Andrea Simonelli

DIS2026

Connecting Regge Factorization and Collins-Soper evolution

SHARP

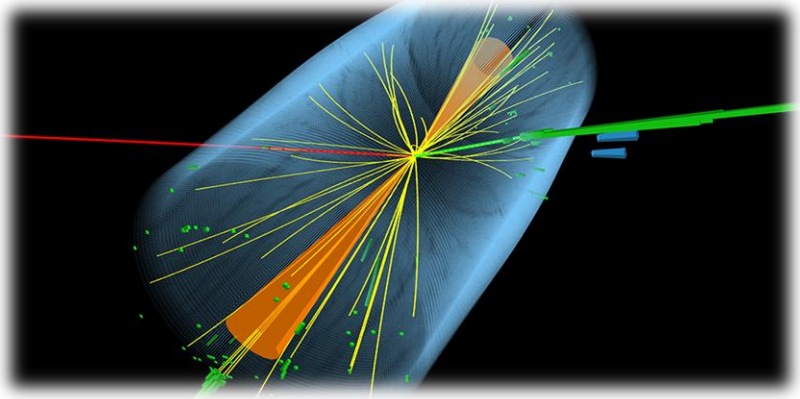
cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Funded by the European Union

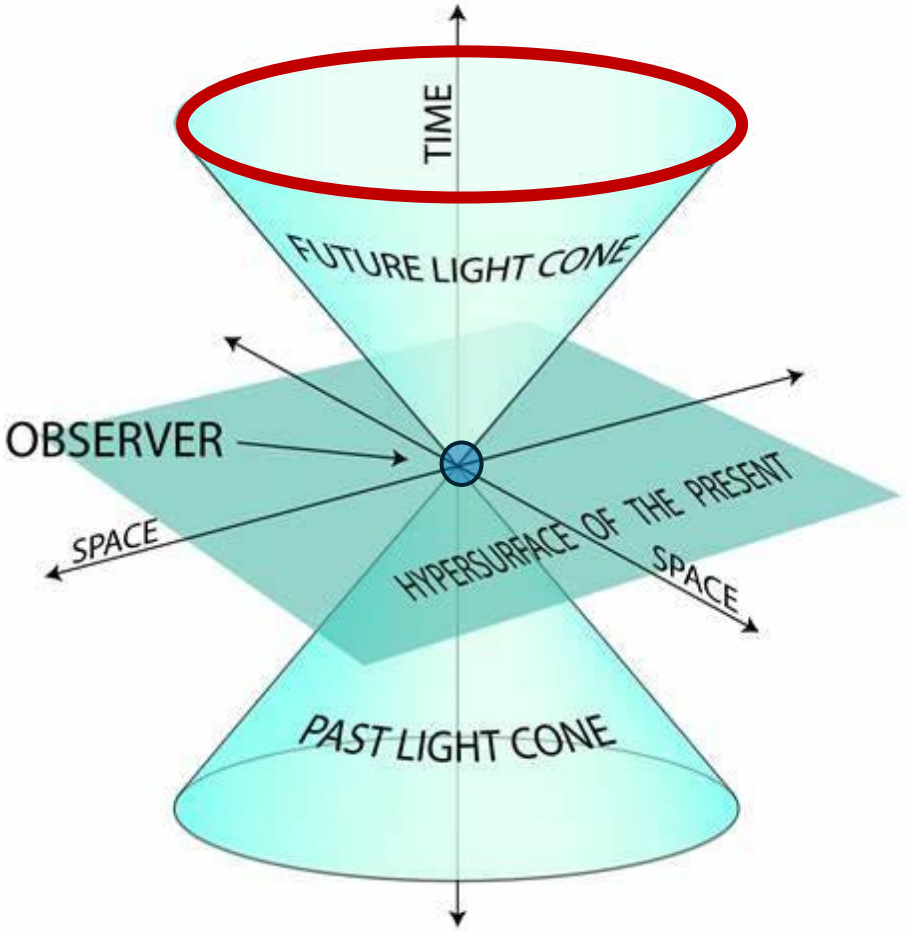
INFN Istituto Nazionale di Fisica Nucleare Sezione di Roma

Factorization

Take a (hadronic) process…



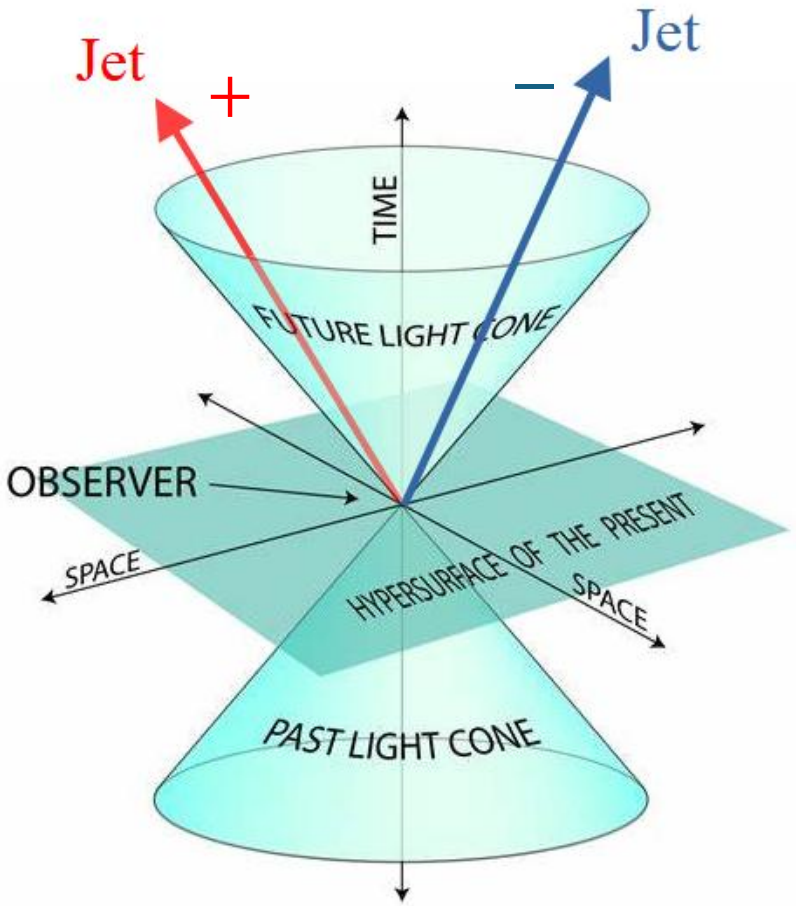
…it has a well-defined underlying spacetime structure fixed by its kinematics



In order to make predictions, short-distance contributions have to be **factorized** from long-distance contributions

Factorization with two dominant opposite LC directions

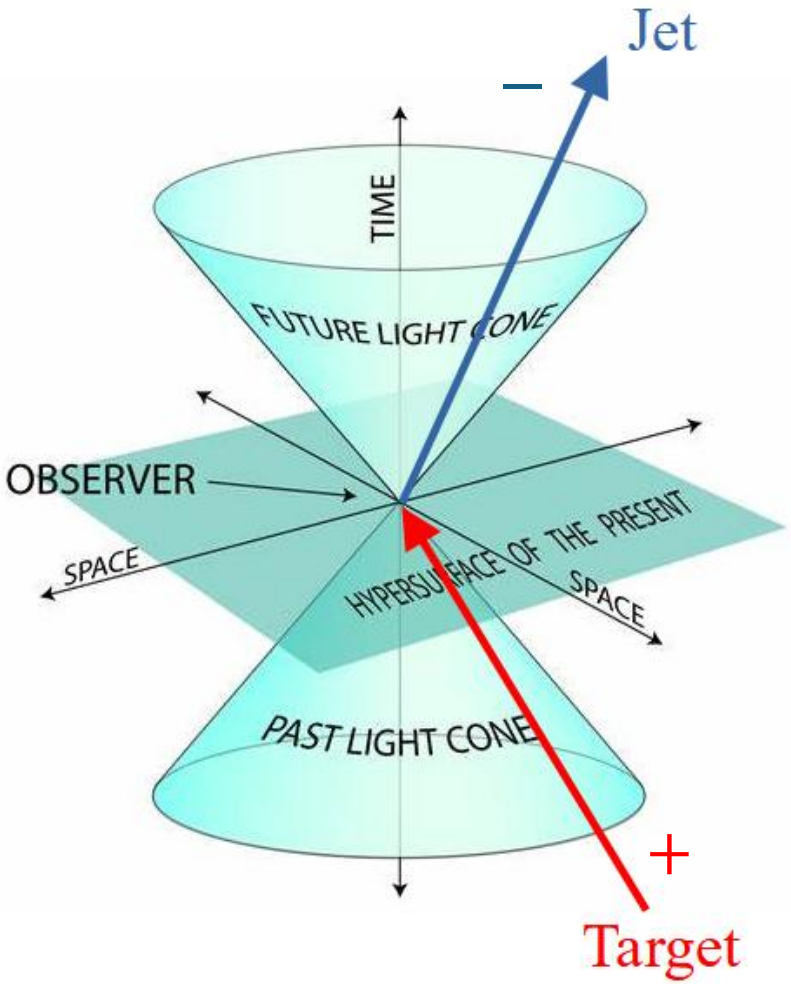
- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ....



Factorization with two dominant opposite LC directions

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ....

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ....

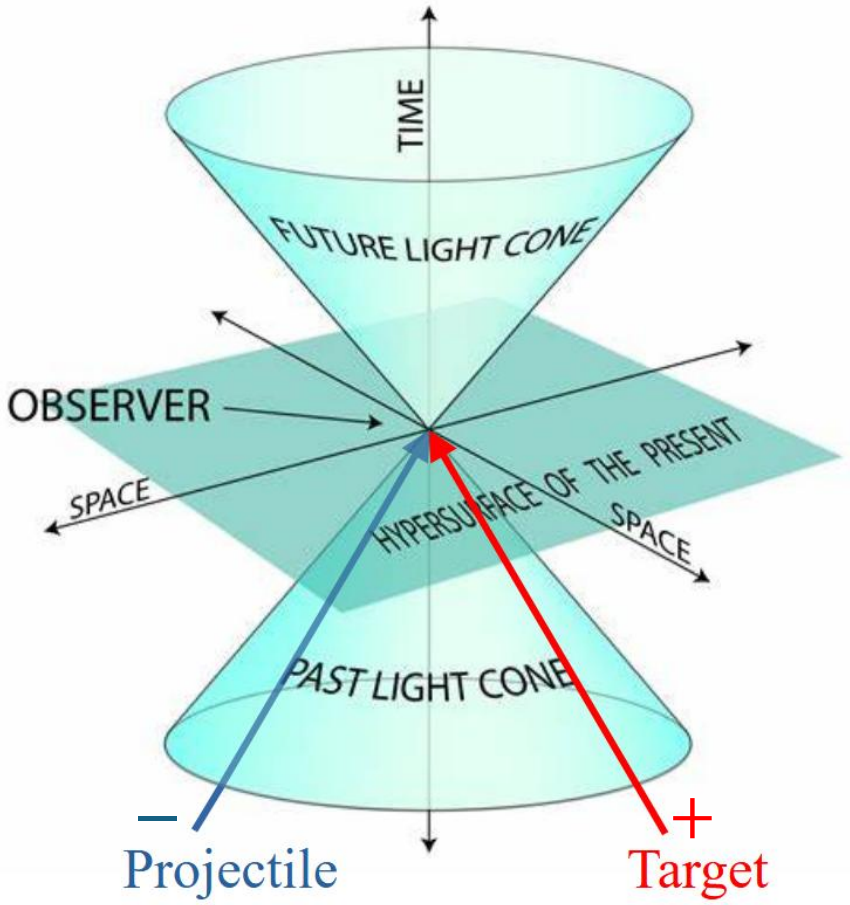


Factorization with two dominant opposite LC directions

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

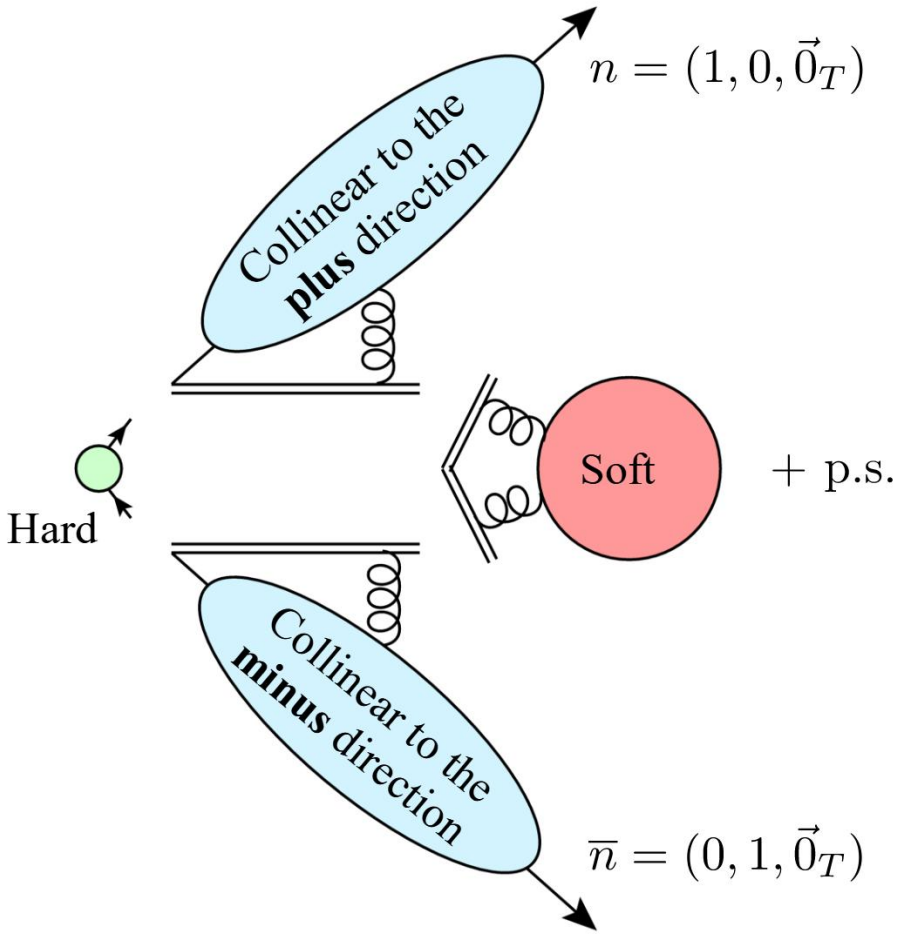
- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- ….



A broader (geometric) meaning of universality

Analogous spacetime structure ⟷ Similar factorization properties



Process-dependent, but under perturbative control

$$O \sim \underbrace{C_+ \cdot S \cdot C_-}_{} \cdot H$$

Non-perturbative, but with universal features

Geometric features across different processes

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- ….

Well established

TMD Factorization
Collins, Soper and Sterman; ‘80s papers

$$H_{process} \underbrace{C_+ \; S \; C_-}_{F_+^{tmd} F_-^{tmd}}$$

Geometric features across different processes

- Di-Jet production from $e^+ e^-$ annihilation
- Thrust distribution
- Pair of hadrons produced back-to-back
- ….

Well established

TMD Factorization
Collins, Soper and Sterman; ‘80s papers

$$H_{process} \underbrace{C_+ \;\; S \;\; C_-}_{F_+^{tmd} F_-^{tmd}}$$

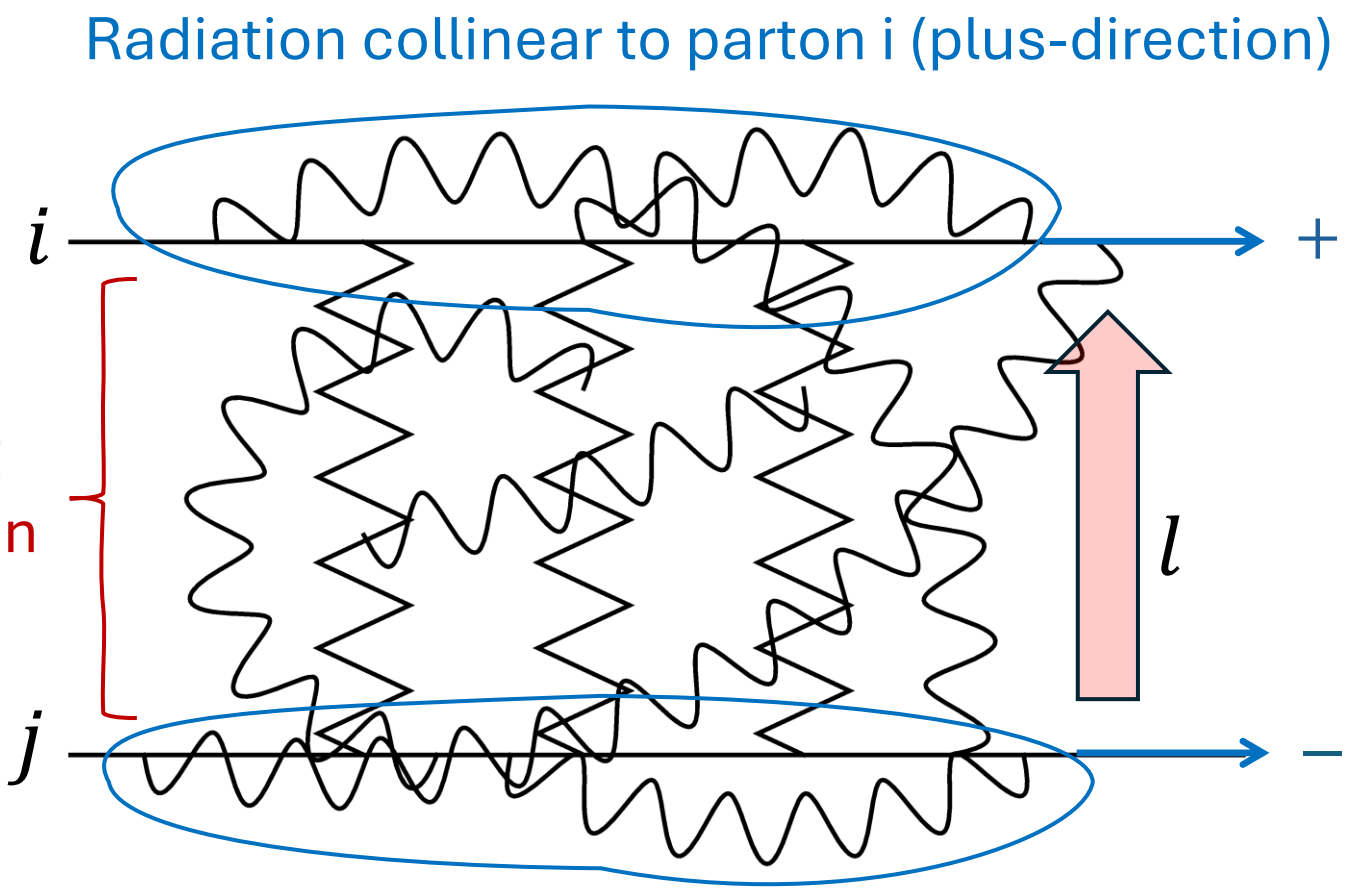
- DIS near threshold
- Semi-inclusive DIS at low q_T
- ….

- Drell-Yan scattering at low q_T
- ….

Sterman 1987; SCET papers;
our recent work 2502.15033 [hep-ph]

$$H_{DIS@thr} \underbrace{C_+ \;\; S \;\; C_-}_{f_+ \;\; J_-}$$

Geometry of $2 \to 2$ forward scattering, i.e. $s \gg |t|$



$$M_{ij} \stackrel{?}{=} C_i^+ \, S \, C_j^- \, H_{ij}$$

Log counting and Reggeon exchanges

$$M_{ij}(s,t) = M_{ij}^{(-)}(s,t) + M_{ij}^{(+)}(s,t)$$

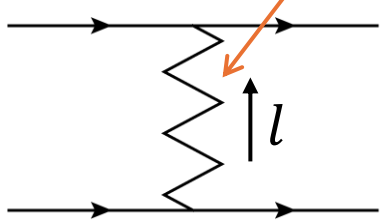
The signature symmetry ($s \leftrightarrow u$) allows the decomposition into $M^{\pm}$, governed by an **odd/even** number of reggeized gluons

$$M_{ij}^{(-)LL}(s,t) = e^{L\, C_A \alpha_G^{LO}(t,\mu^2)}\, M_{ij}^{tree}(s,t)$$

Gluon Regge trajectory (at LO)

$$\alpha_G^{LO}(t,\mu^2) = g^2(\mu)\frac{2}{\epsilon}c(\epsilon)\left(\frac{\mu^2}{-t}\right)^{\epsilon}$$

One reggeized gluon (or Regge pole contribution)



$$\sim \frac{-i\,\eta_{+-}}{-l_T^2 + i\,o}\left(\frac{s}{l_T^2}\right)^{C_A \alpha_G^{LO}(t,\mu^2)}$$

Log counting and Reggeon exchanges

$$M_{ij}(s,t) = M_{ij}^{(-)}(s,t) + M_{ij}^{(+)}(s,t)$$

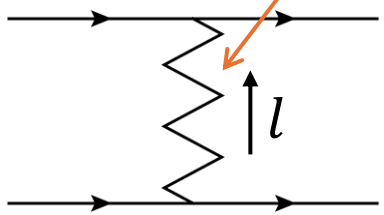
The signature symmetry ($s \leftrightarrow u$) allows the decomposition into $M^{\pm}$, governed by an **odd/even** number of reggeized gluons

$$M_{ij}^{(-)NLL}(s,t) = C_i^{LO}(t,\mu^2)\, e^{L\, C_A \alpha_G^{NLO}(t,\mu^2)} C_j^{LO}(t,\mu^2) M_{ij}^{tree}(s,t)$$

Gluon Regge trajectory (at NLO)

Impact factors of partons i and j (at LO)

One reggeized gluon (or Regge pole contribution)



$$\sim \frac{-i\,\eta_{+-}}{-l_T^2 + i\,o}\left(\frac{s}{l_T^2}\right)^{C_A \alpha_G^{LO}(t,\mu^2)}$$

Log counting and Reggeon exchanges

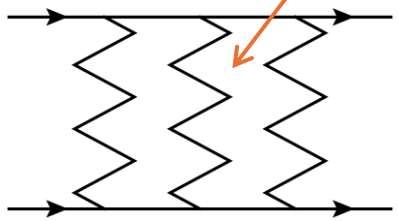
Beyond NLL the game factorization **breaks** due to multiple reggeon exchanges

$$M_{ij}^{(-)NNLL}(s,t) = C_i^{NLO}(t,\mu^2)\, e^{L\, C_A \alpha_G^{NNLO}(t,\mu^2)} C_j^{NLO}(t,\mu^2) M_{ij}^{tree}(s,t) + R_{ij}^{(-)LO}(s,t)$$

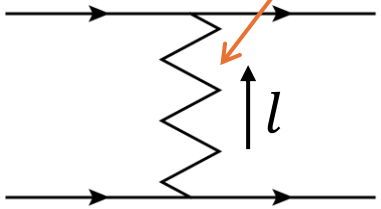
Gluon Regge trajectory* (at NNLO)

Impact factors* of partons i and j (at NLO)

Three reggeized gluons (or Regge cut contribution)



One reggeized gluon (or Regge pole contribution)

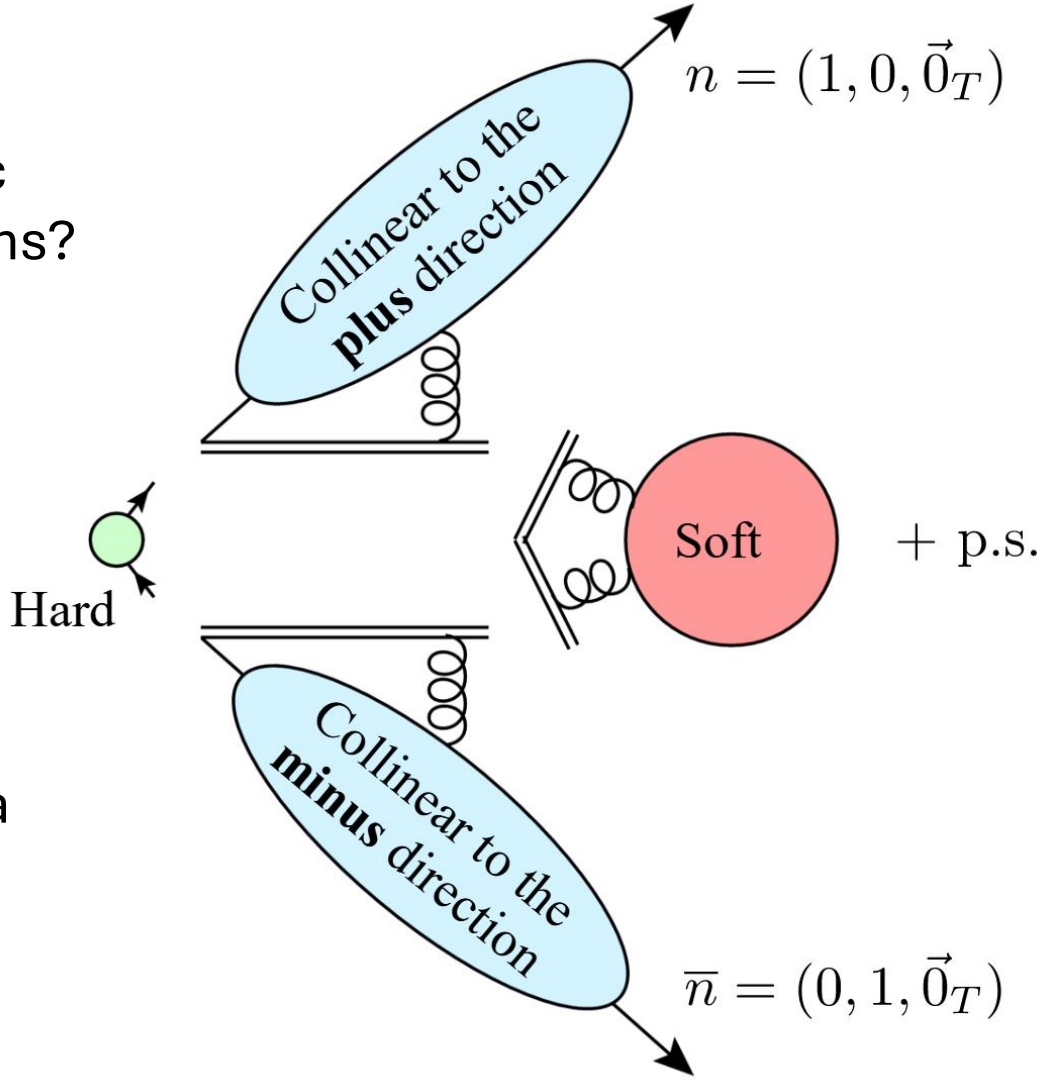


$$\sim \frac{-i\,\eta_{+-}}{-l_T^2 + i\,o}\left(\frac{s}{l_T^2}\right)^{C_A \alpha_G^{LO}(t,\mu^2)}$$

Starting from this accuracy, the definitions of Regge trajectory and impact factors are **not unique**, and depend on how the separation between single and multiple reggeon exchange is implemented

Breakdown of expectations?

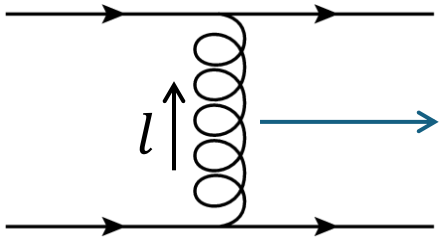
The factorization breaking is surprising:
Are $2 \to 2$ forward amplitudes evading the geometric universality induced by the two opposite LC directions?



How to recover the physical intuition, together with a unique definition of Regge trajectory?

Glauber gluons

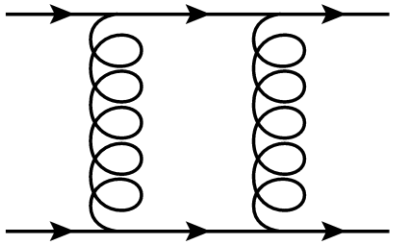
Reggeon exchanges are intimately related to Glauber exchanges

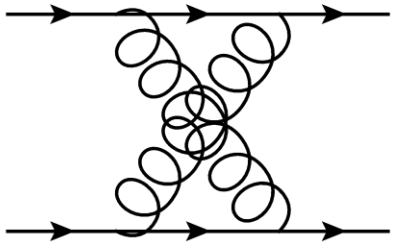


This is a Glauber gluon, i.e. $|l^+\, l^-| \ll l_T^2$.

Kinematics imposes: $l^{\pm} = \pm\frac{t}{\sqrt{2s}}$; $|\vec{l}_T|^2 = -t$

Contributes to **single** Reggeon exchange $M^{LL(-)}, M^{NLL(-)}$





One gluon can be Glauber, while the other can be either collinear to the plus/minus direction or soft.

Or they can both be Glauber.

Contributes to **double** Reggeon exchange $M^{(+)}$

Fixed number of Reggeon exchanges ⟺ Fixed number of Glauber exchange

Factorization with Glaubers

Dealing with Glaubers is usually **fatal**, leading to non-factorizing terms.

Consider a gluon l flowing out of the soft subgraph S and entering into the collinear-to-plus subgraph C.

$$\mathcal{C}^{\alpha}(l,\dots)\mathcal{S}_{\alpha}(l,\dots) = \mathcal{C}^{+}(l,\dots)\mathcal{S}^{-}(l,\dots) + \dots =$$
$$= \left[\mathcal{C}^{+}(l,\dots)l^{-}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l,\dots) + \dots$$

So far, the approximations are valid also if l is Glauber, since they only exploit the effect of boosting C towards the plus direction

If l is *soft, but not Glauber*, then

$$\left[\mathcal{C}^{+}(l,\dots)l^{-}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l,\dots) + \dots = \left[\mathcal{C}(\hat{l},\dots)\cdot\hat{l}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l,\dots) + \dots$$
$$\overset{\text{Ward Id.}}{\mapsto}\mathcal{C}(\dots)\frac{i}{l\cdot n}(-ign^{\rho})\mathcal{S}_{\rho}(l,\dots) + \dots$$

We can approximate l as $\hat{l} = (0, l^{-}, \vec{0}_{T})$ also *inside* the collinear subgraph. This allows Ward identities to be used.

Crucial step

Factorization with Glaubers

Dealing with Glaubers is usually **fatal**, leading to non-factorizing terms.

Consider a gluon l flowing out of the soft subgraph S and entering into the collinear-to-plus subgraph C.

$$\mathcal{C}^{\alpha}(l,\dots)\mathcal{S}_{\alpha}(l,\dots) = \mathcal{C}^{+}(l,\dots)\mathcal{S}^{-}(l,\dots) + \dots =$$
$$= \left[\mathcal{C}^{+}(l,\dots)l^{-}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l,\dots) + \dots$$

So far, the approximations are valid also if l is Glauber, since they only exploit the effect of boosting C towards the plus direction

If instead l is *Glauber*, then

Factorization stops here.

$$\left[\mathcal{C}^{+}(l,\dots)l^{-}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l,\dots) + \dots = \left[\mathcal{C}(l_{G(n)},\dots)\cdot\hat{l}\right]\frac{1}{l\cdot n}n\cdot\mathcal{S}(l_{G},\dots) + \dots$$

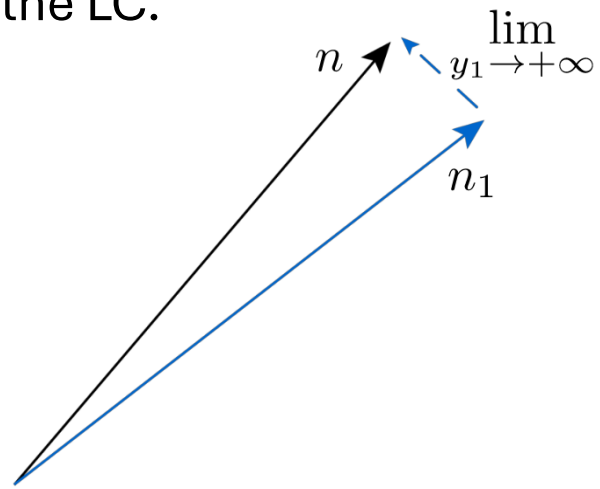
This time transverse momenta counts! We can only approximate l as $l_{G(n)} = (0, l^{-}, \vec{l}_{T})$ *inside* the collinear subgraph. The contraction with $\hat{l}$ prevents the application of Ward identities.

Factorization with Glaubers

In our case, though, the situation is simpler: the number of Glauber is kept fixed, hoping in this way to perform a **solid factorization at a given number of Glauber gluons.**

This requires the introduction of rapidity regulators to deal with the rapidity divergences arising when evaluating the contributions onto the LC.
We adopt the off lightcone tilts

$$n = (1, 0, \vec{0}_T) \mapsto n_1 = (1, e^{-2y_1}, \vec{0}_T)$$
$$\overline{n} = (0, 1, \vec{0}_T) \mapsto n_2 = (e^{2y_2}, 1, \vec{0}_T)$$



With the hope that

$$\mathcal{M}_{ij\to ij}^{n\text{-Glaubers}}(s,t) \stackrel{?}{=} C_i(t,\mu; y_1 - L) S_n(t,\mu; y_1 - y_2) C_j(t,\mu; L - y_2) \mathcal{M}_{ij\to ij}^{(n),\,\text{L.O.}} + \mathcal{O}\left(e^{-2(y_1 - y_2)}\right)$$

Single Glauber exchange

$$M_{ij}^{1-Glauber}(s,t) \stackrel{?}{=} C_i(t,\mu^2; L-y_1) S_1(t,\mu^2, y_1-y_2) C_i(t,\mu^2;\, y_2-L)$$

Where $L = \log(s/-t)$. The equivalence at LL with $M^{(-)LL}$ and at NLL $M^{(-)NLL}$ is ensured if

- $S_1(t,\mu^2, y_1-y_2) = \exp\big((y_1-y_2)\, C_A\, \alpha_G(t,\mu^2)\big)$
- $C_i(\mathrm{t},\mu^2, 0) \equiv C_i(t,\mu^2)$ from the literature.

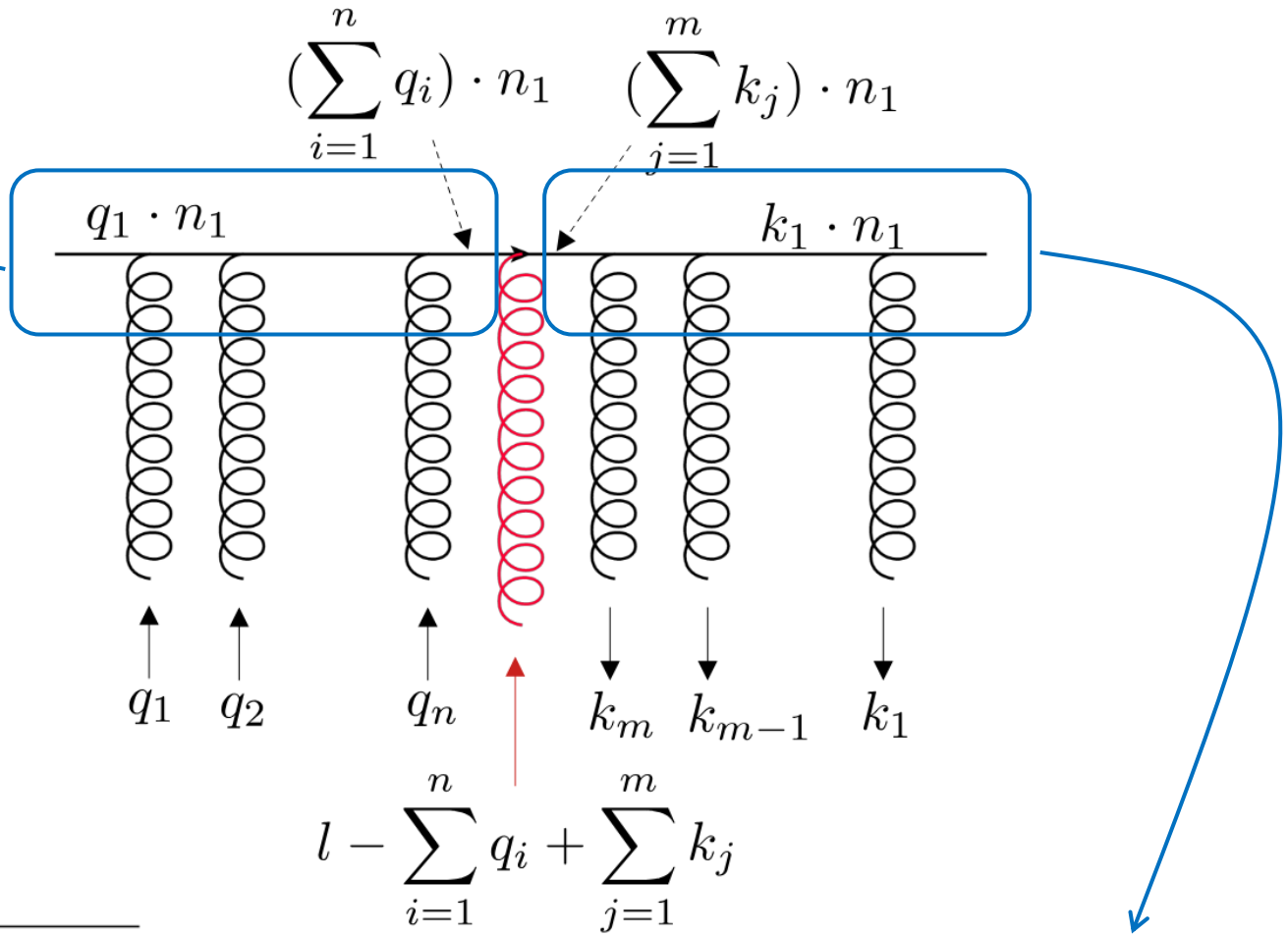
The great improvement comes from the fact that the factorization above should hold to all-orders, allowing for a unique and unambiguous definition of the gluon Regge trajectory as

$$\frac{\partial\ \log(S_1(t,\mu^2, y_1-y_2))}{\partial y_1} = C_A\, \alpha_G(t,\mu^2)$$

✓ Verified at 1-loop;

Working on the full all-order extension.

A sketch of the proof



Wilson line from the Glauber insertion point to $+\infty$, along n_1

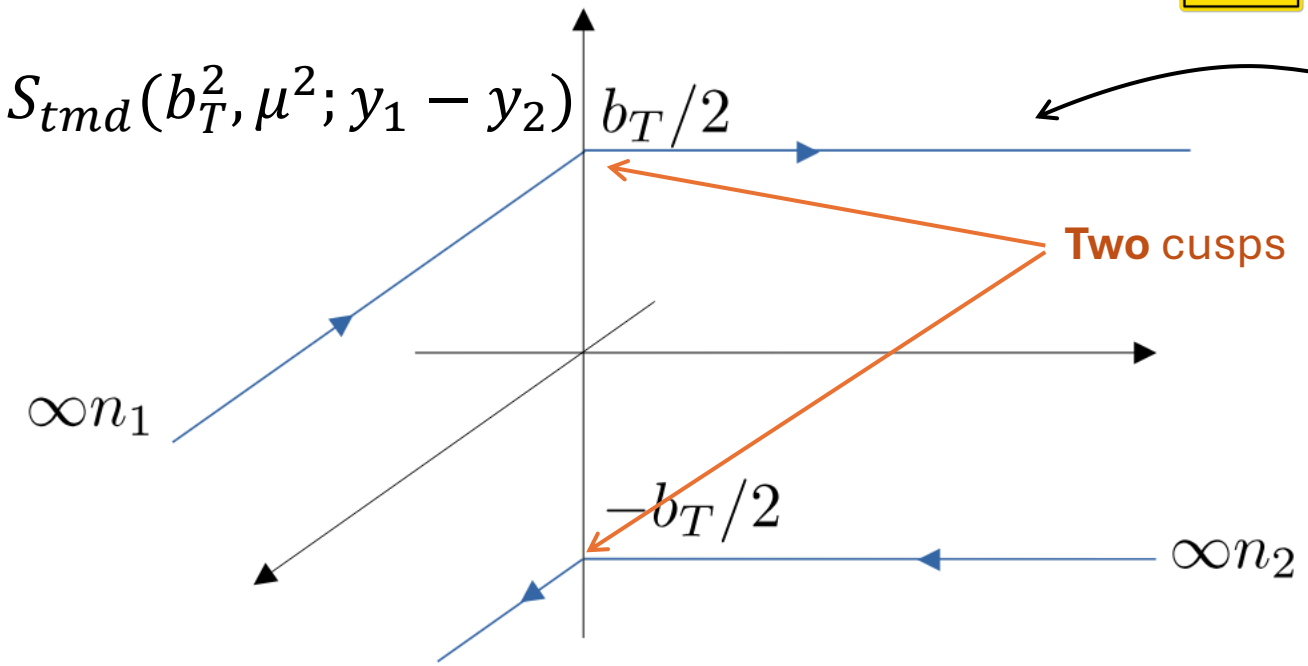
$$(-igt^a n_1^{\mu_1}) \frac{i}{q_1 \cdot n_1 + i\,0} \dots (-igt^a n_1^{\mu_n}) \frac{i}{\sum q_i \cdot n_1 + i\,0}$$

Wilson line from $-\infty$ to the Glauber insertion point, along n_1

$$S_1 \sim \langle W_{n_1}(z_1 \to +\infty) A_+(z_1) W_{n_1}(-\infty \to z_1) \otimes W_{n_2}(z_2 \to +\infty) A_-(z_2) W_{n_2}(-\infty \to z_2) \rangle$$

With $\vec{z}_{1T} - \vec{z}_{2T}$ Fourier conjugate of $\vec{l}_T$

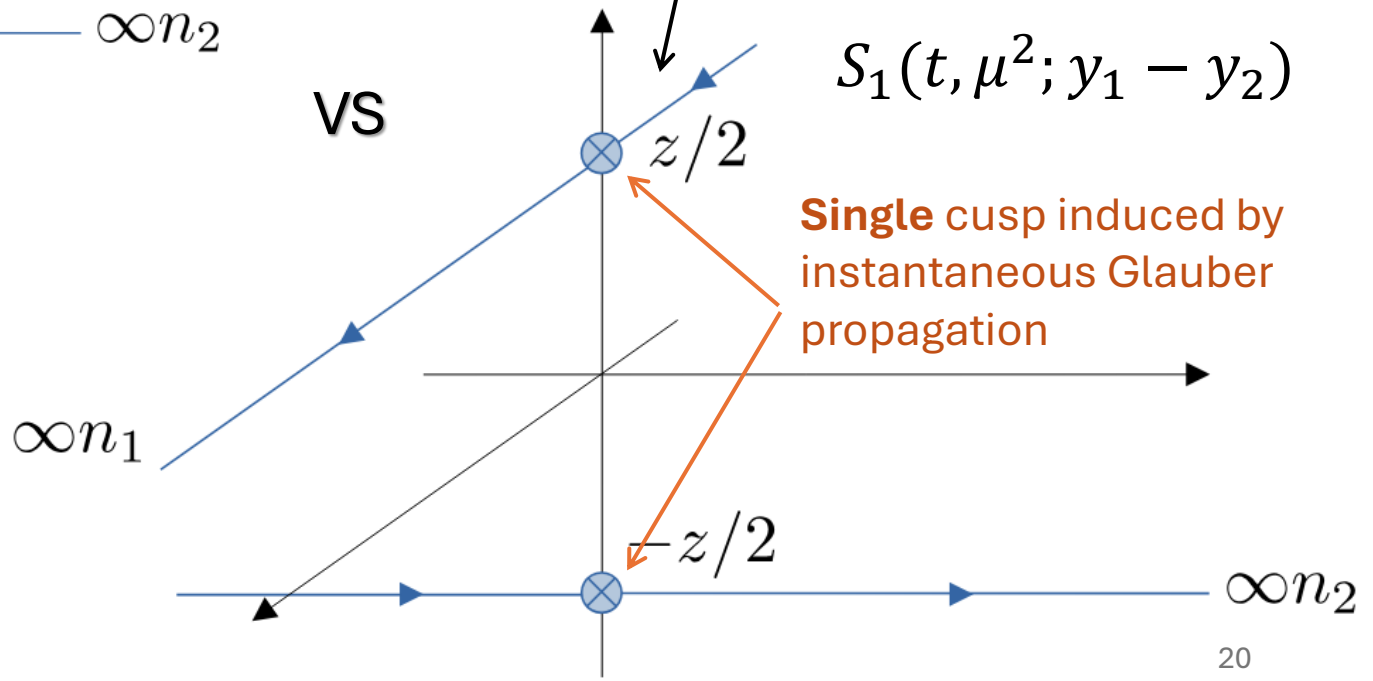
Analogies with the TMD case



Two cusps

Opposite flow in one of the branches

VS



Single cusp induced by instantaneous Glauber propagation

It seems there is an overall $-1/2$ factor between (the exponents) of the two operators

Analogies with the TMD case

$$\frac{\partial \ \log(S_{tmd}(b_T^2, \mu^2, y_1 - y_2))}{\partial y_1} = C_A \ K_G(b_T^2, \mu^2)$$

−1/2 factor **?**

$$\frac{\partial \ \log(S_1(t, \mu^2, y_1 - y_2))}{\partial y_1} = C_A \ \alpha_G(t, \mu^2)$$

Can we unravel a relationship between the Collins-Soper kernel and the gluon Regge trajectory?

From the cusp-structure of the operator S_1, we get the result

$$\frac{d \ \alpha_G(\alpha_S, \mu^2/|t|\,)}{d \ \log \mu} = \Gamma_K^{(8)}(\alpha_S)$$

N.B: right now, this is only a conjecture! Prove it rigorously would be a significant improvement.

To be compared with the known result

$$\frac{d \ K_G(\alpha_S, \mu^2 b_T^2\,)}{d \ \log \mu} = -2 \ \Gamma_K^{(8)}(\alpha_S)$$

Bridging CS-kernel with Regge trajectory

Therefore, we can define the RG-invariant combination:

$$\Delta_G(\alpha_S, \mu^2/|t|) = \alpha_G(\alpha_S, \mu^2/|t|) + \frac{1}{2} K_G(\alpha_S, \mu^2/|t|)$$

$$= \left(\frac{\alpha_t}{4\pi}\right)^2 \Delta_G^{[2]} + \left(\frac{\alpha_t}{4\pi}\right)^3 \Delta_G^{[3]} + \cdots$$

Exploiting the RG-invariance we set $\mu = |t|$ and also $\alpha_S(|t|) \equiv a_t$.

At LO we have explicitly verified that $\Delta_G^{[1]} = 0$.
Inspecting the higher orders, it appears that discrepancy operator Δ is the results *of several delicate cancellations*:

$$\alpha_G^{[2]}(\mu^2 = t) = N^2 \left(\frac{404}{27} - 2\zeta_3\right) - N\, n_f \frac{56}{27}$$

$$-\frac{1}{2} K_G^{[2]}(\mu^2 = t) = N^2 \left(\frac{404}{27} - 14\zeta_3\right) - N\, n_f \frac{56}{27}$$

$$\Delta_G^{[2]} = N^2 12\, \zeta_3$$

- Large-N (planar contributions)
- Max transcendental degree at 2-loops (rational terms cancel out)

Bridging CS-kernel with Regge trajectory

Beyond NLO, using the available prescription-dependent results, the trend seems to keep going:

$$\alpha_G^{[3]}(\mu^2 = t\,)$$
$$= N^3\left(16\,\zeta_5 + \frac{40}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 - \frac{6664}{27}\zeta_3 - \frac{3196}{81}\zeta_2 + \frac{297029}{1458}\right)$$
$$+ N^2\,n_f\left(\frac{412}{81}\zeta_2 + \frac{2}{3}\zeta_4 + \frac{632}{9}\zeta_3 - \frac{171449}{2916}\right) + N\,n_f^2\left(\frac{928}{729} - \frac{128}{27}\zeta_3\right)$$
$$+ n_f\left(-4\zeta_4 - \frac{76}{9}\zeta_3 + \frac{1711}{108}\right)$$

$$\Delta_G^{[3]} = -N^3\left[\frac{4}{27}\left(125 - 304\frac{n_f}{N} + 44\left(\frac{n_f}{N}\right)^2\right)\zeta_3 + 16\,\zeta_2\zeta_3 + 80\,\zeta_5\right]$$

$$-\frac{1}{2}K_G^{[3]}(\mu^2 = t)$$
$$= N^3\left(96\,\zeta_5 + \frac{88}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 - \frac{6164}{27}\zeta_3 - \frac{3196}{81}\zeta_2 + \frac{297029}{1458}\right)$$
$$+ N^2\,n_f\left(\frac{412}{81}\zeta_2 + \frac{2}{3}\zeta_4 + \frac{680}{27}\zeta_3 - \frac{171449}{2916}\right) + N\,n_f^2\left(\frac{928}{729} - \frac{16}{27}\zeta_3\right)$$
$$+ n_f\left(-4\zeta_4 - \frac{76}{9}\zeta_3 + \frac{1711}{108}\right)$$

Conclusions and Future perspectives

$$\alpha_G(\alpha_S, \mu^2/|t|) = -\frac{1}{2} K_G(\alpha_S, \mu^2/|t|) + \Delta_G(\alpha_S, \mu^2/|t|)$$

Easier than Regge trajectory to compute in full QCD (1 loop less)

Can it be obtained in some simpler theory? Hints from SYM N=4 in the planar limit?

There is still work to do. However, if successful, this approach can be extended to higher numbers of Glauber exchanges.
In particular, the two-Glauber case is directly related to the small-x behavior of the DIS cross section, paving the way for extending k_t-factorization beyond leading logarithmic accuracy.

Thank you!

This talk is supported by COST Action CA24159 (SHARP)