The fragmentation mechanism in di-quarkonia production

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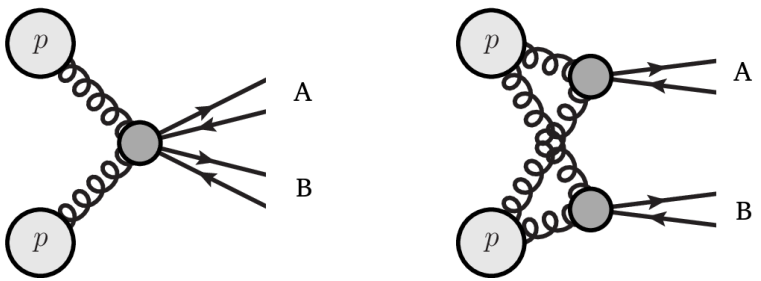
Heavy quarkonia (bound states of $c\bar{c}$ or $b\bar{b}$)

Production mechanism:

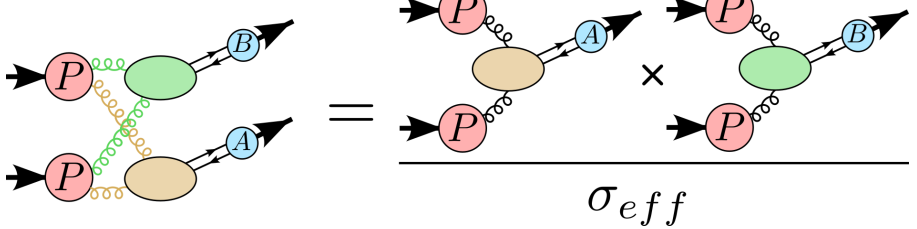
- ⋆ Color evaporation model (CEM)
- ⋆ Non-relativistic QCD (NRQCD)
- ⋆ Color-singlet model (part of NRQCD)

Why double quarkonia production?

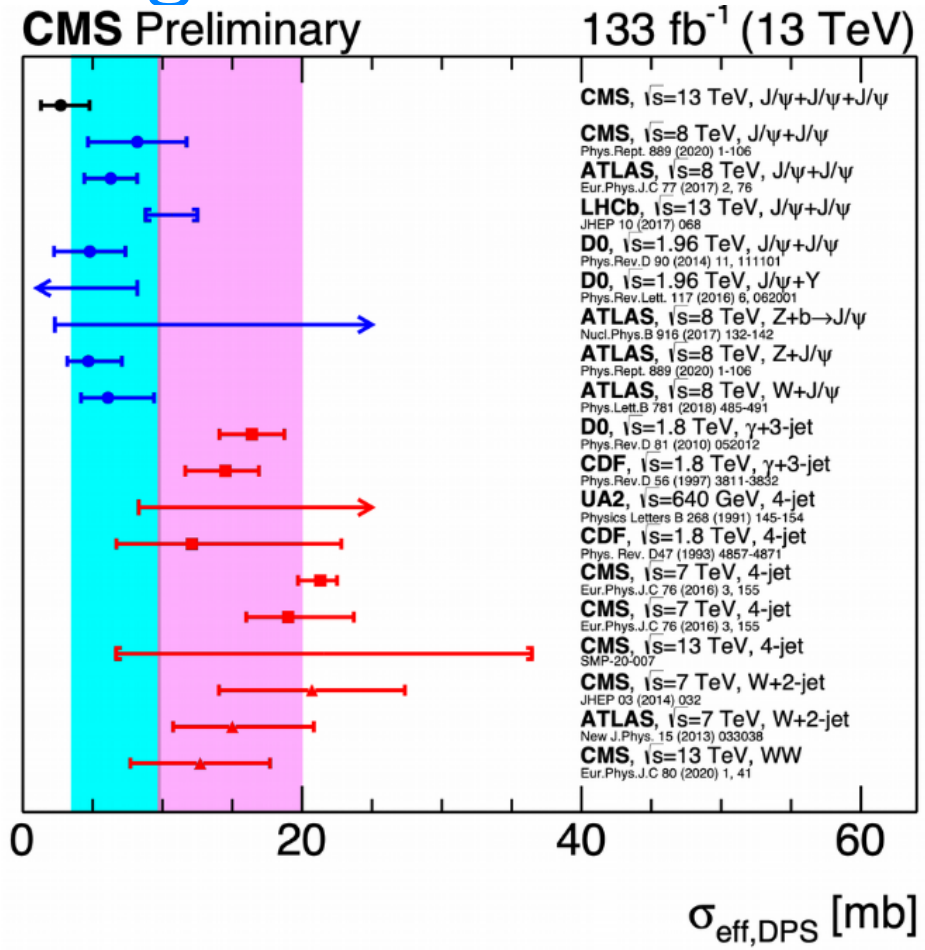
- ⋆ Gives access to double-parton scattering (DPS)
- ⋆ SPS and DPS have different kinematics
- ⋆ Uncertainties on σ_{eff}, pocket formula



$$\sigma^{\text{DPS}} = \frac{\sigma^{\text{SPS}}(pp \to A)\sigma^{\text{SPS}}(pp \to B)}{\sigma_{\text{eff}}}$$



Studying the fragmentation mechanism

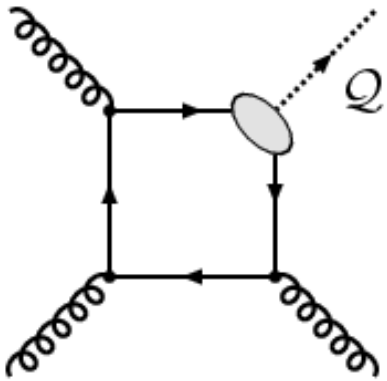


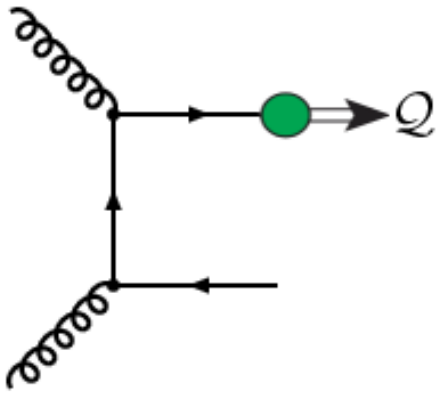
- σ_{eff} smaller for quarkonia.
- Since $\sigma^{\text{SPS}} + \sigma^{\text{DPS}} \simeq \sigma^{\text{exp}}$
- This suggests a missing contribution.

Our main goal: Evaluate the fragmentation contribution

Fragmentation contribution

Direct versus fragmentation production (SPS)





Direct contribution:

Included in estimations of the SPS.

Evaluated with NRQCD.

Fragmentation contribution:

Not included in double-quarkonia production.

Non relativistic QCD

- Top-down effective field theory built from QCD.
- $\mu \sim m_Q$ is the ultraviolet scale.
- Expansion parameters: α_s and v, the relative velocity.
- $\sigma_{Q\bar{Q}(\kappa)}$: Feynman rules + projector for quantum state κ.

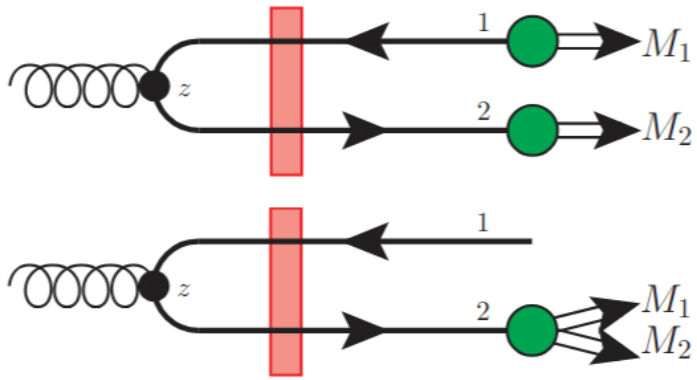
Factorization hypothesis:

$$\sigma^H = \sum_\kappa \sigma_{Q\bar{Q}(\kappa)}(\mu) \otimes \mathscr{O}_\kappa^H(\mu) \tag{1}$$

$\mathscr{O}_\kappa^H$: Long-distance-matrix element (LDME)

	$1,^1S_0$	$1,^3S_1$	$8,^1S_0$	$8,^3S_1$	$1,^1P_1$	$1,^3P_0$	$1,^3P_1$	$1,^3P_2$	$8,^1P_1$	$8,^3P_0$	$8,^3P_1$	$8,^3P_2$
NRQCD Factorization												
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4

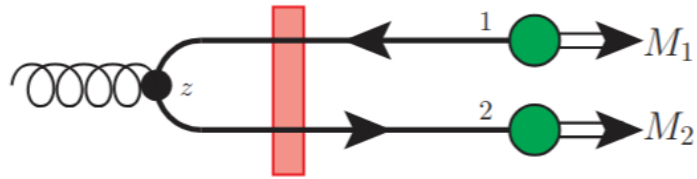
Color Glass Condensate effective theory



Dilute-dense colliding system:

- The gluon comes from the dilute proton.
- The red lines (the shock wave) represent the interaction of the $Q\bar{Q}$ with the dense proton.
- $0+1+2+...$ interactions.
- Standard Feynman rules + specific CGC rules (for the Wilson line).
- Single and double fragmentation.

Single fragmentation

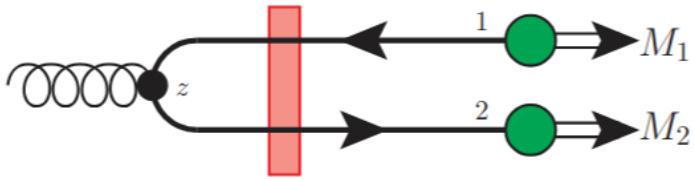


$$\frac{d\sigma_{g+\text{proton}\to H_1+H_2+X}}{dy_1 d^2\boldsymbol{P}_1^\perp dy_2 d^2\boldsymbol{P}_2^\perp} = \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2}\frac{d\hat{\sigma}_{g+P\to Q+\bar{Q}}}{dy_1 d^2\boldsymbol{p}_1^\perp dy_2 d^2\boldsymbol{p}_2^\perp} D_{Q\to H_1}(z_1) D_{Q\to H_2}(z_2)$$

with $\boldsymbol{P}_1^\perp = z_1\boldsymbol{p}_1^\perp$ and $\boldsymbol{P}_2^\perp = z_2\boldsymbol{p}_2^\perp$.

- We considered two cases: $D_{c\to H}$ and $D_{b\to H}$.
- FFs taken from 1311.7078: perturbative + NRQCD.

Single fragmentation



$$\frac{d\sigma_{g+\text{proton}\to H_1+H_2+X}}{dy_1 d^2\boldsymbol{P}_1^\perp dy_2 d^2\boldsymbol{P}_2^\perp} = \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2}\frac{d\hat{\sigma}_{g+P\to Q+\bar{Q}}}{dy_1 d^2\boldsymbol{p}_1^\perp dy_2 d^2\boldsymbol{p}_2^\perp} D_{Q\to H_1}(z_1) D_{Q\to H_2}(z_2)$$

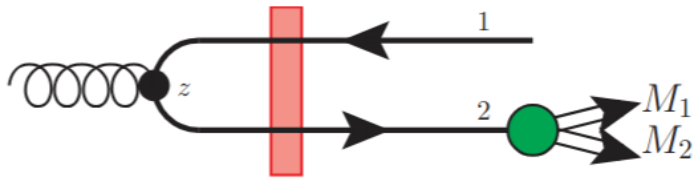
with $\boldsymbol{P}_1^\perp = z_1\boldsymbol{p}_1^\perp$ and $\boldsymbol{P}_2^\perp = z_2\boldsymbol{p}_2^\perp$.

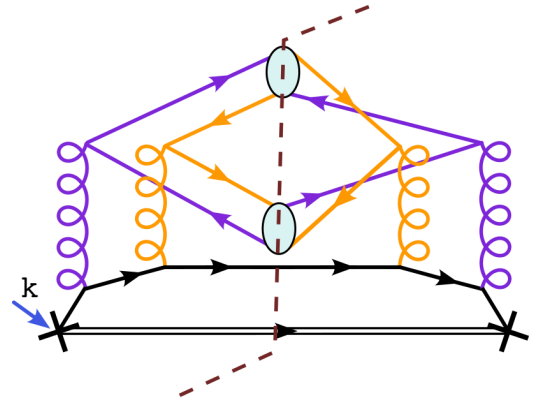
- Heavy numerical simulations for the fully-differential X-section.
- Integrating over transverse momenta simplifies the result:

$$\frac{d\hat{\sigma}_{g+P\to Q+\bar{Q}}}{dy_1 dy_2} = \int d^4\boldsymbol{X}\alpha(1-\alpha)\left|\psi(\alpha, r_{12})\right|^2 \Xi(x_1, x_2),$$

and $\Xi(x_1, x_2)$ a function of the dipole amplitude *only*.

Double fragmentation: bottom contribution



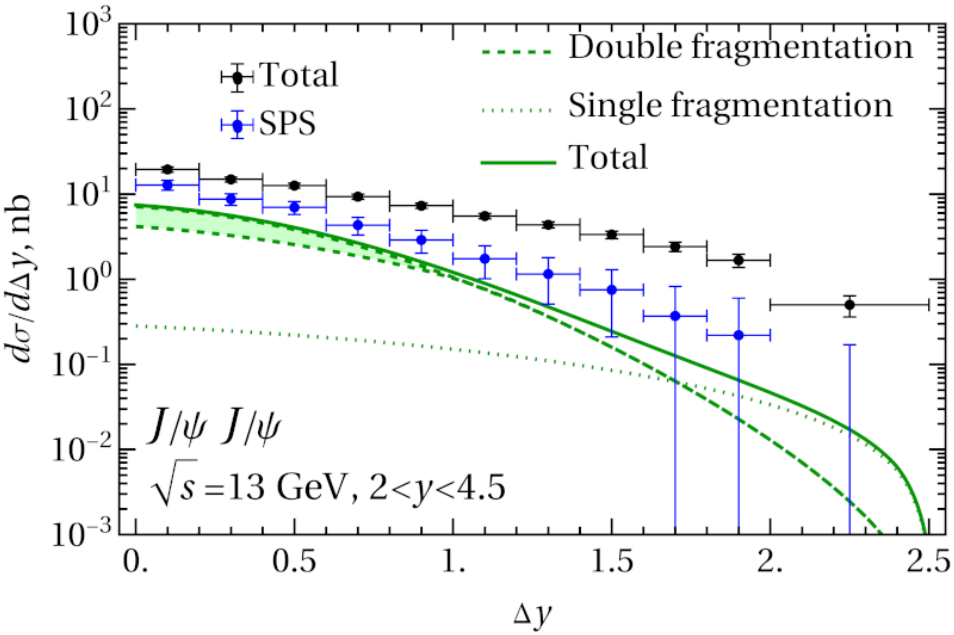


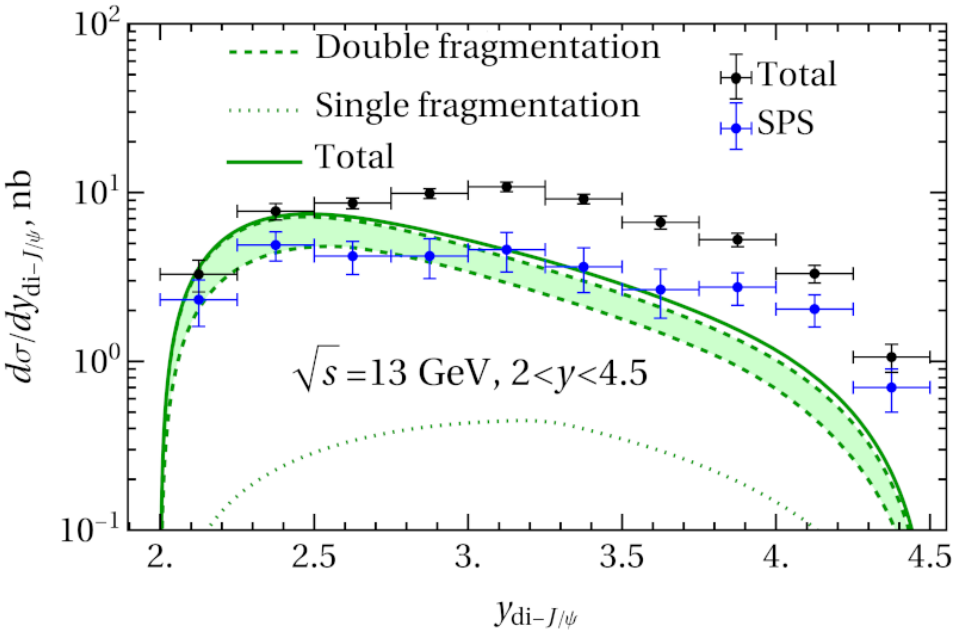
$$\frac{d\sigma_{g+\text{proton}\to H_1+H_2+X}}{dy_1 dy_2} = \int dz \left(\frac{d\hat{\sigma}_{g+P\to Q+\bar{Q}}}{dy_1 dy_2} \int_{M_{\text{min}}} dM_{12} D_{b\to H_1 H_2}(z, \zeta, M_{12}) \right)$$

$$D_{b\to H_1 H_2}(z_1, z_2, M_{12}, R) \propto \frac{1}{4z} \sum_X \text{Tr}\{\gamma^+ \times \text{cut diagram}\}$$

- We did the calculation with Feyncalc.
- There is no UV divergence.
- It is not clear whether our result remains valid at $M_{\text{min}} = 2m_{J/\psi}$ (near-threshold kinematics).

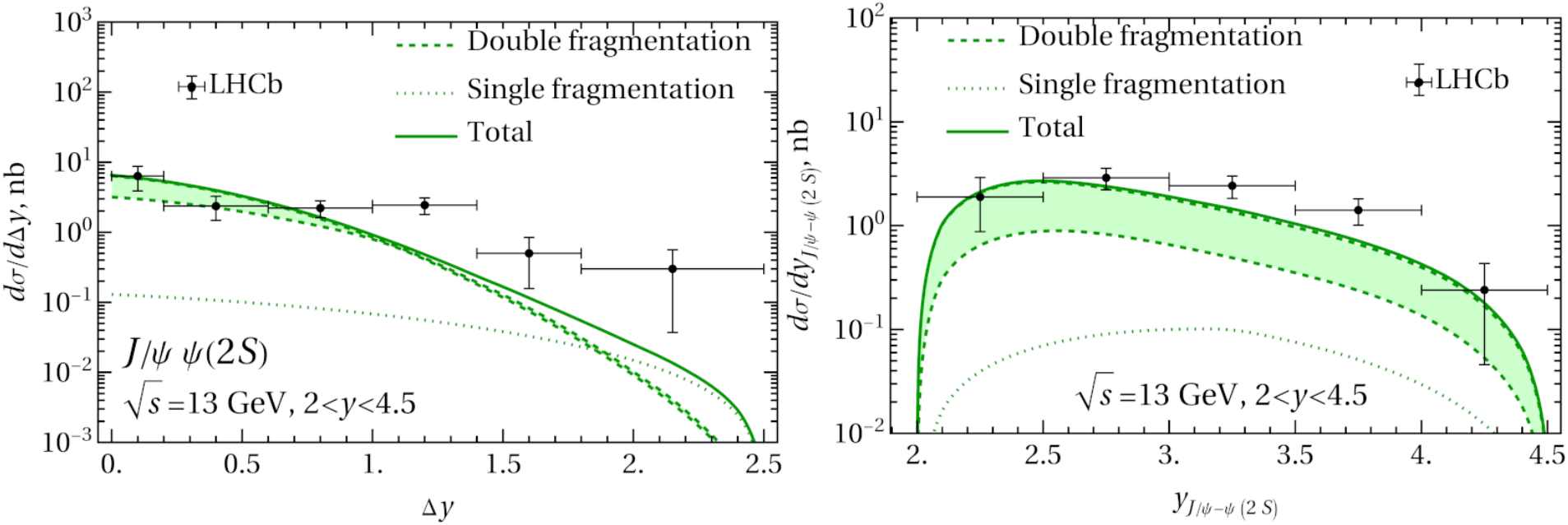
Intermediate result: double J/ψ





- Single fragmentation (SF) dominated by the charm contribution.
- SF negligible, except at large Δy.
- $M_{\mathrm{min}} = 6.2$ and 7 TeV $\rightarrow$ theoretical uncertainty green color band.

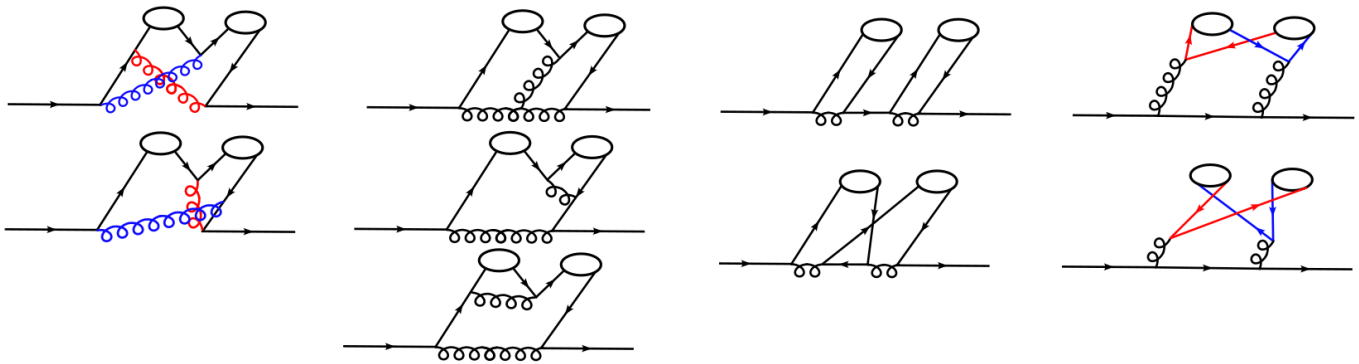
Intermediate result: J/ψ-$\psi(2S)$



Similar conclusions for the J/ψ-$\psi(2S)$ measurement.

Double fragmentation: charm contribution

- More amplitudes...

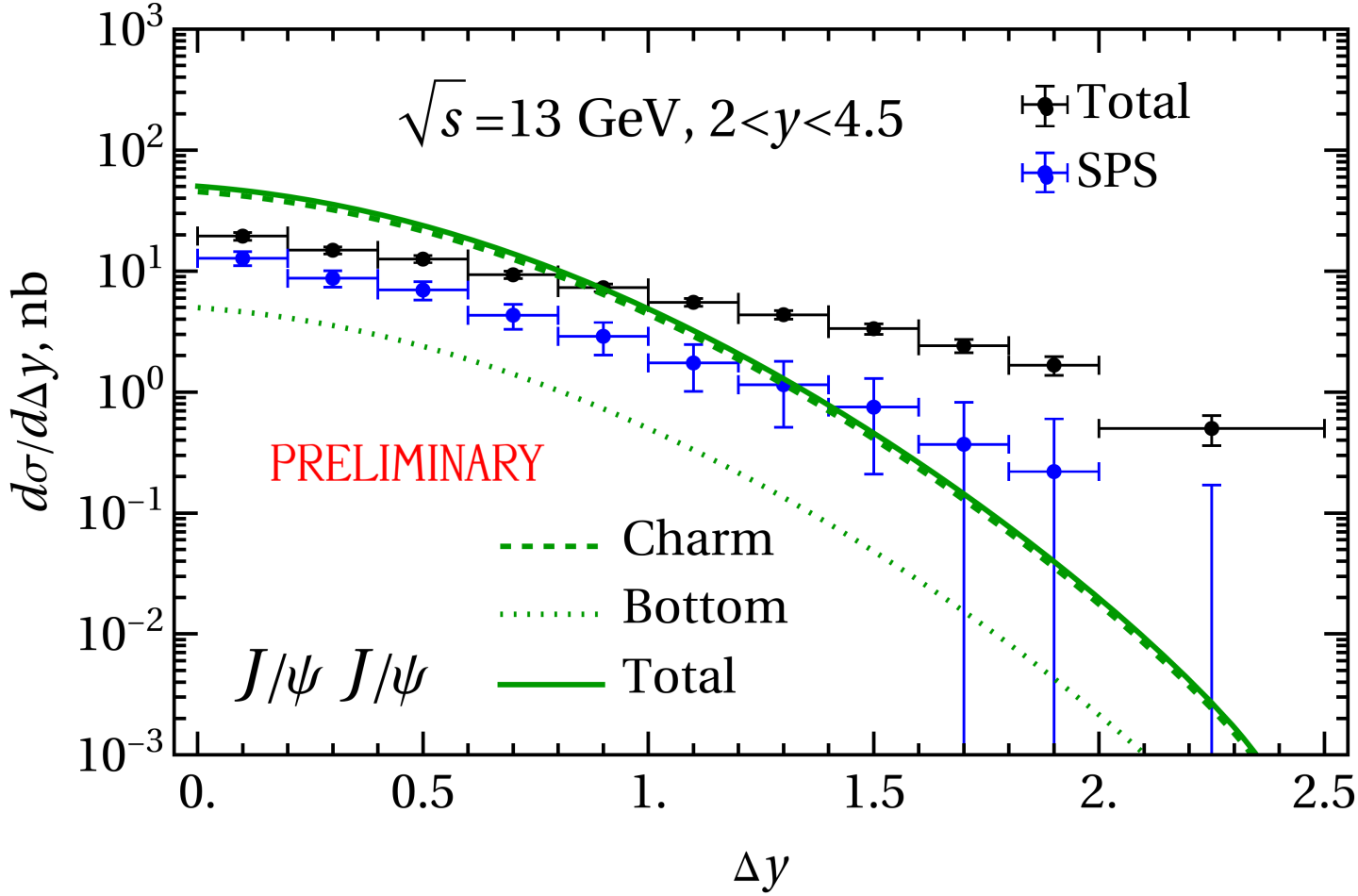


- Longer chain of gamma matrices, e.g., $\mathrm{Tr}\{16\,\gamma\}$ (cross section level) $\Rightarrow$ too complex for Feyncalc.

Our solutions: work at the amplitude level with the helicity formalism:

1. Either with Feyncalc.
2. Or numerically: simple product of matrices with OpenBlas.

Charm + bottom double fragmentation



- Preliminary results: we still have to compare the two different methods in detail.
- This mechanism gives a substantial contribution.

$\Rightarrow$ SPS contribution may be larger than expected $\Rightarrow$ smaller DPS.

Conclusion

- ⋆ We have evaluated the single- and double-fragmentation contribution to double-quarkonia production.
- ⋆ The double fragmentation is significant.
- ⋆ Possible smaller DPS (larger σ_{eff}): may bring agreement with other measurements.
- ⋆ We have computed for the first time the $D_{Q\to H_1H_2}$ di-quarkonia fragmentation function.

Thanks for your attention!