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Heavy quark Charged DIS at NNLO

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Motivation :

Why Heavy-quark Charged-current DIS ?

- **Process :** $\nu(\bar{\nu}) + p \to l^-(l^+) + X, \;\; X \in$ inclusive final states, $l \in \{e, \mu\}$
- **Lowest order :** $W^+ + s \to c, \;\; W^- + \bar{s} \to \bar{c}$

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- **Lowest order :** $W^+ + s \to c, \;\; W^- + \bar{s} \to \bar{c}$
- Direct access to strange quark content of the nucleon

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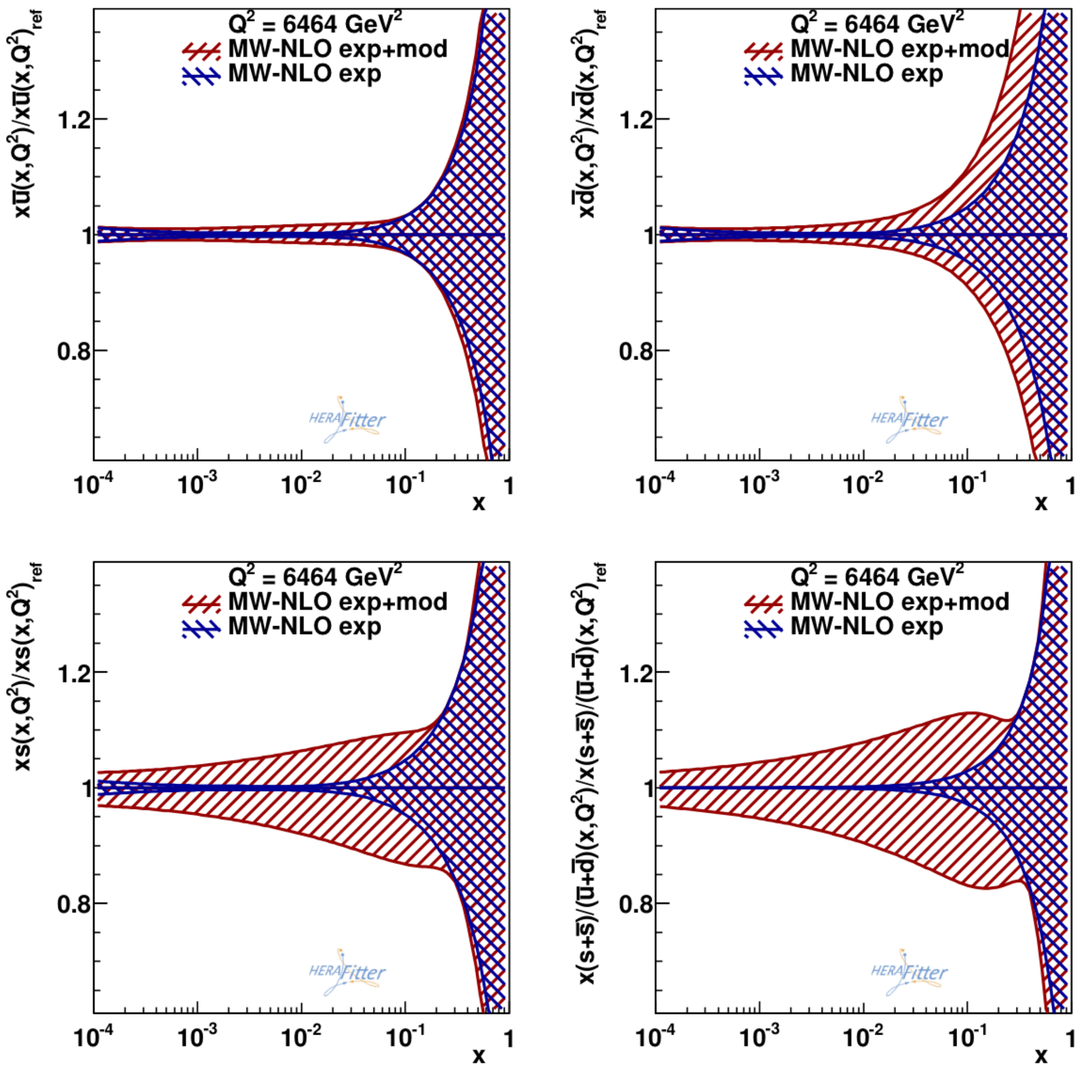
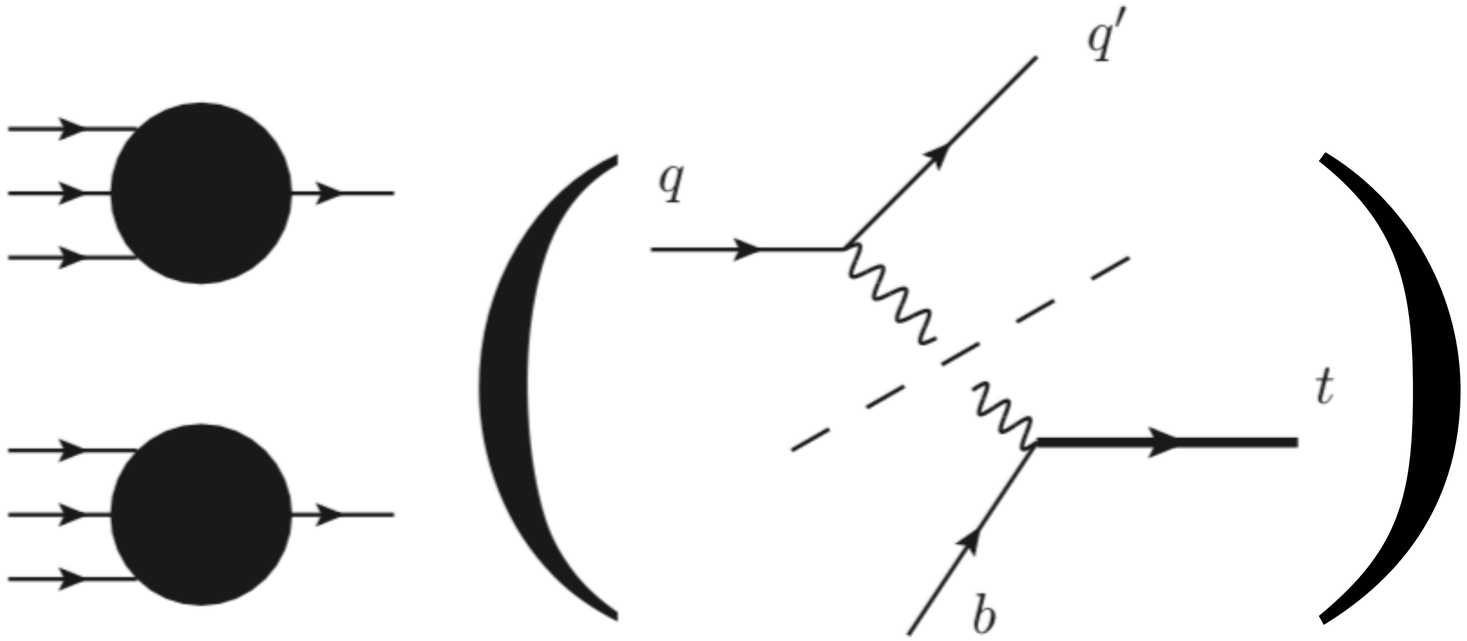


[ATL-PHYS-PUB-2014-015]

- s-quark pdf uncertainty alone can induce ~10MeV uncertainty in M_W measurement!
- NLO PDF set determined from a fit to **HERA I** data
- Inner dashed band : experimental PDF uncertainty (hessian method)
- Outer dashed band : sum of experimental and model uncertainty

Relevance at the Future collider :



▸ Single Top production in *t*-channel in a CC-DIS approach.

Work done so far

- At NLO exact expression with full mass dependency is known

[Gottschalk, 1981] , [Glück et. al. 1996] , [Blümlein, Moch et. al. 2011]

- An implementation in POWHEG is also accomplished

[Glück et. al. 1996] , [Buonocore et. al. 2024]

- At NNLO asymptotic limit for large momentum transfer, $Q^2 \gg m^2$, has been calculated

[Buza et. al. 1997] , [Moch] , [Blümlein et. al. 2014]

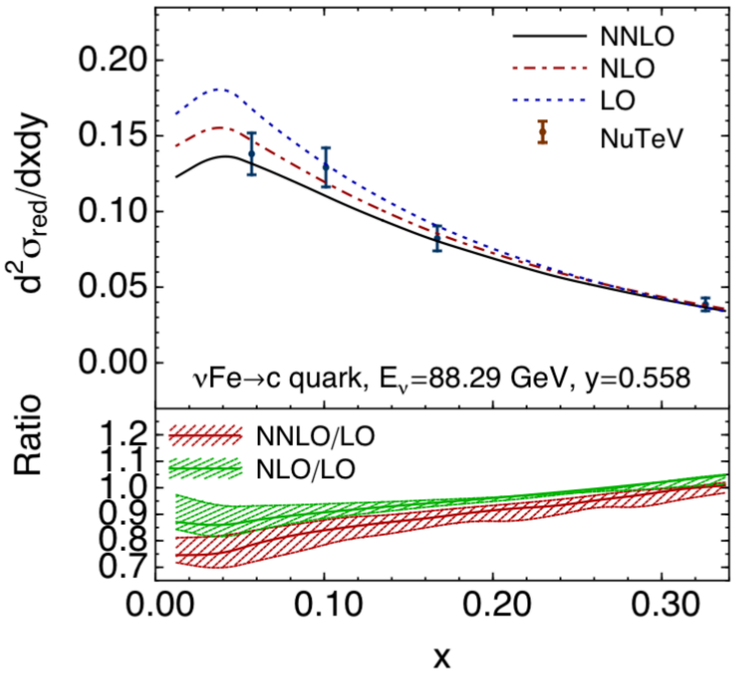
- Exact NNLO computation through phase space slicing method [Berger et. al. 2016]

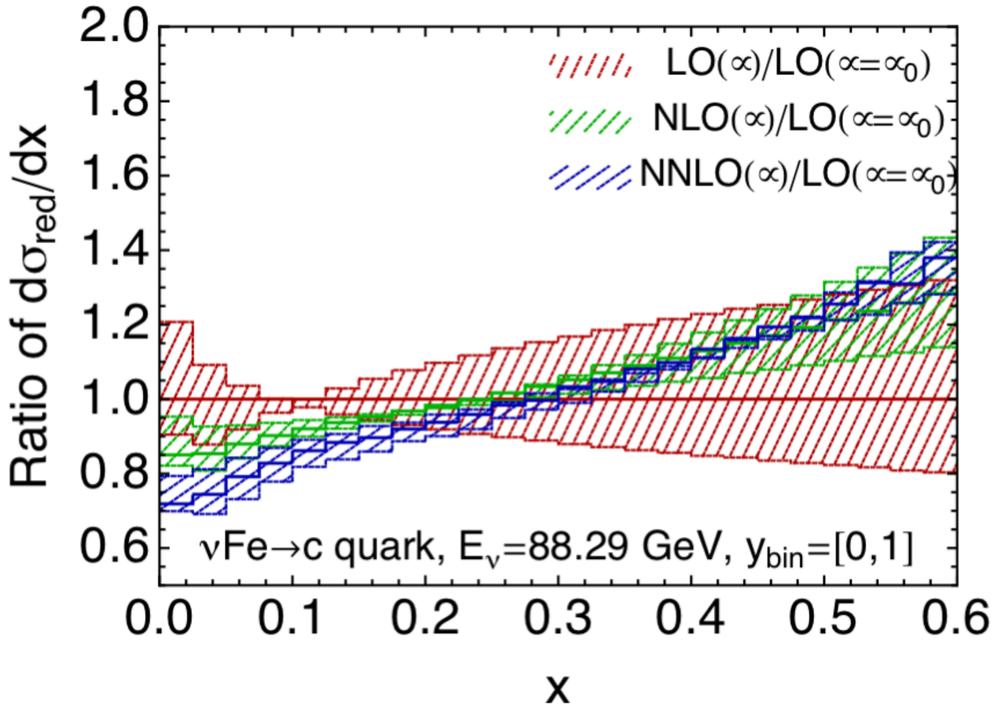
- Even N^{3}LO results for the non-singlet contributions at large momentum transfer are available

[Blümlein et. al. 2015-2016]

Lessons learned

- Asymptotic limits are well justified and sufficiently accurate for HERA data
- For Neutrino DIS: NNLO corrections are negative in the region of the data and can reach up to 10% of NLO
- Reduction of theoretical scale uncertainties





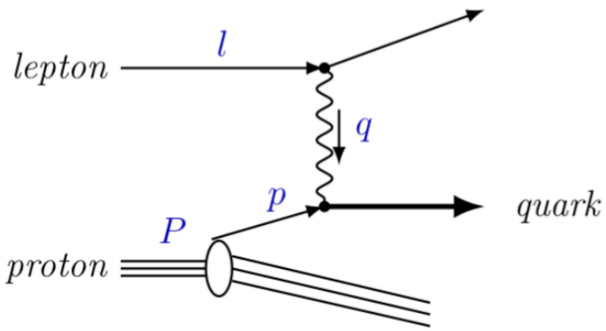
[Berger et. al. 2016]

Objective

- Compute analytically the exact mass dependency for the heavy-quark
- Study numerically the interplay of Q^2 and mass of the heavy-quark

Organization of the calculation

- Hadronic cross–section $\propto L^{\mu\nu} W_{\mu\nu}$
  - Leptonic tensor $L^{\mu\nu}$ for charged current
  - Hadronic tensor $W_{\mu\nu}$ parametrized through structure functions :



$$W_{\mu\nu} = \frac{1}{2x} e_{\mu\nu}\ F_L(x, Q^2, m) + \frac{1}{2x} d_{\mu\nu}\ F_2(x, Q^2, m) + a_{\mu\nu}\ F_3(x, Q^2, m),$$

with tensor structure ,

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \quad , \quad d_{\mu\nu} = -\, g_{\mu\nu} - \frac{4x^2}{q^2} p_\mu p_\nu - \frac{2x}{q^2}(p_\mu q_\nu + q_\mu p_\nu)$$

$$a_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{p \,.\, q} \quad , \quad Q^2 = -\, q^2,\ x = Q^2/(2p \,.\, q)$$

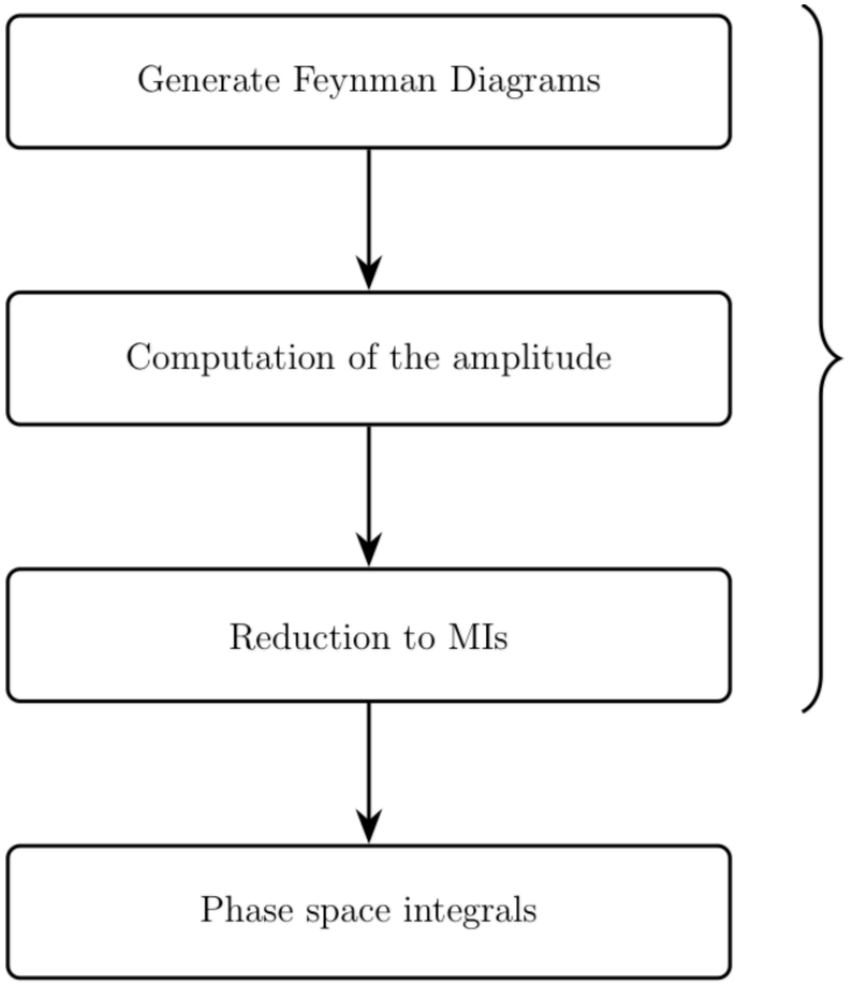
Approach

- Computation in Mellin-space : → Talk by Marco Klann
  - Mellin moments of the coefficient function .
  - For each moment : generate diagrams, perform IBPs and compute MIs in momentum space.
- Computation in x-space : ← This talk
  - Compute with full kinematic x-space dependency .
  - Generate all diagrams, perform IBPs and compute MIs in x-space.

	RR	RV	VV
bottom	$[s + W^* \to c + X]_{X \in gg,\ q\bar{q},\ s\bar{s}}$	$s + W^* \to c + g$	$s + W^* \to c$
gluon	$[g + W^* \to c + \bar{s} + g]$	$g + W^* \to c + \bar{s}$	
singlet	$[q + W^* \to c + \bar{s} + q]$		

VV: [Guido Bell, 2006], [Bonciani et. al., 2008] , [Dutta et. al., 2023-2024], [Fael et. al., 2024].

Computation –I

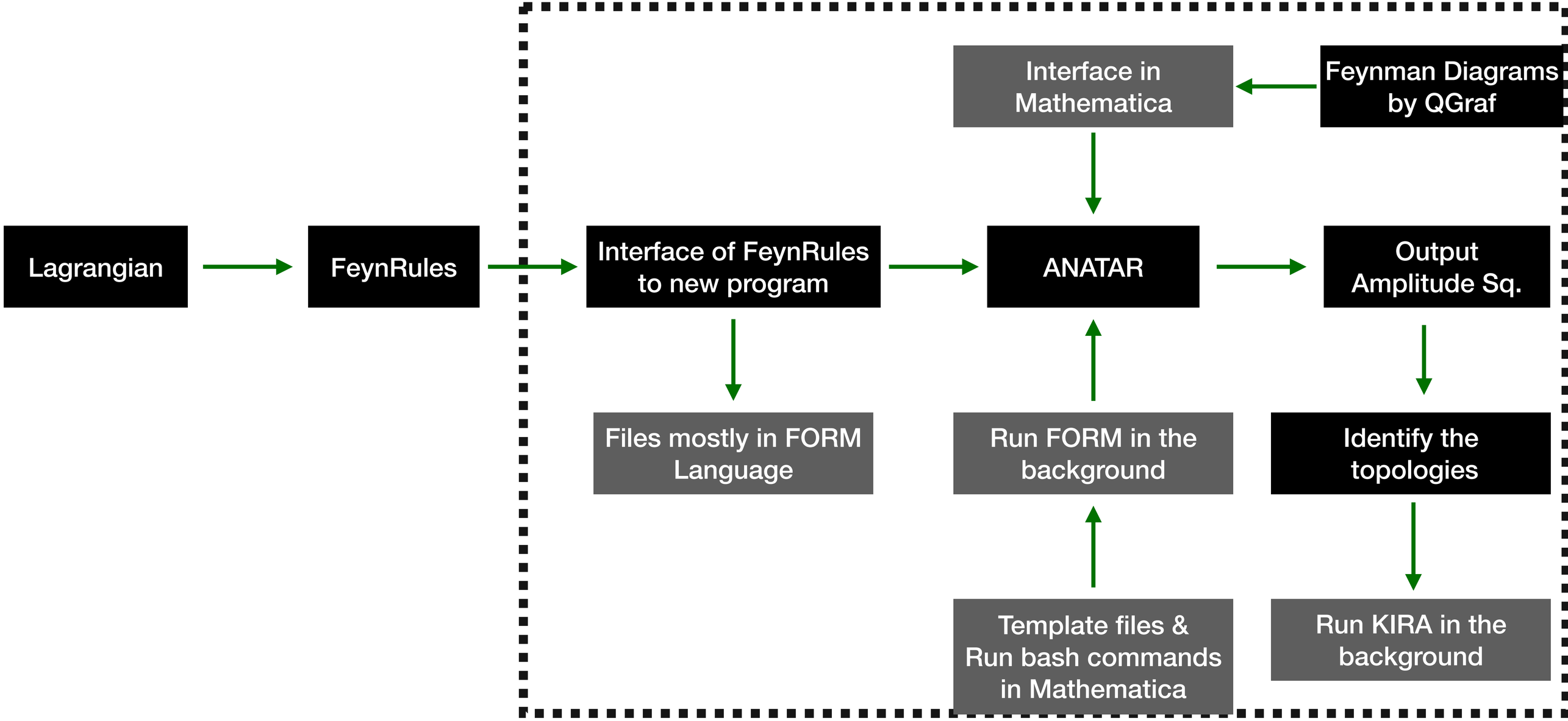


ANATAR

https: //github.com/ANATAR-hep/ANATAR

[Duhr, PM, Vasquez , 2025]

ANATAR - in a nutshell



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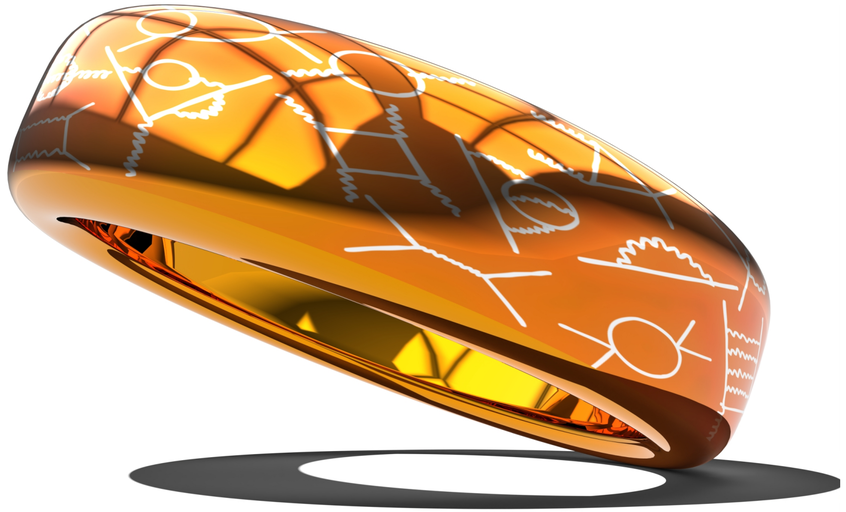


`GenerateAmplitude`
`GenerateAmplitudeConjugate`
`GenerateAmplitudeSquare`

`FindTopology`

`AmplitudeToTopologies`

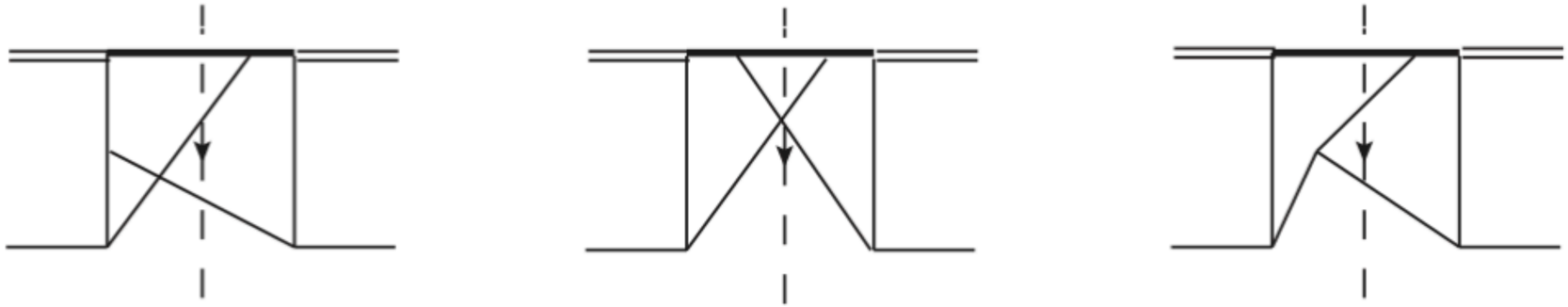
`IntegralReductionKira`

- Options :
  - Trim polarisation sum
  - Contract with projectors
  - Denote cut propagators

Computation –II

- Some of the topologies found :

$s + W^* \rightarrow c + g + g$



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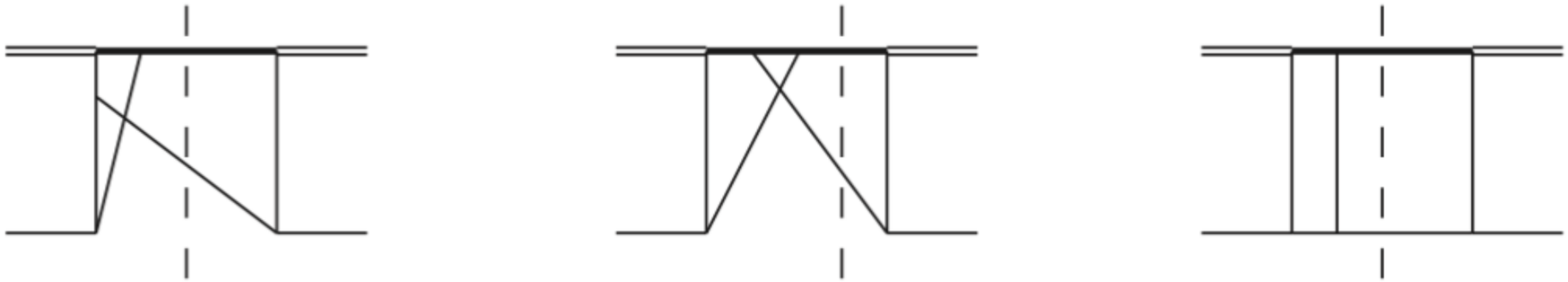

Double line : Q^2 , thick line: massive , thin line : massless, dash line: cuts

Computation –III

- ϵ-form of the differential equation : the MIs are put into a differential form using differential operators

$$d\vec{f}(\epsilon, y, Q^2) = \left(\sum_{i=1}^{N} A_i(\epsilon, y)\ dy \right) \vec{f}(\epsilon, y, Q^2)$$

where A_i are $N_{\text{master}} \times N_{\text{master}}$ -matrices of rational functions in $y \equiv \dfrac{m^2}{s}$ and ϵ and $Q^2 = 1$

- Transforming the basis of MIs with an invertible transformation T,

$$\vec{f} = T(\epsilon, y)\vec{f}'$$

  - sets up the differential equation in canonical form:

$$d\vec{f}'(\epsilon, y) = \epsilon \left(\sum_{i=1}^{N} A_i'(y)\ dy \right) \vec{f}'(\epsilon, y)$$

  - We use the INITIAL and Libra package for this purpose. [Dlapa et. al., 2020] [Lee, 2020]

Computation –IV

- For boundary integrals we use asymptotic expansion by regions .
- Angular integrals involved are of the form of : Neerven Integrals

$$\int d\Omega_{k_1 k_2} \frac{1}{(a + b\cos\theta)^j (A + B\cos\theta + C\sin\theta\cos\phi)^l}$$

[Neervan, 1986]

  - The idea stems from the rotational invariance of the angular integrals [Somogyi, 2011]
  - Use of Feynman parametzation [Smirnov et. al. 2012–2024]
  - Integrate out the delta function and make change of variables
  - Use the integral representation of the one–denominator massive angular integral and feed it to `asy.m`

Computation –V

- Region Integrals are computed using Mellin–Barnes techniques using `MB.m`

[Czakon, 2005]

`MBoptimizedRules` → `MBcontinue` → `MBexpand`

- MBintegrals are then transformed into multiple sums : `MBsums`

[Ochman et.al., 2015]

- The infinite sums are then summed up using HarmonicSums.m and EvaluateMultiSums.m

`MBIntToSum` → `ComputeGeneratingFunction` → `Iterated integrals`

[Schneider , 2004] [Schneider , 2021] [Ablinger , 2021]

- These iterated integrals are then expressed in terms of MPLs and MZVs .

Conclusion

- Using ANATAR , all the coefficient functions and reductions to MIs are done for RV and RR subprocesses.

- Using `asy.m` and `MB.m` , the boundary integrals are still being evaluated.

	RR	RV	VV
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- So far the checks :
  - The general solution of the differential equation satisfies the differential equation.
  - The boundary integrals are consistent with the asymptotic solutions from the differential equation.
  - The boundary integrals are also checked numerically using `pysecdec` [Borowka et. al., 2018]

Next steps

- After all boundary integrals, the following steps needs to be done:
  - Distributional expansion of all the integrals
  - Complete numerical check of all the integrals using AMFlow . [Liu et. al., 2022]
  - Checks with the existing results in the massless limits.
- Numerical implementation of the analytic results and study the interplay of Q^2 vs m^2 in different kinematic regimes
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Thank you !