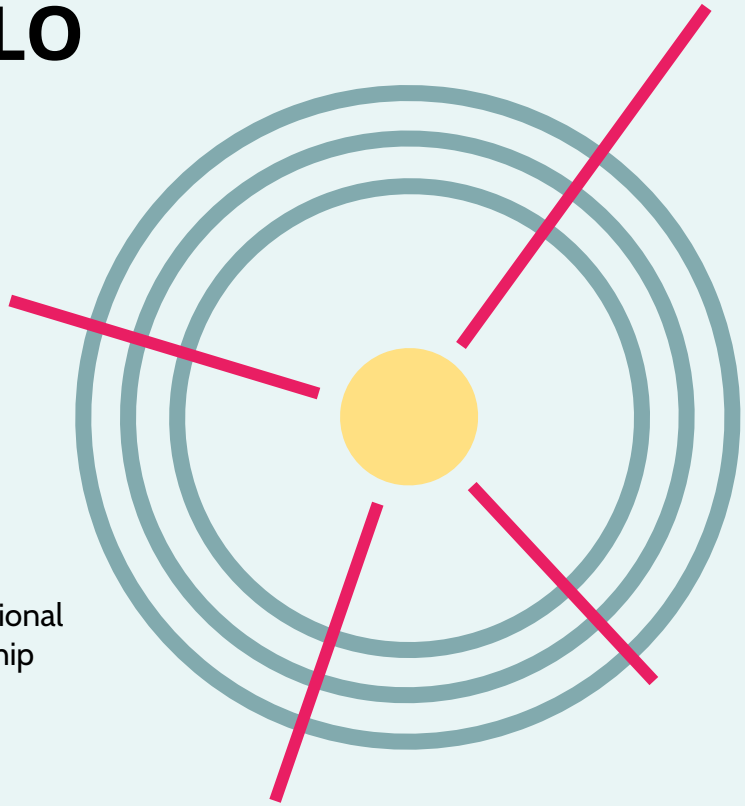
Four-jet production at electron-positron colliders at NNLO

Matteo Marcoli

33rd International Workshop on Deep Inelastic Scattering and Related Subjects

Bologna, 07/05/2026



Durham University

IP3

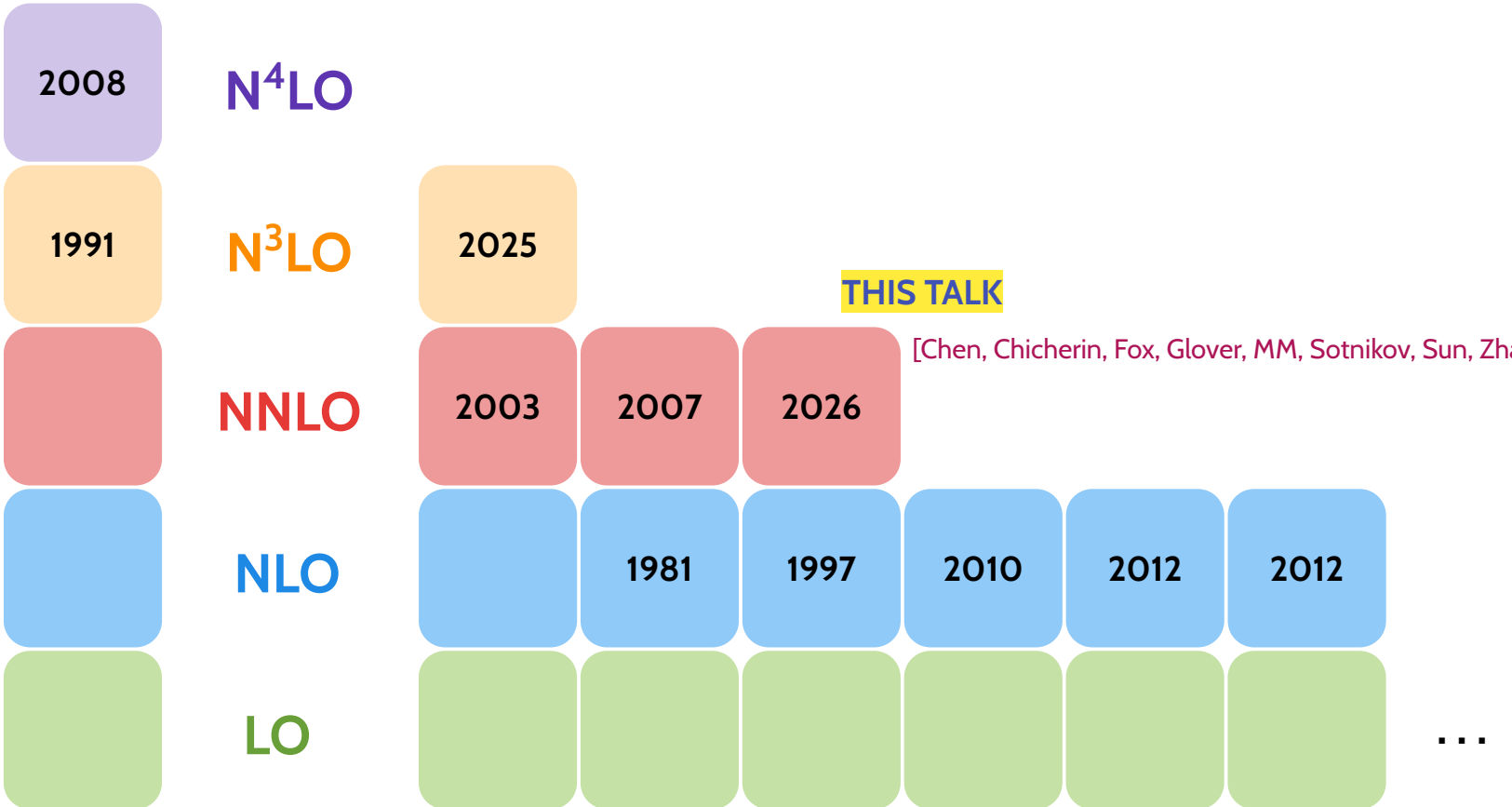
THE ROYAL SOCIETY Newton International Fellowship

work in collaboration with Xuan Chen, Dmitry Chicherin, Elliot Fox, Nigel Glover, Vasily Sotnikov, Huiting Sun, Hantian Zhang and Simone Zoia

hep-ph/2602.18185

INTRODUCTION

Jet production in electron-positron annihilation



References in the backup slides

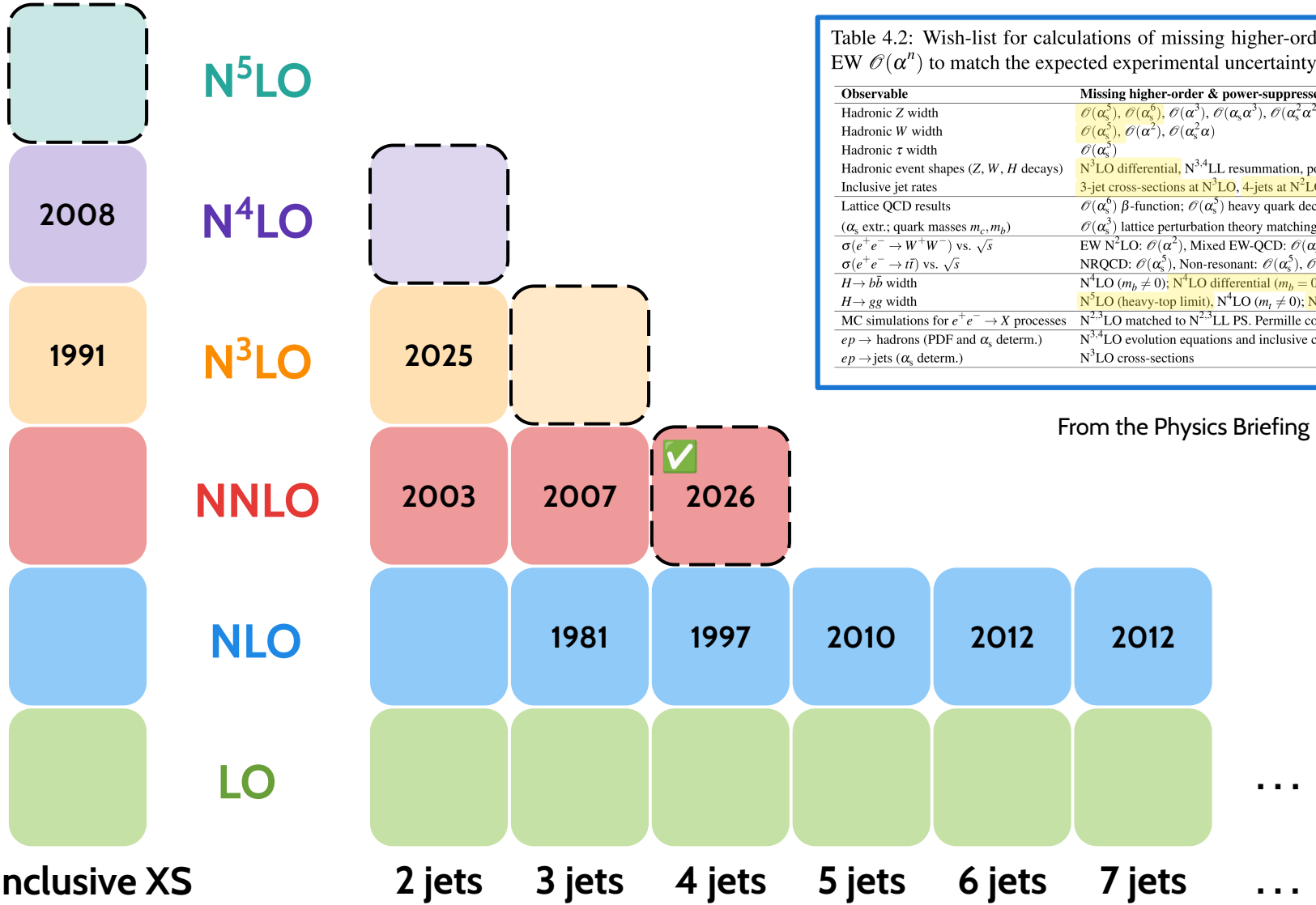
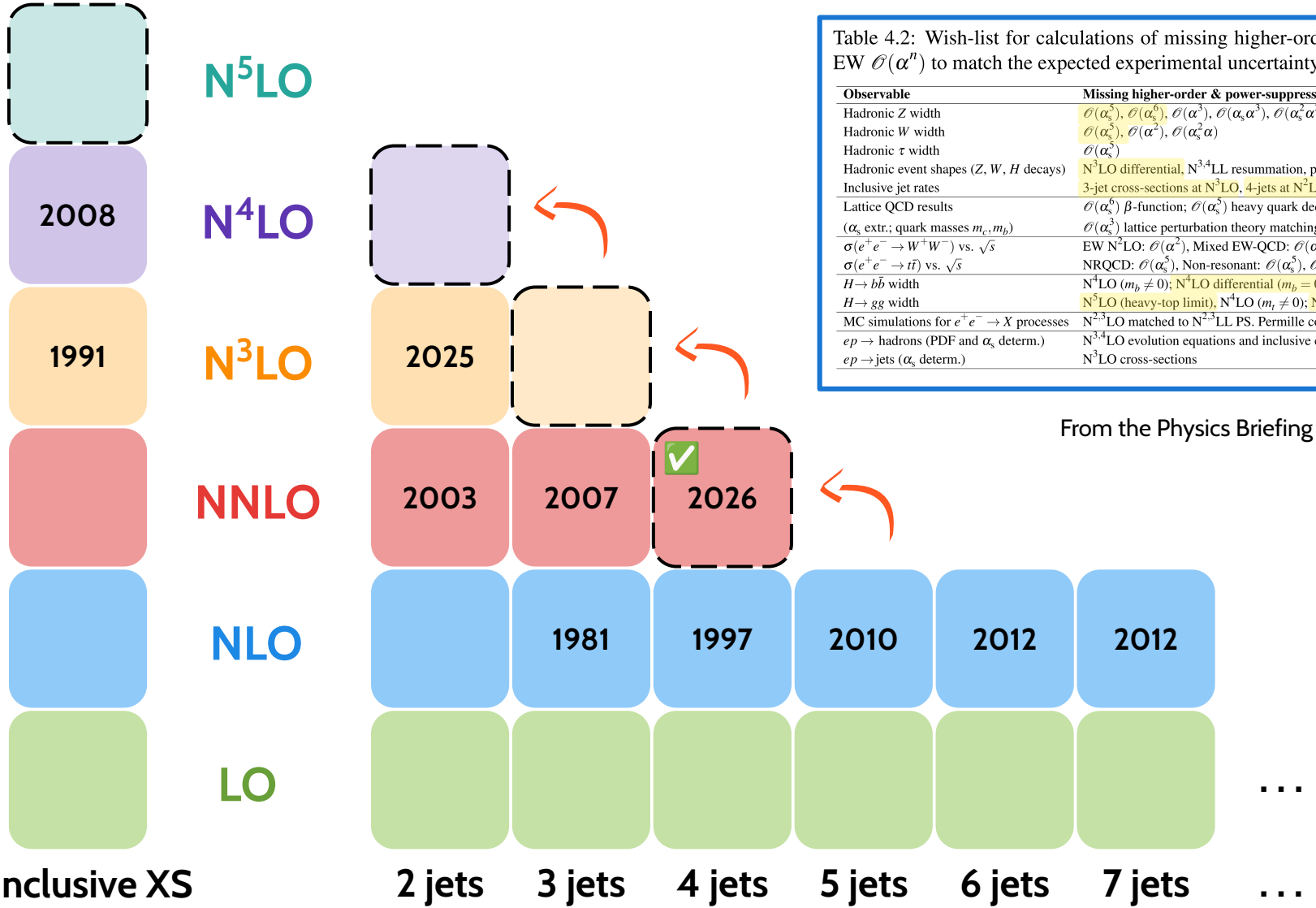


Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and ep colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^6)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha_s^2\alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha_s^2\alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^3)$
Hadronic event shapes (Z, W, H decays)	N^{3}LO differential, N^{3,4}LL resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N^{3}LO, 4-jets at N^{2}LO, 5-jets at NLO
Lattice QCD results	$\mathcal{O}(\alpha_s^6)$ β-function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
(α_s extr.; quark masses m_c, m_b)	$\mathcal{O}(\alpha_s^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
$\sigma(e^+e^- \to W^+W^-)$ vs. $\sqrt{s}$	EW N^{2}LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s\alpha^2)$, $\mathcal{O}(\alpha_s^2\alpha)$
$\sigma(e^+e^- \to t\bar{t})$ vs. $\sqrt{s}$	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
$H \to b\bar{b}$ width	N^{4}LO ($m_b \neq 0$); N^{4}LO differential ($m_b = 0$)
$H \to gg$ width	N^{5}LO (heavy-top limit), N^{4}LO ($m_t \neq 0$); N^{4}LO differential, N^{3}LO differential ($m_t \neq 0$)
MC simulations for $e^+e^- \to X$ processes	N^{2,3}LO matched to N^{2,3}LL PS. Permille control of non-pQCD effects (hadronization, CR, ...)
$ep \to$ hadrons (PDF and α_s determ.)	N^{3,4}LO evolution equations and inclusive cross-sections
$ep \to$ jets (α_s determ.)	N^{3}LO cross-sections

From the Physics Briefing Book [de Blas et al. 2511.03883]

References in the backup slides



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$ep \to$ jets (α_s determ.)	N^{3}LO cross-sections

From the Physics Briefing Book [de Blas et al. 2511.03883]

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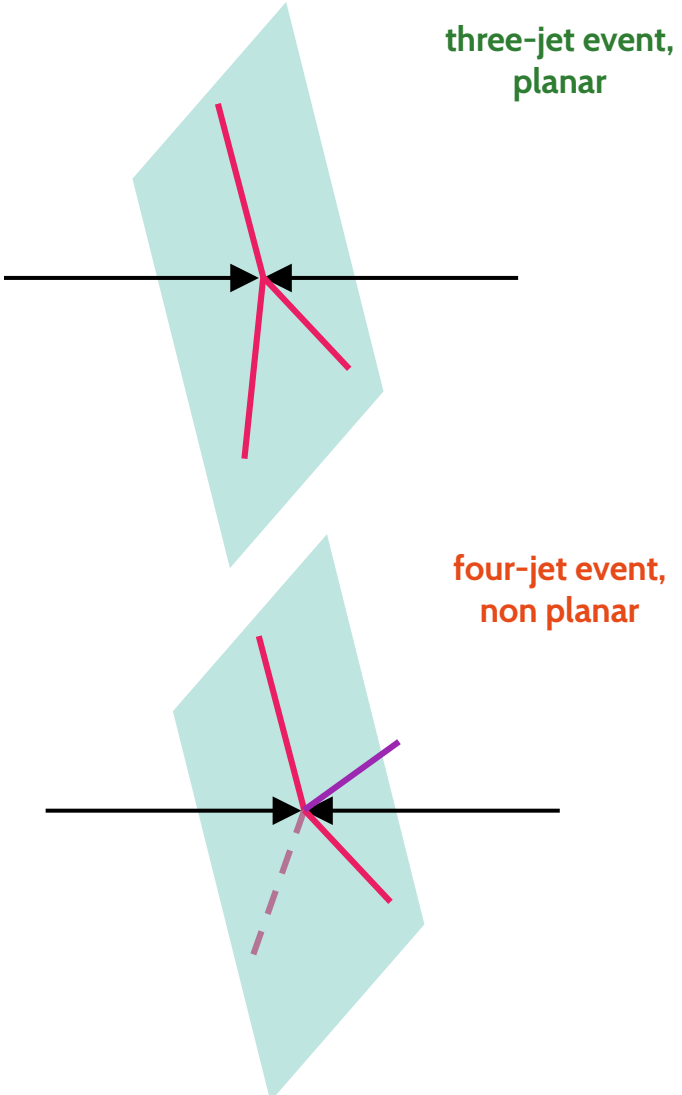
What's interesting about four-jet production?

- Four-jet events deviate from the **planar configuration**.
  Several **event shape variables** have been considered: thrust-minor, aplanarity, D-parameter, light jet mass, narrow jet broadening, ...;

- Four-jet event shapes are significantly harder to investigate than three-jet ones: more challenging fixed-order, resummation, hadronization modelling;

- At NNLO, the complexity is comparable to cutting-edge calculations for hadronic 2→3 processes. **Highest jet-multiplicity** considered so far at NNLO;

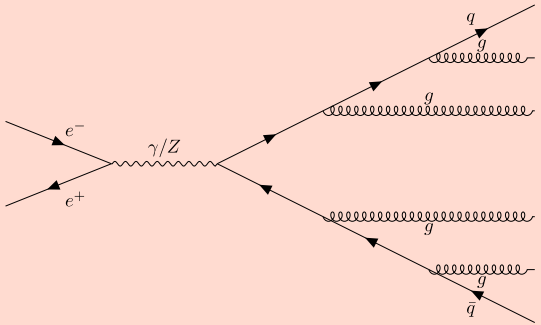
- As a first application, we compute the **four-jet rate at NNLO**;



SETUP OF THE CALCULATION

Setup of the calculation: matrix elements

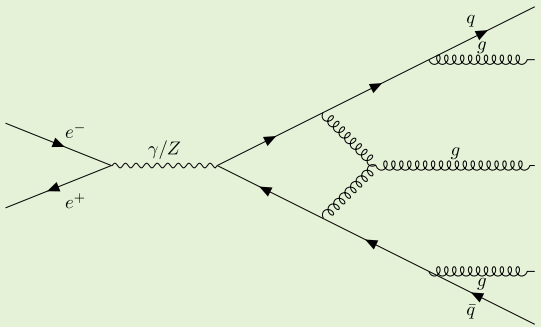
REAL-REAL



[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 1907.13071]

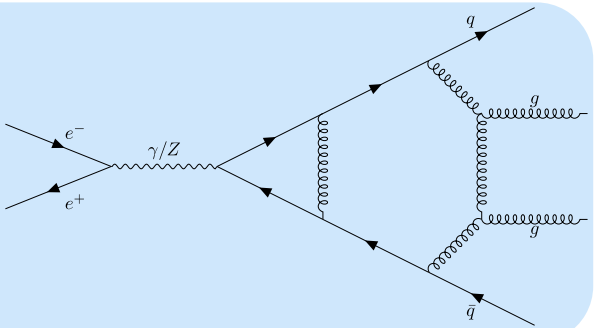
- from **OpenLoops2**
- fully decomposed into colour factors
- double-precision is enough

REAL-VIRTUAL



- from **OpenLoops2**
- **no colour decomposition**
- switch to built-in higher precision mode in the IR limits

VIRTUAL-VIRTUAL



- **NEW!**

Setup of the calculation: two-loop contributions

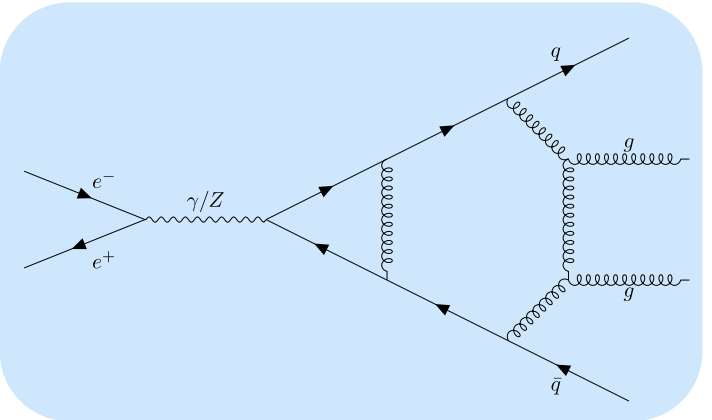
- cutting edge: **five-point** two-loop with **one external massive leg**

$$R^{\vec{a}}(\vec{p}) = \sum_{i,j} r_i(\vec{p})\ M_{ij}^{\vec{a}}\ f_j(\vec{p})$$

one-mass pentagon functions

rational coefficients

[Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia 2009.13917]
[Chicherin, Sotnikov, Zoia 2110.10111]



- crossing of the amplitudes for pp → Vjj [Abreu, Febres Corder, Ita, Klinkert, Page, Sotnikov 2110.07541] [De Laurentis, Ita, Page, Sotnikov 2503.10595]

$$\{f_j^{\text{prod}}\} \to \{f_j^{\text{decay}}\}$$

Complete set of decay pentagon functions available in the **PentagonFunctions++** public library.
Symbolic expressions are available on Zenodo at **https://doi.org/10.5281/zenodo.18630975** .

[Catani 9802439] [Sterman, Tejeda-Yeomans 0210130] [Becher, Neubert 0901.0722]

- we consider the **leading-colour approximation** for the two-loop finite reminder, defined in the **Catani scheme**

$$N_c \to \infty \qquad \text{with} \quad N_f/N_c \quad \text{kept constant}$$

- The computational cost for the integration of the two-loop contributions is negligible compared to the rest of the calculation.

Setup of the calculation: IR subtraction and numerical implementation

[Gehrmann-De Ridder, Gehrmann, Glover 0505111]

- Infrared subtraction performed with the **antenna subtraction method**:
  - **colourful antenna subtraction** + **generalized antenna functions**:
    → **simplicity and computational efficiency**

[Chen, Gehrmann, Glover, Huss, MM 2203.13531]
[Gehrmann, Glover, MM 2310.19757]
[Braun-White, Glover, Preuss 2302.12787, 2307.14999]
[Fox, Glover, MM 2410.12904]

- We integrate IR-subtracted remainders within the parton-level Monte Carlo framework ***NNLOJET***

[NNLOJET Collaboration, A. Huss *et al.* 2503.22804]

  - efficient numerical implementation, **even for challenging regions** of the phase space (three-jet limit);

- We consider:
  - **mZ** = 91.2 GeV, $\alpha_s(mZ)$ = 0.118, N_f = 5 massless quark flavours;
  - no top-loop contributions: expected to be negligible
  - only partial singlet contributions (no axial-singlet, ...): also expected to be negligible

[Knihel, Kuhn Phys.Lett.B 224 (1989) 229-232]
[Hagiwara, Kuruma, Yamada Nucl.Phys.B 358 (1991) 80-96]
[Dixon, Signer 9609460, 9706285]
[Frederix, Frixione, Melnikov, Zanderighi 1008.5313]

- All our results are available in digital format on Zenodo at **https://doi.org/10.5281/zenodo.18630975**

RESULTS

Results: jet rates in electron-positron annihilation

The **n**-jet rate is given by the fraction of events with **n** resolved jets:

$$R_n(y_{\rm cut}) = \frac{\sigma_n(y_{\rm cut})}{\sigma_{\rm inc.}}$$

jet resolution parameter

exclusive n-jet cross section

inclusive hadronic cross section

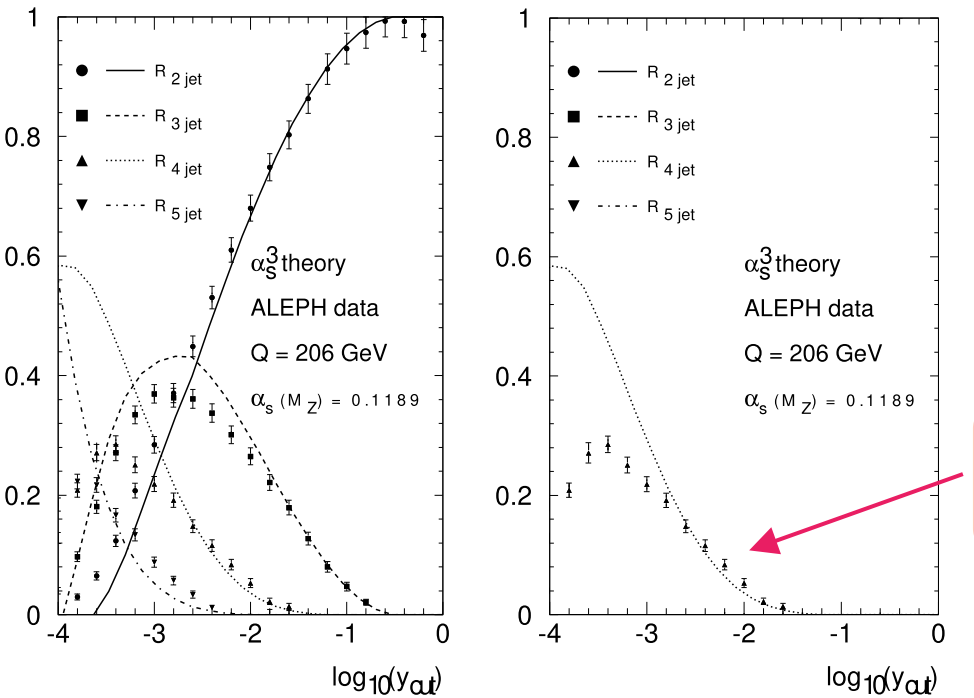
[Brown, Stirling *Phys. Lett.* B 252 (1990)]
[Catani, Dokshitzer, Olsson, Turnock, Webber *Phys. Lett.* B 269 (1991)]

We consider the Durham (k_t) jet clustering algorithm:

$$y_{ij}^D = \frac{2\min(E_i^2, E_j^2)}{Q^2}(1-\cos\theta_{ij}), \quad y_{ij}^D > y_{\rm cut}\ \forall\, ij$$

Jet rates at order α_s^3:

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 0802.0813]



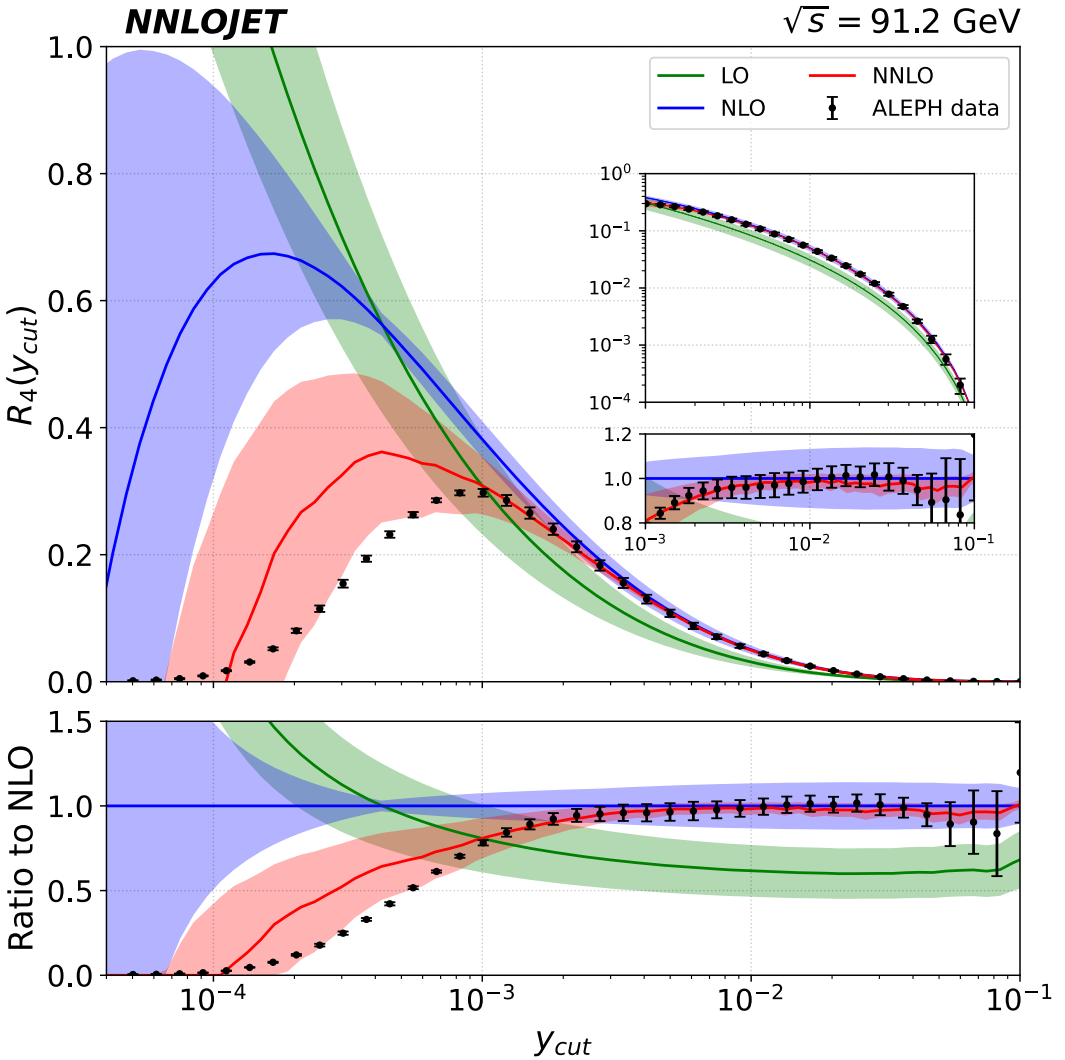
R_4 is only NLO-accurate

Results: the four-jet rate at NNLO

- Data from ALEPH [Heister et al. *Eur.Phys.J.C* 35 (2004) 457-486]

- We identify a **perturbative region**: **0.001 ≤ ycut ≤ 0.1**:
  - Remarkable convergence: scale uncertainties reduce from 15% at NLO to 3%-5% at NNLO (symmetrized);
  - NNLO correction amounts to a negative 2%-5%;
  - Improved agreement with data;

- For **ycut ≤ 0.001**, we observe perturbative breakdown:
  - larger scale uncertainties, non-verlapping scale bands, predictions turning negative
  - All-order resummation is needed;
  - Still NNLO reproduces logarithmic terms and the shape of the peak much better;



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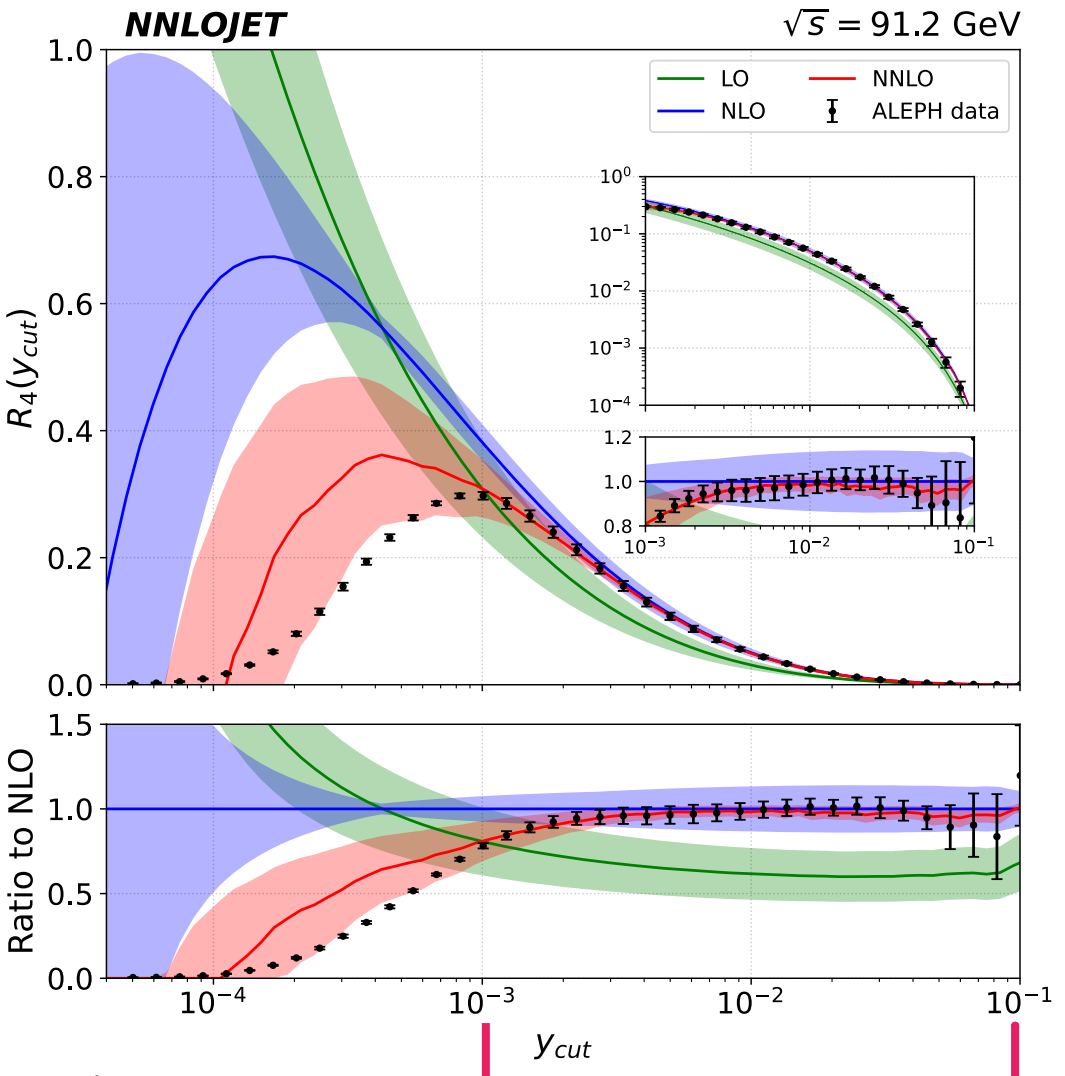

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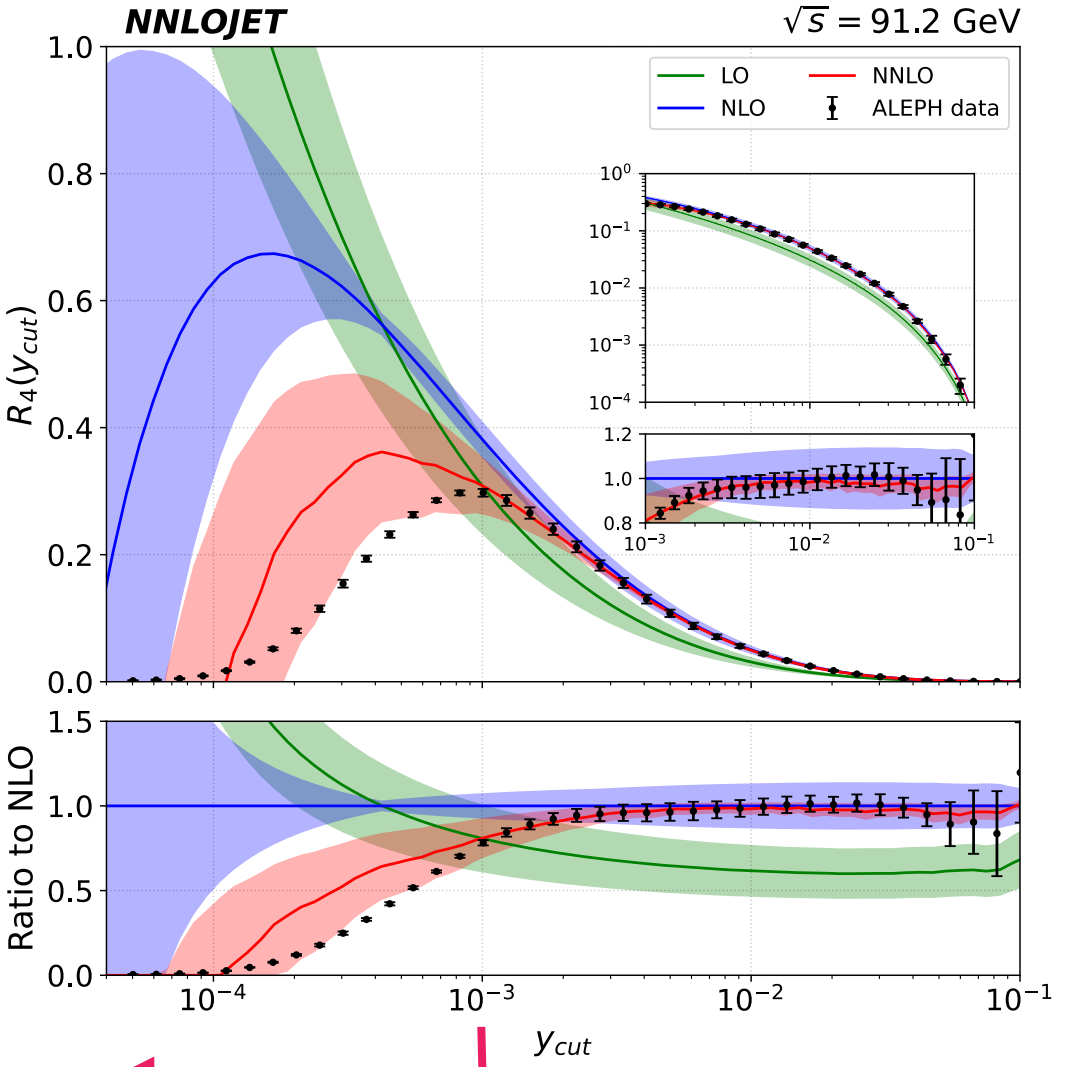
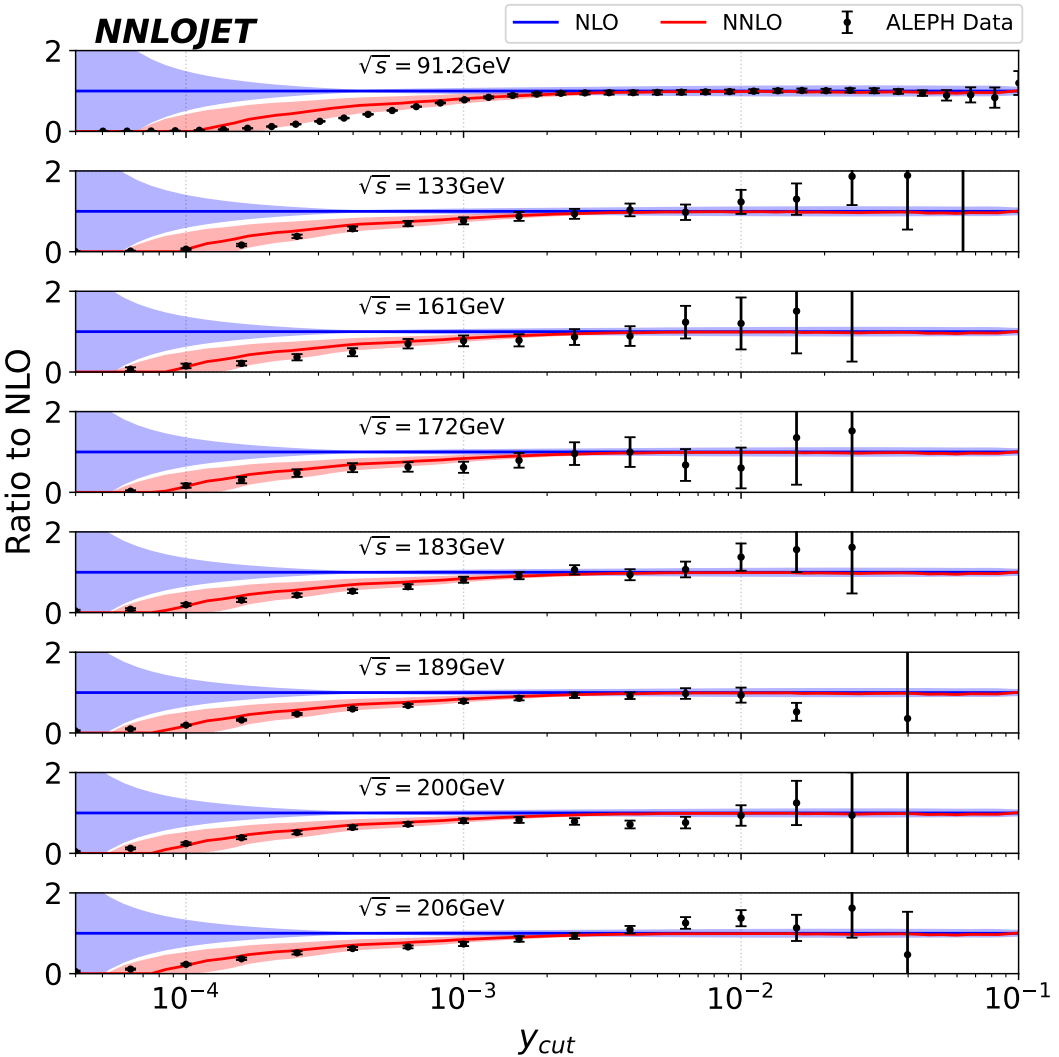


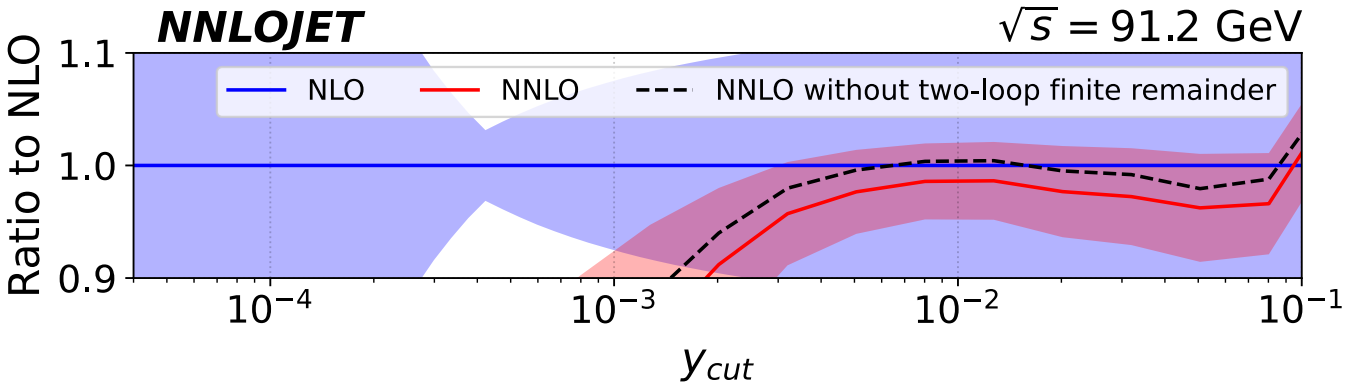
Results: energy scan

- Similar observations as at 91.2 GeV;

- You can find all plots and data at different c.o.m. energies on Zenodo at https://doi.org/10.5281/zenodo.18630975



Results: impact of the two-loop finite remainder

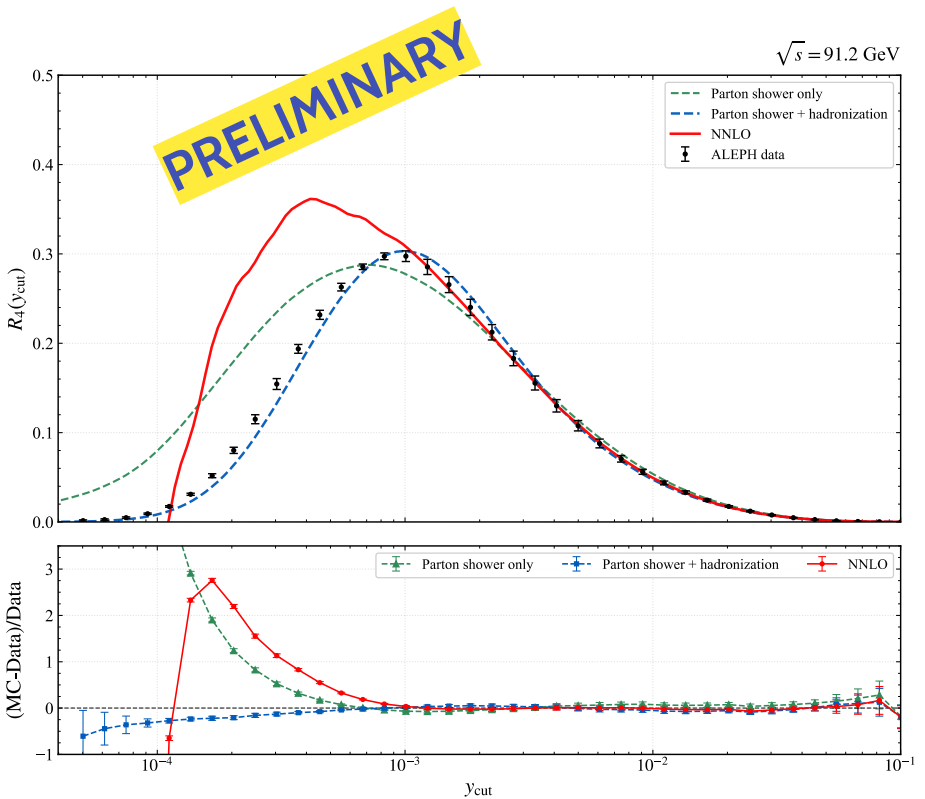


- In the perturbative region, the two-loop finite remainder amounts to a **negative 2%-5% of the full NNLO**;
- Here, the NNLO correction is dominated by two-loop finite-reminder.
  Though peculiar, the overall size of the finite remainder is similar to what observed in hadronic processes;
- One can estimate that the uncertainty due to **missing subleading-colour contributions** is around **1% of the full NNLO**;
- The relative size of the two-loop finite remainder is largely insensitive to the c.o.m. energy;

Impact of hadronization effects

Hadronization corrections are expected to be small in the perturbative region:

- Clustering particles via the Durham algorithm reduces the sensitivity to hadronization effects;
- Fixed-order predictions seems to reproduce data very well;
- We verified explicitly by running Pythia+Vincia.

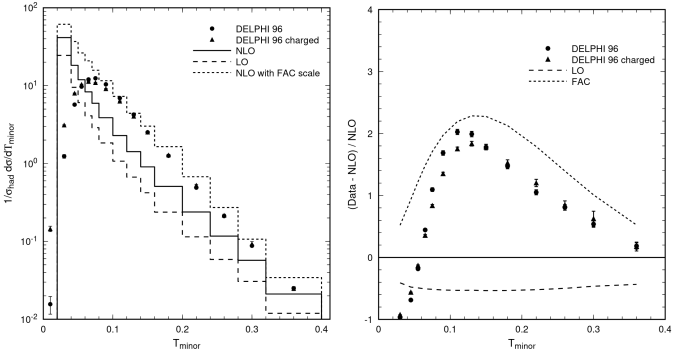


[Thanks to Huiting Sun for producing this plot]

Four-jet event shapes: work in progress

[Campbell, Cullen, Glover 9809429]

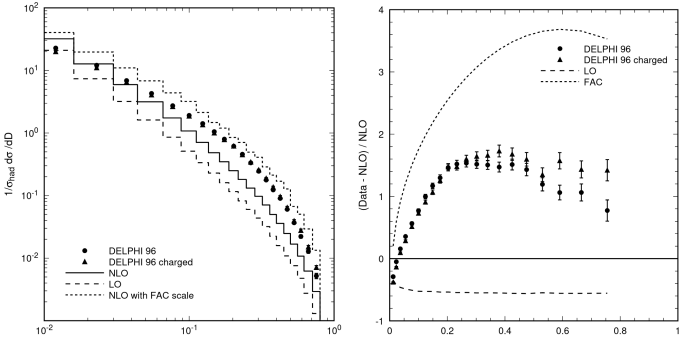
- Fixed-order predictions are quite off data, suggesting large non-perturbative corrections. This can't be fixed by NNLO or higher-orders;



NLO

- Hadronization corrections are challenging to model;

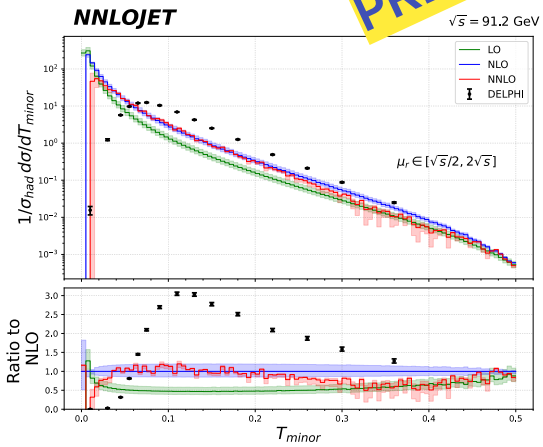
[Banfi, Dokshitzer, Marchesini, Zanderighi 0010267, 0101205, 0104162]
[Burby, Glover 0101226]

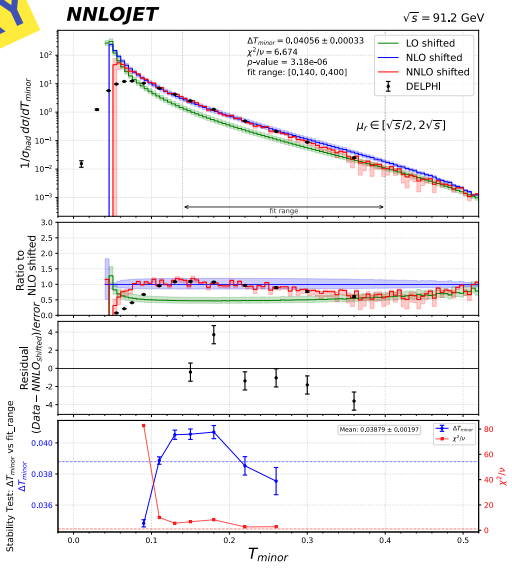


NLO

- Some preliminary results:

PRELIMINARY





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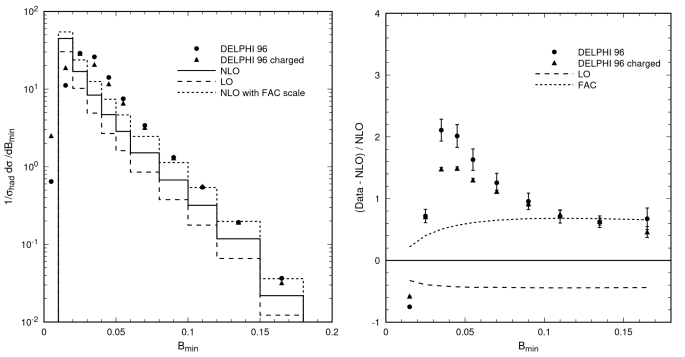


NLO

SUMMARY AND OUTLOOK

Summary:

- We presented **NNLO-accurate results for the four-jet rate** in electron-positron annihilation;
- These were obtained with the **first state-of-the-art NNLO calculation of four-jet production**, efficiently implemented within the ***NNLOJET*** framework;
- The NNLO correction leads to an improved agreement with LEP data and a significant **reduction of theory uncertainties**;
- Effects **beyond fixed-order** seem to be crucial to reproduce four-jet event shape data, even in the four-hard-jet regions.

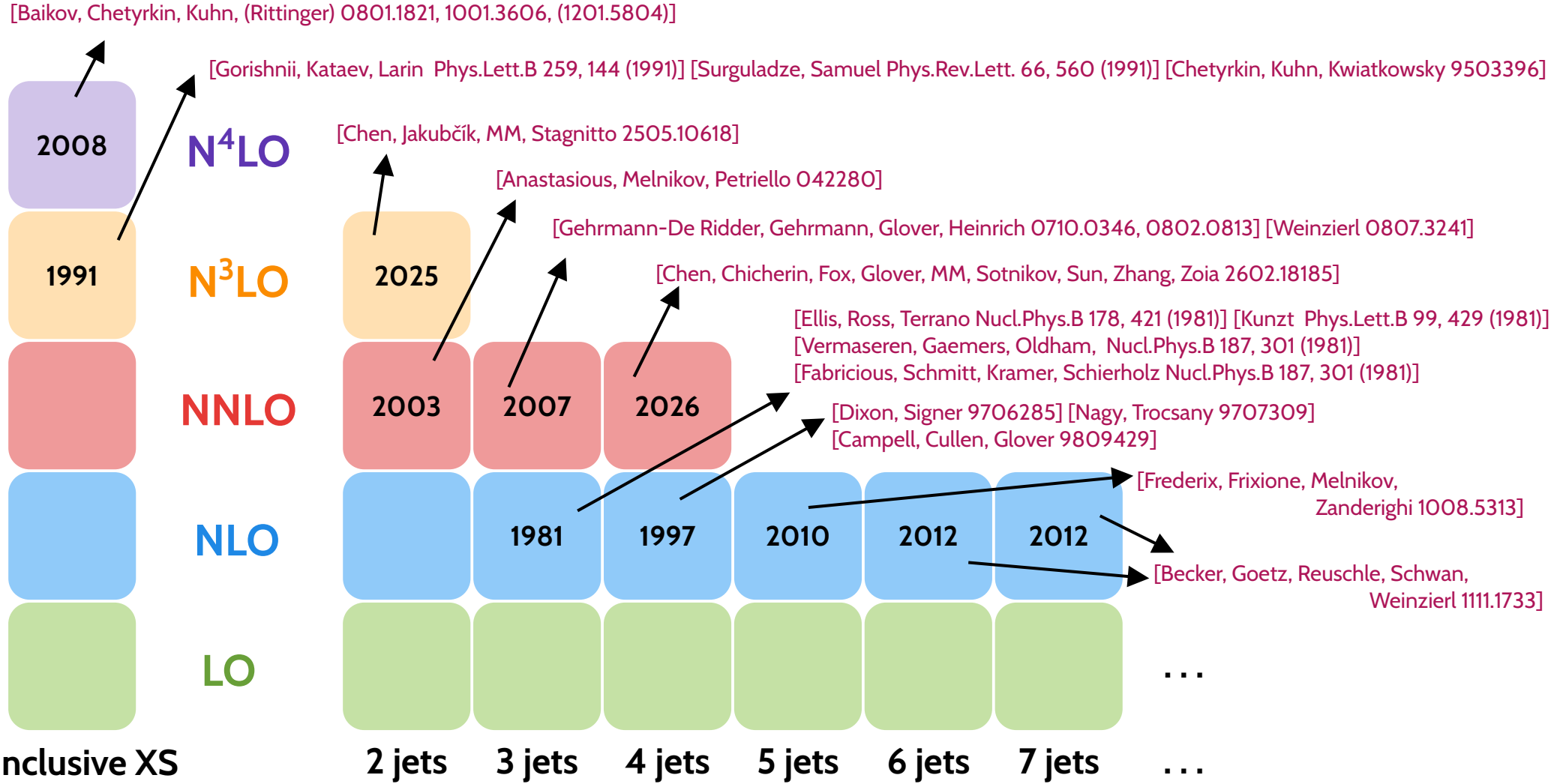
Outlook:

- Investigation of the NNLO correction and non-perturbative effects for **four-jet event shapes**;
- **Resummation** in the three-jet (planar) limit;
- Mass effects, top-loop correction, axial contributions, ...

Thank you for your attention!

Backup slides

Jet production in electron-positron annihilation: references



Setup of the calculation: infrared subtraction scheme

[Gehrmann-De Ridder, Gehrmann, Glover 0505111]

We rely on the **antenna subtraction method** for the removal of infrared divergences.

In particular:

[Gehrmann, Glover, MM 2310.19757]
[Chen, Gehrmann, Glover, Huss, MM 2203.13531]

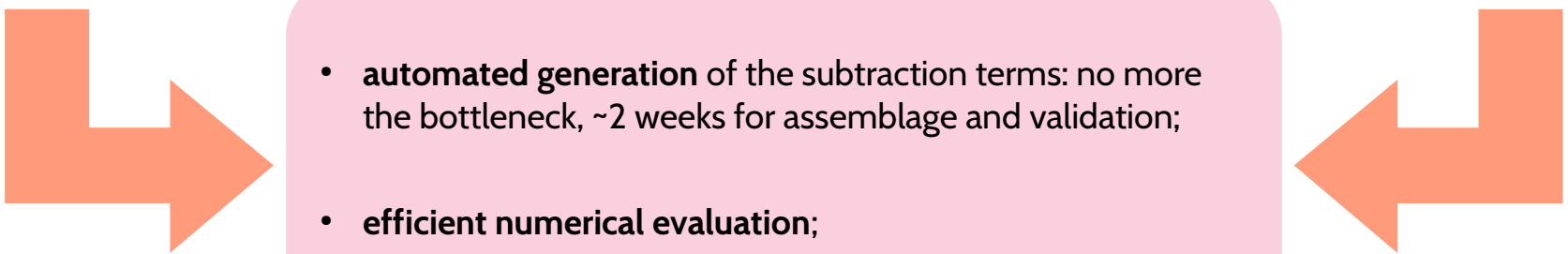
COLOUFUL ANTENNA SUBRACTION

infer real-emission counterterms from the universal structure of infrared virtual singularities

[Braun-White, Glover, Preuss 2302.12787, 2307.14999]
[Fox, Glover, MM 2410.12904]

GENERALISED ANTENNA FUNCTIONS

construction of antenna functions with multiple hard radiators to efficiently address complicated emission topologies



- **automated generation** of the subtraction terms: no more the bottleneck, ~2 weeks for assemblage and validation;
- **efficient numerical evaluation**;