Complete NLO BFKL impact factors for quarkonium hadroproduction in NRQCD

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in collaboration with J.-P. Lansberg,
M. Nefedov, S. Wallon, L. Szymanowski,

[M. F., J.-P. Lansberg, M. Nefedov,
L. Szymanowski, S. Wallon, in press on JHEP (2026)]

Deep Inelastic Scattering 2026, 4-8 of May, Bologna

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THE ULAM PROGRAMME

Collinear and BFKL factorization

- Collinear factorization in hadron-hadron collisions

$$\sigma = \sum_{ij} \int dx_1 \int dx_2 f_i^{H_1}(x_1, Q^2) f_j^{H_2}(x_1, Q^2) \hat{\sigma}_{ij}(x_1 p_1, x_2 p_2, Q^2)$$

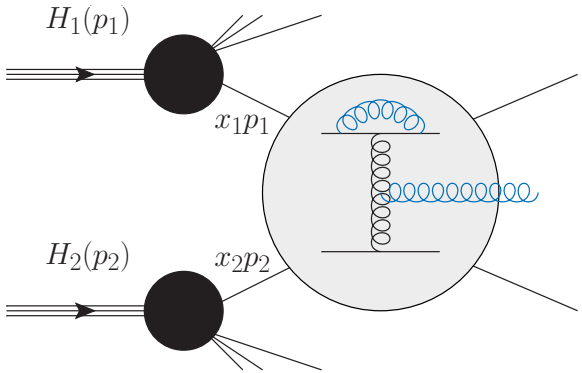
Parton distribution function (PDF)

Renormalization group eq. in Q^2

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi]

Partonic cross section

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_s \hat{\sigma}_{ij}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2)} + \ldots$$



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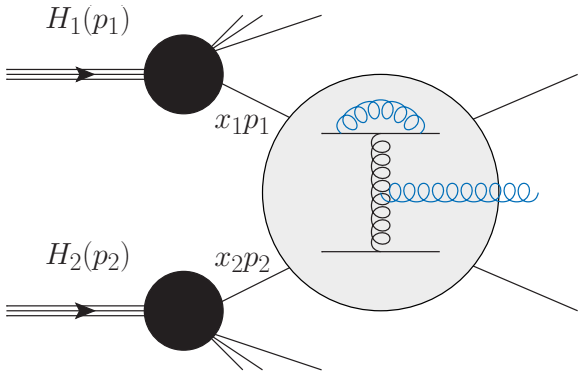


- High-energy limit of QCD: $s \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies \alpha_s \ln(s/Q^2) \sim 1$

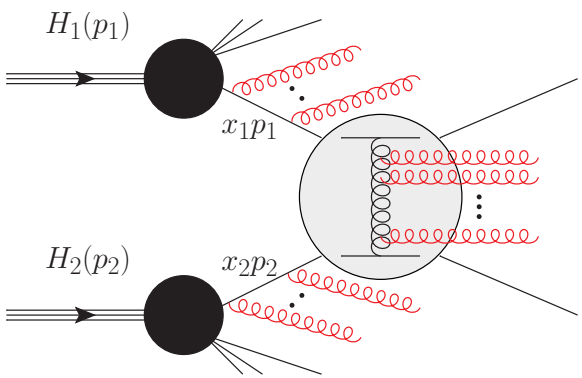
$$\sigma_0 = \sum_n \left[A_n \, \alpha_s^n(Q^2) \ln^n(s/Q^2) + B_n \, \alpha_s^{n+1}(Q^2) \ln^n(s/Q^2) + \ldots \right]$$

Leading logarithmic approximation (LL)

[Balitsky, Fadin, Kuraev, Lipatov]

Next-to-LL (NLL)

(state of the art)



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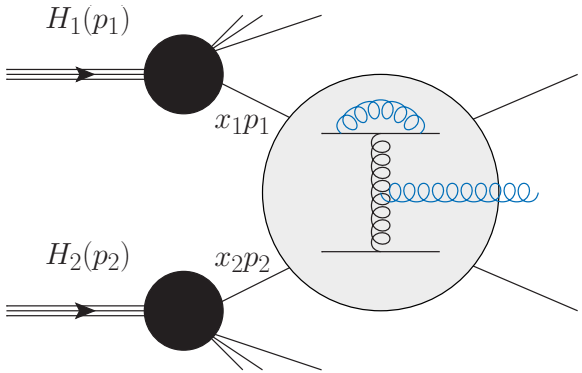

Easily achieved for **heavy flavor** at LHC

- High-energy limit of QCD: $s \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies \alpha_s \ln(s/Q^2) \sim 1$

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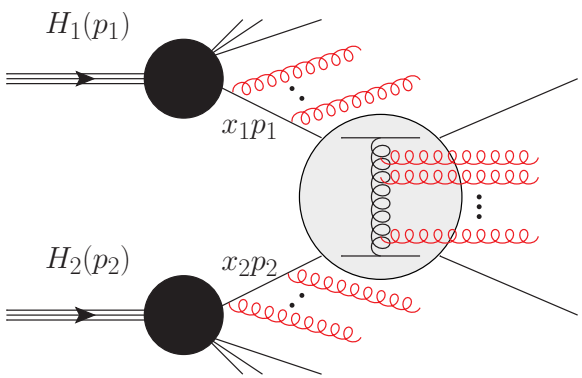


High-energy resummation at colliders

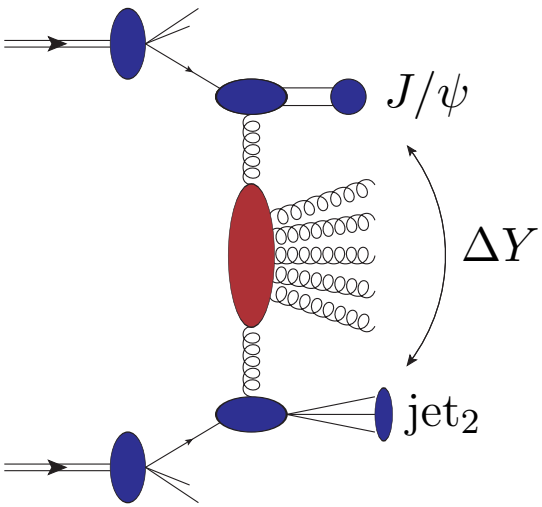
- Proton-proton collision at center-of-mass energy $S = (P_1 + P_2)^2$

$$\ln\left(\frac{S}{Q^2}\right) = \ln\left(\frac{x_1 x_2 S}{Q^2}\right) + \ln\left(\frac{1}{x_2}\right) + \ln\left(\frac{1}{x_1}\right)$$

Not in this talk!

Forward/backward

(e.g. J/Ψ plus jet)



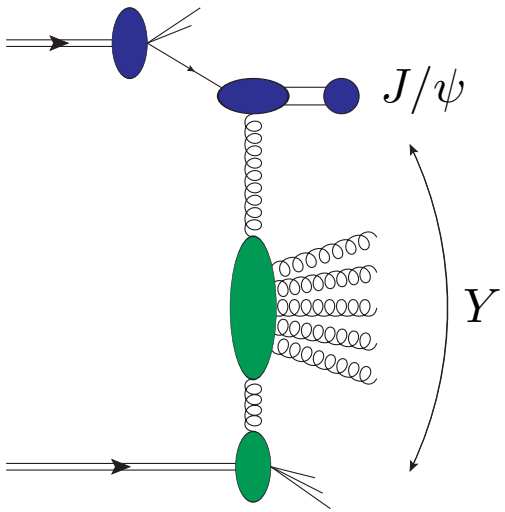
$$\sigma = \Phi_1 \otimes G_{\text{BFKL}} \otimes \Phi_2$$

$$\Phi_i(x_i) \otimes_{x_i} f(x_i)$$

$$\alpha_s \ln\left(\frac{x_1 x_2 S}{|p_{1\perp}||p_{2\perp}|}\right) = \alpha_s \Delta Y \sim 1$$

Single forward

(e.g. forward J/Ψ production)



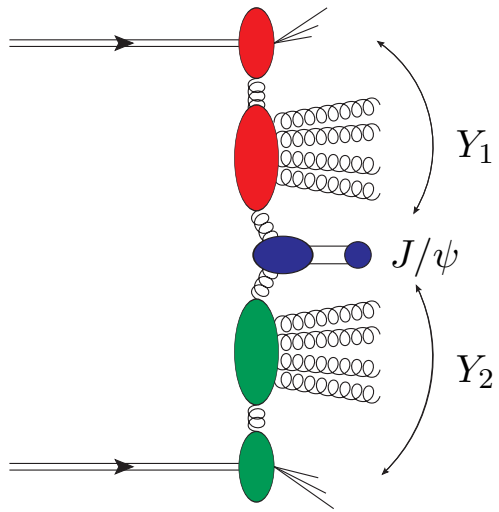
$$\sigma = \Phi \otimes \mathcal{U}_{\text{BFKL}}$$

$$\mathcal{U}_{\text{BFKL}} = \Phi_{\text{prot}} \otimes G_{\text{BFKL}}$$

$$\alpha_s \ln(1/x_2) = \alpha_s Y \sim 1$$

Central

(e.g. central J/Ψ production)



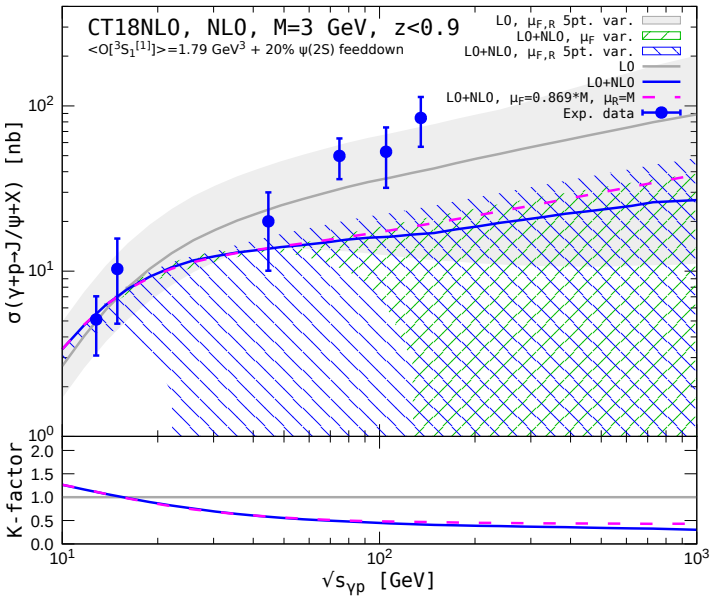
$$\sigma = \mathcal{U}_{\text{BFKL}} \otimes \Xi_{\text{centr.}} \otimes \mathcal{U}_{\text{BFKL}}$$

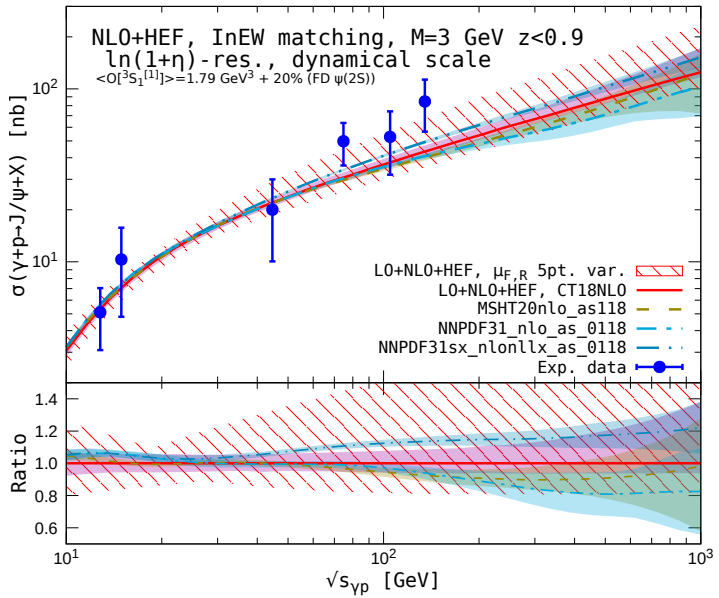
$$\alpha_s \ln(1/x_2) = \alpha_s Y_2 \sim 1$$

$$\alpha_s \ln(1/x_1) = \alpha_s Y_1 \sim 1$$

Curing high-energy instability of quakonium production

- **Instabilities** of the inclusive J/Ψ photoproduction for $\sqrt{s} > 20$ GeV





- Similar pattern observed in the inclusive η_c hadroproduction for $\sqrt{s} > 1$ TeV
- Instabilities are due to a **lack of resummation** of terms $\alpha_s^n \ln^{n-1}(M^2/\hat{s})$
- A leading-logarithmic accurate treatment within High-Energy Factorization improves the stability of the results [Lansberg, Nefedov, Ozelick (2023)]

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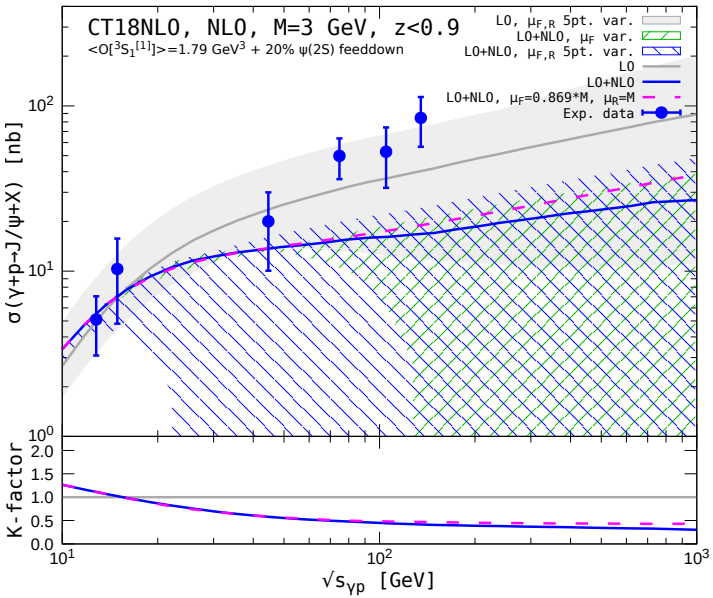

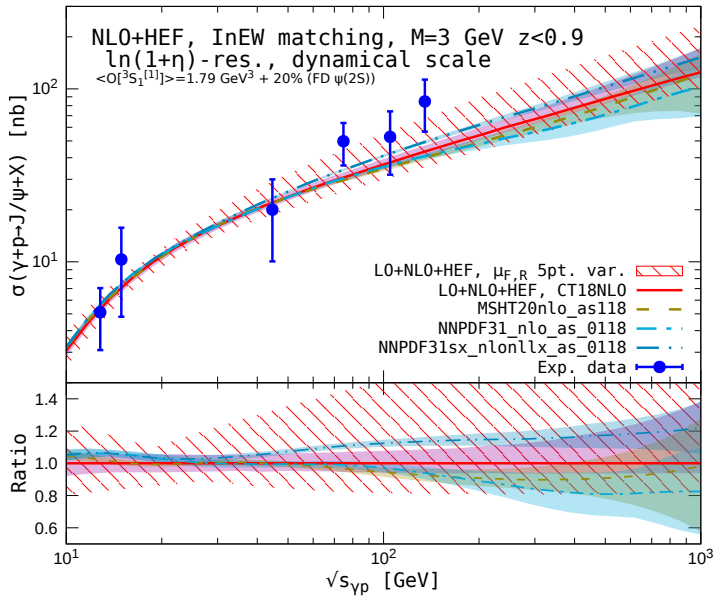

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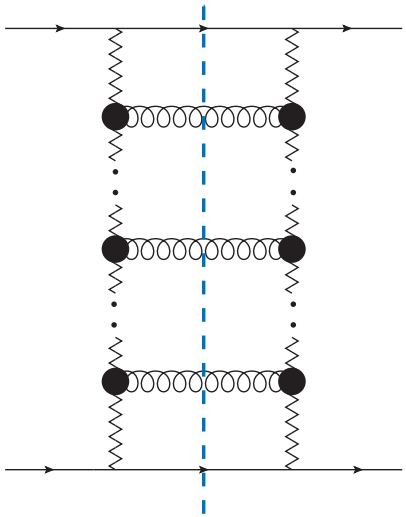
Precision? $\longrightarrow$ **Beyond leading logarithms**

BFKL approach

$$\sigma_{AB\to X} = \frac{1}{2s}\sum_n$$



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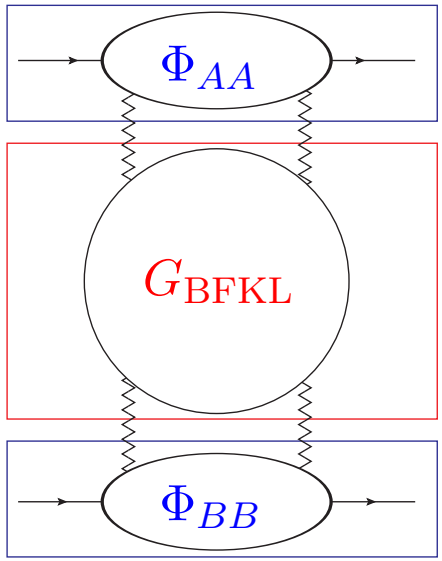


$\longleftarrow$ Impact Factor

Process-dependent, off-shell coefficient function

$\nwarrow$ Green function (universal)

- $G_{\rm BFKL} \longrightarrow$ Iteration of a perturbative kernel ($\mathcal{K}_{\rm BFKL}$) $\longrightarrow$

- $\vec{k}_T$ (transverse momentum) **factorization**

$$\sigma_{AB\to X} = \int \frac{d^2\vec{q}_1}{(2\pi)}\frac{d^2\vec{q}_2}{(2\pi)}\frac{\Phi_{AA}(\vec{q}_1)}{(\vec{q}_1^{\,2})^2}\underbrace{G_{\rm BFKL}(\vec{q}_1,\vec{q}_2;s\,)}\frac{\Phi_{BB}(\vec{q}_2)}{(\vec{q}_2^{\,2})^2} \sim s^{\omega_P}$$

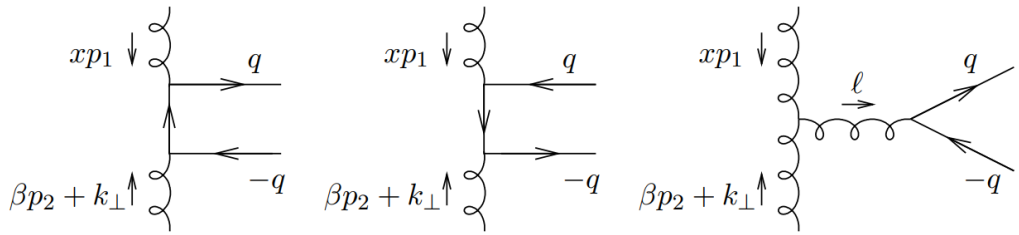
$\swarrow$ Controls the whole energy behavior & satisfies the **BFKL equation**

$\nwarrow$ power-like growth

LO impact factors in NRQCD

- Impact factors for Quarkonium [Boussarie, Ducloué, Szymanowski, Wallon]
- **Color Evaporation Model**

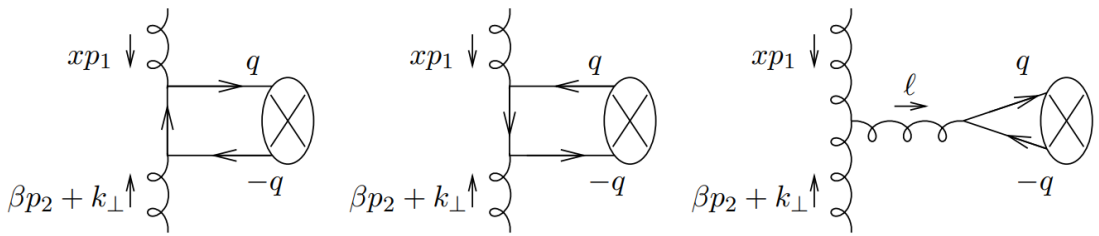
$$\mathcal{V}_{J/\psi} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \, \frac{d\mathcal{V}_{c\bar{c}}}{dM^2}$$



- **NRQCD expansion**

$$\begin{aligned} |J/\psi\rangle \quad &= \quad O(1)\,|c\bar{c}\left[{}^3S_1^{[1]}\right]\rangle + O(v^2)\,|c\bar{c}\left[{}^3P_J^{[8]}\right] + g\rangle \\ &+ \quad O(v^2)\,|c\bar{c}\left[{}^1S_0^{[8]}\right] + g\rangle + O(v^2)\,|c\bar{c}\left[{}^3S_1^{[8]}\right] + gg\rangle + \ldots \end{aligned}$$

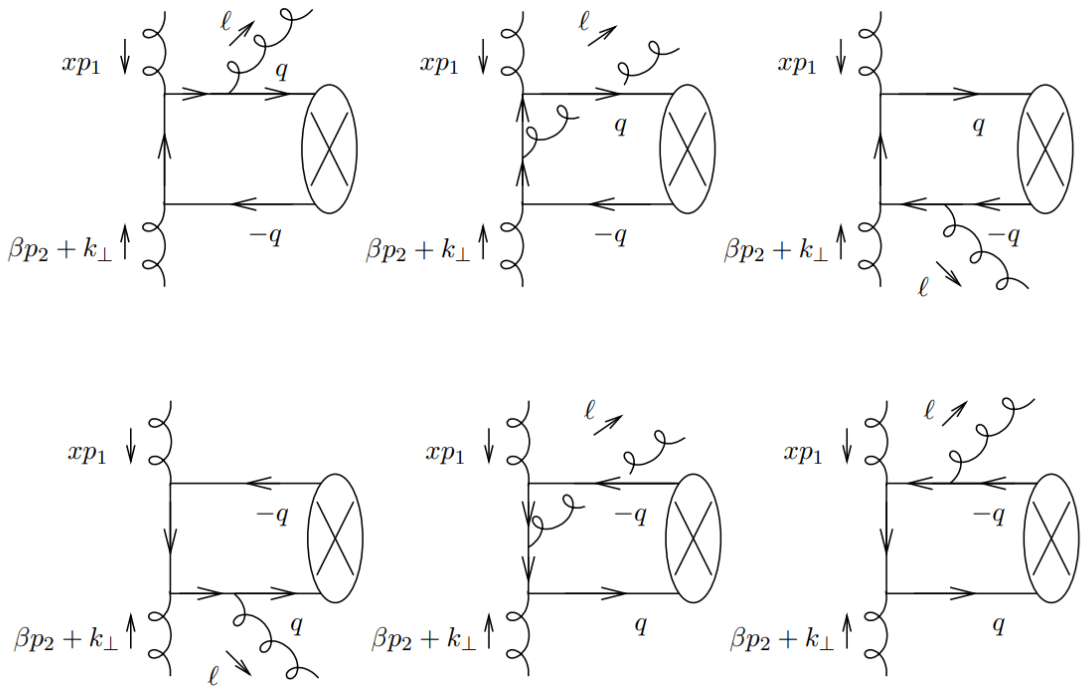
- Impact factor: **Color Octect** case



$$[v\,(q)\,\bar{u}\,(q)]^{ji\to a}_{\alpha\beta} \to t^a_{ji} d_8 \left(\frac{\langle \mathcal{O}_8 \rangle_{J/\psi}}{m}\right)^{\frac{1}{2}} \left[\hat{\varepsilon}^*_{J/\psi}\,(2\hat{q} + 2m)\right]_{\alpha\beta}$$

LO impact factors in NRQCD

- Impact factor in the **Color Singlet** case



$$[v(q)\,\bar{u}(q)]^{ij}_{\alpha\beta} \to \frac{\delta^{ij}}{4N_c}\left(\frac{\langle \mathcal{O}_1\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_{J/\psi}\,(2\hat{q}+2m)\right]_{\alpha\beta}$$

A first encounter with NLO Quarkonium impact factors

- The treatment of heavy-flavor production depends on the kinematic regime

↙ ↘

- High-p_T regime ($p_T^2 \gg m_Q^2$) — ↓ — **Fragmentation** from a light parton — ↓
  - Leading power in m_Q^2/p_T^2
  - Resummation of $\ln(p_T^2/m_Q^2)$

- Low-p_T regime ($p_T^2 \sim m_Q^2$) — ↓ — **Short-distance production** mechanism — ↓
  - All m_Q^2/p_T^2 powers
  - No resummation of $\ln(p_T^2/m_Q^2)$

- So far: NLO only in the fragmentation regime
  [Celiberto, M.F. (2022)][Celiberto, Lonigro (2025)]
- **Main advantage**: still light-quark-to-hadron impact factors
  [Papa, Ivanov (2012)]
- FFs at initial scale from NRQCD computations
  [Braaten, Yuan — Cheung (1993)] [Zheng, Chang, Wu (2019)]
- DGLAP evolution can be performed by using **APFEL++**
  [Bertone (2017)]

BFKL for Quarkonium: high p_T or low p_T?

- Consider double J/Ψ production at large rapidity

Resums $\alpha_s(M_{1,2\perp})\Delta Y = \alpha_s(M_{1,2\perp}) \ln\left(\frac{x_1 x_2 S}{M_{1\perp} M_{2\perp}}\right)$ $\qquad M_{i\perp} = \sqrt{m_Q^2 + p_{T,i}^2}$

Fragmentation $(p_T^2 \gg m_Q^2)$	**Short-distance production** $(p_T^2 \sim m_Q^2)$
• Natural α_s gets **smaller** + • A given ΔY is reached at **larger values of $x_1 x_2$** to compensate the large $M_{1\perp} M_{2\perp}$ product $\downarrow \; f(x_i) \xrightarrow[x_i \to 1]{} 0$ • **PDFs push down** the cross section = **Smaller BFKL effects!** (Collinear Factorization domain)	• Natural α_s is **larger** + • A given ΔY is reached at more **moderate values of $x_1 x_2$** $\downarrow$ • At the same ΔY **PDFs are probed in a region where they are larger** = **Stronger BFKL effects!** (Collinear & k_T Factorizations domain)

- We need a low-p_T treatment

Lipatov effective action

- **Slicing the whole rapidity range into intervals**

$$L_{\text{eff}} = 4\text{tr}\left[R_+\partial_T^2 R_-\right] + \sum_i \left[L_{\text{QCD}}(A_\mu^{[y_i,y_{i+1}]}, \psi_q^{[y_i,y_{i+1}]}) + L_{\text{Rg}}(A_\mu^{[y_i,y_{i+1}]}, R_+, R_-)\right]$$

- Hermitian form of the **effective Lagrangian**

$$L_{Rg}(x) = \frac{i}{g_s}\text{tr}\left[R_+(x)\partial_\rho^2\partial_-\left(W_x[A_-] - W_x^\dagger[A_-]\right) + R_-(x)\partial_\rho^2\partial_+\left(W_x[A_+] - W_x^\dagger[A_+]\right)\right]$$

$$W_x[A_\pm] = P\exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm\left(x_\pm, x'_\mp, \mathbf{x}_T\right)\right] = 1 - ig_s\left(\partial_\pm^{-1}A_\pm\right) + \ldots$$

- Induced interactions generate an infinite tower of Rg^n vertices
- Eikonal propagators $\partial_\pm^{-1} \sim 1/k^\pm$ produce **rapidity divergences**

$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu, \quad \frac{1}{[k_\pm]} \to \frac{1}{[\tilde{k}_\pm]} = \frac{1}{[k_\pm + r k_\mp]}, \quad 0 < r \ll 1$$

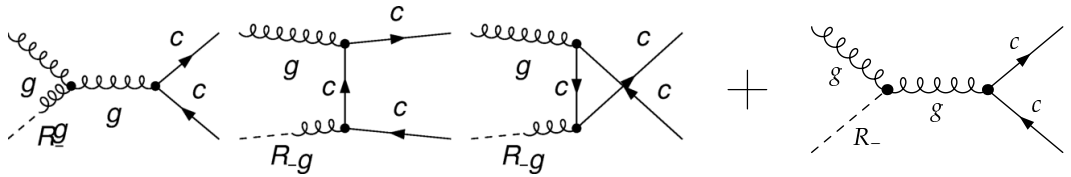
- BFKL $\leftrightarrow$ **Rapidity renormalization group** in $\ln r \sim \ln s$

Quarkonium IF: virtual corrections

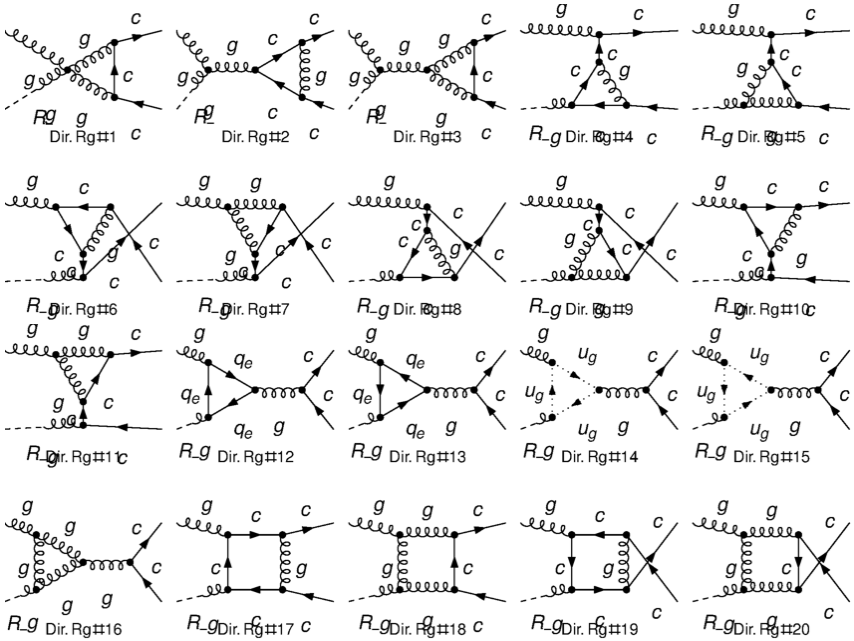
- **States**: $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^1S_0^{[8]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$

[Nefedov (2023)]

Interference with LO:

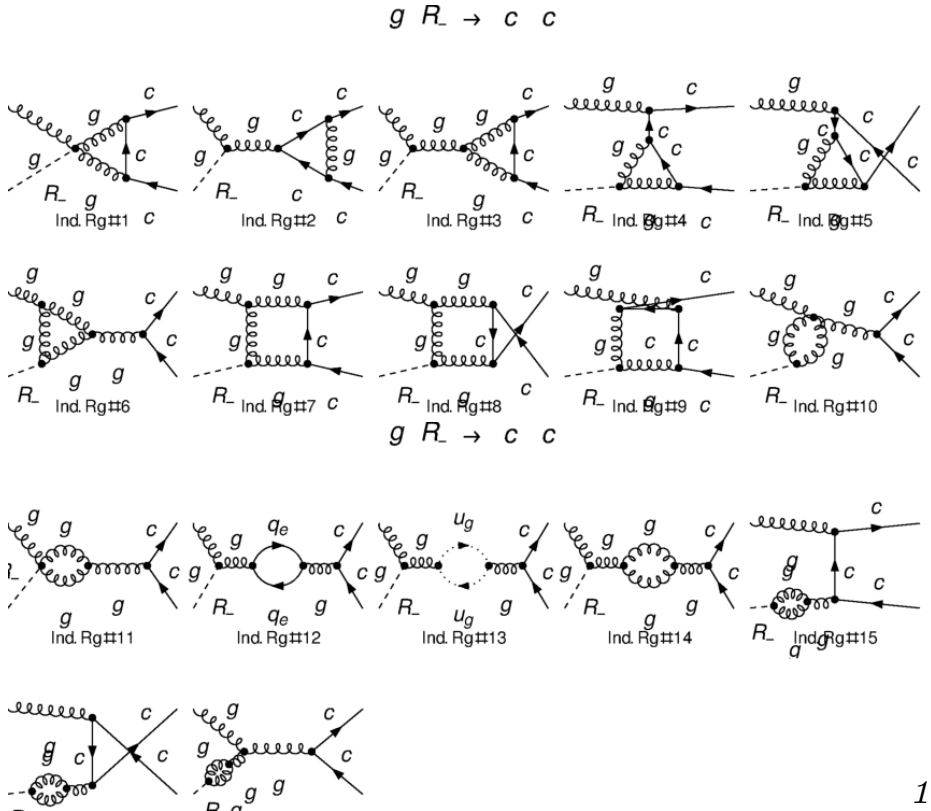


Some Rg-coupling diagrams:



and so on...

Induced Rgg coupling diagrams:



Quarkonium IF: virtual corrections

Strategy in a nutshell

- Diagrams generated using custom `FeynArts` model-file, projectors on the $c\bar{c}\left[{}^1S_0^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}\right]$-state is inserted
- IBP reduction to master integrals performed using `FIRE`
- Master integrals are expanded in $r \ll 1$ using Mellin-Barnes representation.

Structure of the result

- **Color singlet state**

$$\left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\left[-\frac{C_A}{\epsilon^2}+\frac{1}{\epsilon}\left(C_A\ln\frac{p_+^2}{r\mathbf{q}_T^2}+\beta_0+3C_F-C_A\right)\right]-\frac{10}{9}n_F+F_{{}^1S_0^{[1]}}(\mathbf{q}_T^2/M^2)$$

- **Color octet states**

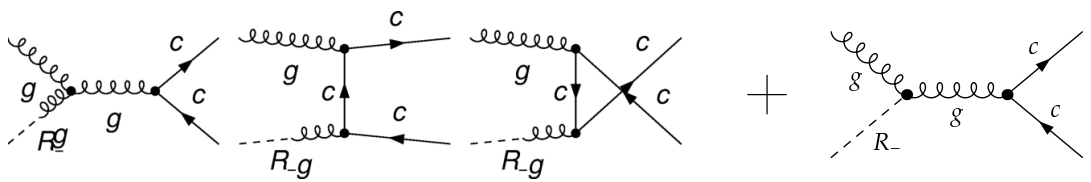
$$\left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\left[-\frac{C_A}{\epsilon^2}+\frac{1}{\epsilon}\left(C_A\ln\frac{p_+^2}{r\mathbf{q}_T^2}+C_A\ln\left(1+\frac{\mathbf{q}_T^2}{M^2}\right)+\beta_0+3C_F-2C_A\right)\right]$$
$$-\frac{10}{9}n_F+F_{(m)}(\mathbf{q}_T^2/M^2)$$

with $m = {}^1S_0^{[8]}, {}^3S_1^{[8]}$

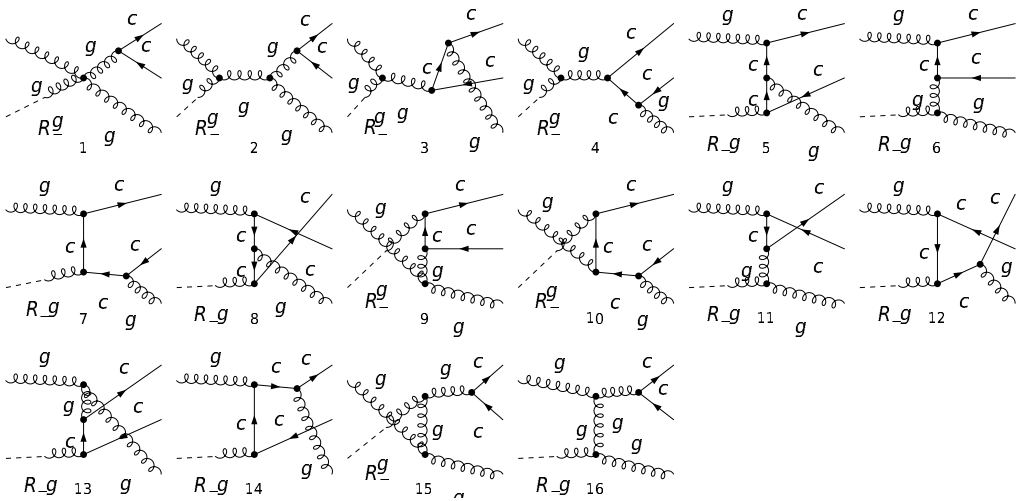
Quarkonium IF: real corrections

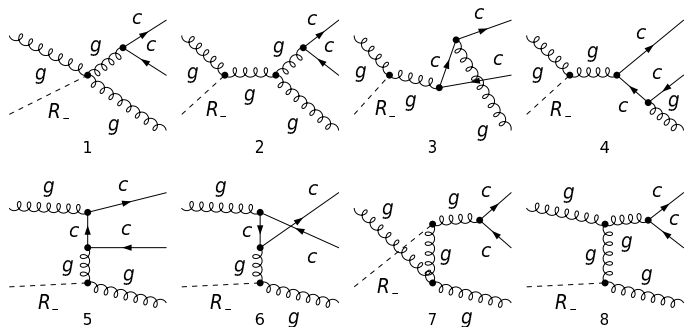
- **States**: $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^1S_0^{[8]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$

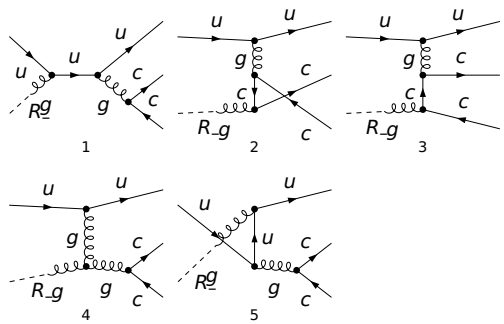
[M.F., Nefedov, Lansberg, Szymanowski, Wallon (2025)]

LO diagrams: 

NLO diagrams (also quarks start to contribute)







Quarkonium IF: real corrections

Strategy in a nutshell

- Diagrams generated using custom `FeynArts` model-file
- `FeynCalc` for algebraical manipulations (projection on the $c\bar{c}\left[{}^1S_0^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}\right]$-state, etc...)
- `FORM` for Lorentz contractions

Structure of the result

[M.F., Nefedov, Lansberg, Szymanowski, Wallon (2025)]

- The result for the $g + R_- \to Q\bar{Q}[\mathcal{Q}] + g$ process is $\tilde{H}_{Rg}^{(\mathcal{Q})}(\mathbf{q}_T, \mathbf{p}_T, z)$

as, e.g. for ${}^1S_0^{[1]}$:

$$\tilde{H}_{Rg}^{({}^1S_0^{[1]})} = \frac{1}{z(1-z)\boldsymbol{q}_T^2} \frac{C_A \left(M^2 + \boldsymbol{p}_T^2\right)^2}{s^2 t^2 u^2 \left(M^2 - s\right)^2 \left(M^2 - t + \boldsymbol{q}_T^2\right)^2 \left(M^2 - u\right)^2} \times \sum_{n=-1}^{3} (1-z)^n w_{g,n}^{({}^1S_0^{[1]})}(s,t,u)$$

where $t_1 = \mathbf{q}_T^2$ and s, t and u are expressed through $\mathbf{p}_T$, $\mathbf{q}_T$, M and z.

Quarkonium IF: real corrections

- The result is huge and contains **IR** and **rapidity** singularities

$$
\begin{aligned}
w_{g,2}^{(^1S_0^{[1]})} = s\Big\{ & s^5\Big(t+t_1+u\Big)^2\Big[t-u\Big]\Big[\,t+u\Big] + s^4\Big[t+t_1 \\
& +u\Big]\Big[-2u^3[t+2t_1] + 2t^2[t+t_1][t+3\,t_1] + u^2[2t^2 - 5tt_1 \\
& -4t_1^2] + tu[6t^2+9tt_1+t_1^2]\Big] + s^2\Big[t\;t_1u^5 + 4t^2t_1(t+t_1)^3[t+3t_1] \\
& +2u^4[t^3+6t^2t_1-2t_1^3] + 2u^3[4t^4+27t^3t_1\;+29t^2t_1^2-6tt_1^3-4t_1^4] \\
& +tu[t+t_1][2t^4+32t^3t_1+90t^2t_1^2+51tt_1^3\;+5t_1^4] + u^2[8t^5+73t^4t_1 \\
& +163t^3t_1^2+84t^2t_1^3-6tt_1^4-4t_1^5]\Big] + s\;\Big[4t^2t_1^2(t+t_1)^4 + 2tt_1u^5[t+t_1] \\
& +u^4[t^4+17t^3t_1+24t^2t_1^2+4tt_1^3-t_1^4] + 2t\;t_1u[t+t_1][2t^4+20t^3t_1 \\
& +36t^2t_1^2+11tt_1^3+t_1^4] + u^3[2t^5+33t^4t_1+\;\;101t^3t_1^2+64t^2t_1^3-4tt_1^4 \\
& -2t_1^5] + u^2[t^6+22t^5t_1+117t^4t_1^2+172t^3\;t_1^3+57t^2t_1^4-4tt_1^5-t_1^6]\Big] \\
& +tt_1u\Big[t+t_1\Big]\Big[t+t_1+u\Big]\Big[2\;t_1+u\Big]\Big[t_1u[-t_1+u] + 4t^2[2t_1 \\
& +u] + tu[6t_1+u]\Big] + s^3\Big[t^6 - 6t_1^2u^2(t_1\;+u)^2 + 4t^5[3t_1+2u] \\
& +2tt_1u[t_1+u][2t_1^2-7t_1u-2u^2] + t^4[34\;t_1^2+53t_1u+15u^2] + t^3[36t_1^3 \\
& +87t_1^2u+61t_1u^2+8u^3] + t^2t_1[13t_1^3+46t_1^2\;u+41t_1u^2+14u^3]\Big]\Big\}
\end{aligned}
$$

- We need a **subtractive approach!**

Quarkonium IF: real corrections

$$V_a^{(m,1,\text{ fin.})}(\boldsymbol{q}_T,\boldsymbol{p}_T,z)=\frac{z|\boldsymbol{p}_T|}{\pi}h_m^{(0)}(\boldsymbol{p}_T^2)\left[\tilde{H}_{Ra}^{(m)}(\boldsymbol{q}_T,\boldsymbol{p}_T,z)-\mathcal{J}_{Ra}^{(m)}(\boldsymbol{q}_T-\boldsymbol{p}_T,\boldsymbol{p}_T,z)|_{r=0,\epsilon=0}\right]$$

- The **subtraction term** $\mathcal{J}_{Ri}^{(\text{sub.}),\mathcal{Q}}$ should remove the non-integrable singularities from the "exact matrix element" $\tilde{H}_{Ri}^{(\mathcal{Q})}(\mathbf{q}_T,\mathbf{p}_T,z)$

$$\begin{aligned}\mathcal{J}_{Rg}^{(m)}(\boldsymbol{k}_T,\boldsymbol{p}_T,z) &= \frac{2C_A}{\boldsymbol{k}_T^2}\left[\frac{1-z}{(1-z)^2+r\frac{\boldsymbol{k}_T^2}{q_+^2}}+\frac{1}{z}+z(1-z)-2+\Delta j_{Rg}^{(m,\text{pol.})}(\boldsymbol{k}_T,\boldsymbol{p}_T,z)\right]\\ &\quad+\Delta\mathcal{J}_{Rg}^{(m,\text{CO})}(\boldsymbol{k}_T,\boldsymbol{p}_T,z)\\ \mathcal{J}_{Rq}^{(m)}(\boldsymbol{k}_T,\boldsymbol{p}_T,z) &= \frac{C_F}{\boldsymbol{k}_T^2}\left[\frac{1+(1-z)^2}{z}-\epsilon z+\Delta j_{Rq}^{(m,\text{pol.})}(\boldsymbol{k}_T,\boldsymbol{p}_T,z)\right],\end{aligned}$$

- **Blue** term: **rapidity, collinear and soft & collinear** singularities

$$\frac{1-z}{(1-z)^2+r\frac{\boldsymbol{k}_T^2}{q_+^2}}=\frac{1}{(1-z)_+}-\delta(1-z)\frac{1}{2}\ln\frac{r\boldsymbol{k}_T^2}{q_+^2}+O(r)$$

- **Red** term: contains the additional **soft singularity** of the color octet
- **Green** term: contains **angular correlations** (only spin singlet)

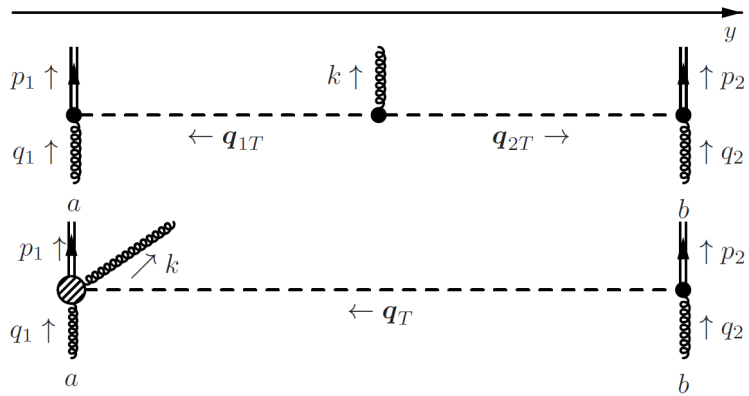
Quarkonium IF: real corrections

- **UV-renormalization**: $\overline{\text{MS}}$ for α_s + on-shell wave-function renormalization for the heavy-quarks
- **Soft div.** cancel in the combination of real and virtual corrections (**KLN**)
- Cancellation of initial states **collinear singularities** with renormalized PDFs

$$-\frac{1}{\epsilon} P_{ab}(z) \to P_{ab}(z) \ln \frac{\boldsymbol{p}_T^2}{\mu_F^2}$$

- Terms $\propto \ln \frac{\boldsymbol{p}_T^2}{m_{\mathcal{Q}}^2} \implies$ When $\boldsymbol{p}_T^2 \gg m_{\mathcal{Q}}^2 \to$ fragmentation limit
- **Rapidity divergence** is $\propto K_{\text{BFKL}}$ and can be subtracted according to

  - Lipatov EFT (r-regulator)
  - **BFKL scheme** ($s_{gR} < s_\Lambda$)
  - Shockwave scheme (k^+ cut-off)



To change scheme:

$$\Delta \mathcal{J}_{RR}(k) = \mathcal{J}_{RR}^{(s_\Lambda)} - \mathcal{J}_{RR}^{(r)} = \frac{2C_A}{\boldsymbol{k}_T^2} \left[\frac{1}{1-z} \theta\left(1 - z > \frac{\boldsymbol{k}_T^2}{s_\Lambda}\right) - \frac{1-z}{(1-z)^2 + \frac{r\boldsymbol{k}_T^2}{q_\perp^2}} \right]$$

Summary and prospects

Summary

- **Instabilities** in fixed order calculation of Quarkonium production
- Problems are generated by the **lack of high-energy resummation**
- NLL accuracy is imperative in small-x physics
- **NLO impact factors** for forward Quarkonium production

[Nefedov (2023)]

[M.F., Lansberg, Nefedov, Szymanowski, Wallon (2025)]

Future prospects

- **J/Ψ-plus-jet** at full NLL
- **Single inclusive quarkonium hadroproduction** at full NLL
- Extension to central J/Ψ hadroproduction

Summary and prospects

Summary

- **Instabilities** in fixed order calculation of Quarkonium production
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Future prospects

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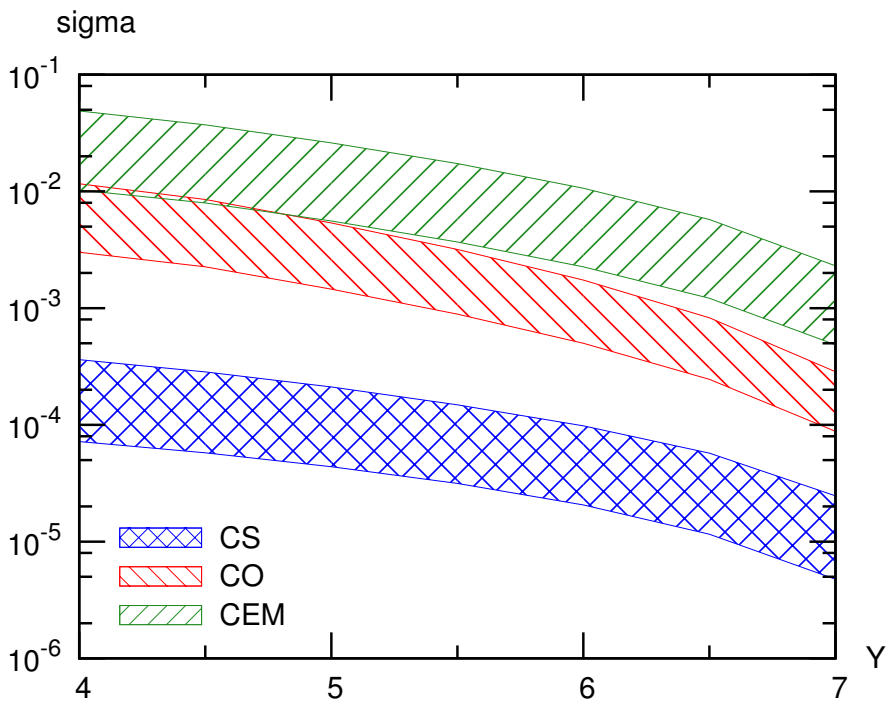
Thank you for the attention!

Backup

J/ψ plus jet production

- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

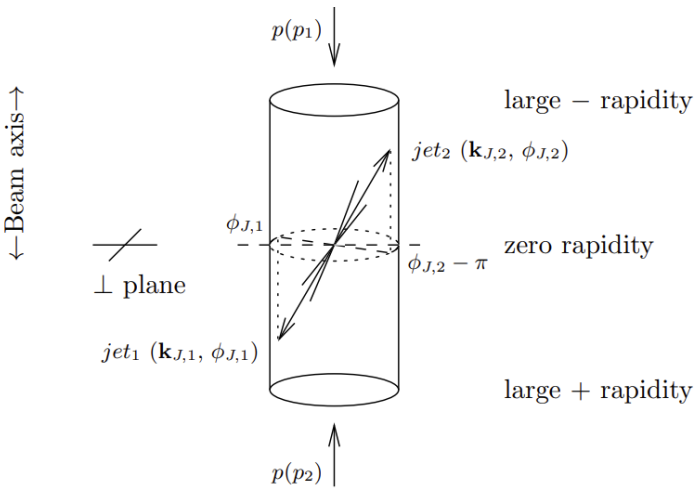
$$\frac{d\sigma}{d|k_{J/\psi}|d|k_{\rm jet}|dY}[\text{nb GeV}^{-2}] \qquad |\mathrm{k_{J/\psi}}| = |\mathrm{k_{jet}}| = 10 \text{ GeV}$$

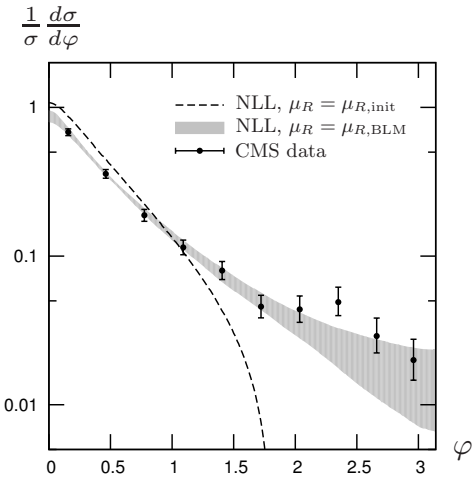


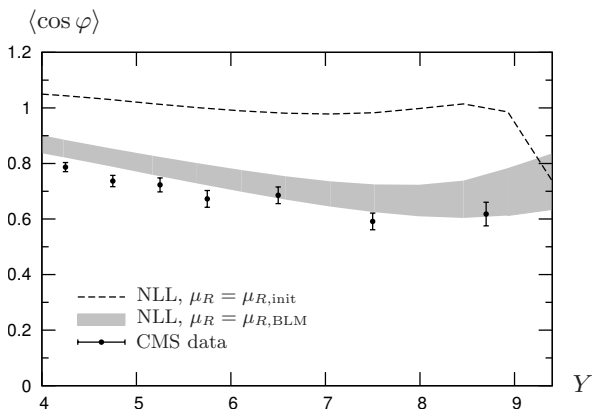
[Boussarie, Ducloué, Szymanowski, Wallon (2018)]

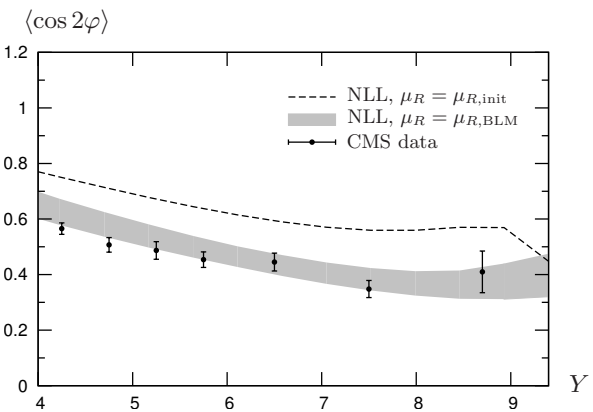
Muller-Navelet: Theory vs Experiment

- Relative angle in the azimuthal plane $\varphi = \phi_{J_1} - \phi_{J_2} - \pi$



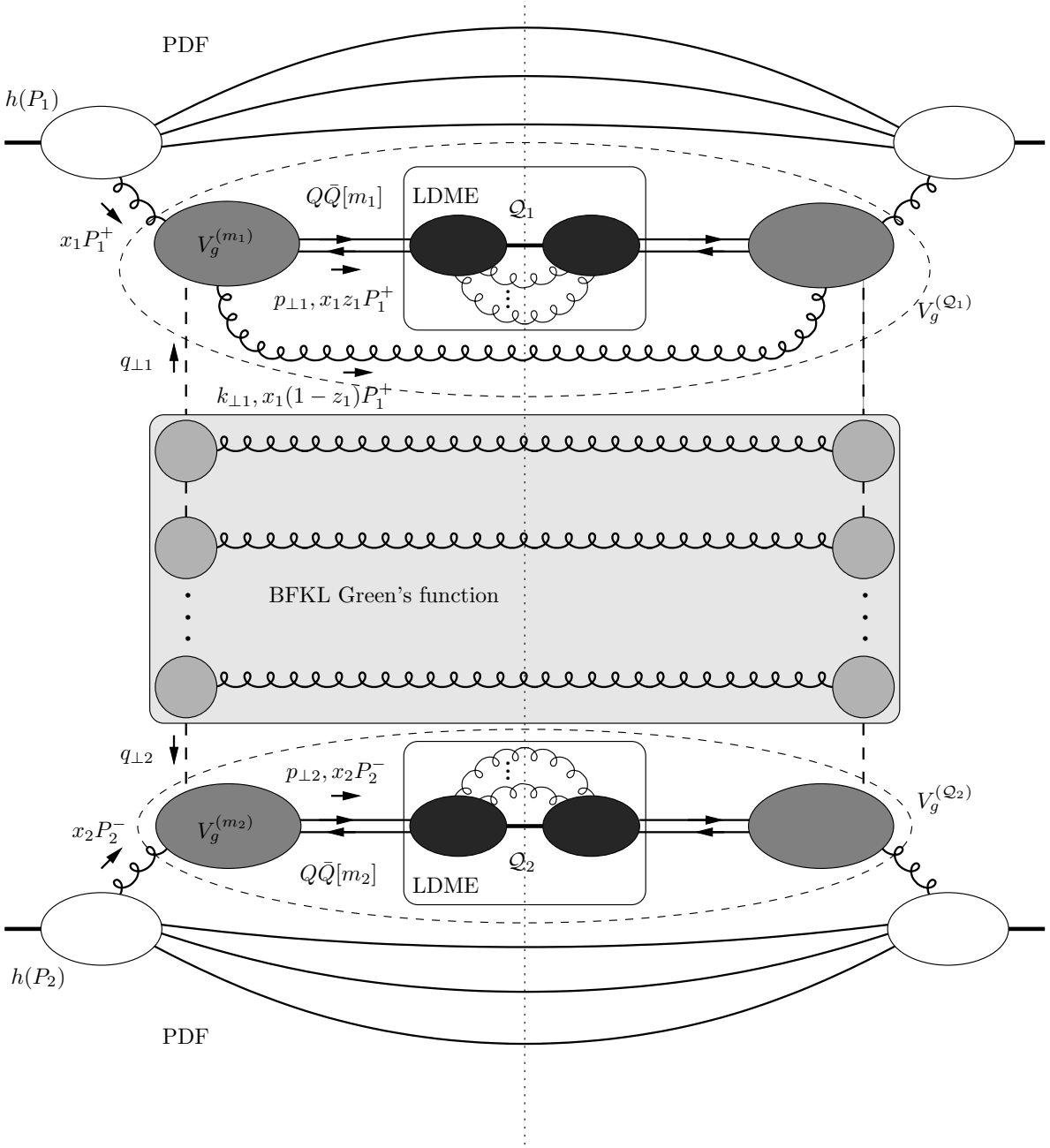






In this slide: [Ducloué, Szymanowski, Wallon (2013)]
[Caporale, Ivanov, Murdaca, Papa (2014)]

Factorization at NLL (double quarkonium case)



J/ψ production from single parton fragmentation

- J/Ψ production from single parton fragmentation

[Celiberto, M.F. (2022)] [Celiberto, Lonigro (2025)]

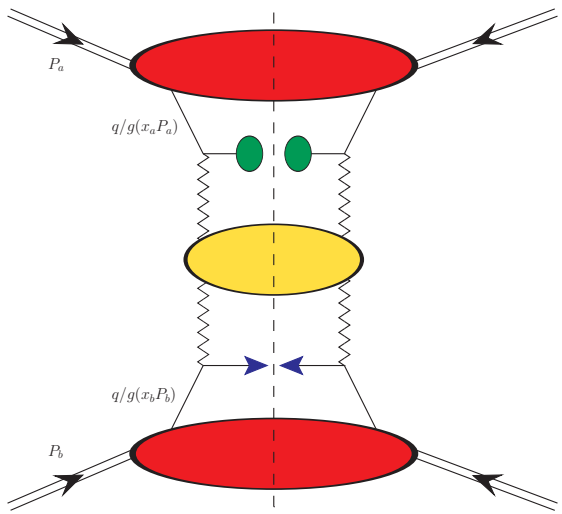
$$p(P_a) + p(P_b) \to \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J)$$

- **Hybrid cross section**

$$\frac{d\sigma}{dy_{\mathcal{Q}} dy_J d^2\vec{p}_{\mathcal{Q}} d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

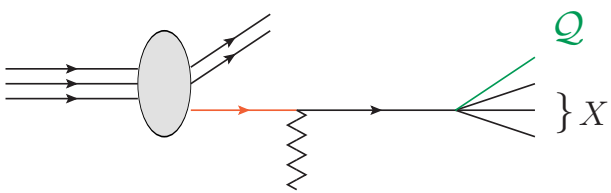
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_{\mathcal{Q}}(\vec{q}_1, x_g, \vec{p}_{\mathcal{Q}}) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_J(\vec{q}_2, x_J, \vec{p}_J)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_{\mathcal{Q}} x_J s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

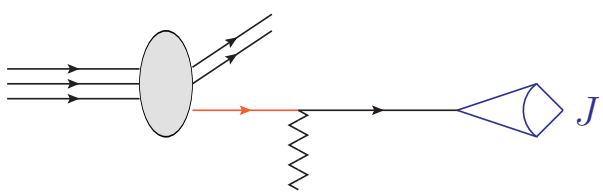


- Impact factors [Ivanov, Papa (2012)]

$$V_{\mathcal{Q}}(\vec{q}_1, x_{\mathcal{Q}}, \vec{p}_{\mathcal{Q}}) = f_{q/g} \otimes H \otimes D^{\mathcal{Q}}_{q/g}$$

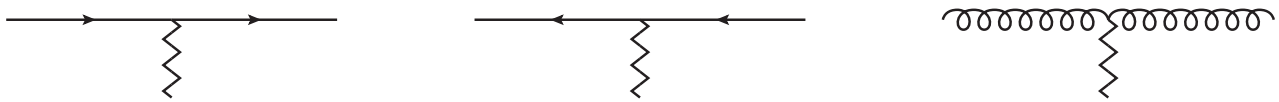


$$V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \cdot \mathcal{J}$$



J/ψ production from single parton fragmentation

- Parton-Parton-Reggeon (PPR) vertices



- LO jet impact factor

$$c_J(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x)+\sum_{\beta=q,\bar{q}}f_\beta(x)\right),$$

- LO light hadron impact factor

$$c_{\mathcal{Q}}(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\int_x^1\frac{d\zeta}{\zeta}\left(\frac{\zeta}{x}\right)^{2i\nu-1}$$
$$\times\left[\frac{C_A}{C_F}f_g(\zeta)D_g^{\mathcal{Q}}\left(\frac{x}{\zeta}\right)+\sum_{\alpha=q,\bar{q}}f_\alpha(\zeta)D_\alpha^{\mathcal{Q}}\left(\frac{x}{\zeta}\right)\right]$$

- Both known at NLO

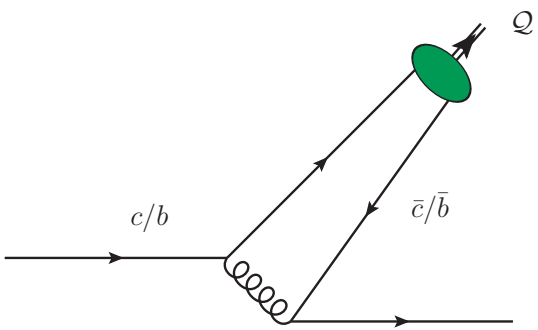
[Bartels, Colferai, Vacca (2003)]
[Caporale, Ivanov, Murdaca, Papa (2014)]
[Ivanov, Papa (2012)]

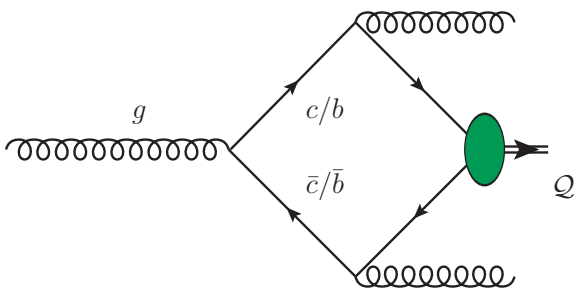
J/ψ production from single parton fragmentation

- J/Ψ fragmentation functions in the **NRQCD** framework

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n])\langle \mathcal{O}^{\mathcal{Q}}([n])\rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{[1]}$
- Initial inputs for the DGLAP evolution





[Braaten, Cheung, Yuan (1993)] [Zheng, Chang, Wu (2019)]
[Braaten, Yuan (1993)]

- Evolution performed by **APFEL++** [Bertone (2017)]

$$\left\{D_Q^{\mathcal{Q}}(z,3m_Q), D_g^{\mathcal{Q}}(z,2m_Q)\right\}_{\text{NRQCD}} \xrightarrow{\textbf{APFEL++}} \boxed{\textbf{Onium FFs}}$$

J/Ψ *fragmentation: heavy-quark channel*

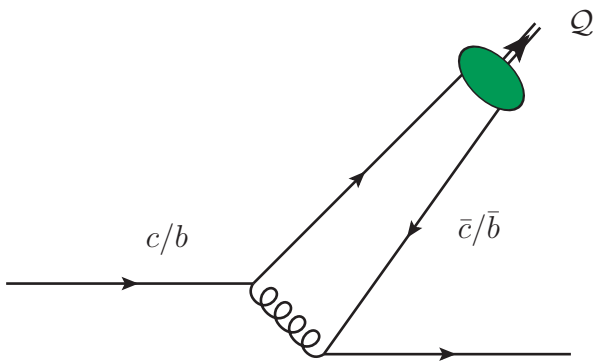
- NRQCD FFs

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n]) \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{[1]}$
- Heavy-quark fragmentation function computed at $\mu_0 = 3m_Q$ in NRQCD

$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z)$$

$$+\frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \, \Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$



- LO fragmentation function

[Braaten, Cheung, Yuan (1993)]

$$D_Q^{\mathcal{Q},\mathrm{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3} \frac{8z(1-z)^2}{27\pi(2-z)^6} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \, (5z^4 - 32z^3 + 72z^2 - 32z + 16)$$

- The NLO correction is given by a polynomial function

[X.C. Zheng, C.H. Chang, and X.G. Wu (2019)]

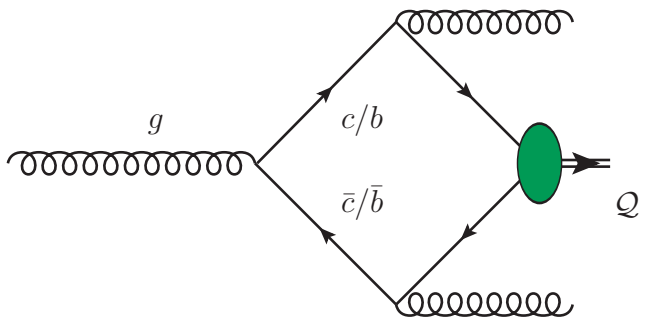
J/Ψ *fragmentation: gluon and light-quark channels*

- Gluon fragmentation function computed at $\mu_0 = 2m_Q$ in NRQCD

[Braaten, Yuan (1993)]

$$D_g^{\mathcal{Q}}(z, 2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$



- FFs evolved from the initial scale through the DGLAP evolution equations
- Light-quarks FFs (LQFFs) are zero at the initial scale: $D_q^{\mathcal{Q}}(z, 2m_Q) = 0$
- At higher scales LQFFs dynamically generated by evolution

$$D_q^{\mathcal{Q}}(z, \mu > 2m_Q) = \underbrace{D_q^{\mathcal{Q}}(z, 2m_Q)}_{0} + \frac{\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{(2m_Q)^2}\right)\int_z^1 \frac{dx}{x} P_{qg}(x) D_g^{\mathcal{Q}}\left(\frac{z}{x}, 2m_Q\right)$$