One-loop renormalization of the quark TMD in the projectile light-cone gauge

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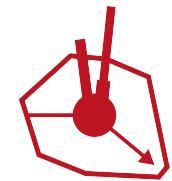


DIS2026

- Correspondence between TMD and CGC factorizations in their common validity region has been studied recently e.g. in 1101.0715, 1710.05698, 2204.11650, 2308.00022, 2405.19404, 2308.02449

- Without relying back-to-back limit, can we utilize CGC tools to study TMDs?

- We calculate NLO corrections to quark TMD with choices common to both approaches
  - Projectile light-cone gauge $A^+ = 0$
  - Background field formalism
  - Pure rapidity regulator (1812.08189 Ebert, Moult, Stewart, Tackmann, Vita, Zhu
    Application to CGC studied in 2510.08550 Altinoluk, Beuf, Penttala)

- Continuation for similar work in target light-cone gauge
  2505.20467 Altinoluk, Beuf, Jalilian-Marian

- Also related 2502.15889 Mukherjee, Skokov, Tarasov, Tiwari

Background Field Formalism

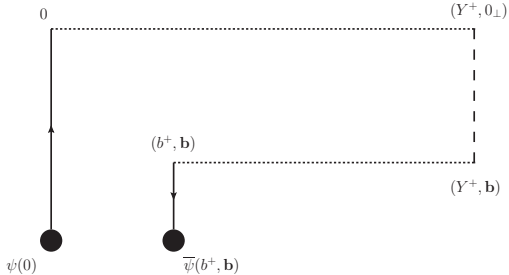
Perturbative expansion in the background field method:

$$\Psi(x) = \psi(x) + \delta\Psi(x)$$
$$A^a_\mu(x) = \mathcal{A}^a_\mu(x) + \delta A^a_\mu(x)$$

First non-zero corrections quadratic in the fluctuation fields i.e. $\sim g^2$

Dilute regime: background fields are small
$\longrightarrow$ keep only terms that are quadratic in the background fields

Quark TMD in Semi-Inclusive DIS



- Left-moving target ($P^- \gg 1$) in the projectile light-cone gauge: $A^+ = 0$
- $Y^+ \to \infty$
- Small transverse separation $\mathbf{b}$

$$q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x}, \mathbf{b}) = \lim_{Y^+ \to +\infty} \int \frac{\mathrm{d}b^+}{2\pi}\, e^{-i\mathrm{x}P^- b^+} \left\langle P \middle| \overline{\Psi}(b^+, \mathbf{b}, 0^-)\, \frac{\gamma^-}{2}\, \overbrace{U(Y^+, \mathbf{b}, 0^-; b^+, \mathbf{b}, 0^-)^\dagger}^{\text{light-like}} \right.$$
$$\times \underbrace{U(Y^+, \mathbf{b}, 0^-; Y^+, 0_\perp, 0^-)}_{\text{space-like}}\, \underbrace{U(Y^+, 0_\perp, 0^-; 0)}_{\text{light-like}}\, \Psi(0) \left| P \right\rangle$$

Here fields and coupling are renormalised, operator definition not

n.r. = not yet renormalized unsub. = rapidity divergences not yet subtracted

Quark TMD in Semi-Inclusive DIS

SIDIS: $\mathrm{d}\sigma \propto \hat{\sigma} \otimes f \otimes D$

Soft factor: removing double counting of soft gluons between TMD PDFs and TMD fragmentation functions

Included in the TMD PDFs and TMD fragmentation functions

Collins-Soper-Sterman equations

UV divergences $\longrightarrow$ Dimensional regularization scale μ

Rapidity divergences $\longrightarrow$ Rapidity regularization scale ζ

Linear evolution for both scales by CSS equations

Quark TMD in the Background Field Formalism

$$q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x},\mathbf{b}) = \lim_{Y^+\to+\infty} \int \tfrac{\mathrm{d}b^+}{2\pi}\, e^{-i\mathrm{x}P^-b^+} \left\langle P\middle|\overline{\Psi}(b^+,\mathbf{b},0^-)\,\tfrac{\gamma^-}{2}\,\overbrace{U(Y^+,\mathbf{b},0^-;b^+,\mathbf{b},0^-)^\dagger}^{\text{light-like}} \right.$$
$$\times \underbrace{U(Y^+,\mathbf{b},0^-;Y^+,0_\perp,0^-)}_{\text{space-like}}\,\underbrace{U(Y^+,0_\perp,0^-;0)}_{\text{light-like}}\,\Psi(0)\left|P\right\rangle$$

light-like:

$$U(Y^+,\mathbf{b},0^-;b^+,\mathbf{b},0^-) =$$
$$\mathcal{P}\exp\left\{-i\mu^\epsilon g\int_{b^+}^{Y^+}\mathrm{d}x^+ t^a A_a^-(x^+,\mathbf{b},0^-)\right\}$$

space-like:

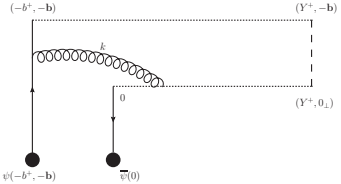
$$U(Y^+,\mathbf{x}+\mathbf{b},0^-;Y^+,\mathbf{x},0^-) =$$
$$\mathcal{P}\exp\left\{-i\mu^\epsilon g\int_0^1 d\tau\,\mathbf{b}^i\,t^a\,A_i^a(Y^+,\mathbf{x}+\tau\mathbf{b},0^-)\right\}$$

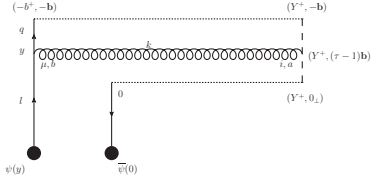
Insert perturbative expansions:

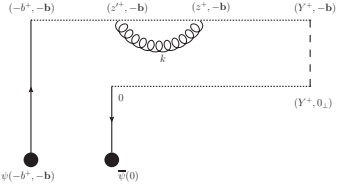
$$\Psi(x) = \psi(x) + \delta\Psi(x)$$

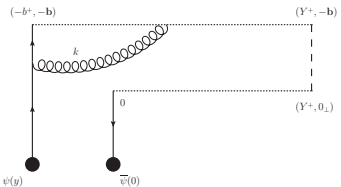
$$A^a_\mu(x) = \mathcal{A}^a_\mu(x) + \delta A^a_\mu(x)$$

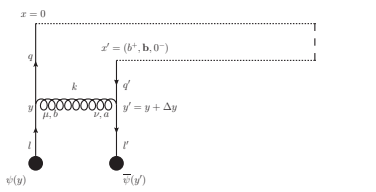
NLO corrections

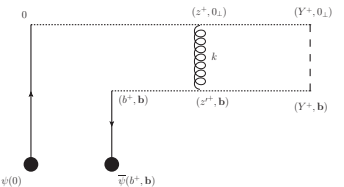


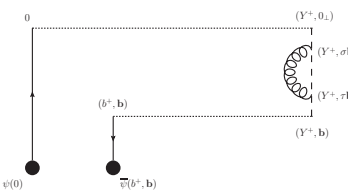


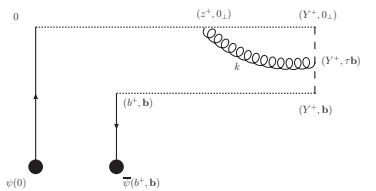






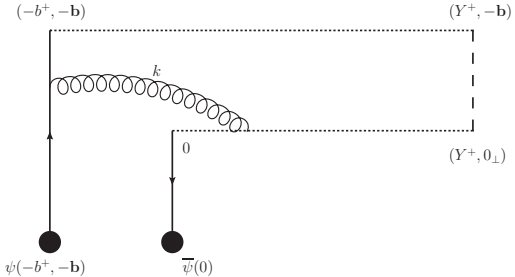






+ symmetric diagrams

Rapidity divergences



$$\tilde{G}^{\mu\nu}_{0,F}(k) = \frac{i}{(k^2+i0)}\left\{-g^{\mu\nu} + \frac{(k^\mu n^\nu + n^\mu k^\nu)}{[n\cdot k]}\right\}$$

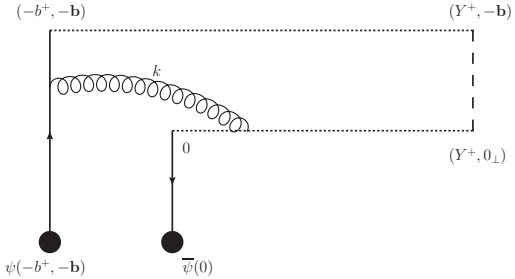
Projectile light-cone gauge: $A^+ = 0$

$\longrightarrow$ light-cone pole $\quad n\cdot k = k^+, \qquad n^- = 1$

Mandelstam-Leibbrandt (ML) prescription:

$$\frac{1}{[k^+]} = \frac{\theta(k^-)}{k^+ + i0} + \frac{\theta(-k^-)}{k^+ - i0}$$

Rapidity divergences



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$\frac{1}{[k^+]} = \frac{\theta(k^-)}{k^+ + i0} + \frac{\theta(-k^-)}{k^+ - i0}$

Rapidity divergence when $\frac{|k^+|}{|k^-|} \to \infty$
Here $k^- \to 0$

Pure-rapidity regulator:

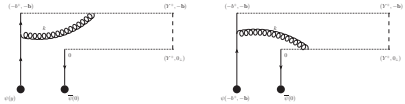
$$\omega^2\left[\frac{|k^-|}{|k^+|}\,\frac{\nu^+}{\nu^-}\right]^{\eta/2} \xrightarrow{k^2=0} \omega^2\left[\frac{2|k^-|^2}{\mathbf{k}^2}\,\frac{\nu^+}{\nu^-}\right]^{\eta/2}$$

which can be added after integration over k^+ $\qquad \zeta \equiv \frac{2(xP^-)^2\nu^+}{\nu^-}$

Bookkeeping factor $\lim_{\eta\to 0}\omega^2 = 1$ and $\frac{\nu^+}{\nu^-}\frac{\partial}{\partial\left(\frac{\nu^+}{\nu^-}\right)}\omega^2 = -\frac{\eta}{2}$ (1812.08189, 1104.0881, 1202.0814 See talk by G. Beuf Wednesday)

Rapidity Divergences

Double pole: $\frac{1}{\epsilon} \times \frac{1}{\eta}$ from



+ symmetric diagrams

$$\frac{\alpha_s C_F}{\pi} (\pi\mu^2 \mathbf{b}^2)^\epsilon \left[\frac{\zeta \mathbf{b}^2}{4}\right]^{\frac{\eta}{2}} \frac{\Gamma\left(-\frac{\eta}{2}-\epsilon\right)}{\Gamma\left(1+\frac{\eta}{2}\right)} \frac{\omega^2}{\eta} \int \frac{\mathrm{d}y^+}{2\pi} e^{ixP^- y^+} \langle P|\mathcal{T}\, \overline{\psi}(0)\, \frac{\gamma^-}{2}\, \psi(y^+, -\mathbf{b}, 0^-)|P\rangle_c$$

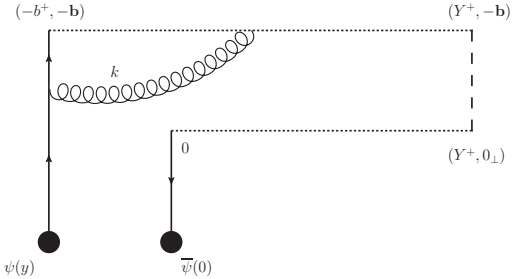
$$= \frac{\alpha_s C_F}{2\pi} \underbrace{\Gamma(-\epsilon)}_{\frac{1}{\epsilon}\text{ pole}} (\pi\mu^2 \mathbf{b}^2)^\epsilon \left[\frac{2\omega^2}{\eta} + \log\left(\frac{\zeta \mathbf{b}^2}{c_0^2}\right) - \underbrace{\Psi(-\epsilon)}_{\frac{1}{\epsilon}\text{ pole}} + \Psi(1)\right] q_{\mathrm{Bckgd}}(x, \mathbf{b}; \mu^2)$$

where $\zeta \equiv \frac{2(xP^-)^2\nu^+}{\nu^-}$, $\quad c_0 = 2e^{-\gamma_E}$

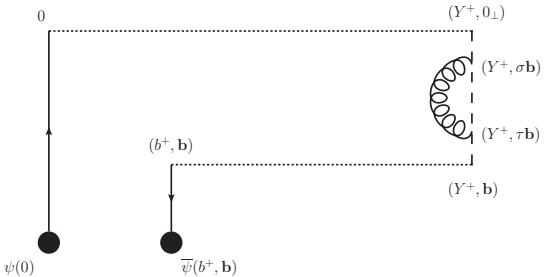
Rapidity renormalization factor: $\quad Z_{\mathrm{rap.}} = 1 - \frac{\alpha_s C_F}{\pi} \Gamma(-\epsilon)(\pi\mu^2 \mathbf{b}^2)^\epsilon \frac{\omega^2}{\eta} + \mathcal{O}(\alpha_s^2)$

here we used Dirac equation: $\alpha_s \int \mathrm{d}^{4-\epsilon} y e^{iy\cdot l} \langle P|\bar{\psi}(0)\gamma^- \not{l} \psi(y)|P\rangle \propto \mathcal{O}(\alpha_s^2)$ for massless quarks

UV Divergences



$$\frac{\alpha_s C_F}{2\pi}\,(4\pi\mu^2)^\epsilon \frac{\Gamma(2-\epsilon)}{\Gamma(1-\epsilon)}\Gamma(\epsilon)\; q_{\mathrm{Bckgd}}(x, \mathbf{b}; \mu^2)$$

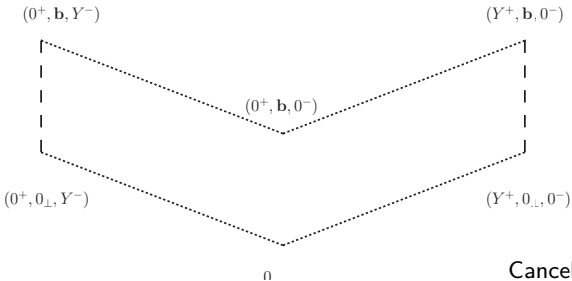


Also UV pole from Wilson line self-energy at infinity
(will cancel with the soft factor)

Soft Factor

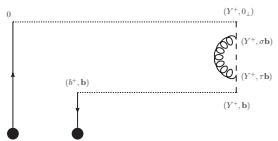
$$S(\mathbf{b}) = 1 + \frac{T_F}{N_c}\langle 0|\mathcal{TP}\frac{1}{2}\left\{-ig\mu^\epsilon \int_l \mathrm{d}x^\mu t^a A^a_\mu(x)\right\}^2 |0\rangle + \mathcal{O}(g^3)$$

$$= 1 - \frac{g^2\mu^{2\epsilon}C_F}{2}\int_l \mathrm{d}x^\mu \int_l \mathrm{d}y^\nu G_{0F,\mu\nu}(x,y)$$



In light-cone gauge only contribution from Wilson line self-energies at opposite infinities

Cancels with diagram:



$$S(\mathbf{b}) = 1 + \frac{\alpha_s C_F}{\pi}(\pi\mu^2\mathbf{b}^2)\frac{\Gamma(1-\epsilon)}{\epsilon(1-2\epsilon)}$$

$$\frac{1}{\sqrt{\mathcal{S}(\mathbf{b})}} q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x},\mathbf{b};\mu^2)$$

Rapidity Renormalization

$$Z_{\text{rap.}} = 1 - \frac{\alpha_s C_F}{\pi}\Gamma(-\epsilon)(\pi\mu^2\mathbf{b}^2)^\epsilon \frac{\omega^2}{\eta} + \mathcal{O}(\alpha_s^2)$$

$$Z_{\text{rap.}} = 1 - \frac{\alpha_s^{(0)} C_F}{\pi}\Gamma(-\epsilon)(\pi\mathbf{b}^2)^\epsilon \frac{\omega^2}{\eta} + \mathcal{O}(\alpha_s^{(0)2}) \quad \text{where} \quad \alpha_s^{(0)} = \mu^{2\epsilon}\alpha_s(\mu^2) + \mathcal{O}(\alpha_s^2)$$

$$q^{\text{n.r.}}_{\text{sub.}}(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \lim_{\eta\to 0} Z_{\text{rap.}} \frac{1}{\sqrt{\mathcal{S}(\mathbf{b})}} q^{\text{n.r.}}_{\text{unsub.}}(\mathrm{x}, \mathbf{b}; \mu^2)$$

Renormalization in Light-Cone Gauge with ML prescription

Quark field renormalization factor $Z_2\tilde{Z}_2 = 1 - \frac{\alpha_s C_F}{4\pi}\frac{S_\epsilon}{\epsilon}$

(for $\frac{\not{n}\not{\bar{n}}}{2\bar{n}\cdot n}\Psi^{(0)}$ in projectile L-C gauge with ML-prescription)

$$q(x,\mathbf{b};\mu^2,\zeta) = Z^{UV}\, Z_2\tilde{Z}_2 \overbrace{q^{\text{n.r}}_{\text{sub.}}(x,\mathbf{b};\mu^2,\zeta)}^{1/\eta \text{ subtracted}}$$

$$Z_2\tilde{Z}_2 q^{\text{n.r}}_{\text{sub.}}(x,\mathbf{b};\mu^2,\zeta) = q_{\text{Bckgd}}(x,\mathbf{b};\mu^2)\left\{1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{S_\epsilon}{\epsilon^2} + \left(\log\left(\frac{\mu^2}{\zeta}\right) + \frac{3}{2}\right)\frac{S_\epsilon}{\epsilon}\right]\right\} + \text{finite NLO} + \mathcal{O}(\alpha_s^2)$$

$$\longrightarrow \quad Z_{\text{UV}} = 1 - \frac{\alpha_s C_F}{2\pi}\left[\frac{S_\epsilon}{\epsilon^2} + \left(\log\left(\frac{\mu^2}{\zeta}\right) + \frac{3}{2}\right)\frac{S_\epsilon}{\epsilon}\right] + \mathcal{O}(\alpha_s^2) \qquad \text{here } S_\epsilon = (4\pi e^{-\gamma_E})^\epsilon$$

$Z_{\text{rap.}}$ and Z_{UV} stay same in target and projectile L-C gauges

CSS equations

$Z_{\text{rap.}} = 1 - \frac{\alpha_s^{(0)} C_F}{\pi} \Gamma(-\epsilon) (\pi \mathbf{b}^2)^{\epsilon} \frac{\omega^2}{\eta} + \mathcal{O}(\alpha_s^{(0)2})$ where $\alpha_s^{(0)} = \mu^{2\epsilon} \alpha_s(\mu^2) + \mathcal{O}(\alpha_s^2)$

$Z_{\text{UV}} = 1 - \frac{\alpha_s C_F}{2\pi} \left[\frac{S_\epsilon}{\epsilon^2} + \left(\log\left(\frac{\mu^2}{\zeta} \right) + \frac{3}{2} \right) \frac{S_\epsilon}{\epsilon} \right] + \mathcal{O}(\alpha_s^2)$ $\zeta \equiv \frac{2(\mathrm{x}P^-)^2 \nu^+}{\nu^-}$

$$\mu \frac{d}{d\mu} \log q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \mu \frac{d}{d\mu} \log Z_{UV}$$

$$\zeta \frac{d}{d\zeta} \log q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \zeta \frac{d}{d\zeta} \log Z_{UV} + \zeta \frac{d}{d\zeta} \log Z_{\text{rap.}}$$

$$\mu^2 \frac{d}{d\mu^2} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \left\{ \frac{\alpha_s C_F}{2\pi} \left[\log\left(\frac{\mu^2}{\zeta} \right) + \frac{3}{2} \right] + O(\alpha_s^2) \right\} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

$$\zeta \frac{d}{d\zeta} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta) = \left\{ -\frac{\alpha_s C_F}{2\pi} \log\left(\frac{\mu^2 \mathbf{b}^2}{c_0^2} \right) + O(\alpha_s^2) \right\} q(\mathrm{x}, \mathbf{b}; \mu^2, \zeta)$$

where $c_0 = 2e^{-\gamma_E}$

Summary

- Goal: a unified framework to understand better the connections between the TMD and CGC approaches
- We calculated NLO corrections to unpolarized quark TMD in SIDIS using:
  - Background field formalism to obtain perturbative corrections
  - Projectile light-cone gauge $A^+ = 0$
  - ML prescription for L-C pole
  - Pure rapidity regulator
- We recovered the Collins-Soper-Sterman equations from rapidity and UV renormalization
- In the future:
  - Other prescriptions (Principal Value, Retarded, Advanced)
  - NLO corrections for gluon TMD

Backup: Other prescriptions

$$\tilde{G}^{\mu\nu}_{0,F}(k) = \frac{i}{(k^2 + i0^+)} \left\{ -g^{\mu\nu} + \frac{1}{2}(k^\mu n^\nu + n^\mu k^\nu) \left[\frac{1}{(n\cdot k) + i\epsilon} + \frac{1}{(n\cdot k) - i\epsilon} \right] \right.$$
$$\left. + i\pi(1-2\beta)\left[k^\mu n^\nu - n^\mu k^\nu\right]\delta(n\cdot k) \right\}$$

Extracting η pole

$$\int_0^1 \mathrm{d}z(1-z)^{\eta-1} f(z) = f(1)\int_0^1 \mathrm{d}z(1-z)^{\eta-1} + \int_0^1 \mathrm{d}z(1-z)^{\eta-1}\left[f(z)-f(1)\right]$$
$$= \frac{f(1)}{\eta} + \text{ rapidity-finite terms}$$

Backup: Diagram 6

$$diag.6 =$$

$$\frac{\alpha_s \mu^{2\epsilon} C_F}{2\pi} \int \mathrm{d}^{4-2\epsilon} y \, \langle P | \mathcal{T} \, \overline{\psi}_\alpha(0) \, \psi_\beta(y) | P \rangle_c \, e^{i \mathrm{x} P^- y^+} \int \frac{\mathrm{d}^{2-2\epsilon} \mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d} l^+}{(2\pi)} \, e^{i(l^+ y^- - \mathbf{l}\cdot(\mathbf{y}+\mathbf{b}))}$$

$$\times \int \frac{\mathrm{d}^{2-2\epsilon} \mathbf{k}}{(2\pi)^{2-2\epsilon}} \, \frac{e^{i\mathbf{k}\cdot\mathbf{b}}}{\mathbf{k}^2 \left[2\mathrm{x} P^- l^+ - (\mathbf{l}-\mathbf{k})^2\right]} \omega^2 \left[\frac{2\nu^+}{\mathbf{k}^2 \nu^-}\right]^{\frac{\eta}{2}} (\mathrm{x} P^-)^\eta \int_0^1 \mathrm{d}z \, (1-z)^{\eta-1} z^{-1-\eta} \, e^{i \mathrm{x} P^- y^+ \frac{1-z}{z}}$$

$$\times \left[-1 + \frac{1-z}{(1-z) - \frac{z\mathbf{k}^2}{[2\mathrm{x} P^- l^+ - (\mathbf{l}-\mathbf{k})^2]}}\right] \left\{ \gamma^- \left[\mathrm{x} P^- \gamma^+ - (\mathbf{l}-\mathbf{k})\cdot\gamma\right] \left[2\gamma^+ \mathrm{x} P^- \frac{1-z}{z} - \mathbf{k}\cdot\gamma\right] \right\}_{\alpha\beta}$$

where $k^- \to z \equiv \mathrm{x} P^- \frac{1-z}{z}$

Backup: Diagram 4

$$
\begin{aligned}
diag.4 = &-\frac{\alpha_s \mu^{2\epsilon} C_F}{2\pi} \int \mathrm{d}^{4-2\epsilon} y \, \langle P | \overline{\psi}_\alpha(0) \, \psi_\beta(y) | P \rangle \, e^{i \mathrm{x} P^- y^+} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d}l^+}{(2\pi)} \\
&\times \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} \, \omega^2 \left[\frac{2\nu^+}{\mathbf{k}^2 \nu^-} \right]^{\frac{\eta}{2}} e^{i l^+ y^-} e^{-i \mathbf{l}\cdot(\mathbf{y}+\mathbf{b})} \int_0^1 \mathrm{d}\xi \frac{1-\xi}{\xi} \, \xi^\eta (\mathrm{x} P^-)^\eta \\
&\times \frac{\left\{ \gamma^- \left[-(\mathbf{l}-\mathbf{k}) \cdot \gamma_\perp (2\mathrm{x} P^- \xi \gamma^+ - \mathbf{k}\cdot\gamma_\perp) - \mathrm{x} P^- (1-\xi) \gamma^+ \mathbf{k}\cdot\gamma_\perp \right] \right\}_{\alpha\beta}}{\left[2\mathrm{x} P^- (1-\xi) l^+ - (\mathbf{l}-\mathbf{k})^2 \right] \left[2\mathrm{x} P^- \xi (1-\xi) l^+ - \xi (\mathbf{l}-\mathbf{k})^2 - (1-\xi)\mathbf{k}^2 \right]}
\end{aligned}
$$

where $k^- \to \xi \equiv k^- / \mathrm{x} P^-$

Backup: Double pole from diags. 4 and 6

$$
-\frac{\alpha_s \mu^{2\epsilon} C_F}{2\pi} [\mathrm{x}P^-]^\eta \left[\frac{2\nu^+}{\nu^-}\right]^{\frac{\eta}{2}} \frac{\omega^2}{\eta} \int \mathrm{d}^{4-2\epsilon} y \, \langle P|\overline{\psi}(0)\gamma^- \psi(y)|P\rangle \, e^{i\mathrm{x}P^- y^+}
$$

$$
\times \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{l}}{(2\pi)^{2-2\epsilon}} \int \frac{\mathrm{d}l^+}{(2\pi)} \, e^{il^+ y^-} e^{-i\mathbf{l}\cdot(\mathbf{y}+\mathbf{b})} \int \frac{\mathrm{d}^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}} \frac{1}{[\mathbf{k}^2]^{1+\frac{\eta}{2}}} \left[1 - e^{i\mathbf{k}\cdot\mathbf{b}}\right]
$$