++}$$

$$W_N^{\mu\nu} = \frac{1}{4\pi M}\int \sum_X \sum_{s_X} J^{\mu,\dagger}(p_X, s_X, , p_N, s_N) J^\nu(p_X, s_X, p_N, s_N)(2\pi)^4 \delta^4(q + p_N - p_X)\delta(p_X^2 - M_X^2)\frac{d^4p_X}{(2\pi)^3}\frac{1}{2(2\pi)}$$



$$A^\mu = \sum_{h_V, h_1} \frac{1}{k_V^+}\frac{1}{k_1^+}\frac{\bar{u}(k_1', h_1')(ie_1\gamma^\mu)u(k_1, h_1)}{\mathcal{D}_1}\frac{\prod_{i=1}^3 \bar{u}(k_i, h_i)\Gamma^{V\to 3q}\chi_V\bar{\chi}_V\bar{\chi}_R\Gamma^{B\to VR}u(p_N, h_N)}{\mathcal{D}_2}$$

Calculation of the scattering amplitude



$$A^\mu = \sum_{h_1,h_V} \frac{1}{x_V}\frac{1}{\beta_1}\frac{\bar{u}(k_1',h_1')(ie_1\gamma^\mu)u(k_1,h_1)}{m_V^2-\sum_{i=1}^3\frac{k_{i,\perp}^2+m_i^2}{\beta_i}}\frac{\prod_{i=1}^3\bar{u}(k_i,h_i)\Gamma^{V\to 3q}\chi_V\bar{\chi}_V\bar{\chi}_R\Gamma^{B\to VR}u(p_N,h_N)}{M^2-\frac{k_{V,\perp}^2+m_V^2}{x_V}-\frac{k_{R,\perp}^2+m_R^2}{x_R}}$$

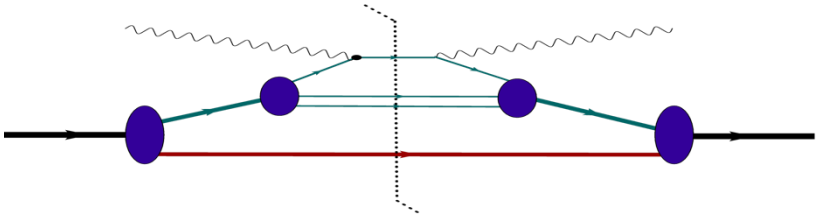
$$\psi_{VR}(x_V,\mathbf{k}_{R,\perp},x_R,\mathbf{k}_{V,\perp}) = \frac{\bar{\chi}_V\bar{\chi}_R\Gamma^{B\to VR}u(p_N,h_N)}{m_N^2-\frac{k_{V,\perp}^2+m_V^2}{x_V}-\frac{k_{R,\perp}^2+m_R^2}{x_R}}$$

$$\psi_{3q}(\{\beta_i,\mathbf{k}_{i,\perp},h_i\}_{i=1}^3) = \frac{\prod_{i=1}^3\bar{u}(k_i,h_i)\Gamma^{V\to 3q}\chi_V}{m_V^2-\sum_{i=1}^3\frac{k_{i,\perp}^2+m_i^2}{\beta_i}}$$

where $\{\beta_i,\mathbf{k}_{i,\perp},h_i\}_{i=1}^3$ denotes the LC momenta and helicities of the three valence quarks in the wave function.

$$A^\mu = \sum_{h_1,h_V}\bar{u}(k_1,h_1)(ie_1\gamma^\mu)u(k_1,h_1)\frac{\psi_{VR}(x_V,\mathbf{k}_{R,\perp},x_R,\mathbf{k}_{V,\perp})}{x_V}\frac{\psi_{3q}(\{\beta_i,\mathbf{k}_{i,\perp},h_i\}_{i=1}^3)}{\beta_1}$$

Calculation of the Structure Function



$$W_N^{\mu\nu}(x,Q^2)=\frac{1}{4\pi M_N}\sum_{\{h_i,\tau_i\}}\int\delta(1-\sum_{i=1}^{3}x_i-x_R)\frac{dx_R}{x_R}\prod_{i=1}^{3}\frac{dx_i}{x_i}16\pi^3\delta^{(2)}(\sum_{i=1}^{3}\mathbf{k}_{i,\perp}+\mathbf{k}_{R,\perp})\frac{d^2\mathbf{k}_{R,\perp}}{16\pi^3}\prod_{i=1}^{3}\frac{d^2\mathbf{k}_{i,\perp}}{16\pi^3}A^{\mu\dagger}A^{\nu}$$

$$F_2(x,Q^2)\equiv\sum_i e_i^2 x f_i(x,Q^2),=\frac{MQ^2}{2x(p_N^+)^2)}W_N^{++}$$

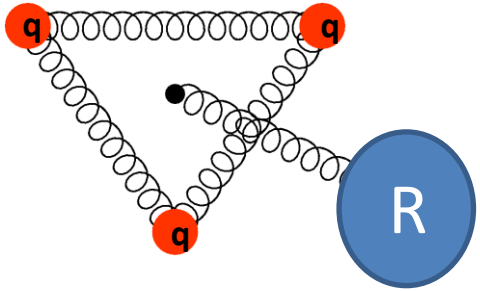
$$f_q(x_B)=\sum_{h_i}\int\delta(1-\sum_{i=1}^{3}x_i-x_R)\frac{dx_R}{x_R}\prod_{i=1}^{3}\frac{dx_i}{x_i}[d^2\mathbf{k}_\perp]16\pi^3\delta^{(2)}(\sum_{i=1}^{3}\mathbf{k}_{i,\perp}+\mathbf{k}_{R,\perp})\frac{d^2\mathbf{k}_{R,\perp}}{16\pi^3}\prod_{i=1}^{3}\frac{d^2\mathbf{k}_{i,\perp}}{16\pi^3}$$

$$\times\delta(x_1-x_B)|\psi_{3q}(\{\beta_i,\mathbf{k}_{i,\perp},h_i\}_{i=1}^{3})|^2|\psi_V(x_V,\mathbf{k}_{R,\perp},x_R,\mathbf{k}_{V,\perp})|^2.$$

Model – Light Front Wave Functions

- ψ_{3q} → relativistic mutually-coupled LF Harmonic Oscilator model

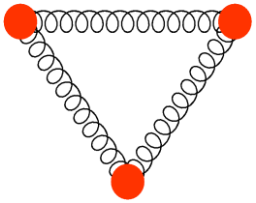
- ψ_{VR} → use Gaussian with non-relativistic kinematics



$$\psi_{3q} \sim \exp\left[-\frac{B_R}{2}\sum_{i=1}^{3}\frac{\tilde{k}_{i,\perp}^2 + m^2}{x_i/x_V}\right]\sqrt{x_2 x_3}$$

$$\psi_{VR} \sim \exp\left[-\frac{B_R}{2}\left(k_{R\perp}^2 + k_{R,z}^2\right)\right]\sqrt{x_R}$$

Modeling Wave Functions



Wave function of 3q valence system: Relativistic coupled Harmonic Oscillator

$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp\left[-\frac{B_V}{2}(k^2_{12,cm} + k^2_{23,cm} + k^2_{31,cm})\right]\sqrt{x_2 x_3}, \tag{1}$$

where A_V and B_V are parameters and $x_i, k_{i,\perp}$, $(i \neq j = 1,2,3)$ are LC momentum fractions and transverse momenta of each valence quark in the reference considered frame. The $k^2_{ij,cm}$s, $(i \neq j = 1,2,3)$ in the exponent of the wave function represent relative three momenta in the CM system of i,j pairs defined as follows:

$$k^2_{ij,cm} = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{4s_{ij}}, \tag{2}$$

where the invariant energy of the i,j pair is:

$$s_{ij} = (k_i + k_j)^+(k_i + k_j)^- - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2 = (x_i + x_j)\left(\frac{k^2_{i,\perp} + m_i^2}{x_i} + \frac{k^2_{j,\perp} + m_j^2}{x_j}\right) - (\mathbf{k}_{i,\perp} + \mathbf{k}_{j,\perp})^2. \tag{3}$$

$$\tilde{\mathbf{k}}_{i,\perp} = \mathbf{k}_{i,\perp} - \frac{x_i}{x_V}\mathbf{k}_{V,\perp}, \quad (\mathrm{i} = 1,2,3)$$

$$\sum_{i=1}^{3} \tilde{k}_{i,\perp} = 0.$$

$$\psi_{3q}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1}^3) = 16\pi^3 m_N A_V \exp\left[-\frac{B_V}{8}\left(\sum_{i=1}^3 x_V \frac{\tilde{k}^2_{i,\perp} + m^2}{x_i} - 9m^2\right)\right]$$

Modeling Cluster-Residual Nucleon Wave Functions

Wave function of V-R system: Model in a Gaussian form

$$\psi_R(x_R, \mathbf{p}_{R,\perp}) = \sqrt{16\pi^3 m_N} A_R e^{-B_R p_R^2} \sqrt{x_R}$$

x_R – light-cone momentum fraction of the recoil system

p_R – relative momentum between CMs of V and R system

- Considering a non-relativistic approximation for recoil system $p_R < m_R$

$$p_{R,z} \approx (x_R m_N - m_R).$$

$$f_q(x_B) = \sum_{h_i} \int \delta(1 - \sum_{i=1}^{3} x_i - x_R) \frac{dx_R}{x_R} \prod_{i=1}^{3} \frac{dx_i}{x_i} [d^2\mathbf{k}_\perp] 16\pi^3 \delta^{(2)}(\sum_{i=1}^{3} \mathbf{k}_{i,\perp} + \mathbf{k}_{R,\perp}) \frac{d^2\mathbf{k}_{R,\perp}}{16\pi^3} \prod_{i=1}^{3} \frac{d^2\mathbf{k}_{i,\perp}}{16\pi^3}$$
$$\times \delta(x_1 - x_B) |\psi_{3q}(\{\beta_i, \mathbf{k}_{i,\perp}, h_i\}_{i=1}^{3})|^2 |\psi_V(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})|^2.$$

$$\begin{aligned} f_q(x_B, Q^2) &= \mathcal{N} \int_0^{1-x_B} dx_2 \int_0^{1-x_B-x_2} dx_3 \exp\left[-\frac{B_V x_V}{4} \sum_{i=1}^{3} \frac{m_i^2}{x_i} - B_R M_N^2 (x_V - (1 - \frac{M_R}{M_N}))^2 \right] \\ &\times \frac{x_2 x_3}{x_V^3} \left(1 - e^{-a_{cm} Q_{cm}^{max2}}\right) \left(1 - e^{-a_{rel} Q_{rel}^{max2}}\right) \left(1 - e^{-B_R Q^2}\right) \end{aligned}$$

where $a_{cm} = \frac{B_V x_V}{4} \frac{x_V}{x_3(x_1+x_2)}$ and $a_{rel} = \frac{B_V x_V}{4} \frac{x_1+x_2}{x_1 x_2}$.

considering large Q^2 and $m_q \to 0$ limit

$$f_q(x_B, Q^2) = \frac{\mathcal{N}}{6} \int_{x_B}^{1} dx_V \exp\left[-B_R m_N^2 \left(x_V - (1 - \frac{m_R}{m_N})\right)^2\right] \frac{(x_V - x_B)^3}{x_V^3}$$

where $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2} e^{\frac{9}{4} B_V m_q^2}$

Qualitative Features of the Model

- Evaluating the integral at the maximum of exponent: $f_q(x_B, Q^2) \sim (1 - x_B - \frac{m_R}{m_N})^3$

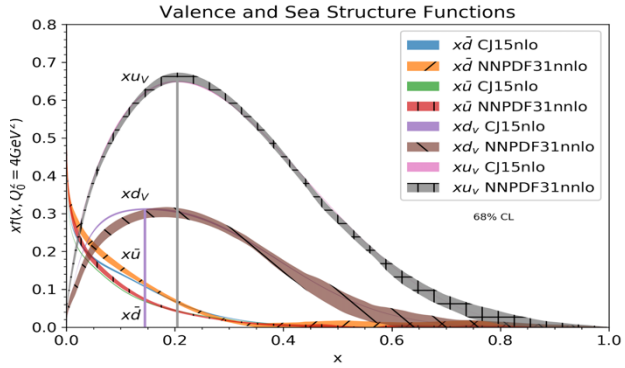
In this case $h(x_B, t) = x_B f_q(x_B, Q^2) \sim x_B \left(1 - x_B - \frac{m_R}{m_N}\right)^3$

which peaks at $x_p \approx \frac{1}{4}(1 - \frac{m_R}{m_N})$.

At moderate Q^2 (M_c^2) characteristic $x_p \sim 0.2$ resulting in $m_R \sim m_\pi$.

In the model: $m_R(u) < m_R(d)$:

Explains $x_p^d < x_p^u$



Nucleon – Valence q Peak

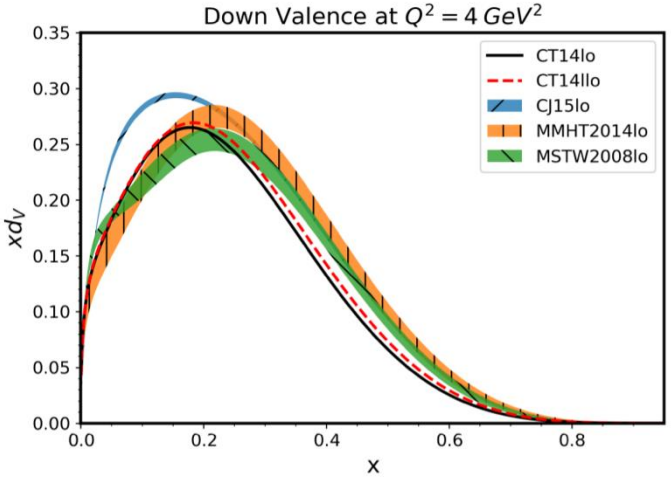
$$x_p \approx \frac{1}{4}\left(1 - \frac{m_R}{m_N}\right)$$

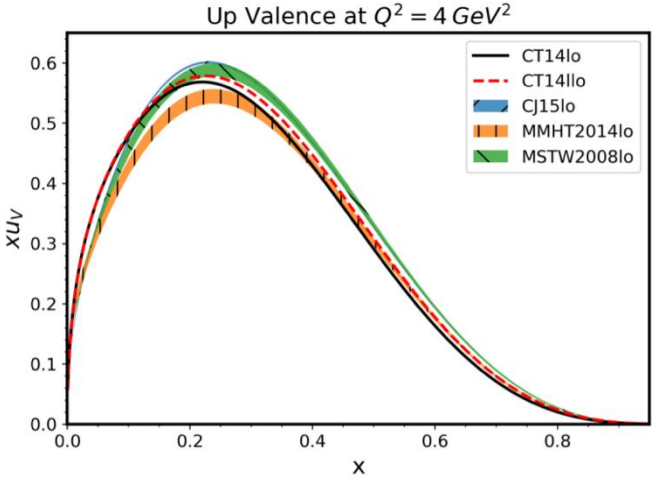
- For $m_R \approx m_\pi \rightarrow x_p \approx 0.2$
- Massless valence quarks

- Different from diquark (Close & Thomas (1988)):

$$x_p = 1 - \frac{m_{dq}}{m_N}$$

- $m_{dq} \approx 800\ MeV$, $x_p \leq 1$



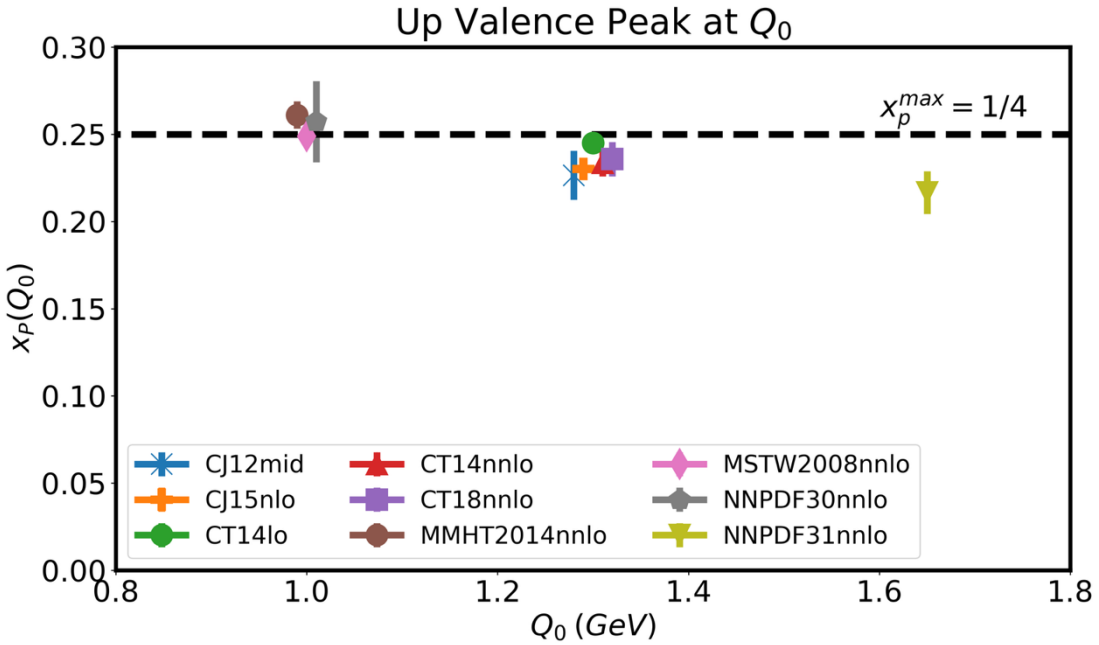


Possible Universal Limit

$$x_p \approx \frac{1}{4}\left(1 - \frac{m_R}{m_N}\right).$$

Universal $x_p = 1/4$ limit

- Tested against PDF sets
- Test at lowest starting Q_0
- Universal $x_p = 1/4$ limit



Leon, C. and Sargsian, M. *Eur. Phys. J. C* **82, 309 (2022)**

Leon, C. and Sargsian, M. *Nuclear Physics A, (2025)*

Numerical Estimates: choosing the parameters of the model

the model has five parameters A_V, B_V, A_R, B_R and m_R.

- For valence quarks we assume that characteristic separations in the 3q system in the impact parameter space is $\langle b_{i,j}^2 \rangle \sim (0.3\mathrm{Fm})^2$. This allows us to evaluate $B_V = 4\langle b_{i,j}^2 \rangle \frac{x_i}{x_V} \approx \frac{4}{3}\langle b_{i,j}^2 \rangle$.
- We assume that this parameter does not change with the QCD evolution.

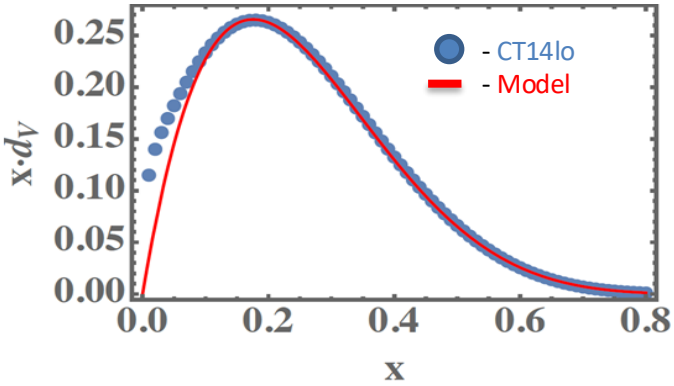
- For the recoil system, because of the use of we can relate $A_R = \left(\frac{B_R}{\pi}\right)^{\frac{3}{4}}$.
- We expect the parameter B_R, which characterizes the size of the residual system

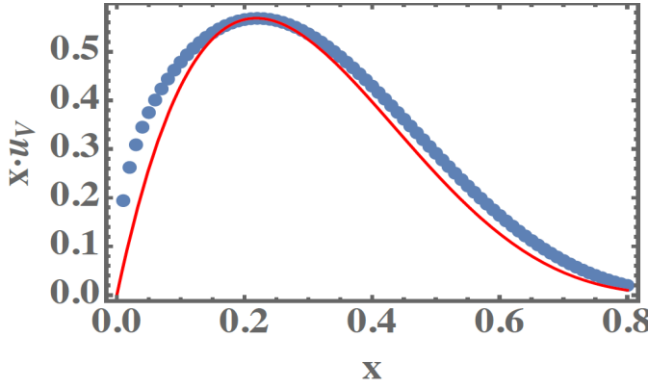
- the parameter A_V is fixed through the normalization factor, $\mathcal{N}$ using: $\mathcal{N} = \frac{16\pi^3 A_V^2 A_R^2 m_N^3}{B_R B_V^2}$

-the remaining parameters $\mathcal{N}$, m_R and B_R are evaluated by fitting to empirical PDFs

Nucleon - Fitting Results

- Fit at peak to CT14lo and minimizes χ^2 – Soft dominates
- d_V fits well at high x
- $u_V \rightarrow$ correlations important, less Soft





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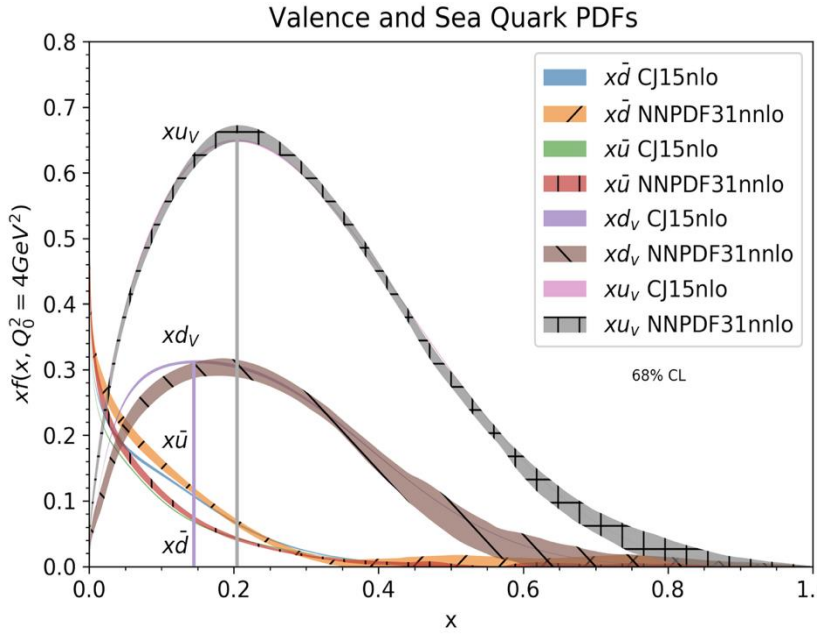
Once parameters are fixed we can evaluate the strength of the missing high momentum component

- For the normalization one obtains for d-quark $N_d = 0.8$ and for u-quark $N_u = 1.54$. These results also indicates that one expects that Regge mechanism at $x < x_p$ and qq-correlations to contribute $\sim 20\%$ and $\sim 23\%$ of total normalizations for d- and u- quarks respectively.

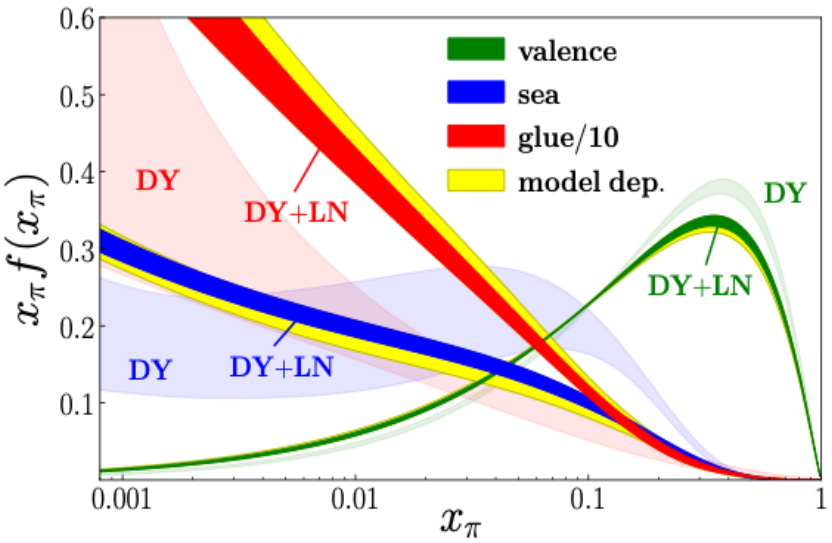
- The evaluation of the momentum sum rule ($P_q = \int x q_V(x) dx$) yields $P_d = 0.1$ and $P_u = 0.246$ compared to $P_d = 0.108$ and $P_u = 0.264$ of CTNN distribution. Because of lesser contribution from the Regge mechanism in this case these estimates evaluate better the contribution from qq correlations. Here for qq-correlations one obtains for the d-quark, $\sim 1\%$ and for the u-quark is $\sim 11\%$.

Pion Valence Quark Distribution

Nucleon



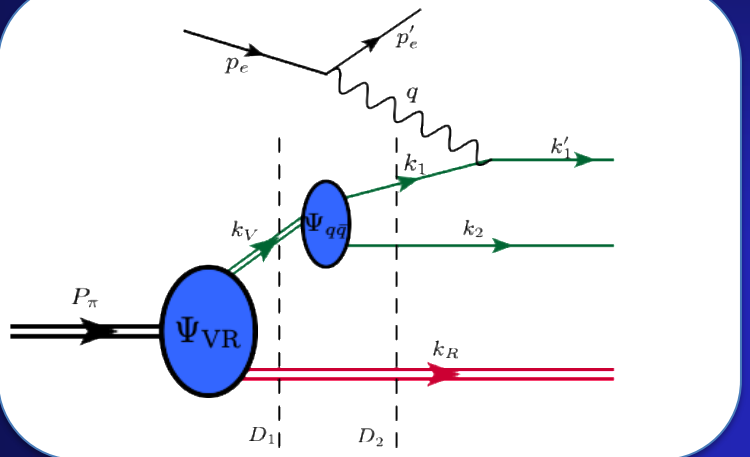
Pion



$xf(x)$- distributions

“Calculation of Pion valence PDF”

$$A^{\mu}(k_1',h_1',k_2,h_2,k_R,h_R)=\sum_{h_V}\frac{\bar{u}(k_1',h_1')ie_q\gamma^{\mu}(\not{k}_1+m_q)v(k_2,h_2)\Gamma^{V\to q\bar{q}}\chi_V}{k_1^+D_2}\frac{\chi_V^{\dagger}\chi_R^{\dagger}\Gamma^{\pi\to VR}\chi_{\pi}}{k_V^+D_1},$$



$$D_1=p_\pi^- - k_R^- - k_V^- = \frac{1}{p_\pi^+}\left(m_\pi^2-\frac{k_{R\perp}^2+m_R^2}{x_R}-\frac{k_{V\perp}^2+m_V^2}{x_V}\right) \quad (1)$$

$$D_2=p_V^- - k_1^- - k_2^- = \frac{1}{K_V^+}\left(m_V^2-\frac{k_{1\perp}^2+m^2}{\beta_1}-\frac{k_{2\perp}^2+m^2}{\beta_2}\right). \quad (2)$$

where $\beta_i=\frac{x_i}{x_V}$ for $i=1,2$ quark and anti-quark.

Phenomenological LF wave functions for valence $q\bar{q}$ cluster:

$$\psi_{VR}(x_V,\mathbf{k}_{R,\perp},x_R,\mathbf{k}_{V,\perp})=\frac{\bar{\chi}_V\bar{\chi}_R\Gamma^{\pi\to VR}}{m_\pi^2-\frac{k_{V,\perp}^2+m_V^2}{x_V}-\frac{k_{R,\perp}^2+m_R^2}{x_R}}$$

and residual system

$$\psi_{q\bar{q}}(\beta_1,\beta_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp})=\frac{\bar{u}(k_1,h_1)\Gamma^{V\to q\bar{q}}v(k_2,h_2)\chi_V}{m_V^2-\sum_{i=1}^{2}\frac{k_{i,\perp}^2+m_i^2}{\beta_i}}$$

$$(\not{k}_1+m_q)=\sum_{h1}u(k_1,h_1)\bar{u}(k_1,h_1)+\frac{\gamma^+(k^2-m^2)}{k^+}$$

Maerovitz, Leon, Sosa, Sargsian, ArXiV 2025

$$A^+(k_1', h_1, k_2, -h_1, k_R) = \left[2k_1^+ ie_q \frac{\psi_{q\bar{q}}(\beta_1, \beta_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp})}{\sqrt{2}\beta_1} - 2k_1^+ ie_{\bar{q}} \frac{\psi_{q\bar{q}}(\beta_1, \beta_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp})}{\sqrt{2}\beta_1}\right] \times \frac{\psi_{VR}(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp})}{x_V}$$

$$W^{++}(x, Q^2) = \frac{1}{4m_\pi} \sum_{\{h_i, \tau_i\}} \int [dx][d^2\mathbf{k}_\perp] \frac{x_1}{p_\pi q} \delta(x_1 - x_B) |A^+|^2$$

$$F_2(x, Q^2) = \frac{m_\pi Q^2}{2x(p_\pi^+)^2} W_\pi^{+,+}$$

and within the leading approximation from the parton model

$$F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2),$$

$$f_V(x_B, Q^2) = \int^{Q^2} \frac{dx_1 d^2\mathbf{k}_{1,\perp}}{16\pi^3 x_1} \frac{dx_2 d^2\mathbf{k}_{2,\perp}}{16\pi^3 x_2} \frac{dx_R d^2\mathbf{k}_{R,\perp}}{16\pi^3 x_R} \delta(x_1 - x_B) 16\pi^3 \delta(1 - x_1 - x_2 - x_R) \times \delta^{(2)}\left(\sum_{i=1,2,R} \mathbf{k}_{i,\perp}\right) |\Psi_{q\bar{q}}(\beta_1, k_{1,\perp}, \beta_2, k_{2,\perp})|^2 |\Psi_{VR}(x_R, k_{R,\perp})|^2$$

$\tilde{\mathbf{k}}_{i,\perp} = \mathbf{k}_{i,\perp} - \beta_i \mathbf{k}_{V,\perp}$ (i=1,2).

$$f_V(x_B, Q^2) = \int_0^{1-x_B} \frac{dx_R}{(16\pi^3)^2 x_B (1 - x_B - x_R) x_R}$$

$$\times \left[\int^{Q^2} d^2\mathbf{k}_\perp |\Psi_{q\bar{q}}(\beta, \mathbf{k}_\perp)|^2 \right]_{q\bar{q}} \left[\int^{Q^2} d^2\mathbf{k}_{R,\perp} |\psi_{VR}(x_R, \mathbf{k}_{R,\perp})|^2 \right]_{VR}$$

$[\cdots]_{q\bar{q}}$ and $[\cdots]_{VR}$ we can interpret as response functions of valence - cluster and residual systems respectively.

Two-Body Light-Front Wave Functions

$$\int \mid \Psi^{LF}(k_{12}) \mid^2 \frac{dy}{y(1-y)} \frac{d^2k_{12,\perp}}{16\pi^3} = 1 \qquad \text{where } y = \frac{k_1^+}{p_T^+}$$

$$k_{12}^2 = \frac{(s_{12} - (m_1 - m_2)^2)\,(s_{12} - (m_1 + m_2)^2)}{4s_{12}}$$
$$= \frac{m_1^2(1-y) + m_2^2 y + k_{12,\perp}^2}{4y(1-y)} - \frac{m_1^2 + m_2^2}{2} + \frac{(m_1^2 - m_2^2)^2 y(1-y)}{4(m_1^2(1-y) + m_2^2 y + k_{12\perp}^2)}$$

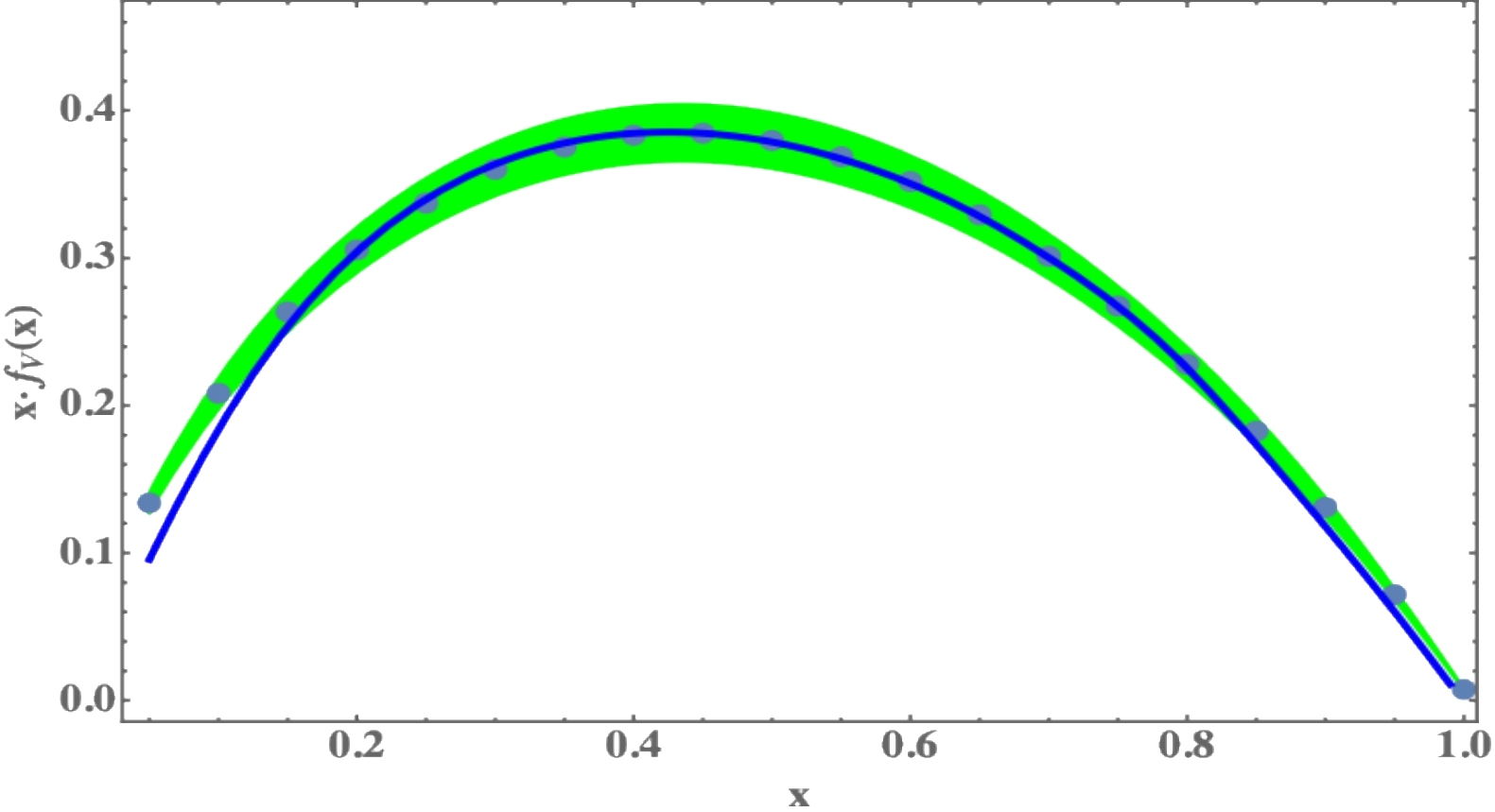
and

$$s_{12} = \frac{m_1^2(1-y) + m_2^2 y + k_{12,\perp}^2}{y(1-y)}$$

$$\Psi_{12}^{LF}(k_{12}) = \Psi_{12}^{NR}(k_{12}) \sqrt{\frac{E_1 E_2}{E_1 + E_2} 16\pi^3} \qquad \int \mid \Psi^{NR}(k_{12}) \mid^2 d^3k_{12} = 1$$

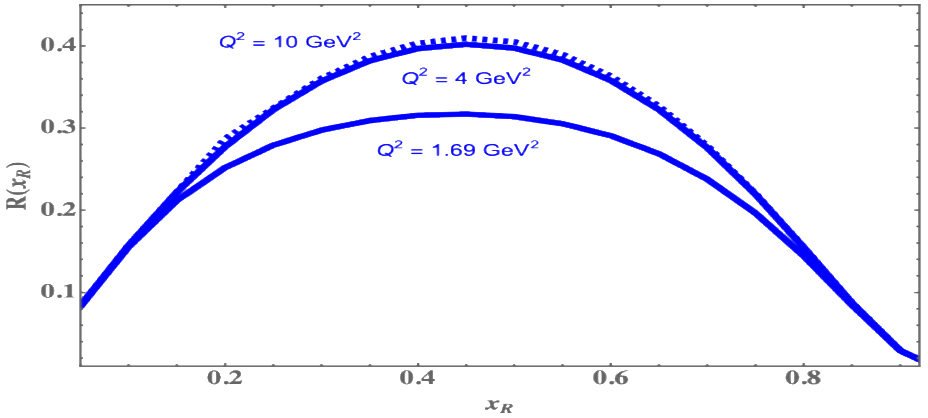
$$\Psi_{q\bar{q}}^{LF}(\beta, k_{12,\perp}) = \sqrt{8\pi^3 E_k} A_V \exp\left[-\frac{B_V}{2} k_{12}^2\right]$$

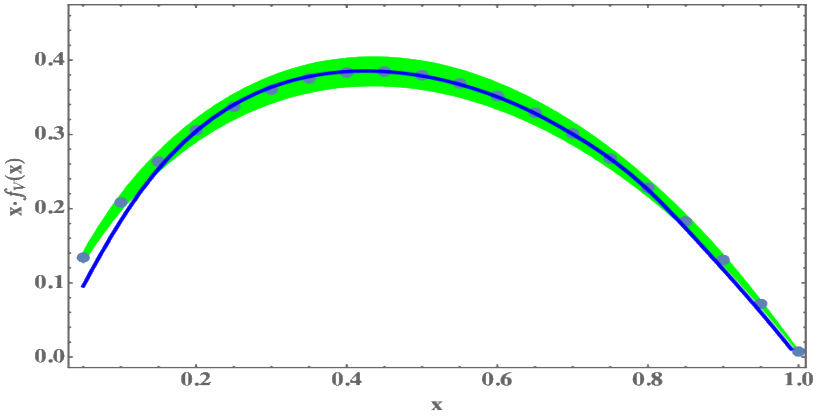
$$\Psi_{12}^{LF}(x_R, k_{R,\perp}) = A_R \sqrt{16\pi^3} \sqrt{\frac{E_V E_R}{E_V + E_R}} e^{-\frac{B_R}{2} k_R^2}$$



The best fit of the JAM18 pion PDF obained for the parameters of $m_R = 0$, $m_V = 0.95$ GeV and $B_V = 1.2\text{GeV}^{-2}$. Note that the qualitative agreement of the data requires $m_V \sim Q_0$ and $m_R \sim 0$

Diffuse Residual System

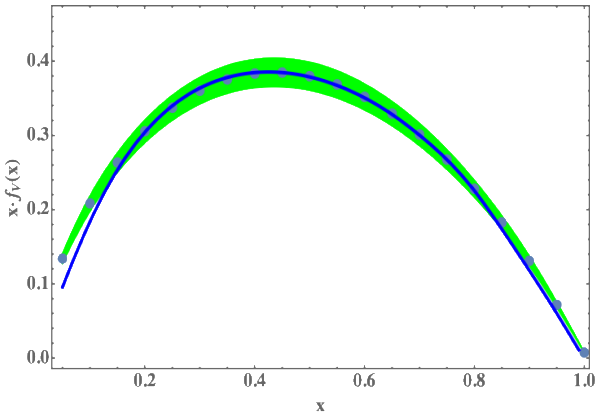




$$\beta_1 = \frac{x}{x_R} \approx 1 \qquad m_V \sim Q_0$$

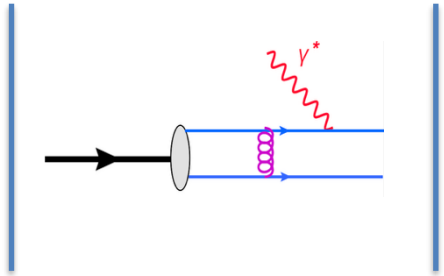
Feynman Mechanism

Hard Contribution



Hard Contribution

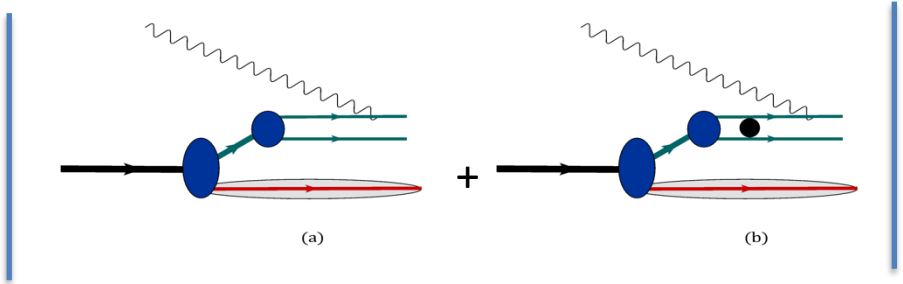
Before



2

$$(1-x)^2$$

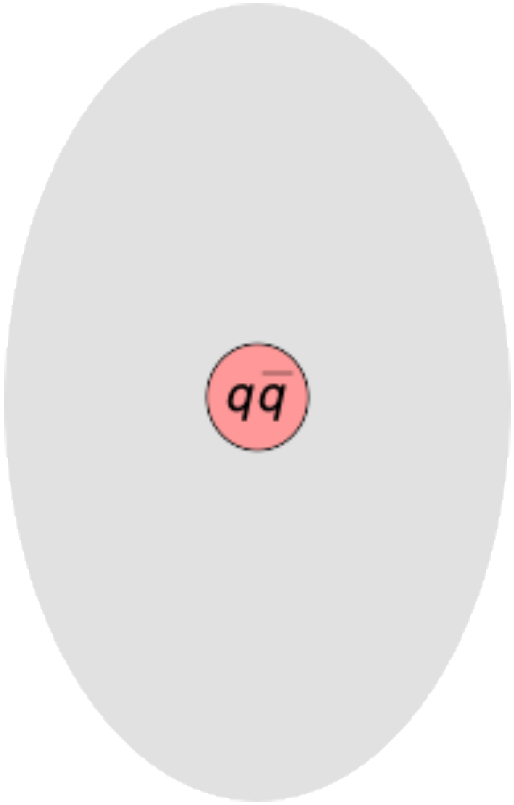
To Conserve Unitarity



2

$$(1-x)^\beta \qquad \beta < 2$$

pion



- Diffuse Residual System

- Highly Virtual Valence Quark Cluster

Some Empirical Observation

valence quarks in nucleon peak at ¼ - this requires $x f_V^N(x) \sim x(1-x)^3$

valence quarks in pion peak at **1/2** - this requires $x f_V^\pi(x) \sim x(1-x)^1$

But then what is observed is "experimentally": $f_V^N(x)|_{x\to 1} \sim (1-x)^3$ and $f_V^\pi(x)|_{x\to 1} \sim (1-x)^1$

Somehow peak positions of $x f_V(x)$ at $x < 0.5$

Define the asymptotic behavior at $x \to 1$

Outlook

- Based on phenomenological wave functions -> Spectral Function

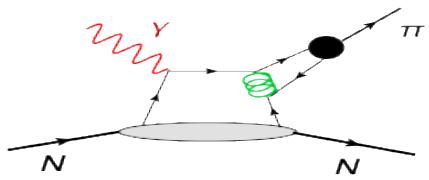
- Add qq and qqq correlations
- Check Universality of residual state wave function
- With LFWF parameters fitted from PDFs, check validity by calculating predictions for other processes
- Predictions for TMDs GPDs

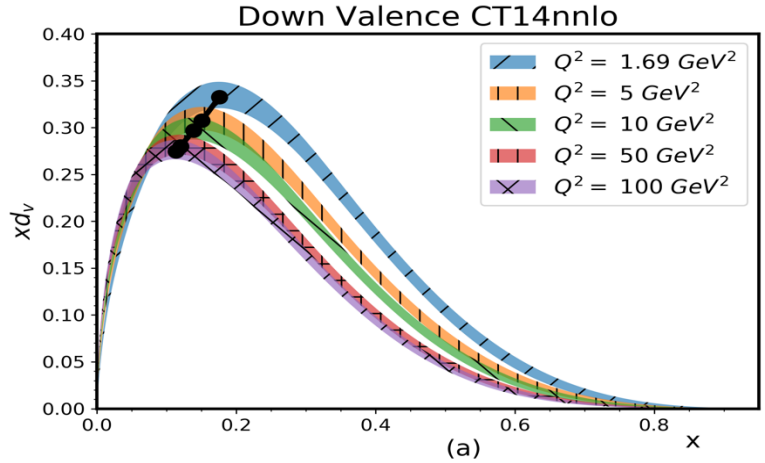
- It seems the model reproduces pion data
- Indicates a diffuse character of pion
- Can Reconcile two pictures of pion
- Any chance to measure PDF of a non-Goldstone meson

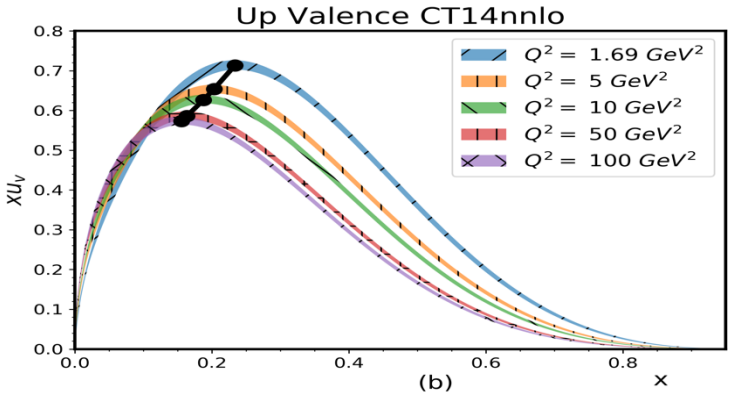
- It seems there is a relation between peak position of xf_V and f_V at $x \to 1$

- Other explanations why $x_p = 1/4$?

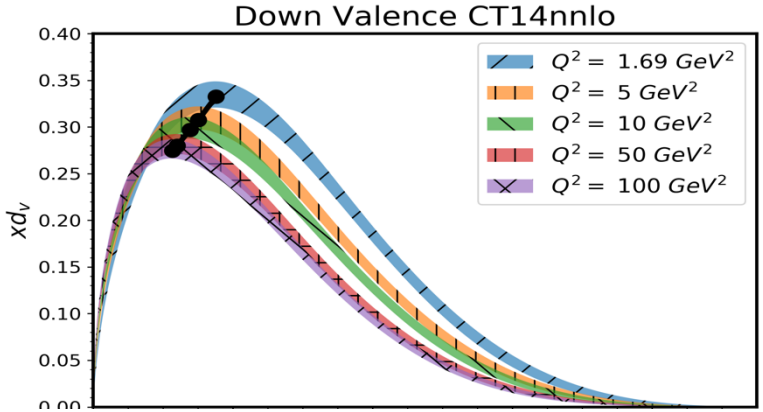
- Calculating other processes: $\gamma + N \to \pi + N$

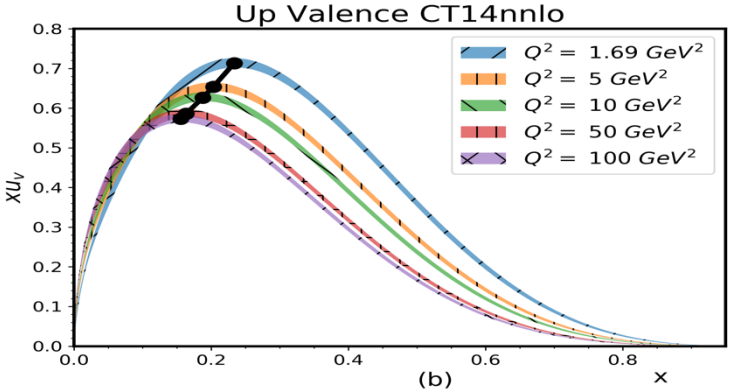


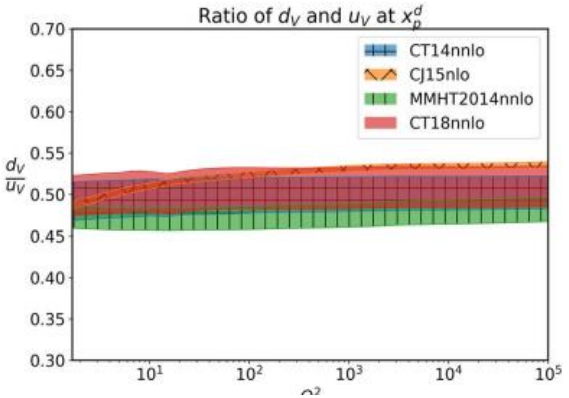

Down Valence CT14nnlo
$Q^2 = 1.69\ GeV^2$
$Q^2 = 5\ GeV^2$
$Q^2 = 10\ GeV^2$
$Q^2 = 50\ GeV^2$
$Q^2 = 100\ GeV^2$
xd_v
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
x
(a)

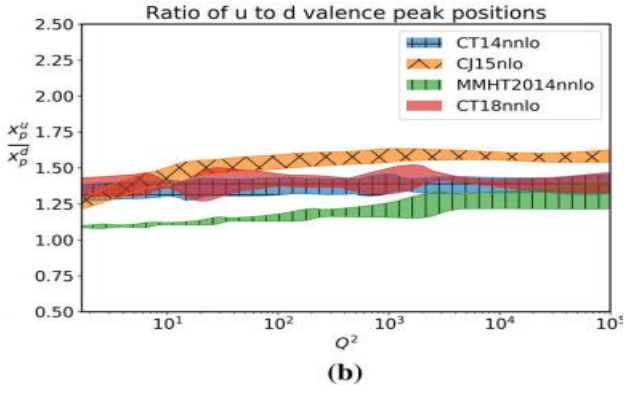

Up Valence CT14nnlo
$Q^2 = 1.69\ GeV^2$
$Q^2 = 5\ GeV^2$
$Q^2 = 10\ GeV^2$
$Q^2 = 50\ GeV^2$
$Q^2 = 100\ GeV^2$
xu_v
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
x
(b)

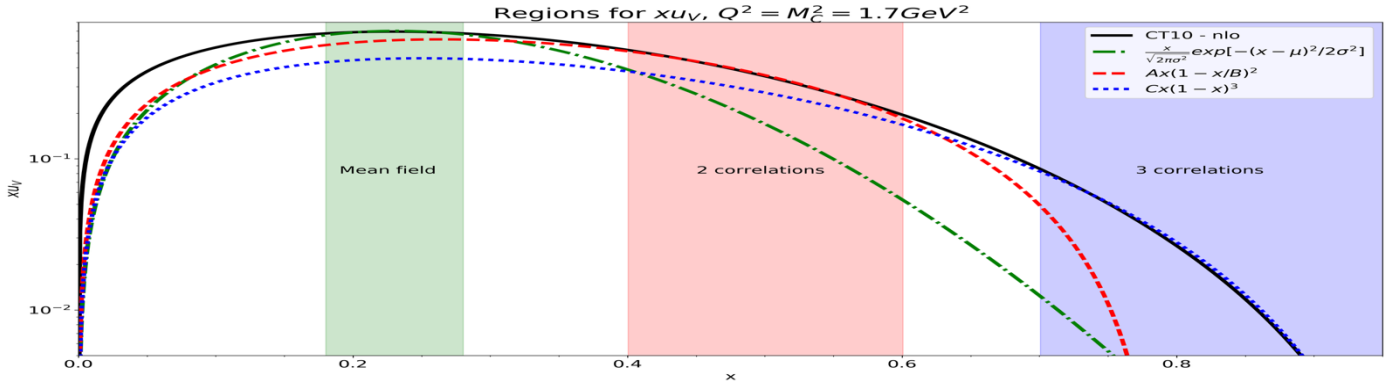
- **Treating the height of the peak $h(x_p,Q^2)$ and position of the peak x_p as "physical observables":**

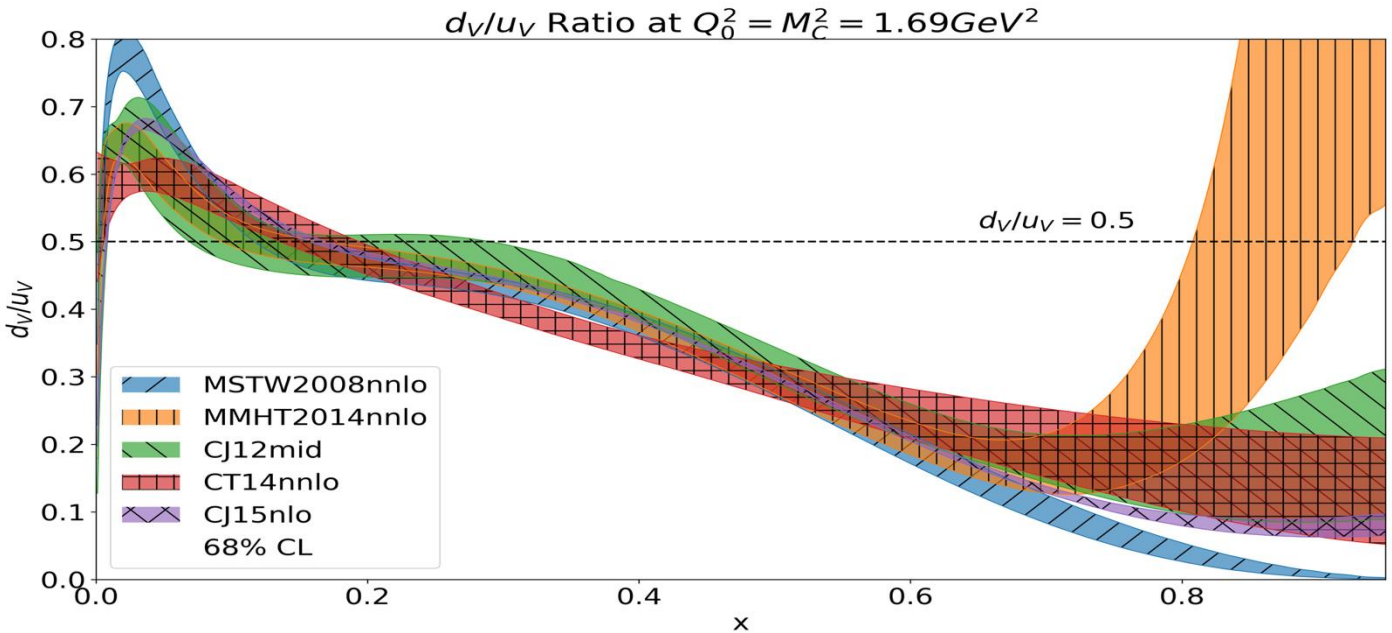









Regions for xu_V, $Q^2 = M_C^2 = 1.7GeV^2$
CT10 - nlo
$\frac{x}{\sqrt{2\pi\sigma^2}} exp[-(x-\mu)^2/2\sigma^2]$
$Ax(1-x/B)^2$
$Cx(1-x)^3$
Mean field
2 correlations
3 correlations
xu_V
10^{-1}
10^{-2}
0.0
0.2
0.4
0.6
0.8
x


d_V/u_V Ratio at $Q_0^2 = M_C^2 = 1.69GeV^2$
$d_V/u_V = 0.5$
MSTW2008nnlo
MMHT2014nnlo
CJ12mid
CT14nnlo
CJ15nlo
68% CL
d_V/u_V
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.2
0.4
0.6
0.8
x