Electroproduction structure function $F_2(Q^2, x)$ in the low Q^2, low x region - an update

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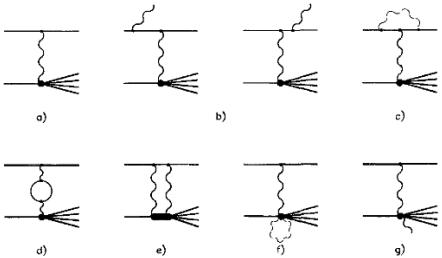
DIS2026

Royal Hotel Carlton, Bologna, Italy
4-8 May, 2026

Why ? Radiative corrections in DIS BB, Bardin, Kurek, Scholz, Z. Phys. C66 (1995) 591



Measurement goal: a); lowest order RC: b) – g)

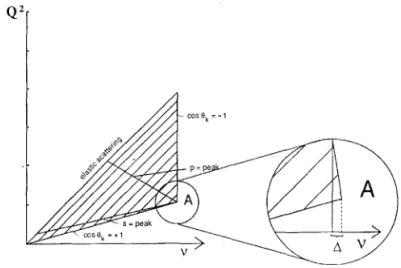


Fig. 2. Range of kinematical variables from which the radiative tails contribute to the cross section measured at the point $A(Q^2, \nu)$

OBSERVE: if we measure at point A(Q^2, ν) then $F_2(Q^2, x)$ has to be known at

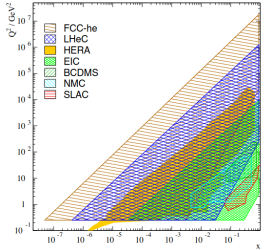
$$0 < Q^2 < Q^2_{\mathrm{max}} \quad \text{and} \quad x_{\mathrm{meas}} < x < 1$$

Why ? Low Q^2 and low x region is very interesting

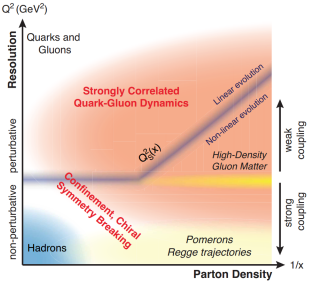
- Scaling is not a valid concept at low Q^2;
  kinematic conditions at $Q^2 \to 0$, any ν:
  $F_2 = \mathcal{O}(Q^2), \quad R = \mathcal{O}(Q^2), \quad F_{\rm L} = \mathcal{O}(Q^4).$
  (from kinematic singularites in hadronic tensor)

  Which degrees of freedom
  describe transition from high to low Q^2 ???
  Connection to the photoproduction limit.

  Possible dynamic model description:
  Generalised Vector Meson Dominance.



P. Agostini *et al.*, J. Phys. G48 (2021) 110501

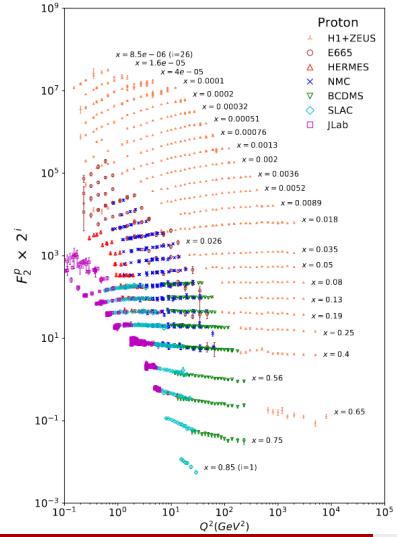


E.C. Ashenauer *et al.*, hep-ph 1708.01527v3

- PDFs grow with decreasing x (albeit slower than at large Q^2).

- At low x and high energy, cross-sections dominated by low Q^2 physics. Regge limit: $s = W^2 \gg Q^2$.
  Traditionally: Regge parametrisation as function of energy. Meaning ???

Measurements of $F_2^{\mathrm{p}}(Q^2, x)$



- Data courtesy of Shujie Li, LBL and A. Accardi, JLab (CJ Collaboration). THANK YOU!
- Only data with $W^2 > 3.5$ GeV^2 are included
- For model comparison we used $F_2(Q^2, x)$ data at $\boldsymbol{Q^2 < 2}$ GeV^2 and $\boldsymbol{x < 0.2}$...
- ...in 4 bins of x: 0.0089, 0.018, 0.035, 0.13

Figure: S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024) and 2025 update

Basic concepts

- Limiting conditions for $F_{1,2}$ at $Q^2 \to 0$ come from kinematic singularities in hadronic tensor.
- OPE in the Bjorken limit: $F_2(Q^2, x) = \sum_{n=0}^{\infty} \frac{C_n(Q^2, x)}{(Q^2)^n}, \qquad C_n(^2) = C_n(\log Q^2)$ (**)

  ($n = 0 \to$ leading twist, LT, (=2); $n \geq 1 \to$ higher twist, HT, (=4,6,8,...))

  OBS! HTs are only corrections to leading twist (approx. scaling). They cannot describe low-Q^2 region!
- Nonperturbative region description demands summing (**) beforehand and then continuing it to $Q^2 \sim 0$.
- This is automatically accomplished e.g. in VMD (GVMD) model which can be represented as (**) and gives (approximate) scaling at large Q^2. Origin of VMD: γ interactions are similar to those of a hadron:

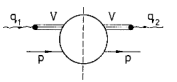
  $$F_2\left(Q^2, x = \frac{Q^2}{s + Q^2 - M^2}\right) = \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2}$$

  If only a finite number of VM then $F_2 \to \frac{1}{Q^2}$ at $Q^2 \to \infty$ and there is no LT here.

  Scaling can be introduced by including infinite number of VMs $\Longrightarrow$ GVMD
- GVMD is proper for fixed, large s and $Q^2 \ll s$, i.e. low x

Model combining GVMD and dynamical parton approach

- Original model: J. Kwiecinski & BB, Z.Phys. C43 (1989) 251; double dispersion relations with $q_1^2 = q_2^2 = -Q^2$
- $F_2(Q^2,x) = \underbrace{\frac{Q^2}{4\pi}\sum_v^{\rho,\omega,\phi}\frac{\sigma_{vp}(s)}{(Q^2+M_v^2)^2}\frac{M_v^4}{\gamma_v^2}}_{F_2^{\rm VMD}} + \underbrace{Q^2\int_{Q_0^2}^{\infty}{\rm d}Q'^2\frac{\Phi(Q'^2,s)}{(Q'^2+Q^2)^2}}_{F_2^{\rm partons}}$



$q_1^2 = q_2^2 = -Q^2$

where $\Phi(Q'^2,s) = -\frac{Im}{\pi}\int^{-Q'^2}\frac{{\rm d}Q''^2}{Q''^2}F_2^{\rm AS}(Q''^2,x')$ and by construction $\lim_{Q^2\to\infty}F_2(Q^2,x) = F_2^{\rm AS}(Q^2,x)$

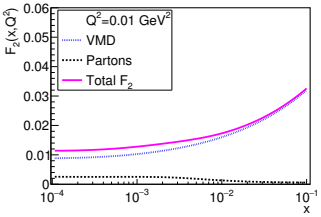
- This representation is valid at fixed $s \equiv W^2, s \gg Q^2$ (i.e. low x) but arbitrary value of Q^2, including Q^2 =0
- It is **NOT A FIT**; Q_0^2 is well constrained; $Q_0^2 > (M_v^2)_{\rm max}$ where $(M_v^2)_{\rm max}$ is the highest VM mass in the sum
- Photoproduction: $\sigma_{\gamma p} = \lim_{Q^2\to 0} 4\pi^2\alpha\frac{F_2}{Q^2}$
- Further developement: $\boldsymbol{F_2^{\rm partons}(Q^2,x) = \frac{Q^2}{Q^2+Q_0^2}F_2^{\rm AS}(Q^2+Q_0^2,\bar{x})}$ where $\boldsymbol{\bar{x} = \frac{Q^2+Q_0^2}{s+Q^2-M^2+Q_0^2}}$

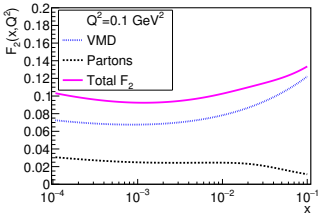
  which retains all properties of the original $\boldsymbol{F_2^{\rm partons}}$; BB & J. Kwiecinski, Phys.Lett. B295 (1992) 263;
  This form will be used in the present work.

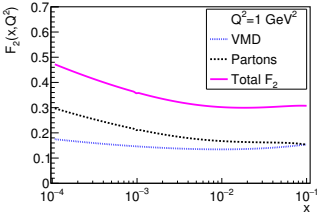
Input to the model

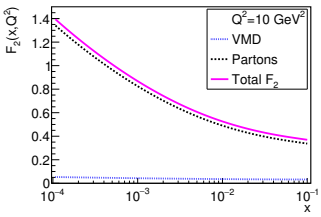
- Q_0^2 value is well constrained: $Q_0^2 > (M_v^2)_{\max}$; here $(M_v)_{\max} = M_\phi \approx 1.02$ GeV.
  We have assumed Q_0^2 = 1.2 GeV^2.
- As the asymptotic function $F_2^{\rm AS}$ we took the NLO expression with CT18 parton distribution functions.
- Coupling constants, γ_v^2 determined from $\Gamma_{v\to e^+e^-}$: $\gamma_v^2/\pi = \alpha^2 M_v/(3\Gamma_{v\to e^+e^-}) \quad \Longrightarrow$ PDG data
- Vector meson – proton total cross sections, $\sigma_{\rm vp}$ were related to $\pi^\pm$p and K$^\pm$p, assuming the additive QPM:
  $\sigma_{\rho\rm p} = \sigma_{\omega\rm p} = (\sigma_{\pi^+\rm p} + \sigma_{\pi^-\rm p})/2$ and $\sigma_{\phi\rm p} = \sigma_{\rm K^-p} + \sigma_{\rm K^+p} - \sigma_{\pi^-\rm p}$. Possible non-diagonal terms,
  e.g.: ρ^0p $\to \omega^0$p, etc estimated negligible.
- Energy dependent parameterization of $\sigma_{\pi^\pm\rm p}, \sigma_{\rm K^\pm p}$:
  $(\sigma_{\rm M^-p} - \sigma_{\rm M^+p})/2 = \sigma_2\nu^{-\alpha_2}, \quad (\sigma_{\rm M^-p} + \sigma_{\rm M^+p})/2 = \sigma_1\nu^{-\alpha_1} + \sigma_\infty$
  here: $\sigma_\infty = \sigma_0[1 + c(\ln\gamma)^2], \quad \gamma = \nu/m_{\rm M}$, M = π, K

Results of the calculations: $F_2(x)$, Q^2 = const



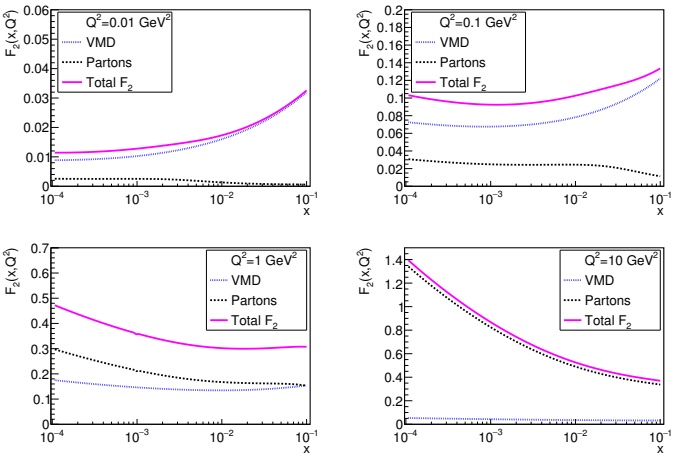


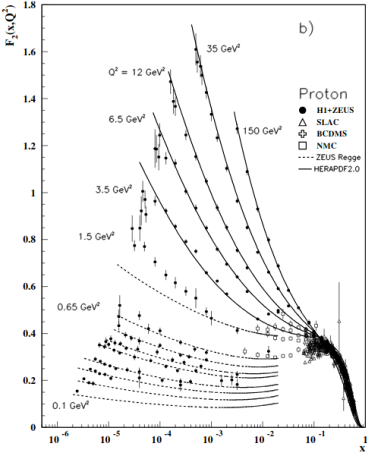




- x dependence at fixed Q^2 reflects $\sigma_{\rm vp}(s)$ and $F_2^{\rm AS}(x = Q^2/(s + Q^2 - M^2))$;
  $\Longrightarrow$ increase of $F_2(Q^2, x)$ with decreasing x
- increase of $F_2(Q^2, x)$ with decreasing x weaker at small Q^2 than at large Q^2 due to the QCD evolution and the fact that $\bar{x} > x$
- s dependence of $\sigma_{\rm vp}(s)$ relatively weak here
- Due to all the above reasons x dependence of $F_2(Q^2, x)$ relatively weak for low Q^2

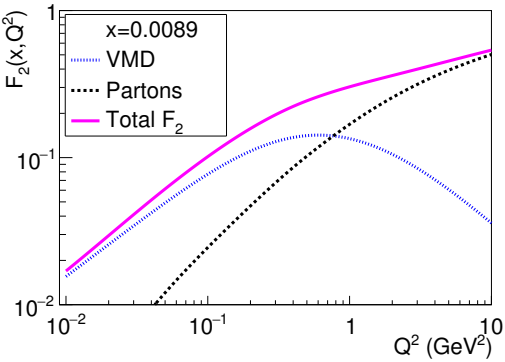
Results of the calculations: $F_2(x)$, Q^2 = const





S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024) and 2025 update

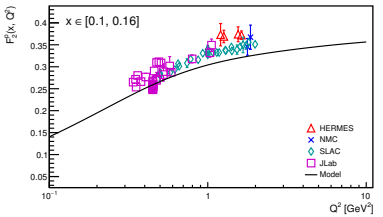
Results of the calculations: $F_2(Q^2)$, x = const = 0.0089

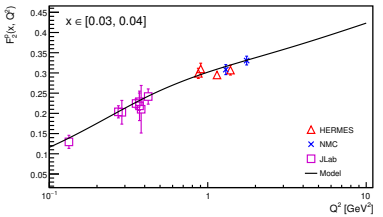


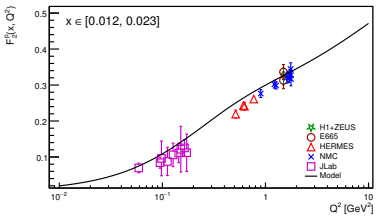
- An “approach to scaling” clearly visible at $Q^2 \lesssim 1$ GeV^2 (due to Q^2 factor in the eq. for F_2)
- Magnitude of this change controlled by F_2^{VMD} contrib.
- Partonic contrib., F_2^{partons} still significant (∼20-30%) there
- At Q^2 = 10 GeV^2 the model F_2 differs from F_2^{AS} by less than 3%

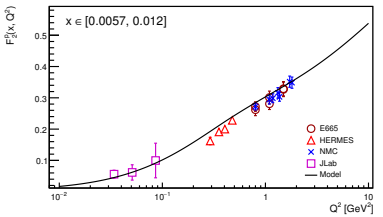
Comparison with world data: $F_2^{\mathrm{p}}(Q^2)$, bins of x

$$Q^2 < 2\ \mathrm{GeV}^2, \quad x < 0.2, \quad W^2 > 3.5\ \mathrm{GeV}^2$$









Reasonable description of $F_2^{\mathrm{p}}(Q^2, x)$ data

Summary

- Precise measurements at EIC need a good knowledge of radiative corrections.
- Radiative correction procedure demands parameterisations of DIS structure functions valid at $\mathbf{0 < Q^2 < Q^2_{meas}}$ and $\boldsymbol{x_{meas} < x < 1}$.
- We revisit and update an $F_2(Q^2, x)$ representation (not a fit), Phys. Lett. B295 (1992) 263, based on GVMD and valid at low x and any Q^2.
- The model incorporates the vector meson dominance dynamics and the QCD improved parton model, suitably extended to the low Q^2 region.
- A relevance of the vector meson dominance dynamics clearly emerges at low Q^2.
- The model reasonably reproduces the $F_2(Q^2, x)$ world measurements at low x and Q^2.
- **Work in progress:** the PDF dependence, photoproduction limit, heavier VMs and non-diagonal terms in VMD,...
- We have already proposed a model of $F_{\rm L}(Q^2, x)$ valid at low x and any Q^2 (Phys.Lett. B 829 (2022) 137086); a code is available upon request.