Next to Leading Order corrections to the Next to Eikonal DIS structure functions

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accepted for JHEP publication

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

DIS2026
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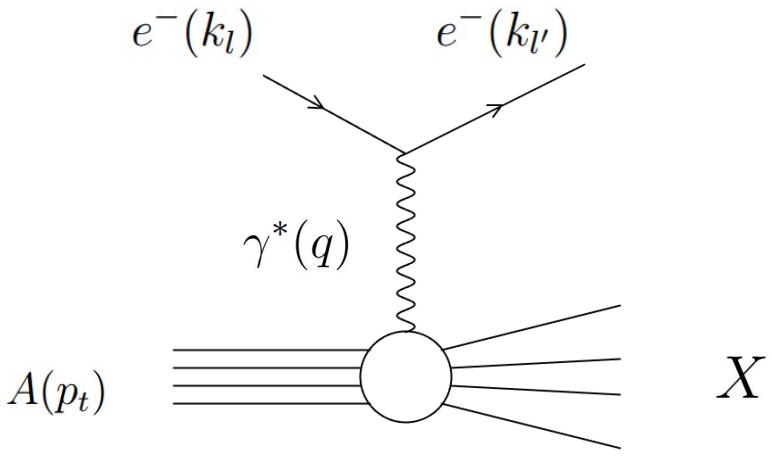
33rd INTERNATIONAL WORKSHOP ON DEEP INELASTIC SCATTERING AND RELATED SUBJECTS

Deep Inelastic Scattering (DIS)



- Photon virtuality $Q = \sqrt{-q^2}$

The transverse resolution within the nucleus scale as $x_\perp \sim \frac{1}{Q}$

- Bjorken "x" $x_{Bj} \simeq \frac{Q^2}{2q \cdot p_t}$

Partonic momentum fraction of the nucleus interacting with the photon.

- Unpolarized DIS cross section $$\frac{d^2\sigma^{l+A\to l'+A}}{dx_{Bj}dQ^2} = \frac{\alpha_e}{\pi x_{Bj} Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma^*}\left(x_{Bj},Q^2\right)+(1-y)\,\sigma_L^{\gamma^*}\left(x_{Bj},Q^2\right)\right]$$

- "T" for transversely polarized photon ;
- "L" for longitudinaly polarized photon.

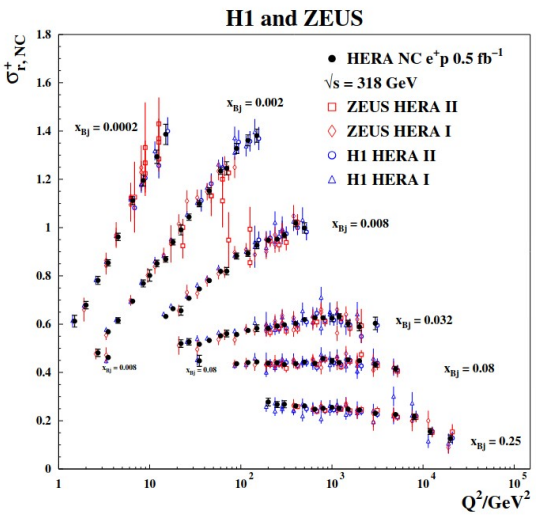
Centre of mass : $s = (k_l + p_t)^2$

Inelasticity : $y = \frac{2q \cdot p_t}{s}$

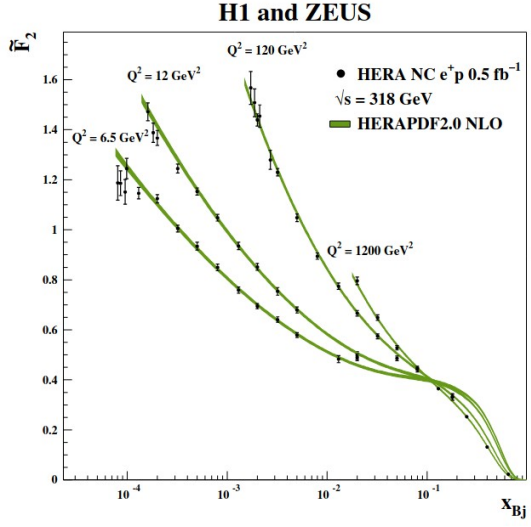
Deep Inelastic Scattering (DIS)

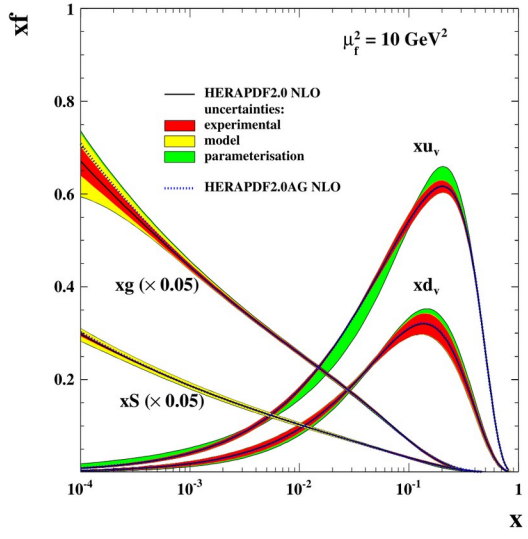
- Unpolarized DIS Structure Functions (SF) : $\sigma^{\gamma^*}_{T,L}(x_{Bj}, Q^2) = \frac{(2\pi)^2 \alpha_{\text{em}}}{Q^2} F_{T,L}(x_{Bj}, Q^2)$

[H1 and ZEUS collaborations, H. Abramowicz et al. Eur.Phys.J.C 75 (2015) 12, 580]



- As x_{Bj} decreases the Q^2 the dependence of the SF increases.

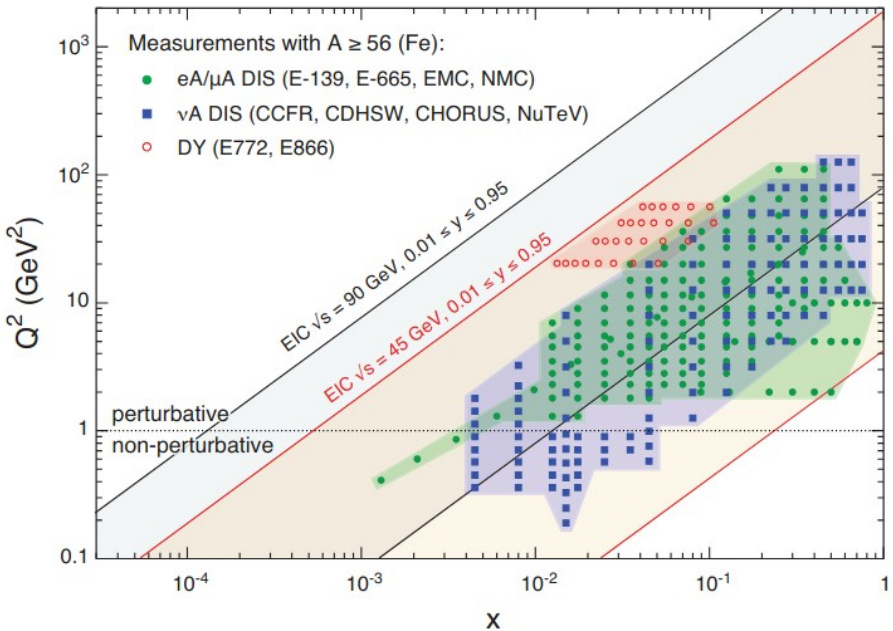




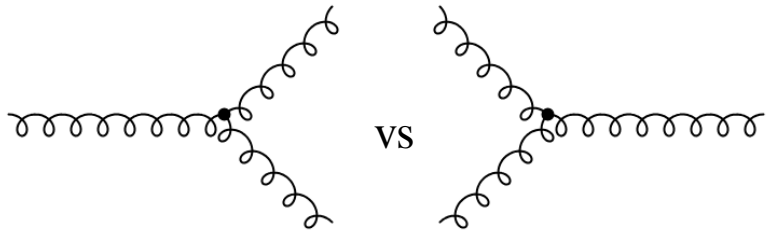
- Likely the growth of the gluon distribution at small x_{Bj} the SF grows as x_{Bj} decreases.

Benchmark of the talk

- Future of DIS, the Electron Ion Collider,
  "Understanding the glue that binds us all".



[A. Accardi et al. Eur.Phys.J.A 52 (2016) 9, 268]



- The saturation scale $Q_s^2 \sim \left(\frac{A}{x}\right)^{\frac{1}{3}} \Lambda_{\mathrm{QCD}}^2$ is in the perturbative regime of QCD.

[A. Stasto, K. J. Golec-Biernat, Kwiecinski, Phys.Rev.Lett. 86 (2001) 596-599]

- Towards the understanding of SFs evolutions in the (x_{Bj}, Q^2)- plan.

The Bjorken limit of QCD : improved parton model

In the limit $Q^2 \to \infty$ and $x_{Bj} = \text{cst}$

- *In this limit quark and gluon are quasi-free.*
- *The nuclei A is at hight energy the sum of its constituent*

- Collinear factorization theorem :

Separation of the hard scale effects from non-perturbative Parton Distribution Function (PDF)

$$F_{L/T}\left(x_{Bj}, Q^2\right) = \sum_{a \in A} \int_{x_{Bj}}^{1} \frac{dz}{z} f_{a/A}\left(z, \mu\right) \; \hat{\sigma}^{a}_{L/T}\left(\frac{x_{Bj}}{z}, \frac{Q}{\mu}, \alpha_s\right) + O\left(\frac{\mu^2}{Q^2}\right)$$

PDF — Partonic cross section

- The Leading Order (LO) :

$$F_T\left(x_{Bj}, Q^2\right) = \sum_f e_f^2 \; x_{Bj} \left(q_f\left(x_{Bj}\right) + \overline{q}_f\left(x_{Bj}\right) \right) + O(\alpha_s)$$

$$F_L\left(x_{Bj}, Q^2\right) = 0 + O(\alpha_s)$$

: Bjorken scaling & parton model

[J. D.Bjorken, PhysRev.179.1547(1969)]
[J. D.Bjorken, E. A.Paschos, Phys.Rev.185.1975(1969)]
[R. D.Feynman, Phys.Rev.Lett.23.1415 (1969)]

- Beyond LO accuracy :

$$xq^S(x, \{Q^2, \mu^2, \epsilon\})\Big|_{\text{LO+NLO}} \simeq xq^S(x)\Big|_{\text{LO}} + \frac{\alpha_s}{2\pi}\left[-\frac{1}{\epsilon} + \log\left(\frac{Q^2}{\mu^2}\right) \right] \int_x^1 dz \left\{ P_{qq}(z) \, \frac{x}{z} q^S\left(\frac{x}{z}\right)\Big|_{\text{LO}} + P_{qg}(z) \, \frac{x}{z} g\left(\frac{x}{z}\right)\Big|_{\text{LO}} \right\}$$

- Low "x" behavior :

$$xq^S(x)\Big|_{\text{LO}} = C_q \, x + O(x^2)$$

Quark distribution is the LO but it is the subleading contribution in energy.

$$xg\left(x\right)\Big|_{\text{LO}} = C_g + C'_g \, x + O(x^2)$$

Gluon distribution are α_s supressed, but it is the leading contribution in energy.

Parton Distribution Functions (PDFs)

- **Standard PDFs :** Partonic fields have a longitudinal support on $]-\infty^+, \infty^+[$.

Quark distribution : $$q(x) = \int_{-\infty^+}^{\infty^+} \frac{db^+}{2\pi} e^{-ib^+ xp_t^-} \langle p_t | \overline{\psi}\left(b^+, \mathbf{0}_\perp\right) \frac{\gamma^-}{2} \mathcal{U}_F\left(b^+, 0^+, \mathbf{0}_\perp\right) \psi\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle$$

Gluon distribution : $$g(x) = \frac{1}{xp_t^-} \int_{-\infty^+}^{\infty^+} \frac{db^+}{2\pi} e^{-ib^+ xp_t^-} \langle p_t | F_j^{a\,-}\left(b^+, \mathbf{0}_\perp\right) \mathcal{U}_A\left(b^+, 0^+, \mathbf{0}_\perp\right)_{ab} F_j^{b\,-}\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle$$

- Background PDFs and their fluctionations :

$$\psi(b^+, \mathbf{0}_\perp) \to \Psi\left(b^+, \mathbf{0}_\perp\right) + \delta\psi\left(b^+, \mathbf{0}_\perp\right) \qquad A^\mu\left(b^+, \mathbf{0}_\perp\right) \to \mathcal{A}^\mu\left(b^+, \mathbf{0}_\perp\right) + \delta A^\mu\left(b^+, \mathbf{0}_\perp\right)$$

$$q(x) = \mathbb{Q}(x) + \delta q(x) \qquad g(x) = \mathbb{G}(x) + \delta g(x)$$

[A. V.Belisky, X.Ji, F.Yuan, Nucl.Phys.B 656 (2003)165198]
[S.Bhattacharya, C.He, Z.Kang, D.Padilla, J.Penttala, JHEP04(2026)079]
[G.Chirilli, 2603.30000[hep-ph]]
[JF work in progress]

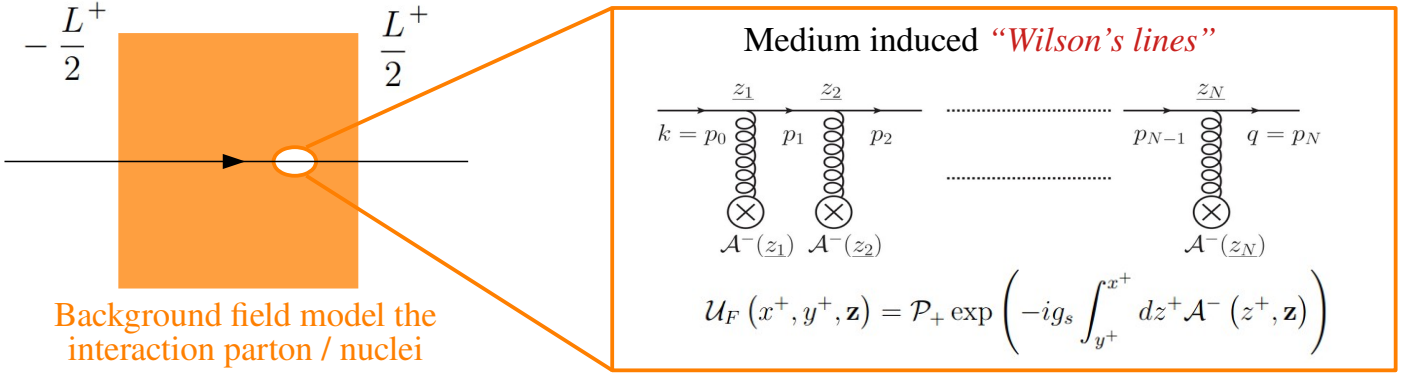
- **Background PDFs :** Background fields have drastic decay beyond the target width $\left[-\frac{L^+}{2}, \frac{L^+}{2}\right]$.

Quark distribution : $$\mathbb{Q}(x) = \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} \frac{db^+}{2\pi} e^{-ib^+ xp_t^-} \langle p_t | \overline{\Psi}\left(b^+, \mathbf{0}_\perp\right) \frac{\gamma^-}{2} \mathcal{U}_F\left(b^+, 0^+, \mathbf{0}_\perp\right) \Psi\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle \qquad \overline{\mathbb{Q}}(x) = -\mathbb{Q}(-x)$$

Gluon distribution : $$\mathbb{G}(x) = \frac{1}{xp_t^-} \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} \frac{db^+}{2\pi} e^{-ib^+ xp_t^-} \langle p_t | \mathcal{F}_j^{a\,-}\left(b^+, \mathbf{0}_\perp\right) \mathcal{U}_A\left(b^+, 0^+, \mathbf{0}_\perp\right)_{ab} \mathcal{F}_j^{b\,-}\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle$$

Partonic Background fields

We apply a large Lorentz boosted along the photon propagation to resolve the target stucture,



- Eikonal & NEik quark propagators

[T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska 10.1103/PhysRevD.104.014019 (2021)]

- Eikonal & NEik gluon propagators

[T. Altinoluk, G. Beuf, and S. Mulani, Phys.Rev.D111(2025) 3, 034028]

[G.Chirilli, JHEP01(2019)118]

- Boosted gluon background : $\mathcal{A}^\mu\left(x^+, x^-, \mathbf{x}\right) \approx \mathcal{A}^-\left(x^+, \mathbf{x}\right) \propto O(\gamma_t)$; $\mathcal{A}^\perp\left(x^+, \mathbf{x}\right) \propto O(\gamma_t^0)$; $\mathcal{A}^+\left(x^+, \mathbf{x}\right) \propto O(1/\gamma_t)$

  Leading contribution — Sub-leading contribution — Subsub-leading contribution (even discard in LC-gauge)

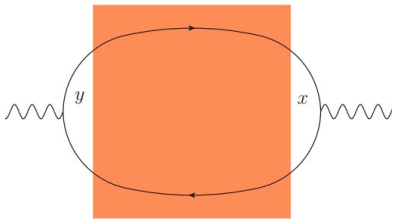
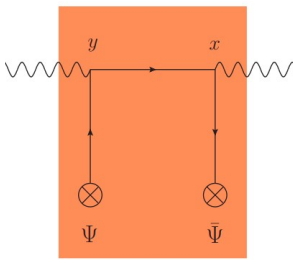
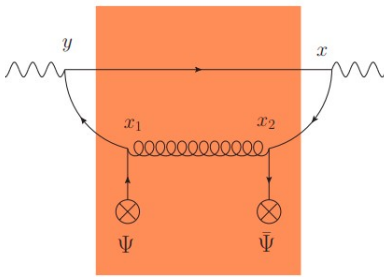
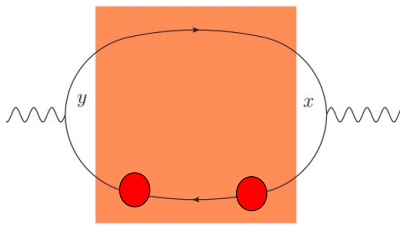
- Boosted quark background : $\Psi(x^+, x^-, \mathbf{x}_\perp) \approx \frac{\gamma^+\gamma^-}{2}\Psi\left(x^+, \mathbf{x}\right) \propto O(\sqrt{\gamma_t})$; $\frac{\gamma^-\gamma^+}{2}\Psi(x^+, \mathbf{x}_\perp) \propto O(1/\sqrt{\gamma_t})$

$\frac{\gamma^+\gamma^-}{2} + \frac{\gamma^-\gamma^+}{2} = 1_4$

Sub-leading contribution (the so called "good" target's spinor)

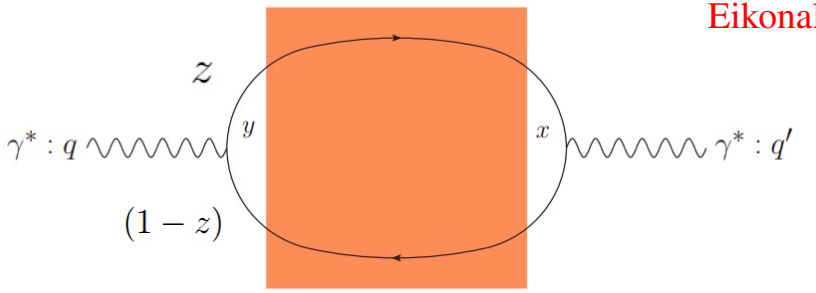
Subsub-leading contribution (the so called "bad" target's spinor)

Power Counting

	Leading Order : $O(\alpha_s^0)$ dilute regime ; $O(1/\alpha_s)$ dense regime.		Next to Leading Order : $O(\alpha_s)$ dilute regime ; $O(\alpha_s^0)$ dense regime.	
	Quark background	Gluon background	Quark background	Gluon background
Eikonal	**NA**	**NA**	**NA**	
Next to Eikonal		**NA**		

(+ the anti-quark contributions) ● : Field strength $\mathcal{F}_j^-$ insertion

Dipole picture of DIS in the Eikonal approximation



Eikonal = Leading power in x_{Bj} due to a pure gluon background interaction.

- Gluon background operator, the Dipole :

$$d_F(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{N_c} \left\langle \mathrm{Tr}_c \left[\mathcal{U}_F(\mathbf{x}_1) \mathcal{U}_F^\dagger(\mathbf{x}_2) \right] \right\rangle_A$$

- Eikonal DIS SF :

$$F_{T/L}(x_{Bj}, Q^2) = 4N_c \sum_f e_f^2 \int_0^1 dz \int \frac{d^4\mathbf{x}}{(2\pi)^4} H_{T/L}\left(z, |\mathbf{x}_1 - \mathbf{x}_2|, Q^2\right) \left(1 - d_F(\mathbf{x}_1, \mathbf{x}_2)\right)$$

$$H_T\left(z, |\mathbf{x}_1 - \mathbf{x}_2|, Q^2\right) = z(1-z)\left(z^2 + (1-z)^2\right) Q^4 K_1\left(\sqrt{z(1-z)Q^2}|\mathbf{x}_1 - \mathbf{x}_2|\right)^2$$

$$H_L\left(z, |\mathbf{x}_1 - \mathbf{x}_2|, Q^2\right) = 4z^2(1-z)^2 Q^4 K_0\left(\sqrt{z(1-z)Q^2}|\mathbf{x}_1 - \mathbf{x}_2|\right)^2$$

- Formaly obtained at $x_{Bj} = 0$;
- $F_L \neq 0$ NLO SF with respect to the naive parton model ;
- Beyond LO rapidity divergences are resummed with BK-JIMWLK equations.

[J. Jalilian-Marian, A. Kovner, H. Weigert, Phys.Rev.D 59(1998)014015]
[Y. V. Kovchegov, Phys. Rev. D60 (1999)]

- Equivalence between forward target's Fock states & CGC average :

[F.Dominguez, C.Marquet, B.Xiao, F.Yuan, Phys.Rev.D 83 (2011) 105005]

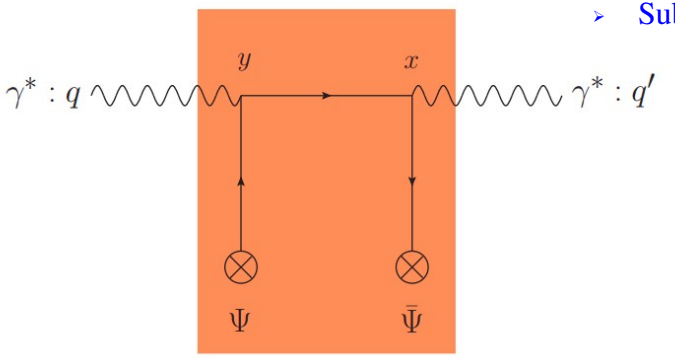
$$\langle O \rangle_A = \lim_{p_f \to p_i} : \frac{\langle p_f | \hat{O} | p_i \rangle}{\langle p_f | p_i \rangle}$$

- Correspondence between CGC operator and parton distribution

[T.Altinoluk, G.Beuf, A.Czajka, C.Marquet, Phys.Rev.D111(2025)1, 014010]
[T.Altinoluk, R.Boussarie, P.Kotko, JHEP 05 (2019)156]

Leading Order NEik DIS cross section : Quark background

- eg. Quark background channel,



- Subeikonal quark operator : $\int dx^+ dy^+ \, \overline{\Psi}(x^+, \mathbf{x}_\perp) \frac{\gamma^-}{2} \Psi(y^+, \mathbf{y}_\perp) \propto O(\gamma_t^{-1}) \quad \left(\text{Eik Op. } \mathcal{D}_F(\mathbf{x}, \mathbf{y}) \propto O(\gamma_t^0) \right)$

$$\sigma^{\mathrm{T}}_{\mathrm{LO},\Psi} = \pi e_f^2 \frac{x_{Bj}}{Q^2} \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} \frac{db^+}{2\pi} \, \langle p_t | \overline{\Psi}\left(b^+, \mathbf{0}_\perp\right) \frac{\gamma^-}{2} \mathcal{U}_F\left(b^+, 0^+, \mathbf{0}_\perp\right) \Psi\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle$$

$$x_{Bj} \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} \frac{db^+}{2\pi} \langle p_t | \overline{\Psi}\left(b^+, \mathbf{0}_\perp\right) \frac{\gamma^-}{2} \mathcal{U}_F\left(b^+, 0^+, \mathbf{0}_\perp\right) \Psi\left(0^+, \mathbf{0}_\perp\right) | p_t \rangle = x_{Bj} \mathbb{Q}(x_{Bj}) \; + \; \mathrm{NNEik}$$

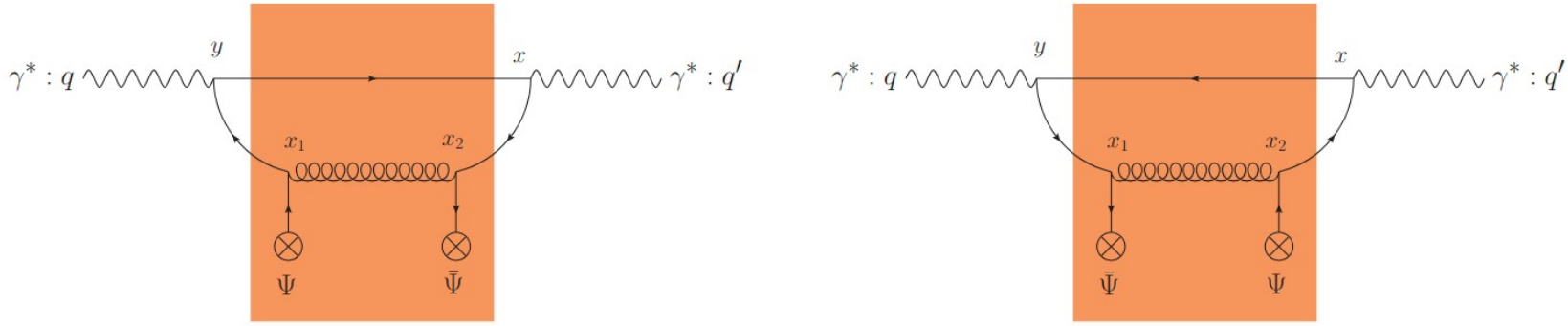
With the sum of the quark and anti-quark background channels we recover *"Quark background contribution to the parton model"*:
[T. Altinoluk, G. Beuf, and S. Mulani, Phys.Rev.D 113 (2026)]

- LO DIS SF :

$$F_T\left(x_{Bj}, Q^2\right)\Big|_{\mathrm{LO},\Psi+\overline{\Psi}} = \sum_f e_f^2 \, x_{Bj} \left(\mathbb{Q}_f(x_{Bj}) + \overline{\mathbb{Q}}_f(x_{Bj})\right) \; + \; \mathrm{NNEik}$$

$$F_L\left(x_{Bj}, Q^2\right)\Big|_{\mathrm{LO},\Psi+\overline{\Psi}} = 0 \; + \; \mathrm{NNEik}$$

Next to Leading Order NEik DIS cross section : Quark background



- NLO DIS *un-substracted* cross section : It has rapidity & UV divergences

$$\sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}} = \frac{e_f^2 g^2 g_s^2}{2}\frac{x_{Bj}}{Q^2}\int_0^1 \frac{dz}{z}\int \frac{d^{2-2\epsilon}\mathbf{r}}{(2\pi)^{2-2\epsilon}}\frac{1}{(\mathbf{r}^2)^{-\epsilon}}\Big[\big[z^2+(1-z)^2\big]-\epsilon\Big]\big(\bar{Q}^2\big)^{1-\epsilon}K^2_{1-\epsilon}\big(\bar{Q}|\mathbf{r}|\big)$$
$$\times\, C_F\,\langle p_t|\hat{\mathbb{Q}}^S\left(x_{Bj}\right)\Big(1-\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})\Big)|p_t\rangle \;+\; O\left(N_c^{-1}\right) \;+\; \mathrm{NNEik}$$

- Quark background operator : $\mathbb{Q}^S(x) = \mathbb{Q}(x) + \overline{\mathbb{Q}}(x)$

We used the notation $O = \langle p_t|\hat{O}|p_t\rangle$.

- Gluon background operator : $\mathcal{P}_F(\mathbf{r},\mathbf{0}) = \frac{1}{2}\Big[2 - d_F(\mathbf{r},\mathbf{0}) - d_F^\dagger(\mathbf{r},\mathbf{0})\Big]$ $\qquad \mathcal{P}_F\left(\mathbf{0},\mathbf{0}\right) = 0\,.$

Extraction of the rapidity and UV divergences

- Identification of rapidity and UV div. :

$$\sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}} = \left(\sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}} - \sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}}\Big|_{z\to 0}\right) + \sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}}\Big|_{z\to 0}$$

Pure UV div. + finite Rapidity & UV div.

Rapidity divergence can be regulated like,
[T.Altinoluk, G.Beuf. J.Penttala, JHEP 04 (2026)125]

$$\int_0^1 \frac{dz}{z} \to \int_0^1 \frac{dz}{z}\left[\frac{zq^+}{\nu^+}\right]^\eta$$

$$q^+ = \frac{Q^2}{2x_{Bj}p_t^-}$$

- Pure UV div. + finite :

$$\left(\sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}} - \sigma^{\mathrm{T}}_{\mathrm{NLO},\Psi+\overline{\Psi}}\Big|_{z\to 0}\right) = \frac{e_f^2 g^2 g_s^2}{2} C_F \frac{x_{Bj}}{Q^2}\int_0^1 \frac{dz}{z}\int \frac{d^{2-2\epsilon}\mathbf{r}}{(2\pi)^{2-2\epsilon}}\frac{1}{(\mathbf{r}^2)^{-\epsilon}}$$
$$\times\left\{(1-\epsilon)\left[(\bar{Q}^2)^{1-\epsilon}K^2_{1-\epsilon}(\bar{Q}|\mathbf{r}|) - \frac{2^{-2\epsilon}\Gamma^2(1-\epsilon)}{(\mathbf{r}^2)^{1-\epsilon}}\right]\langle p_t|\hat{\mathbb{Q}}^S(x_{Bj})\left(1-\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})\right)|p_t\rangle\right.$$
$$\left. - 2z(1-z)(\bar{Q}^2)^{1-\epsilon}K^2_{1-\epsilon}(\bar{Q}|\mathbf{r}|)\langle p_t|\hat{\mathbb{Q}}^S(x_{Bj})\left(1-\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})\right)|p_t\rangle\right\} + O\left(N_c^{-1}\right) + \mathrm{NNEik}$$

UV div. the rest is finite.

Substraction of the rapidity and UV divergences

- The NLO quark background contribution to the SF is decomposed in a finite piece and divergent piece,

$$F_T\left(\{x_{Bj},\nu^+,\eta\};\{Q^2,\mu_R^2,\epsilon\}\right)\Big|_{\text{NLO},\Psi+\overline{\Psi}} = F_T\left(x_{Bj},Q^2\right)\Big|^{\text{finite}}_{\text{NLO},\Psi+\overline{\Psi}} + F_T\left(\{x_{Bj},\nu^+,\eta\};\{Q^2,\mu_R^2,\epsilon\}\right)\Big|^{\text{div.}}_{\text{NLO},\Psi+\overline{\Psi}}$$

Depends only on DIS variables

Depends on renormalization scales and UV & rapidity regulator

- *UV scale :* $\mu^2 = \dfrac{\mu_R^2\, e^{\gamma_e}}{4\pi}$

- The divergent contributions are splited into : Pure rapidity, mixed UV and rapidity and pure UV divergences. $+\ O(N_c^{-1}) + \text{NNEik}$

- Pure rapidity :

$$F_T\left(\{x_{Bj},\nu^+,\eta\};Q^2\right)\Big|^{\text{rap. div.}}_{\text{NLO},\Psi+\overline{\Psi}} = -\frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\eta}+\log\left(\frac{1}{x_{Bj}}\right)+\log\left(\frac{Q^2}{2\nu^+ p_t^-}\right)+O(\eta)\right]$$
$$\times\ \langle p_t|\,\hat{F}_T\left(x_{Bj},Q^2\right)\Big|_{\text{LO},\Psi+\overline{\Psi}}\int\frac{d^2\mathbf{r}}{\pi\,\mathbf{r}^2}\left(\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})-\theta\left(\mathbf{r}^2-b_0^2e^{-2\gamma_e}\right)\right)|p_t\rangle$$

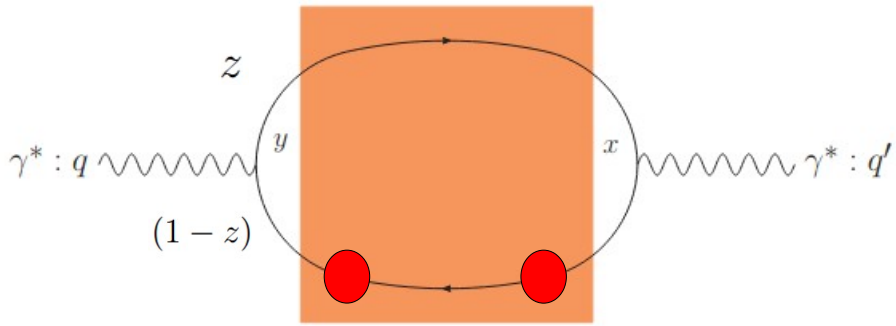
- UV & rapidity :

$$F_T\left(\{x_{Bj},\nu^+,\eta\};\{Q^2,\mu_R^2,\epsilon\}\right)\Big|^{\text{UV \& rap. div.}}_{\text{NLO},\Psi+\overline{\Psi}} = \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\epsilon}+\log\left(\frac{\mu_R^2 b_0^2}{4}\right)-1+O(\epsilon)\right]\left[\frac{1}{\eta}+\log\left(\frac{1}{x_{Bj}}\right)+\log\left(\frac{Q^2}{2\nu^+p_t^-}\right)+O(\eta)\right]F_T\left(x_{Bj},Q^2\right)\Big|_{\text{LO},\Psi+\overline{\Psi}}$$

- Pure UV :

$$F_T\left(x_{Bj};\{Q^2,\mu_R^2,\epsilon\}\right)\Big|^{\text{UV div.}}_{\text{NLO},\Psi+\overline{\Psi}} = -\frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\epsilon}+\log\left(\frac{1}{x_{Bj}}\right)+\log\left(\frac{\mu_R^2}{2q^+p_t^-}\right)+1+O(\epsilon)\right]F_T\left(x_{Bj},Q^2\right)\Big|_{\text{LO},\Psi+\overline{\Psi}}$$

Next to Eikonal contribution from the gluon background



- Subeikonal correction that contributes to the observable :

The second "decoration"

$$\mathcal{U}_F^{(2)}(\mathbf{z}) = (ig_s)^2 \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} dv^+ \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} dw^+ \, \theta\left(v^+ - w^+\right) \left(v^+ - w^+\right) \mathcal{U}_F\left(\frac{L}{2}^+, v^+, \mathbf{z}\right)$$
$$\times \, \mathcal{F}_j^-\left(v^+, \mathbf{z}\right) \mathcal{U}_F\left(v^+, w^+, \mathbf{z}\right) \mathcal{F}_j^-\left(w^+, \mathbf{z}\right) \mathcal{U}_F\left(w^+, -\frac{L}{2}^+, \mathbf{z}\right)$$

- Gluon background operator :

$$G\left(x_{Bj}, \mathbf{r}\right) = 2\mathrm{Re} : (-i) \int \frac{db^+}{\pi} \theta(b^+) b^+ e^{-ixp_t^- b^+} \langle p_t | \mathrm{Tr_c} \left[\mathcal{U}_F^\dagger(\mathbf{r}) \mathcal{U}_F(\infty^+, b^+, \mathbf{0}) \, \mathcal{F}_j^-\left(b^+, \mathbf{0}\right) \mathcal{U}_F(b^+, 0^+, \mathbf{0}) \mathcal{F}_j^-\left(0^+, \mathbf{0}\right) \mathcal{U}_F(0^+, -\infty^+, \mathbf{0}) \right] | p_t \rangle$$

- Same divergence structure as the NLO quark background contribution :

$$C_F \, \mathbb{Q}(x_{Bj}) \to \frac{2}{D-2} T_R \frac{\partial}{\partial x} x\mathbb{G}(x)\Big|_{x=x_{Bj}} \quad ; \quad C_F \langle p_t | \hat{\mathbb{Q}}^S\left(x_{Bj}\right) \left(1 - \hat{\mathcal{P}}_F(\mathbf{r}, \mathbf{0})\right) | p_t \rangle + O(N_c^{-1}) \to G\left(x_{Bj}, \mathbf{r}\right)$$

Structure Functions at NLO : Quark & gluon backgrounds

- NLO DIS SF quark & gluon backgrounds contributions :

$$F_T\left(x_{Bj},Q^2\right)\Big|^{\text{finite}}_{\text{NLO},\Psi+\overline{\Psi}} = \frac{\alpha_s C_F}{2\pi}\sum_f e_f^2\, x_{Bj}\int_0^1\frac{dz}{z}\int\frac{d^2\mathbf{r}}{\pi}\left\{\left[\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)-\frac{1}{\mathbf{r}^2}\right]\langle p_t|\hat{\mathbb{Q}}_f^S\left(x_{Bj}\right)\left(1-\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})\right)|p_t\rangle\right.$$
$$\left. + 2z(1-z)\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)\langle p_t|\hat{\mathbb{Q}}_f^S\left(x_{Bj}\right)\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})|p_t\rangle\right\} \;+\; O(N_c^{-1}) + \text{NNEik}$$

$$F_T\left(x_{Bj},Q^2\right)\Big|^{\text{finite}}_{\text{NLO},G} = \frac{\alpha_s}{2\pi}\sum_f e_f^2\, x_{Bj}\int_0^1\frac{dz}{z}\int\frac{d^2\mathbf{r}}{\pi}\left\{\left[\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)-\frac{1}{\mathbf{r}^2}\right]G\left(x_{Bj},\mathbf{r}\right)\right.$$
$$\left. -2z(1-z)\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)\left[G\left(x_{Bj},\mathbf{r}\right)-G\left(x_{Bj},\mathbf{0}\right)\right]\right\} \;+\text{NNEik}$$

$$F_L\left(x_{Bj},Q^2\right)\Big|_{\text{NLO},\Psi+\overline{\Psi}} = \frac{\alpha_s C_F}{2\pi}\sum_f e_f^2\, x_{Bj}\int_0^1\frac{dz}{z}\int\frac{d^2\mathbf{r}}{\pi}$$
$$\times\, 4z^2(1-z)^2Q^2K_0^2(\bar{Q}|\mathbf{r}|)\langle p_t|\hat{\mathbb{Q}}_f^S\left(x_{Bj}\right)\left(1-\hat{\mathcal{P}}_F(\mathbf{r},\mathbf{0})\right)|p_t\rangle \;+\; O(N_c^{-1}) + \text{NNEik}$$

$$F_L\left(x_{Bj},Q^2\right)\Big|_{\text{NLO},G} = \frac{\alpha_s}{2\pi}\sum_f e_f^2\, x_{Bj}\int_0^1\frac{dz}{z}\int\frac{d^2\mathbf{r}}{\pi}\,4z^2(1-z)^2Q^2K_0^2(\bar{Q}|\mathbf{r}|)G\left(x_{Bj},\mathbf{r}\right) \;+\text{NNEik}$$

Conclusion & perspectives

[T.Altinoluk, G.Beuf, JF, M.Fucilla, 2512.16788 [hep-ph](2025)]

- We have computed the Quark and Gluon background NEik NLO correction to DIS Structure Functions.
- These corrections are written in term of finite corrections and exibit rapidity and UV divergences.
- Divegences indicate that at NLO DIS SFs develop single and double logarithmic scale depences :

$$\log\left(\frac{\mu_R^2}{2q^+p_t^-}\right) \quad ; \quad \log\left(\frac{Q^2}{2\nu^+p_t^-}\right) \quad ; \quad \log\left(\frac{\mu_R^2 b_0^2}{4}\right)\log\left(\frac{Q^2}{2\nu^+p_t^-}\right)$$

UV Rapidity UV & Rapidity

- The work in progress with T. Altinoluk, G. Beuf and M. Fucilla focus on :
- Find the evolutions that regularized these divergences. Starting from the operator definition of the Background quark PDFs we would be able to understand the low "x" region the evolution of transverse DIS SF in the (x_{Bj}, Q^2) -plan.
- Then we want to adress this formalism to quark TMD in hadron production for Semi-Inclusive-DIS observable.

When there is no more, there is again

- Full Results : beyond the large N_c and massless limits ;
- The shockwave approach to PDF ;
- Parton distribution beyond Eikonal accuracy acquire rap. div. : eg. Dijet ;
- Parton propagators beyond the Eikonal accuracy.

Full Results : Beyond the large number of color and the massless limits

- e.g, Quark background operator :

$$f_q(x_{Bj},\mathbf{r}) = \int \frac{dz^+}{2\pi}\,\theta(z^+)\,e^{-ix_{Bj}p_t^- z^+}\langle p_t|\Big[\overline{\Psi}(z^+,\mathbf{0})\,\frac{\gamma^-}{2}\,\mathcal{U}_F\left(z^+,0^+,\mathbf{0}\right)\Psi(0^+,\mathbf{0})\mathrm{Tr_c}\left[\mathcal{U}_F(\mathbf{r})\mathcal{U}_F^\dagger(\mathbf{0})\right]$$

$$-\frac{1}{N_c}\overline{\Psi}(z^+,\mathbf{0})\,\frac{\gamma^-}{2}\,\mathcal{U}_F\left(z^+,\infty^+,\mathbf{0}\right)\mathcal{U}_F(\mathbf{r})\,\mathcal{U}_F\left(-\infty^+,0^+,\mathbf{0}\right)\Psi(0^+,\mathbf{0})\Big]|p_t\rangle$$

$$\mathrm{Re}: f_q(x_{Bj},\mathbf{0}) = C_F\,\mathbb{Q}(x)$$

- e.g Divergences for the quark background channel :

$$\sigma^{\mathrm{T},m}_{\mathrm{NLO},\Psi}\Big|_{\text{rap. div.}} = e_f^2 g^2\pi\frac{x_{Bj}}{Q^2}\frac{\alpha_s}{\pi}\left(\frac{m^2}{\mu^2}\right)^{-\epsilon}(1-\epsilon)\,m^2\,\mathrm{Re}:\int_0^1\frac{dz}{z}\int\frac{d^{2-2\epsilon}\mathbf{r}}{(2\pi)^{1-2\epsilon}}\frac{1}{(\mathbf{r}^2)^{-\epsilon}}\left\{K^2_{1-\epsilon}(m|\mathbf{r}|)\left[f_q(x_{Bj},\mathbf{r})-f_q(x_{Bj},\mathbf{0})\right]+K^2_\epsilon(m|\mathbf{r}|)\,f_q(x_{Bj},\mathbf{r})\right\}$$

$$\sigma^{\mathrm{T},m}_{\mathrm{NLO},\Psi}\Big|_{\text{rap. \& UV div.}} = e_f^2\,g^2\,\pi\,\frac{x_{Bj}}{Q^2}\frac{\alpha_s}{2\pi}\,C_F\,q_f(x_{Bj})\frac{(1-\epsilon)\Gamma(1+\epsilon)}{\epsilon}\left(\frac{m^2}{4\pi\mu^2}\right)^{-\epsilon}\int_0^1\frac{dz}{z}$$

$$\sigma^{\mathrm{T},m}_{\mathrm{NLO},\Psi}\Big|_{\text{UV div.}} = -e_f^2\,g^2\,\pi\,\frac{x_{Bj}}{Q^2}\,\frac{\alpha_s}{2\pi}\,C_F\,q_f(x_{Bj})\,\frac{(1-\epsilon)\Gamma(1+\epsilon)}{\epsilon}\left(\frac{m^2}{4\pi\mu^2}\right)^{-\epsilon}F_1\left(2,\epsilon,\epsilon,3,\frac{2}{1+\sqrt{1+4m^2/Q^2}},\frac{2}{1-\sqrt{1+4m^2/Q^2}}\right)$$

- e.g Finite cross section in the quark background channel :

$$\sigma^{\mathrm{T},m}_{\mathrm{NLO},\Psi}\Big|_{\text{fin.}} = e_f^2\,g^2\,\pi\frac{x_{Bj}}{Q^2}\frac{\alpha_s}{2\pi}\,\mathrm{Re}:\int_0^1\frac{dz}{z}\int\frac{d^2\mathbf{r}}{\pi}\,\Big\{\Big[\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)-m^2K_1^2(m|\mathbf{r}|)+m^2\Big(K_0^2(\bar{Q}|\mathbf{r}|)-K_0^2(m|\mathbf{r}|)\Big)\Big]f_q(x_{Bj},\mathbf{r})$$

$$-\,2z(1-z)\,\bar{Q}^2K_1^2(\bar{Q}|\mathbf{r}|)\left[f_q(x_{Bj},\mathbf{r})-f_q(x_{Bj},\mathbf{0})\right]\Big\}$$

The shockwave approach to PDF

$$\bar{q}(x) = -q(-x) \implies \bar{q}(0) + q(-0) = \bar{q}(0) + q(0) = 0\ ?$$

Shockwave Background PDF [A.V. Belitsky, X. Ji, F. Yuan, Nucl.Phys. B656 (2003) 165-198]

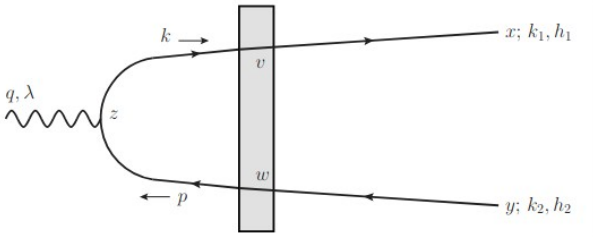
$$\delta q^*(x) \simeq \frac{1}{x}\frac{N_c\,\Gamma(D-2)^2}{\pi^D\Gamma(2(D-2))}\Gamma\left(\frac{D-2}{2}\right)\Gamma\left(\frac{3D-6}{2}\right)\int\frac{d^{D-2}\mathbf{w}d^{D-2}\mathbf{v}}{(\mathbf{w}-\mathbf{v})^{2((D-2)-2\epsilon)}}\left(1-\mathcal{D}_F\left(\mathbf{v},\mathbf{w}\right)\right)$$

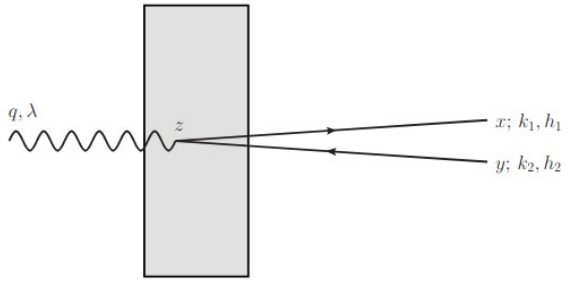
$$\implies 0 < x$$

- Shockwave Background PDF evolution beyond Eikonal : Cancel mixed UV & rapidity div.

$$\begin{aligned} q^S\left(x\,;\{\zeta,\eta\}\,;\{\mu_R,\epsilon\}\right) &= \delta\, q^S\left(x\,;\mu_R\right)\Big|_{\text{Eik}} + \mathbb{Q}^S(x) \\ &+\frac{\alpha_s^{(0)}}{2\pi}\left[\frac{1}{\epsilon}+\log\left(\frac{\mu_R^2 b_0^2}{4}\right)-1\right]\left[\frac{1}{\eta}+\log\frac{1}{\zeta}\right]\left[C_F\mathbb{Q}^{NS}(x)+\frac{\partial}{\partial x}x\mathbb{G}(x)\right] \\ &-\frac{\alpha_s^{(0)}C_F}{2\pi^2}\left[\frac{1}{\eta}+\log\frac{1}{\zeta}\right]\left\{\langle p_t|\int\frac{d^2\mathbf{r}}{\mathbf{r}^2}\hat{\mathbb{Q}}^S(x)\left(\hat{\mathcal{P}}_F\left(\mathbf{r},\mathbf{0}\right)-\Theta\left(\mathbf{r}^2-b_0^2e^{-2\gamma_e}\right)\right)|p_t\rangle\right. \\ &\left.+\langle p_t|\,2\mathrm{Re}:\int\frac{d^2\mathbf{r}}{\mathbf{r}^2}\left(\hat{G}(x\,;\mathbf{r},\mathbf{0})-\Theta\left(b_0^2e^{-2\gamma_e}-\mathbf{r}^2\right)\hat{G}(x\,;\mathbf{0},\mathbf{0})\right)|p_t\rangle\right\} \end{aligned}$$

Parton distribution beyond Eikonal accuracy acquire rap. div. : Dijet





$$\text{for} \quad Q^2 \ll \frac{P_\perp^2}{z(1-z)}$$

$$x_{Bj} \simeq \frac{P_\perp^2}{z(1-z)\, 2q\cdot p_t}$$

- In the back-to-back limit of DIS dijet in a pure gluon background. Integrate over the tranverse momentun exchange with the target,

[T. Altinoluk, G.Beuf, A.Czajka, C.Marquet, Phys.Rev.D111(2025)1,014010]

$$\frac{d^2\sigma_{\gamma_T^*\to q\bar{q}}}{d^2P_\perp}\bigg|_{\text{Eik+NEik}}^{\text{g. Bckdg}} = \frac{\alpha_s}{P_\perp^2}\int_0^1 dz\, P_{\gamma q}(z,e_f^2)\int d^2k_\perp\, x_{Bj}G^{(1)}\left(x_{Bj},k_\perp^2\right) + O\left(x_{Bj}^2\right) + O\left(\frac{k_\perp^2}{P_\perp^2}\right) + O\left(\frac{Q^2}{P_\perp^2}\right)$$

- Do the phase expansion at NEik accuracy,

$$= \frac{\alpha_s}{P_\perp^2}\int_0^1 dz\, P_{\gamma q}(z,e_f^2)\int \frac{2db^+}{(2\pi)p_t^-}\left[1-\frac{1}{z(1-z)}\frac{ib^+P_\perp^2}{2q\cdot p_t}\right]\langle p_t|\mathcal{F}_a^{-j}\left(b^+,\mathbf{0}\right)\mathcal{U}_A\left(b^+,0^+,\mathbf{0}\right)_{ab}\mathcal{F}_b^{-j}\left(0^+,\mathbf{0}\right)|p_t\rangle$$

- Appears the characteristic NEik rapidity divergence occures.

Parton propagators beyond the Eikonal accuracy

- Eikonal Quark propagator :

$$S_F(x,y)|_{\rm Eik} = \int \frac{d_4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i(\not{k}+m)}{[k^2-m^2+i\epsilon]}$$
$$+ \int \frac{d_3\underline{q}}{(2\pi)^3} \int \frac{d_3\underline{k}}{(2\pi)^3} 2\pi\delta(q^+-k^+) e^{-ix\cdot q+iy\cdot k} \frac{(\not{q}+m)\gamma^+(\not{k}+m)}{(2k^+)^2}$$
$$\times \int d_2\mathbf{z} e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Big\{\theta(k^+)\theta(x^+-y^+)[\mathcal{U}_F(y^+,x^+,\mathbf{z})-1] - \theta(-k^+)\theta(y^+-x^+)[\mathcal{U}_F^\dagger(y^+,x^+,\mathbf{z})-1]\Big\}$$

Transverse momentum exchange with the target

- Quark propagator at Next-to-Eikonal, accuracy from *before to after* $x^+ \le -\frac{L^+}{2}$, $\frac{L^+}{2} \le y^+$ the target :

$$S^{b.a.}_{F,q}(x,y) = \int \frac{d^{D-1}\underline{p}}{(2\pi)^{D-1}} \int \frac{d^{D-1}\underline{k}}{(2\pi)^{D-1}} \frac{\theta(p^+)}{2p^+} \frac{\theta(k^+)}{2k^+} e^{-ix\cdot\check{p}} e^{iy\cdot\check{k}} (2\pi)\delta(p^+-k^+)$$
$$\times \int d^{D-2}\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{p}-\mathbf{k})} (\not{\check{p}}+m)\,\gamma^+\,\mathcal{U}^\star_F(\mathbf{z})\,(\not{\check{k}}+m)$$

3 types of Wilson's lines *"decorations"* :

$$\mathcal{U}^\star_F(\mathbf{z}) = \mathcal{U}_F(\mathbf{z}) - \frac{(\mathbf{p}^j+\mathbf{k}^j)}{2(p^++k^+)} \mathcal{U}^{(1)}_{F,j}(\mathbf{z}) - \frac{i}{(p^++k^+)} \mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4(p^++k^+)} \mathcal{U}^{(3)}_{F,ij}(\mathbf{z})$$

Wilson's lines "*decorations*" at NEik accuracy

- *First decoration :*

$$\mathcal{U}^{(1)}_{F,j}\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{v}\right)=2ig_s\int_{-\frac{L}{2}^+}^{\frac{L}{2}^+}dv^+\,v^+\,\mathcal{U}_F\left(\frac{L^+}{2},v^+,\mathbf{v}\right)F_j{}^-\left(v^+,\mathbf{v}\right)\mathcal{U}_F\left(v^+,-\frac{L^+}{2},\mathbf{v}\right)$$

The only decoration that contributes to the *genuine twist-2* gluon TMDs.

- *Second decoration :*

$$\mathcal{U}^{(2)}_{F}\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{v}\right)=(ig_s)^2\int_{-\frac{L}{2}^+}^{\frac{L}{2}^+}dv^+\int_{-\frac{L}{2}^+}^{\frac{L}{2}^+}du^+\theta(v^+-u^+)(v^+-u^+)\,\mathcal{U}_F\left(\frac{L^+}{2},u^+,\mathbf{v}\right)$$
$$\times\,F_j{}^-\left(u^+,\mathbf{v}\right)\mathcal{U}_F\left(u^+,v^+,\mathbf{v}\right)F_j{}^-\left(v^+,\mathbf{v}\right)\mathcal{U}_F\left(v^+,-\frac{L^+}{2},\mathbf{v}\right)$$

The only decoration that contributes to the unpolarized DIS SF in the NEik gluon background.

- *Third decoration :*

$$\mathcal{U}^{(3)}_{F,ij}\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{v}\right)=g_s\int_{-\frac{L}{2}^+}^{\frac{L}{2}^+}dv^+\,\mathcal{U}_F\left(\frac{L^+}{2},v^+,\mathbf{v}\right)F_{ij}\left(v^+,\mathbf{v}\right)\mathcal{U}_F\left(v^+,-\frac{L^+}{2},\mathbf{v}\right)$$

Decoration that contributes polarized scattering.

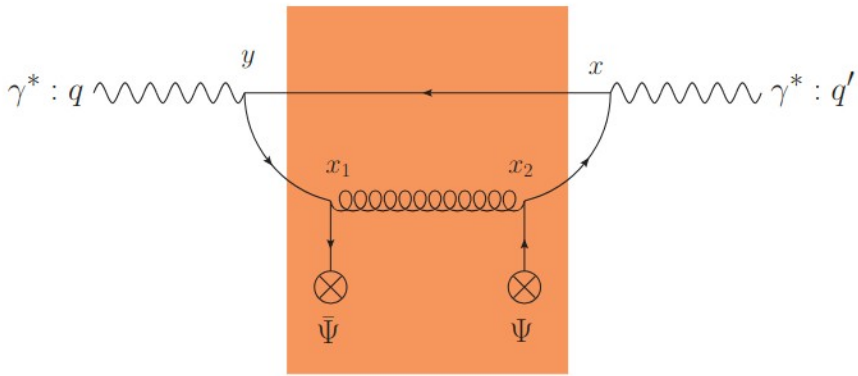
Parton propagators beyond the Eikonal accuracy

- Quark propagator at Next-to-Eikonal accuracy from *before to after* the target :

$$S^{\Psi}_{F,q}(x,y) = -g_s^2 \int_{-\frac{L}{2}^+}^{\frac{L}{2}^+} d^D x_1 \int_{x_1^+}^{\frac{L}{2}^+} d^D x_2 \Big\{ S^{i.a.}_{F,q}(x,x_2)\, t^a \gamma^\sigma \Psi^{(-)}(\underline{x_2}) \left[G_{F,\rho\sigma}(x_2,x_1)^{\text{i.i.}} \right]^{ba} \overline{\Psi}^{(-)}(\underline{x_1})\, t^b\, \gamma^\rho\, S^{b.i.}_{F,q}(x_1,y) \Big\}$$

- Quark propagator at Next-to-Eikonal accuracy from *inside to after* the target :

$$S^{i.a.}_{F,q}(x,y) = \int \frac{d^{D-1}\underline{k}}{(2\pi)^{D-1}} \frac{\theta(k^+)}{2k^+} e^{-ix\cdot\check{k}} (\check{\not{k}} + m)\, \mathcal{U}_F\left(x^+, y^+, \mathbf{y}\right)$$



- Quark propagator at Next-to-Eikonal accuracy from *before to inside* the target :

$$S^{b.i.}_{F,q}(x,y) = \int \frac{d^{D-1}\underline{k}}{(2\pi)^{D-1}} \frac{\theta(k^+)}{2k^+} e^{iy\cdot\check{k}} e^{-ix^- k^+} e^{i\mathbf{x}\cdot\mathbf{k}} \times \left[1 - \frac{\gamma^+\gamma^i}{2k^+} i\overrightarrow{D^F_{\mathbf{x}^i}} \right] (\check{\not{k}} + m)\mathcal{U}_F\left(x^+, y^+, \mathbf{x}\right)$$

- Gluon propagator at Next-to-Eikonal accuracy from *inside to inside* the target :

$$G_F^{\mu\nu}(x,y)^{\text{i.i.}} = \int \frac{dk^+}{2\pi} \frac{1}{2k^+} e^{-ik^+(x^- - y^-)} \Big\{ \frac{2i}{k^+} g^{\mu+} g^{\nu+} \delta\left(x^+ - y^+\right) \delta^{(D-2)}(\mathbf{x}-\mathbf{y}) + \left[g_i^\mu - \frac{ig^{\mu+}}{k^+} \vec{D}^A_{\mathbf{x}^i} \right] \int d^{D-2}\mathbf{z}\, \delta^{(D-2)}(\mathbf{x}-\mathbf{z})\, \delta^{(D-2)}(\mathbf{z}-\mathbf{y}) \times \left[\theta\left(x^+ - y^+\right) \theta\left(k^+\right) \mathcal{U}_A\left(x^+, y^+, \mathbf{z}\right) - \theta\left(y^+ - x^+\right) \theta\left(-k^+\right) \mathcal{U}^\dagger_A\left(y^+, x^+, \mathbf{z}\right) \right] \left[g_i^\nu + \frac{ig^{\nu+}}{k^+} \overleftarrow{D}^A_{\mathbf{y}^i} \right] \Big\}.$$