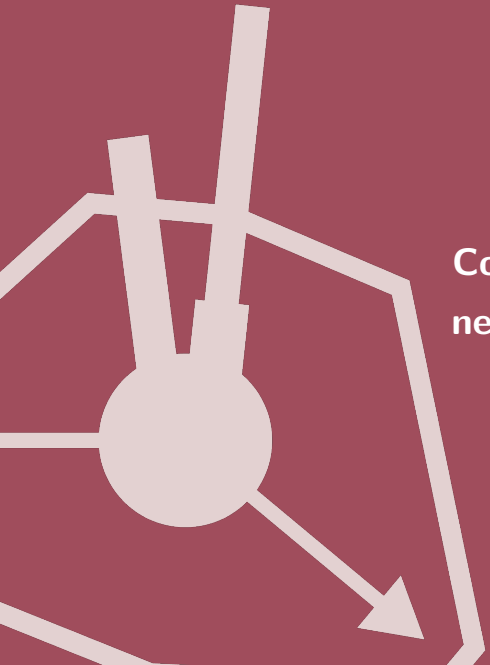
Confronting CGC at NLO with HERA data & Probing gluon saturation through SIDIS at the EIC

Speakers: Carlisle Casuga & Swaleha Mulani



Confronting Color Glass Condensate at next-to-leading order with HERA data

C. Casuga, H. Mäntysaari

ArXiv: 2604.22332

Inference of the Universal Dipole Amplitude

- Promote previous analyses to NLO+NLL (NLO hard factors + NLO BK with resummation of large double and single logs)
- Use Bayesian inference setup to provide statistically rigorous estimates of the BK initial condition uncertainty
- Constrain to both total inclusive and heavy quark data and account for experimental correlated systematic uncertainties

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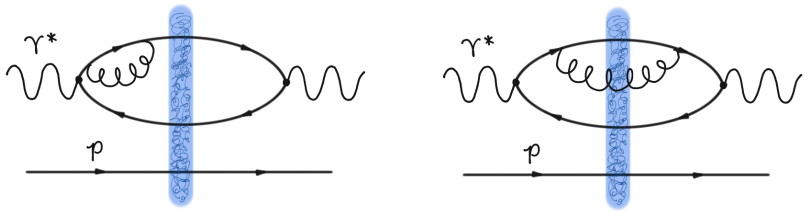
DIS at NLO+NLL



In the unsubtracted scheme,

$$\sigma_{\mathrm{L,T}}^{\mathrm{NLO}} = \sigma_{\mathrm{L,T}}^{\mathrm{IC}} + \sigma_{\mathrm{L,T}}^{\mathrm{dip}} + \sigma_{\mathrm{L,T}}^{qg} \longrightarrow \sigma \sim \mathcal{N}_{\mathrm{NLO}} \otimes \left[\begin{array}{c} \mathrm{NLO} \\ \mathrm{Hard} \\ \mathrm{Factor} \end{array} \right]$$

NLO cross section: implementation using 2604.09071 based on G. Beuf, H. Hänninen, T. Lappi, H. Mäntysaari (2020) 2007.01645; and other papers
NLO BK numerical solution: T. Lappi and H. Mäntysaari (2016) 1601.06598; and other papers

DIS at NLO+NLL

- Dipoles are evolved with the NLO BK which includes resummation of enhanced double and single logarithms + other finite α_s^2 terms

Initial Condition: MV model - inspired parametrization

$$\mathcal{N}(\mathbf{r}, x = x_0) = 1 - \exp\left[-\frac{1}{4}(\mathbf{r}^2 Q_{s0}^2)^\gamma \ln(1/\mathbf{r}\Lambda + e_c \cdot e)\right]$$

Running coupling in coordinate space: $\alpha_s(\mathbf{r}) \sim [\ln(C^2/\mathbf{r}^2\Lambda^2)]^{-1}$

- scale set by Balitsky + smallest dipole (Bal + SD) or parent dipole scheme

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Bayesian Methodology

Gaussian Process Emulator

- Emulator learns dependence of model on parameters:

$$\vec{\theta} \;=\; [Q_{s0}^2, \gamma, C^2, \sigma_0/2, m_c]$$

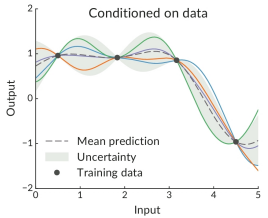

Image from: J. Bernhard Ph.D. Thesis (2018)

Markov Chain Monte Carlo

- Walkers converges to areas of high posterior

$$P(\theta|\text{data}) \;=\; \text{likelihood} \times \text{prior}$$

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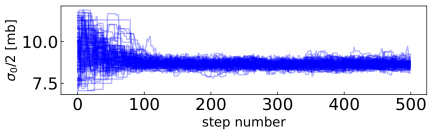
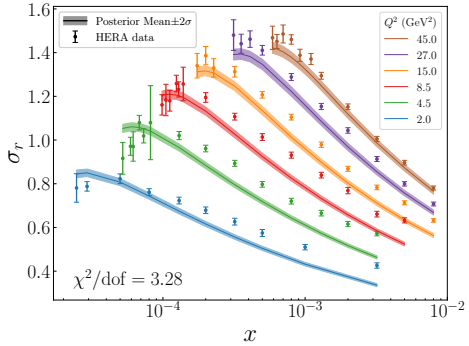
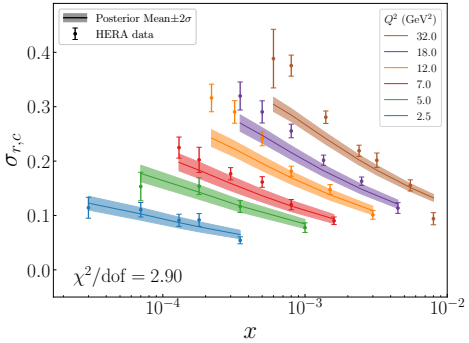


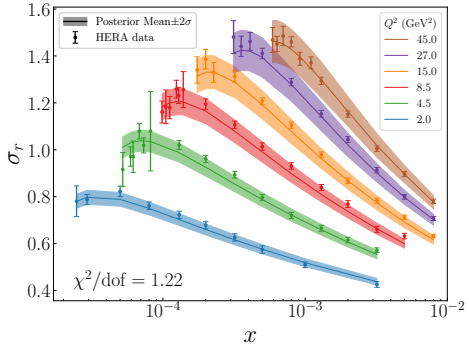
Results: MV initial condition & Bal + sd running coupling

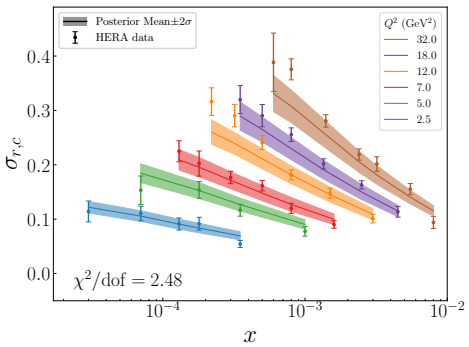




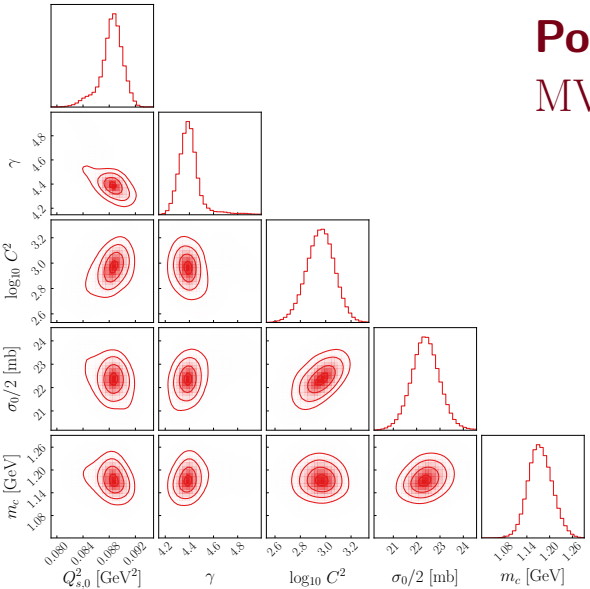
- Parametrization not flexible enough to describe total inclusive (left) and charm contribution (right) data

Results: MV^γ initial condition & $\mathrm{Bal}+\mathrm{sd}$ running coupling





- Meanwhile, better agreement achieved with additional γ parameter



Posterior distribution for MV^γ initial condition

- Well-constrained parameters
- Datasets are well-described
- Large values of C^2: evolution has to be significantly slowed down.

* $\mathrm{Bal} + \mathrm{SD}$ running coupling scale

This study

. . . **is the first global analysis of the dipole model at NLO, including double and single log resummation in the NLO BK.**

. . . provides probability distribution of the parameters in the BK initial condition

! necessary input for propagating uncertainty for all NLO CGC calculations e.g. observables for SIDIS γ^*-hadron scattering

. . . obtains a good description of small-x DIS data.

In the future:

- Effect of finite energy constraint to DIS to dipole initial condition (see M. Bertilsson's talk, Wed 11:00 am)
- Fourier positive initial condition?

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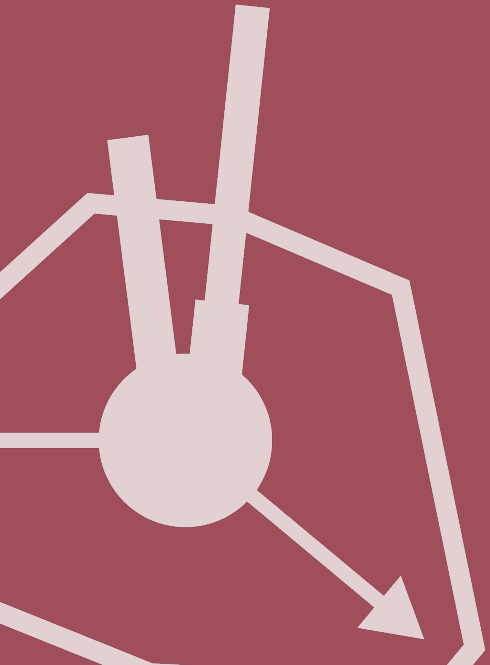

Probing Gluon Saturation through Semi-inclusive Hadron Production at the EIC

S. Mulani, C. Casuga, H. Mäntysaari

(in preparation)

Motivation

- Signatures of gluon saturation, Color Glass Condensate (CGC): one of the main goals for Electron Ion Collider (EIC).
- Semi-inclusive deep inelastic scattering (SIDIS)
  - More differential : potentially powerful probe for saturation signals.
  - Allows differential studies in x, Q^2 and additional scale hadron transverse momenta $P_\perp$.
- Till now, Leading Order (LO) SIDIS cross section not studied numerically in dipole picture.
- Study if SIDIS analysis brings complimentary constraints to initial fit conditions.

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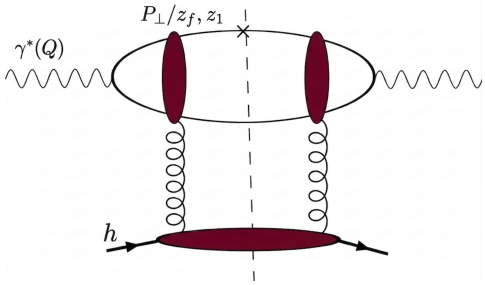
Cross Section in Coordinate Space

- Differential cross section for SIDIS γ^*-hadron scattering is

$$\frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^* h}}{\mathrm{d}^2 P_\perp \, \mathrm{d}y} \propto \int \mathrm{d}z_f \, \frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^* + h \to q + X}}{\mathrm{d}(P_\perp / z_f) \, \mathrm{d}y} \otimes \mathcal{D}(P_\perp, z_f)$$



Marquet-Xiao-Yuan [arXiv:0906.1454]

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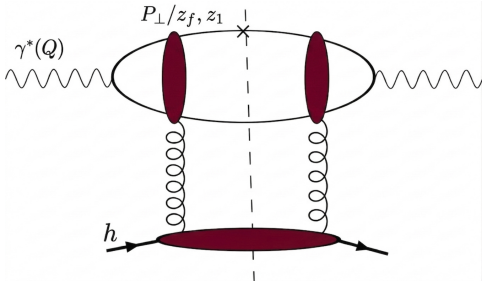

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LO FF : Bertone-Carrazza-Hartland-Nocera-Rojo [arXiv:1706.07049v1 [hep-ph]]

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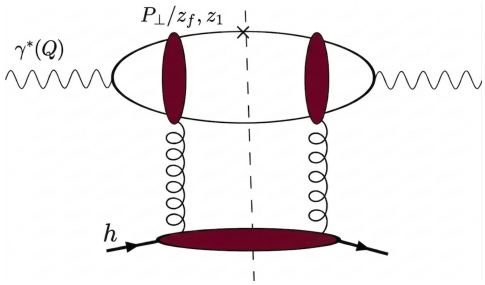
partonic cross section



Marquet-Xiao-Yuan [arXiv:0906.1454]

In coordinate space,

$$\frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^* + h \to q + X}}{\mathrm{d}\mathbf{p} \, \mathrm{d}y} \propto \int \mathrm{d}^2\mathbf{b} \, \mathrm{d}^2\mathbf{r} \, \mathrm{d}^2\mathbf{r}' \, e^{i(\mathbf{r}-\mathbf{r}')\cdot\mathbf{p}} \, \mathcal{H}_{\mathrm{L,T}}(\mathbf{r}, \mathbf{r}', z_1) \otimes \mathcal{N}(\mathbf{r}, \mathbf{b})$$

Hard Factor — Dipole

Cross Section in momentum Space

- Presence of modified Bessel functions in $\mathcal{H}$ give rise to numerical instability at large $P_\perp$

$$\frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^*+h\to q+X}}{\mathrm{d}\mathbf{p}\,\mathrm{d}y} \propto \int \mathrm{d}^2\mathbf{b}\,\mathrm{d}^2\mathbf{r}\,\mathrm{d}^2\mathbf{r}'\,e^{i(\mathbf{r}-\mathbf{r}')\cdot\mathbf{p}}\,\mathcal{H}_{\mathrm{L,T}}(\mathbf{r},\mathbf{r}',z_1)\otimes\mathcal{N}(\mathbf{r},\mathbf{b})$$

- **Solution** : Taking Fourier Transform $\rightarrow$ Isolate numerically challenging momentum space dipole $\tilde{S}$.
- Partonic cross section in momentum space given as :

$$\frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^*+h\to q+X}}{\mathrm{d}\mathbf{p}\,\mathrm{d}y} \propto \sum_f e_f^2 \int d^2\mathbf{b} \int d^2\mathbf{k}\;\tilde{S}(\mathbf{k},\mathbf{b})\otimes\mathcal{H}_{\mathrm{L,T}}(\mathbf{p},\mathbf{k},z_1)$$

Dipole Amplitude

In momentum space

$$\tilde{S}(\mathbf{k},\mathbf{b}) = \int \mathrm{d}^2\mathbf{r}\ \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}}\ [1-\mathcal{N}(\mathbf{r},\mathbf{b})]$$

- $\mathcal{N}(\mathbf{r},Y)$ fitted at different rapidities Y to $F(\mathbf{r}) = 1-\exp\left[\left(-\left(\frac{(\mathbf{r}^2Q_s^2)^\gamma}{4}\ln\left(\frac{1}{\mathbf{r}\Lambda}+e\cdot e_c\right)\right)^p\right)^{\frac{1}{p}}\right]$.
- For analytical $\mathcal{N}$, numerically stable Fourier transform computed using standard Hankel transform algorithm.
- e_c, infrared cut-off in the MV model
- $e_c = 1$ **for MV model** and $e_c \neq 1$ for MV^e model.

MV, MV^e model fit :T. Lappi and H. Mäntysaari [arXiv:1309.6963], MV^e Posterior : C. Casuga, M. Karhunen and H. Mäntysaari [arXiv :2311.10491]

Partonic Cross Section

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γ^*-Proton Scattering

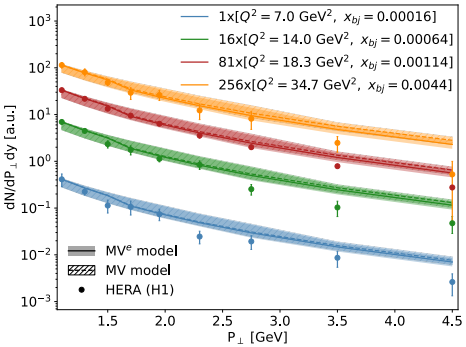
- proton transverse area, $2\int \mathrm{d}^2b \to \sigma_0$
- $\tilde{S}(\mathbf{k},\mathbf{b}) \to \tilde{S}(\mathbf{k})$

γ^*-Nucleus Scattering

$$\frac{\mathrm{d}\sigma_{\mathrm{L,T}}^{\gamma^*+h\to q+X}}{\mathrm{d}\mathbf{p}\,\mathrm{d}y} \propto 2\pi \int_0^{b_0} \mathrm{d}\mathbf{b}\ \mathbf{b}\tilde{S}(\mathbf{k},\mathbf{b}) + 2\pi \int_{b_0}^{\infty} \mathrm{d}\mathbf{b}\ \mathbf{b} A T_A(\mathbf{b}) \frac{\sigma_0}{2} \tilde{S}(\mathbf{k})$$

- $Q_s^2(b_0) = Q_s^2(\text{Proton})$

Comparison with HERA



band $\equiv 2\ \sigma$ uncertainty w.r.t. FF scale

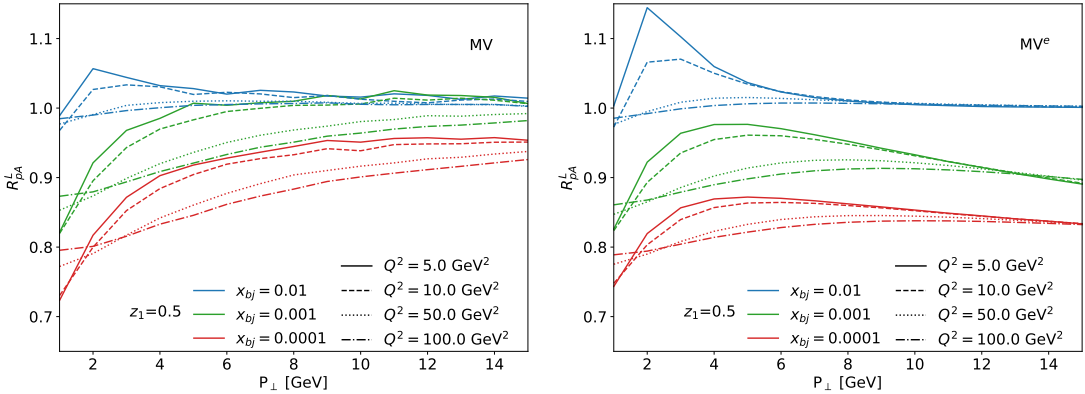
- Compared with HERA data of H1 collaboration from hep-ex/9610006 [hep-ex].
- MV and MV^e models are used as initial condition for the BK evolution : Both are compatible at LO to SIDIS HERA data.

Nuclear Modification Factor

Nuclear modification factor $$R_{pA} = \frac{\dfrac{\mathrm{d}\sigma^{\gamma^* A}}{\mathrm{d}^2 P_\perp \, \mathrm{d}z_1}}{A \times \dfrac{\mathrm{d}\sigma^{\gamma^* P}}{\mathrm{d}^2 P_\perp \, \mathrm{d}z_1}}$$

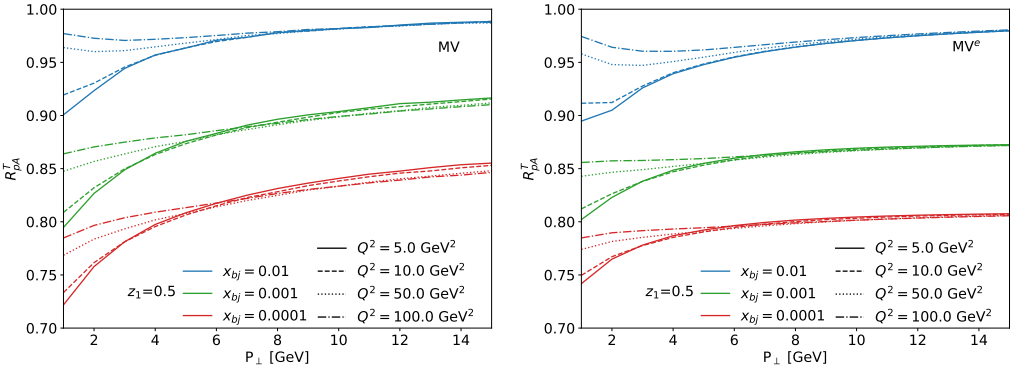
- Good indicator of saturation effects in nucleus.
- At the initial condition, in dilute limit for large $P_\perp$ values $R_{pA} \to 1$.
- We consider $A = 197$ (Au).

Predictions for EIC kinematics : Longitudinal



- Strong x_{bj} dependent suppression predicted for EIC.
- Different behavior w.r.t. initial condition parametrization
  - Due to MV^e dipole evolving to asymptotic shape
  - MV dipole is much closer to BK solution

Predictions for EIC kinematics : Transverse



- More suppression at higher $P_\perp$ in MV^e model
- Total R_{pA} is dominated by transversely polarized photon : Similar behaviour

Summary

- Semi-inclusive hadron production cross section computed at leading order for both proton and Au in dipole picture.
- Both MV and MV^e models independently fitted to inclusive HERA data.
- Both MV and MV^e models at LO are compatible with H1 SIDIS spectra from H1.
- Gluon saturation at small-x studied in case of SIDIS at LO for EIC kinematics.
- Different functional forms of BK equation results in different predictions for R_{pA} in SIDIS that provide constraint to BK initial conditions.

Initial Stages 2027 at Jyväskylä

- The 9th Initial Stages 2027 conference (IS2027)
- **June 7-11, 2027**, Jyväskylä Finland
- International flights to Helsinki + domestic connection
- Organized by Jyväskylä QCD theory + ALICE groups
- https://indico.global/event/15638/

Back-up slides

Parameters

$$\mathcal{N}(\mathbf{r}, x_0) \approx 1 - \exp\left[\frac{-(\mathbf{r}^2 Q_{s,0}^2)^{\gamma}}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

- $\sigma_0/2$, half of normalization to the cross section, proton transverse area, $2\int \mathrm{d}^2 b \to \sigma_0$
- $Q_{s,0}^2$, **related** to the saturation scale at initial x.
- C^2, connects the running coupling in $\mathbf{r}$ to its Fourier transform
- γ, anomalous dimension, controlling the steepness of the cross section related to its fall-off for small dipoles
- e_c, infrared cut-off in the MV model
- m_c, charm quark mass

Cross Section in momentum Space

Partonic cross section in momentum space is given as :

For Longitudinal Polarization :

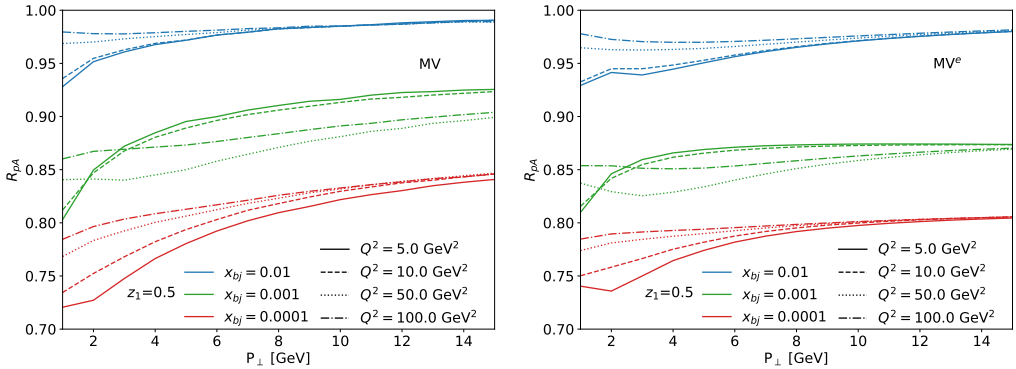
$$\frac{d^2\sigma_{\rm L}}{d^2\mathbf{p}} \propto \sum_f e_f^2 \int d^2\mathbf{b} \int d^2\mathbf{k} \int dz_1 \; \epsilon^2 \left[8z_1(1-z_1)\right] \tilde{S}(\mathbf{k},\mathbf{b}) \left[\frac{1}{\epsilon^2+\mathbf{p}^2} - \frac{1}{\epsilon^2+(\mathbf{p}-\mathbf{k})^2}\right]^2$$

For Transverse Polarization :

$$\frac{d^2\sigma_{\rm T}}{d^2\mathbf{p}} \propto \sum_f e_f^2 \int d^2\mathbf{b} \int d^2\mathbf{k} \int dz_1 \; \tilde{S}(\mathbf{k},\mathbf{b}) \Bigg\{ 2\left(z_1^2 + (1-z_1)^2\right) \times \left[\frac{\mathbf{p}}{\epsilon^2+\mathbf{p}^2} - \frac{\mathbf{p}-\mathbf{k}}{\epsilon^2+(\mathbf{p}-\mathbf{k})^2}\right]^2 + m_f^2 \left[\frac{1}{\epsilon^2+\mathbf{p}^2} - \frac{1}{\epsilon^2+(\mathbf{p}-\mathbf{k})^2}\right]^2 \Bigg\}$$

Predictions for EIC kinematics

Total Nuclear Modification Factor (L + T) :



Predictions for EIC kinematics

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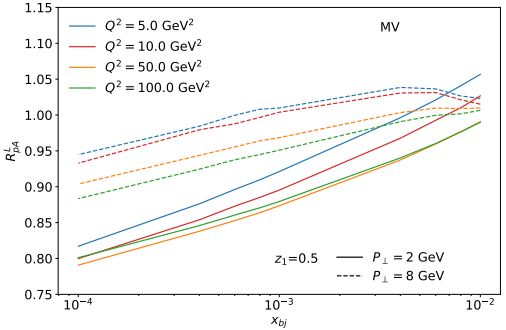
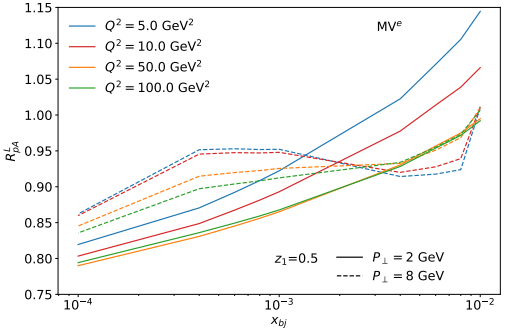
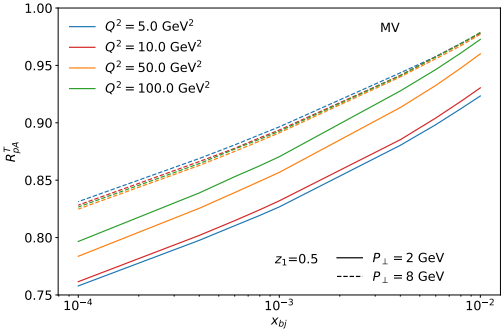


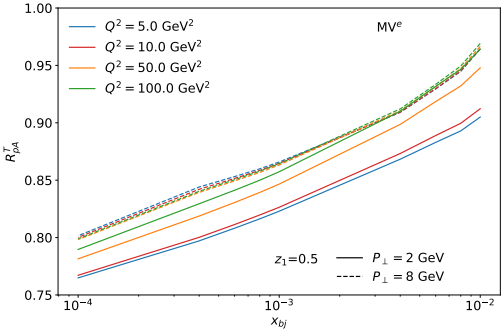


- At higher $P_\perp$ change in the evolution of dipole is more visible.

Predictions for EIC kinematics

For Transverse Polarization :





- Q^2 dependence is very weak with increasing $P_\perp$