The Drell–Yan Structure Functions in k_T Factorization

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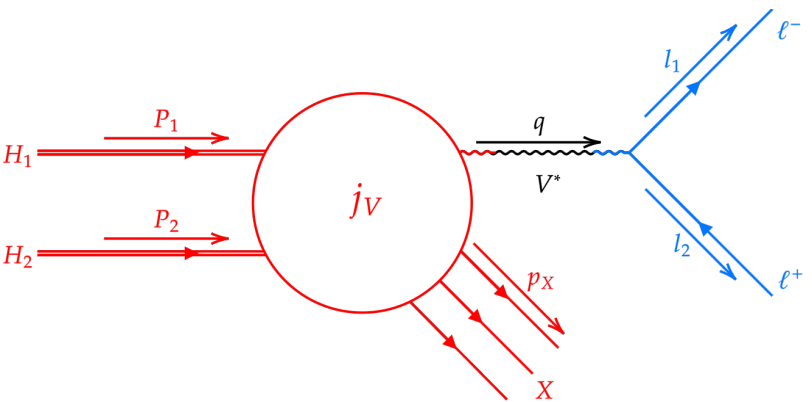
Outline

Main goal of the study — use Drell–Yan observables to constrain the shape of the gluon TMD

- The Drell–Yan process
- Structure Functions
- Factorization
- Lam–Tung relation
- TMD models overview
- Results

The Drell–Yan process as a probe of the hadrons structure

$$H_1(P_1) + H_2(P_2) \longrightarrow V^*(q) + X(p_X) \longrightarrow \ell^-(l_1) + \ell^+(l_2) + X(p_X)$$



$$\mathcal{A}\left(H_1H_2 \to \ell^+\ell^- X\right) = \overline{u}(l_1) ie\Gamma_V^\mu v(l_2) D_V(q)_{\mu\nu} \langle X | ie j_V^\nu | H_1H_2 \rangle$$

The differential cross section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}M^2\mathrm{d}\Omega\mathrm{d}Y\mathrm{d}^2q_T} = \frac{|\mathcal{A}|^2}{128\pi S} = \frac{\alpha_e^2}{8S}\left|D_V\left(M^2\right)\right|^2 W_{\mu\nu}L^{\mu\nu}$$

$L^{\mu\nu}$ - Standard QED calculation

$W_{\mu\nu}$ - Sensitive for an internal hadron structure

Structure Functions are measurable source of information about hadrons

The lepton angular decomposition is often parameterized in the following way

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M^2\mathrm{d}Y\mathrm{d}^2q_T\mathrm{d}\Omega} \propto \Big[1+\cos^2\vartheta+\frac{1}{2}A_0\Big(1-3\cos^2\vartheta\Big)+A_1\sin(2\vartheta)\cos\phi+\frac{1}{2}A_2\sin^2\vartheta\cos(2\phi)$$
$$+A_3\sin\vartheta\cos\phi+A_4\cos\vartheta+A_5\sin^2\vartheta\sin(2\phi)+A_6\sin(2\vartheta)\sin\phi+A_7\sin\vartheta\sin\phi\Big],$$

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$$\underbrace{+A_3\sin\vartheta\cos\phi+A_4\cos\vartheta+A_5\sin^2\vartheta\sin(2\phi)+A_6\sin(2\vartheta)\sin\phi+A_7\sin\vartheta\sin\phi}_{\text{present for }Z^0\text{ and }W^\pm}\Bigg],$$

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where the structure functions are given by the polarized cross sections

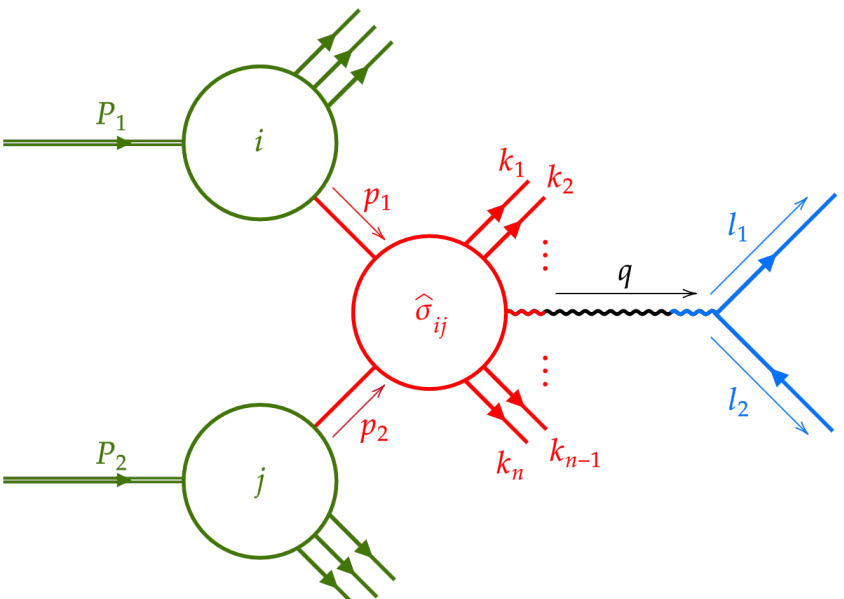
$$A_0 = \frac{2\mathrm{d}\sigma^{(00)}}{\mathrm{d}\sigma^{U+L}}, \qquad A_1 = \frac{\mathrm{d}\sigma^{(+0)}+\mathrm{d}\sigma^{(0+)}-\mathrm{d}\sigma^{(-0)}-\mathrm{d}\sigma^{(0-)}}{\sqrt{2}\mathrm{d}\sigma^{U+L}}, \qquad A_2 = 2\frac{\mathrm{d}\sigma^{(+-)}+\mathrm{d}\sigma^{(-+)}}{\mathrm{d}\sigma^{U+L}},$$
$$A_3 = \sqrt{2}c_l\frac{\mathrm{d}\sigma^{(+0)}+\mathrm{d}\sigma^{(0+)}+\mathrm{d}\sigma^{(-0)}+\mathrm{d}\sigma^{(0-)}}{\mathrm{d}\sigma^{U+L}}, \qquad A_4 = 2c_l\frac{\mathrm{d}\sigma^{(++)}-\mathrm{d}\sigma^{(--)}}{\mathrm{d}\sigma^{U+L}},$$

with

$$\mathrm{d}\sigma^{U+L} = \sum_{r\in\{0,+,-\}}\mathrm{d}\sigma^{(rr)}, \qquad c_l = \frac{2v_la_l}{v_l^2+a_l^2}.$$

THE STRUCTURE FUNCTIONS CAN BE CALCULATED IN QCD

Factorization:
Hadronic cross section = (Parton Distributions) $\otimes$ (Partonic cross section within the pQCD)



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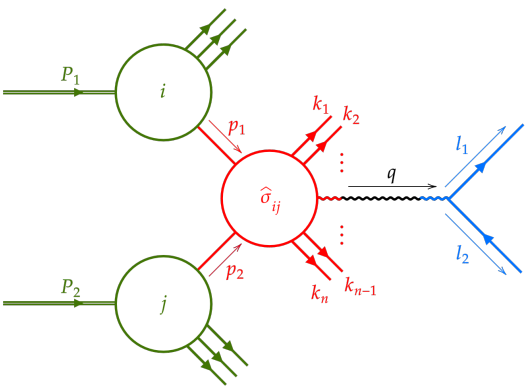


$$\mathrm{d}\sigma \propto \begin{cases} L_{(rr')} \sum\limits_{i,j} \int_0^1 \mathrm{d}x_1 f_i\left(x_1, \mu_F^2\right) \int_0^1 \mathrm{d}x_2 f_j\left(x_2, \mu_F^2\right) \mathrm{d}\hat{\sigma}_{ij}^{(rr')} & \text{Collinear factorization} \\ L_{(rr')} \sum\limits_{i,j} \int_0^1 \frac{\mathrm{d}x_1}{x_1} \int \frac{\mathrm{d}^2 p_{1T}}{\pi} \mathcal{F}_i\left(x_1, \mathbf{p}_{1T}^2, \mu_F^2\right) \int_0^1 \frac{\mathrm{d}x_2}{x_2} \int \frac{\mathrm{d}^2 p_{2T}}{\pi} \mathcal{F}_j\left(x_2, \mathbf{p}_{2T}^2, \mu_F^2\right) \mathrm{d}\hat{\sigma}_{ij}^{*(rr')} & k_T\text{-factorization} \end{cases}$$

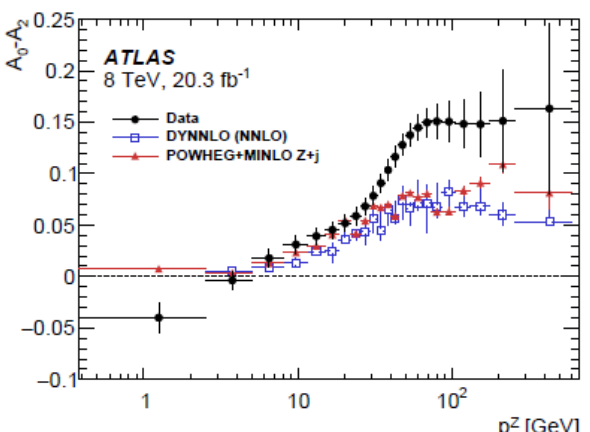
The Lam–Tung relation breaking is not completely explained

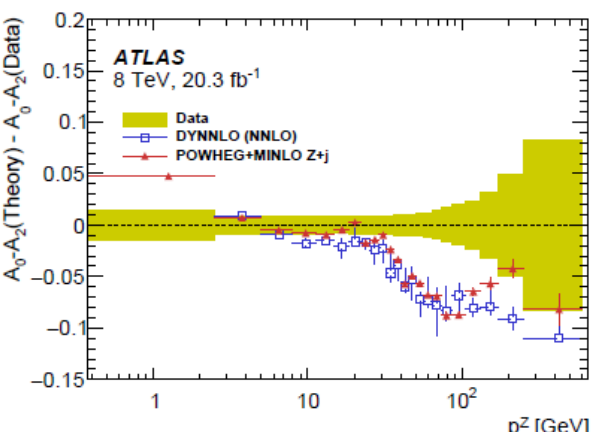
- The Lam–Tung relation is of the form:

$$\boxed{A_{LT} := A_0 - A_2 = 0.}$$

- Fulfilled up to the NLO (for V+jet) in collinear perturbative QCD

- Breaking of Lam–Tung relation may be traced back to a difference between partonic and hadronic collision planes $\longrightarrow$ sensitivity to partons' transverse momenta





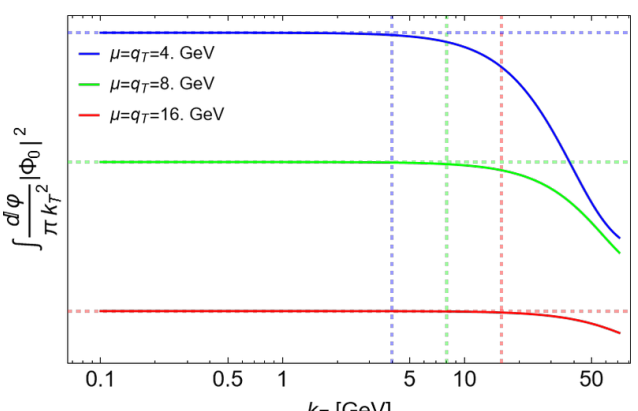
Motivation for including the partons' transverse momenta

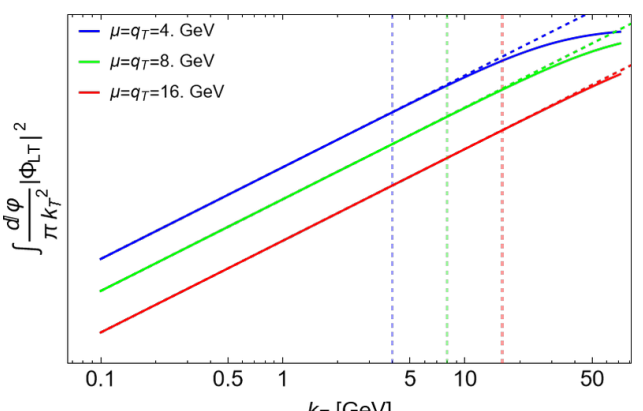
Consider the cross section with one off-shell parton

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 f\left(x_1, \mu^2\right) \int \mathrm{d}x_2 \int \frac{\mathrm{d}^2 k_T}{\pi k_T^2} \mathcal{F}\left(x_2, k_T^2, \mu^2\right) |\Phi|^2 \mathrm{d}PS.$$

The impact factor Φ is related to the collinear amplitude $\mathcal{A}_{\text{coll}}$ by

$$\int \frac{\mathrm{d}\varphi}{\pi} |\Phi|^2 \approx \theta\left(\mu^2 - k_T^2\right) |\mathcal{A}_{\text{coll}}|^2 k_T^2 + \mathcal{O}\left(\left(\frac{k_T}{\mu}\right)^4\right).$$





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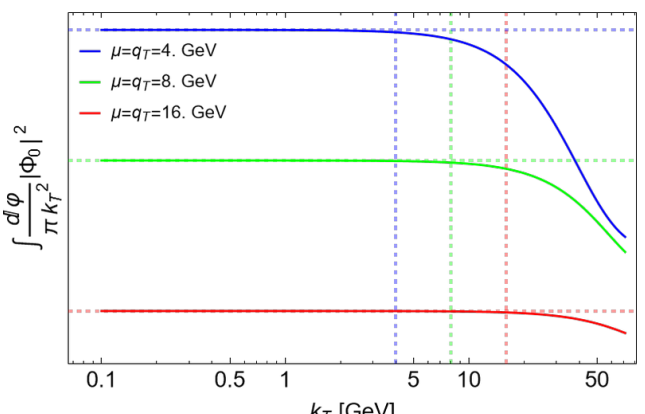
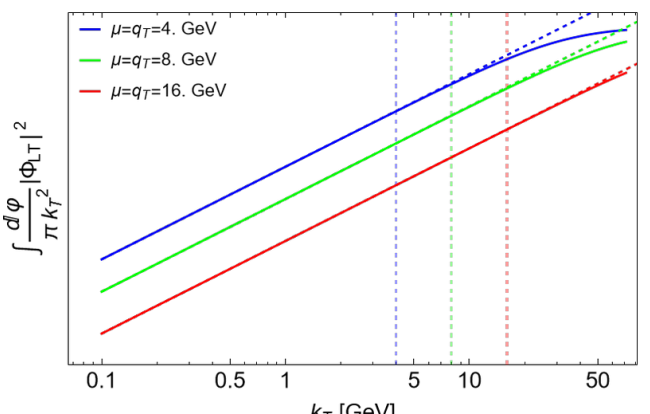


$$\int \mathrm{d}\varphi |\Phi_0|^2 \propto k_T^2$$



$$\int \mathrm{d}\varphi |\Phi_{LT}|^2 \propto k_T^4$$

k_T-FACTORIZATION APPROACH

- We want to take into account the effects of partons' transverse momenta $\Longrightarrow$ **k_T-factorization**
- The leading contribution comes from the gluons

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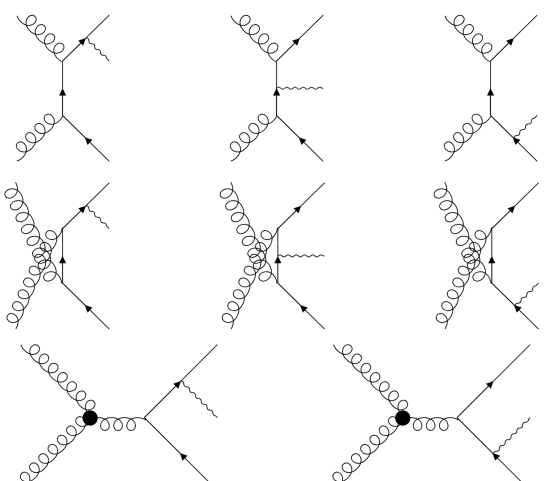
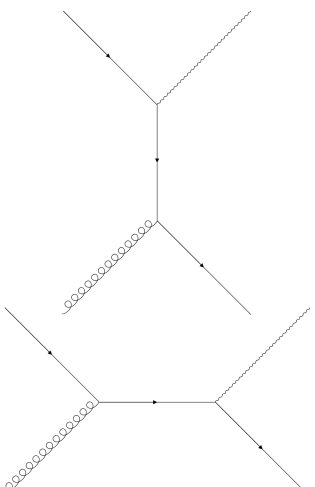
$g^*g^* \longrightarrow q\bar{q}V^*$



k_T-factorization

$q_{\mathrm{val}}g^* \longrightarrow qV^*$



Hybrid factorization

Considered gluon TMD models

- Gaussian model — Golec-Biernat–Wüsthoff (GBW) saturation model [arXiv:hep-ph/9903358]

$$\mathcal{F}_G\left(x, k_T^2\right) = N_2(1-x)^7 \exp\left[-\left(\frac{x}{x_0}\right)^{\lambda} \frac{k_T^2}{k_0^2}\right].$$

- Jung–Hautmann (JH) model

- Kimber–Martin–Ryskin (KMR) model

- "Weizsäcker–Williams" (WW) model (in analogy with photon flux) + modified "Weizsäcker–Williams" (WW′) model

Considered gluon TMD models

- Gaussian model
- Jung–Hautmann (JH) model — Catani–Ciafaloni–Fiorani–Marchesini (CCFM) evolution equation - 3 versions from different fits (2013, 2018, 2023) from `TMDlib`. [`arXiv:1312.7875v2`, `arXiv:1804.11152v2`, `arXiv:2312.08655v3`]
- Kimber–Martin–Ryskin (KMR) model
- "Weizsäcker–Williams" (WW) model + modified "Weizsäcker–Williams" (WW') model

CONSIDERED GLUON TMD MODELS

- Gaussian model
- Jung–Hautmann (JH) model
- Kimber–Martin–Ryskin (KMR) model — k_T dependence generated from the last splitting of the DGLAP evolution.
  [arXiv:hep-ph/0101348v1, arXiv:hep-ph/0209029v2, arXiv:1803.06246v1]
- "Weizsäcker–Williams" (WW) model + modified "Weizsäcker–Williams" (WW') model

Considered gluon TMD models

- Gaussian model
- Jung–Hautmann (JH) model
- Kimber–Martin–Ryskin (KMR) model
- "Weizsäcker–Williams" (WW) model (in analogy with photon flux) [arXiv:1609.04300v2]

$$\mathcal{F}_{\mathrm{WW}}\left(x, k_T^2\right) = \frac{N_1}{k_0^2}(1-x)^7 x^{-\lambda} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \dfrac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases}.$$

Modified "Weizsäcker–Williams" (WW′) model

$$\mathcal{F}'_{WW}\left(x, k_T^2, \mu^2\right) = \frac{x f_g\left(x, \mu^2\right)}{k_0^2\left[1 + \log\left(\frac{\mu^2}{k_0^2}\right)\right]} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \dfrac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases}.$$

where $f_g\left(x, \mu^2\right)$ is a collinear gluon PDF.

Considered gluon TMD models

- Gaussian model

$$\mathcal{F}_G\left(x, k_T^2\right) = N_2(1-x)^7 \exp\left[-\left(\frac{x}{x_0}\right)^{\lambda} \frac{k_T^2}{k_0^2}\right] \quad - \quad \text{Narrow in } k_T$$

- Jung–Hautmann (JH) model

- Kimber–Martin–Ryskin (KMR) model

- "Weizsäcker–Williams" (WW) model

$$\mathcal{F}_{\mathrm{WW}}\left(x, k_T^2\right) = \frac{N_1}{k_0^2}(1-x)^7 x^{-\lambda} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \dfrac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases} \quad - \quad \text{Wide in } k_T$$

  + modified "Weizsäcker–Williams" (WW') model.

Normalization to the total Drell–Yan cross section

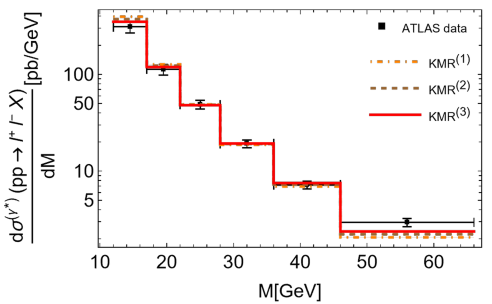
Problem: Some models do not describe the cross-section data well.

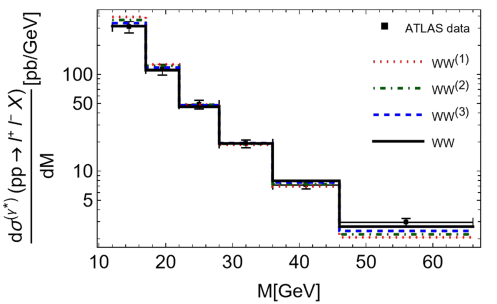
Phenomenological solution: Normalization + x rescaling (Motivated by different kinematics with and without k_T)

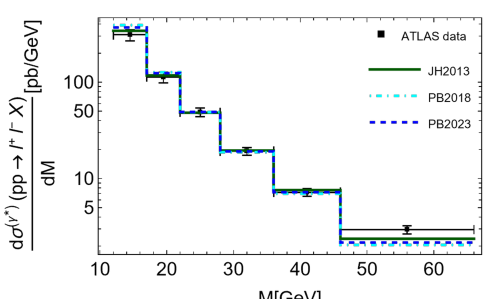
We considered three modifications of KMR and WW′ models in the form

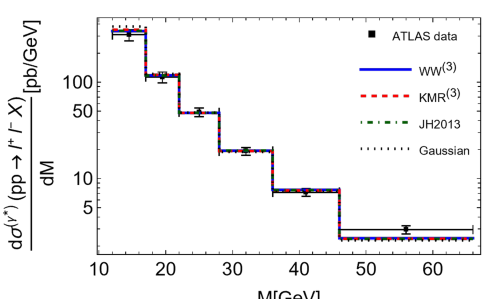
$$\mathcal{F}^{(1)}\left(x, k_T^2, \mu^2\right) = N^{(1)}\mathcal{F}\left(x, k_T^2, \mu^2\right),$$
$$\mathcal{F}^{(2)}\left(x, k_T^2, \mu^2\right) = N^{(2)}\mathcal{F}\left(x/2, k_T^2, \mu^2\right),$$
$$\mathcal{F}^{(3)}\left(x, k_T^2, \mu^2\right) = N^{(3)}\mathcal{F}\left(x/4, k_T^2, \mu^2\right).$$

Normalization to the total Drell–Yan cross section



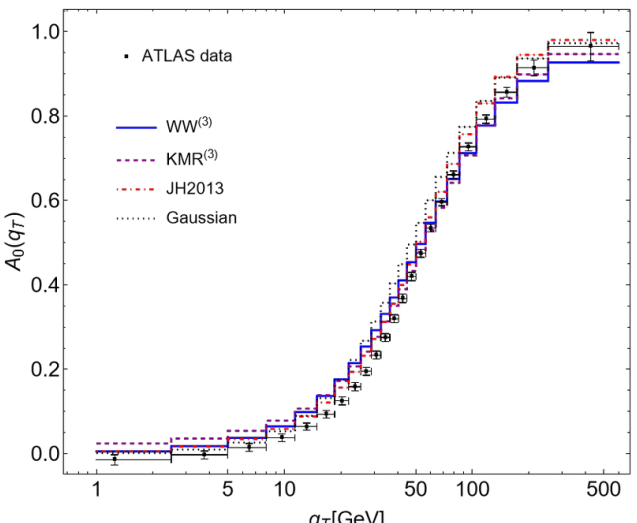


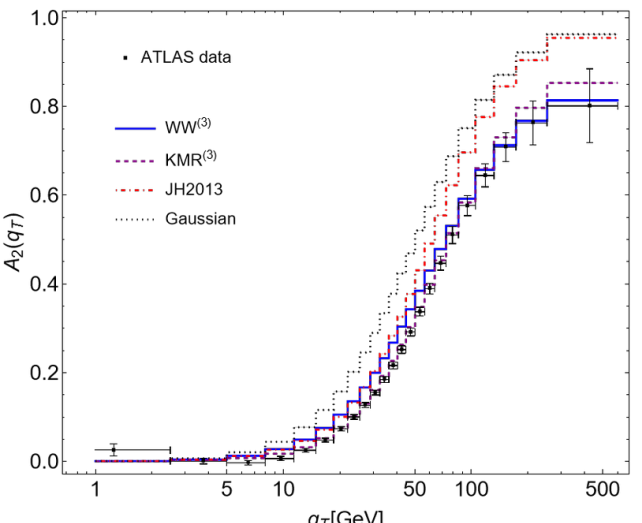




Model	$WW^{(3)}$	$KMR^{(3)}$	JH2013	Gauss.
χ^2/d.o.f	0.98	1.13	1.07	1.74

Results for Parity Conserving Structure Functions — A_0 and A_2





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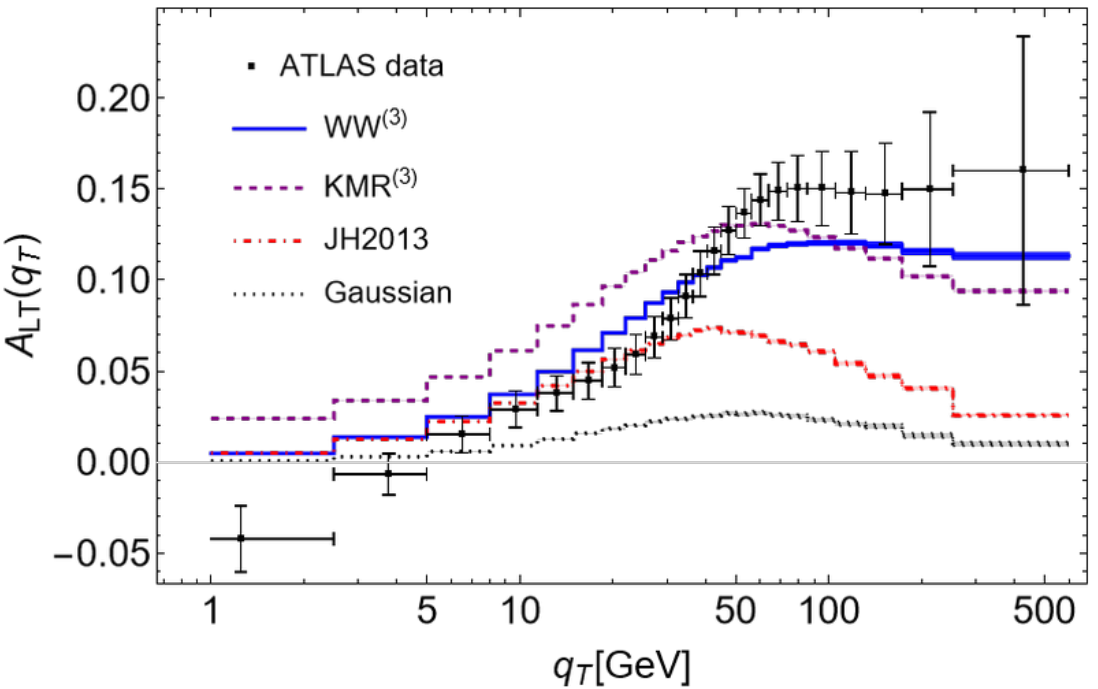
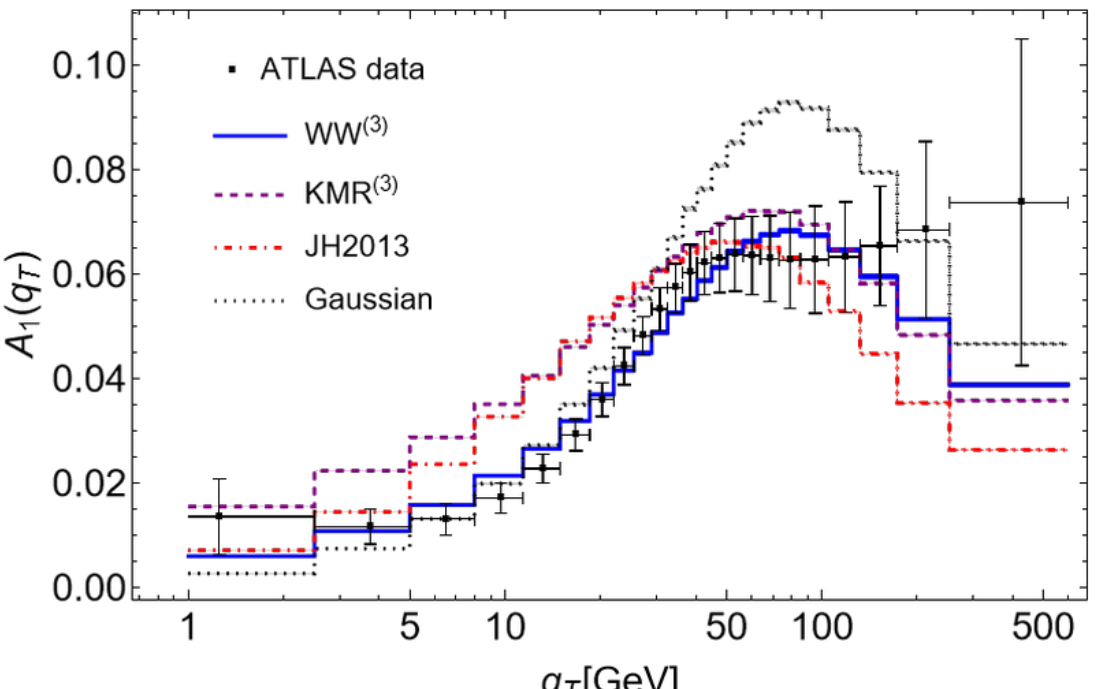
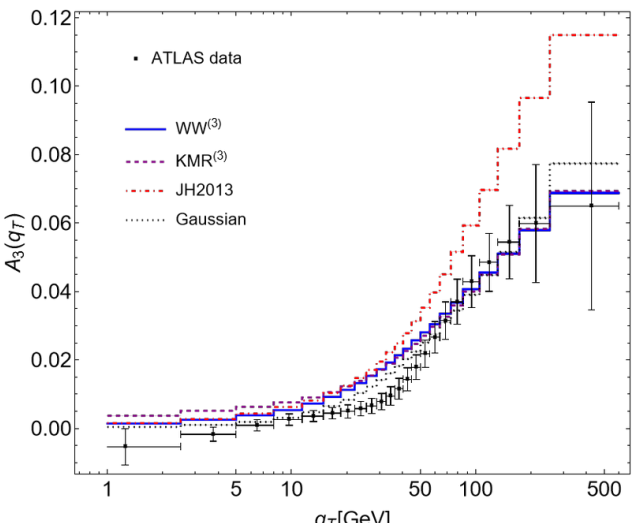
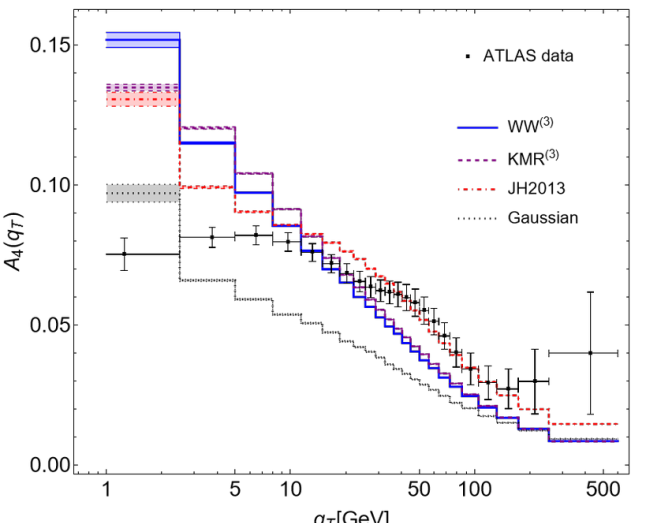


Results for Parity Conserving Structure Functions — A_1



Results for Parity Violating Structure Functions — A_3 and A_4





Comparison of the Results for Different TMD Models

Reduced χ^2 for the structure function data

$$\chi^2_n/\text{d.o.f} = \frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \frac{\left(A^{\text{th}}_{n,i} - A^{\text{data}}_{n,i}\right)^2}{\sigma^2_{i,\text{data}} + \sigma^2_{i,\text{th}}} \qquad \text{where} \qquad \sigma_{i,\text{th}} = 0.12 \cdot A^{\text{th}}_{n,i}.$$

$$\chi^2_{\text{min}}/\text{d.o.f} = \min\left\{\chi^2/\text{d.o.f}\right\} \approx \boxed{2.50} \qquad \text{where} \qquad \chi^2/\text{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi^2_n/\text{d.o.f.}$$

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χ^2/d.o.f for the structure function data:

Model	WW	$\text{WW}^{(1)}$	$\text{WW}^{(2)}$	$\text{WW}^{(3)}$	$\text{KMR}^{(1)}$	$\text{KMR}^{(2)}$	$\text{KMR}^{(3)}$	JH2013	PB2018	PB2023	Gauss.
χ^2/d.o.f	3.83	4.50	2.84	2.50	4.34	3.02	2.78	2.91	4.77	4.28	5.56

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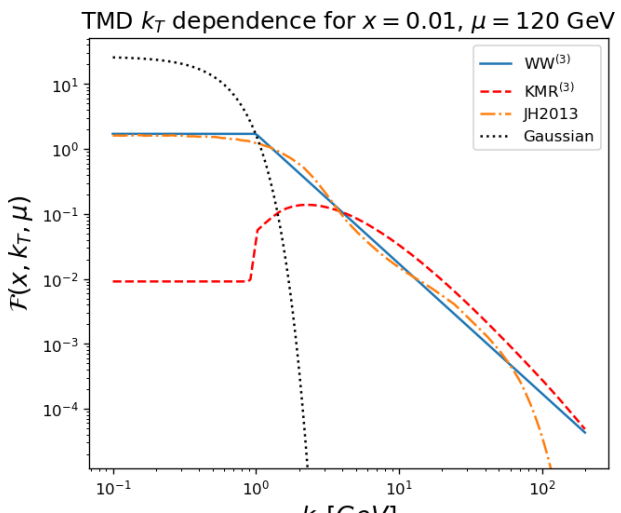
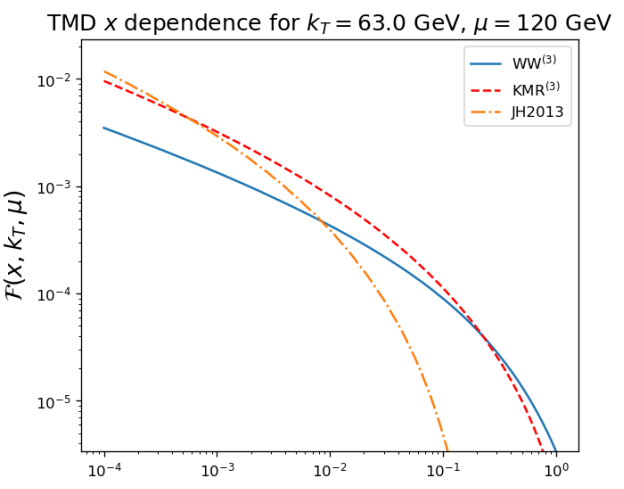
$$\chi^2_{LT,\text{min}}/\text{d.o.f} = \min\left\{\chi^2_{LT}/\text{d.o.f}\right\} \approx \boxed{1.43}$$

Model	$\text{WW}^{(3)}$	$\text{KMR}^{(3)}$	JH2013	Gauss.
χ^2_{LT}/d.o.f	1.43	4.02	7.37	25.33

Comparison of the Results for Different TMD Models

TMD models with lowest χ^2





SUMMARY

- The Drell–Yan process is an excellent probe of internal hadron structure: five non-vanishing structure functions, three kinematical variables dependence (q_T, Y, M^2).
- Wide in k_T Weizsäcker-Williams TMD model, among the models we considered, was found to give the best overall description of the data within the presented formalism.
- In particular, an even better description was found for the Lam–Tung combination.
- We continue the program of constraining gluon TMD with Drell–Yan observables by:
  - fitting TMD to a wider set of data,
  - extending the formalism to include off-shell quarks,
  - A future goal: Inclusion of the NLO corrections in k_T-factorization

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THANK YOU FOR YOUR ATTENTION!

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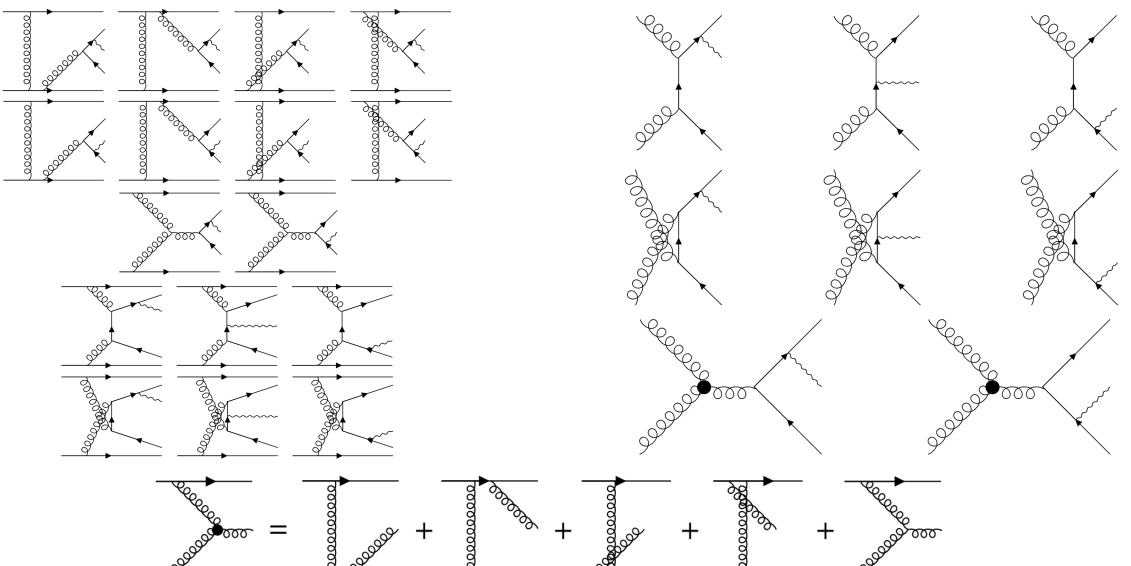
ADDITIONAL MATERIAL

Leading Diagrams

Lipatov Effective Vertex



$$\Gamma^{\mu\nu\rho} = \frac{2}{S}\left[\left(x_1 + \frac{2\mathbf{k}_{1T}^2}{x_2 S}\right)P_1^\mu + \left(x_2 + \frac{2\mathbf{k}_{2T}^2}{x_1 S}\right)P_2^\mu - (k_{1T} - k_{2T})^\mu\right]P_1^\nu P_2^\rho$$

Relations between collinear and high energy channels

The $q_{\text{val}}g^* \to qV$ channel

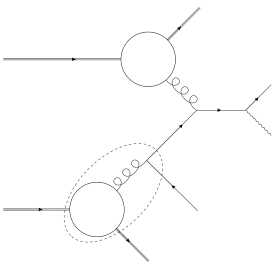
- contains $g \to \bar{q}$ LO DGLAP splitting for $q_{\text{val}}\bar{q} \to V$ matrix element and the NLO $qg \to qV$ contribution.
- neglects the NLO loop corrections for $q\bar{q} \to V$.

The $g^*g^* \to q\bar{q}V$ channel

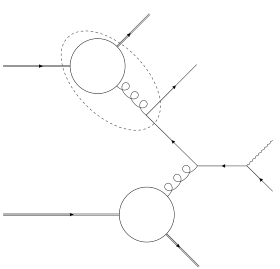
- contains $g \to q_{\text{sea}}$, $g \to \bar{q}$ NLO DGLAP splittings for the $q_{\text{sea}}\bar{q} \to V$ matrix element
- and $g \to q_{\text{sea}}$, $g \to \bar{q}$ LO DGLAP splittings for $q_{\text{sea}}g \to qV$ and $\bar{q}g \to \bar{q}V$ respectively.
- contains the leading contribution to the NNLO collinear $gg \to q\bar{q}V$ matrix element.
- neglects the loop corrections for $q_{\text{sea}}g \to qV$, $\bar{q}g \to \bar{q}V$ and $q\bar{q} \to V$.

Relations between collinear and high energy channels

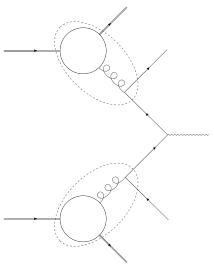
(a)



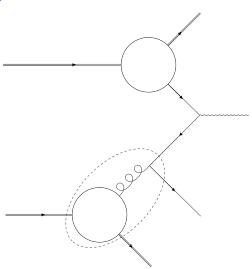
(b)



(c)



(d)



(a) $g \to q_{\text{sea}}$ splitting

(b) $g \to \overline{q}$ splitting

(c) $g \to q_{\text{sea}}$ and $g \to \overline{q}$ splittings

(d) $g \to \overline{q}$ splitting

The Amplitudes Squared

Full amplitude $\mathcal{A}^\mu$ is given by the sum of all diagrams.

- For the $q_{\text{val}}g^* \to qV^*$ there are two diagrams with the same color structure which gives the amplitude

$$\mathcal{A}^\mu = \mathcal{A}_1^\mu + \mathcal{A}_2^\mu = -igT_{ij}^a \left(A_1^\mu + A_2^\mu\right),$$

so the amplitude square is

$$\mathcal{M}^{\mu\nu} = \frac{1}{N\left(N^2-1\right)} \sum_{i,j,a} \mathcal{A}^\mu \overline{\mathcal{A}}^\nu = \frac{g^2}{2N} A^\mu \overline{A}^\nu.$$

- For the $g^*g^* \to q\bar{q}V^*$ the amplitude can be decomposed into symmetric (S) and antisymmetric (A) parts in the adjoint color indices

$$\mathcal{A}^\mu := \sum_{n=1}^{8} \mathcal{A}_n^\mu = \mathcal{A}_S^\mu + \mathcal{A}_A^\mu, \qquad \text{where} \qquad \mathcal{A}_S^\mu := -ig^2 \left(\frac{1}{N}\delta^{ab}\delta_{ij} + d^{abc}T_{ij}^c\right) A_S^\mu, \quad \mathcal{A}_A^\mu := g^2 f^{abc} T_{ij}^c A_A^\mu.$$

In the amplitude squared, the symmetric and antisymmetric parts in the adjoint color indices do not interfere:

$$\mathcal{M}^{\mu\nu} = \frac{1}{\left(N^2-1\right)^2} \sum_{i,j,a,b} \mathcal{A}^\mu \overline{\mathcal{A}}^\nu = \frac{g^4\left(N^2-2\right)}{2N\left(N^2-1\right)} A_S^\mu \overline{A}_S^\nu + \frac{g^4 N}{2\left(N^2-1\right)} A_A^\mu \overline{A}_A^\nu = \mathcal{M}_S^{\mu\nu} + \mathcal{M}_A^{\mu\nu}.$$

The Amplitudes Squared

The spinorial matrix $\mathfrak{A}_n^\mu$ appearing in the amplitude A_n^μ is defined by:

$$A_n^\mu = \overline{u}_{\sigma_3}(p_3)\mathfrak{A}_n^\mu v_{\sigma_4}(p_4).$$

In an analogous way we define the matrices $\mathfrak{A}_S^\mu, \mathfrak{A}_A^\mu$.
The amplitudes squared can be computed as traces

$$\sum_{\sigma_3,\sigma_4} \mathcal{M}_S^{\mu\nu} = \frac{g^4\left(N^2-2\right)}{2N\left(N^2-1\right)} \operatorname{Tr}\Big[\left(\not{p}_3 + m_3\right)\mathfrak{A}_S^\mu\left(\not{p}_4 - m_4\right)\mathfrak{A}_S^{\dagger\nu}\Big],$$

$$\sum_{\sigma_3,\sigma_4} \mathcal{M}_A^{\mu\nu} = \frac{g^4 N}{2\left(N^2-1\right)} \operatorname{Tr}\Big[\left(\not{p}_3 + m_3\right)\mathfrak{A}_A^\mu\left(\not{p}_4 - m_4\right)\mathfrak{A}_A^{\dagger\nu}\Big].$$

Using the trace formulas one can check numerically the helicity structure functions of the form

$$\mathcal{M}_{rr'} = \varepsilon_\mu^{(r)}\mathcal{M}^{\mu\nu}\overline{\varepsilon}_\nu^{(r')},$$

where $r, r' \in \{+,-,0\}$ are basis polarizations of V^*.

Polarized Cross Sections

From the obtained squared amplitudes, one can derive the polarized cross sections by integrating over the phase space with the parton distributions.

- $q_{\mathrm{val}} g^* \to q V^*$

$$\frac{\mathrm{d}\sigma_{r_1 r_2}^{(q_{\mathrm{val}} g^*)}}{\mathrm{d}M^2 \mathrm{d}Y \mathrm{d}^2 q_T} = \sum_f \int \mathrm{d}x_{q/\mathcal{O}f,\mathrm{val}}(x_q, \mu_F) \int \frac{\mathrm{d}^2 k_T}{\pi \mathbf{k}_T^2} \mathcal{F}\left(x_g, \mathbf{k}_T^2, \mu_F\right) \frac{2}{(8\pi)^2 x_q (1 - x_F) S^2} \mathcal{M}_{r_1 r_2}^{(q_{\mathrm{val}} g^*), f},$$

- $g^* g^* \to q \bar{q} V^*$

$$\frac{\mathrm{d}\sigma_{r_1 r_2}^{(g^* g^*)}}{\mathrm{d}M^2 \mathrm{d}Y \mathrm{d}^2 q_T} = \int \mathrm{d}x_1 \int \frac{\mathrm{d}^2 k_{1T}}{\pi \mathbf{k}_{1T}^2} \mathcal{F}\left(x_1, \mathbf{k}_{1T}^2, \mu_F\right) \int \mathrm{d}x_2 \int \frac{\mathrm{d}^2 k_{2T}}{\pi \mathbf{k}_{2T}^2} \mathcal{F}\left(x_2, \mathbf{k}_{2T}^2, \mu_F\right) \frac{(2\pi)^4}{2S} \mathcal{M}_{r_1 r_2}^{(g^* g^*)} \frac{\mathrm{d}z \mathrm{d}\phi_\kappa}{8(2\pi)^9}.$$

GLUON TMD MODELS

- Gaussian model — GBW saturation model

$$\mathcal{F}_G\left(x, k_T^2\right) = N_2(1-x)^7 \exp\left[-\left(\frac{x}{x_0}\right)^{\lambda} \frac{k_T^2}{k_0^2}\right],$$

  with parameters $k_0 = 1$ GeV, $N_2 = 68.4$ GeV^{-2}, $\lambda = 0.29$ and $x_0 = 3 \cdot 10^{-4}$.
- The Jung–Hautmann model

- The Kimber–Martin–Ryskin model

- The "Weizsäcker–Williams" model (in analogy with photon flux) + modified "Weizsäcker–Williams" model

GLUON TMD MODELS

- Gaussian model

- Jung–Hautmann model — CCFM

$$\mathcal{F}\left(x, k_T, \mu\right) = \mathcal{F}_0\left(x, k_T, \mu_0\right) \Delta_s(\mu, \mu_0) + \int \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}q^2}{q^2} \theta(\mu - zq) \Delta_s\left(\mu, zq\right) P\left(z, q, k_T\right) \mathcal{F}\left(\frac{x}{z}, k_T + (1-z)q, \mu\right),$$

  where the Sudakov form factor is of the form

$$\Delta_s(\mu, \mu') = \exp\left[- \int_{\mu'^2}^{\mu^2} \frac{\mathrm{d}q^2}{q^2} \int_0^{1-\mu'/q} \mathrm{d}z \frac{C_A \alpha_s\left((1-z)^2 q^2\right)}{\pi} \frac{1}{1-z} \right],$$

  and $P(z, Q, k_T)$ is the unintegrated splitting function.

- Kimber–Martin–Ryskin model

- "Weizsäcker–Williams" model + modified "Weizsäcker–Williams" model

GLUON TMD MODELS

- Gaussian model
- Jung–Hautmann model
- Kimber–Martin–Ryskin model

$$\mathcal{F}_{\mathrm{KMR}}\left(x,k_T^2,\mu^2\right)=\frac{T_g\left(k_T,\mu\right)}{k_T^2}\frac{\alpha_s\left(k_T^2\right)}{2\pi}\int_x^{\mu/(k_T+\mu)}\mathrm{d}z\left[P_{gg}(z)\frac{x}{z}f_g\left(\frac{x}{z},k_T^2\right)+\sum_q P_{gq}(z)\frac{x}{z}f_q\left(\frac{x}{z},k_T^2\right)\right],$$

with the Sudakov factor of the form

$$T_g(Q,\mu)=\exp\left[-\int_{Q^2}^{\mu^2}\frac{\mathrm{d}p^2}{p^2}\frac{\alpha_s(p^2)}{2\pi}\int_0^{\mu/(p+\mu)}\mathrm{d}z z\left(P_{gg}(z)+\sum_q P_{qg}(z)\right)\right],$$

where $P_{ij}(z)$ are the DGLAP splitting functions and $f_i(x,\mu^2)$ are the collinear PDFs. Below the scale $k_0=1\mathrm{GeV}$ that parametrizes the onset of non-perturbative physics, it is constant in k_T

$$\mathcal{F}_{\mathrm{KMR}}\left(x,k_T^2<k_0^2,\mu^2\right)=xf_g(x,\mu^2)T_g(k_0,\mu)/k_0^2.$$

- "Weizsäcker–Williams" model + modified "Weizsäcker–Williams" model

GLUON TMD MODELS

- Gaussian model
- Jung–Hautmann model
- Kimber–Martin–Ryskin model
- "Weizsäcker–Williams" model

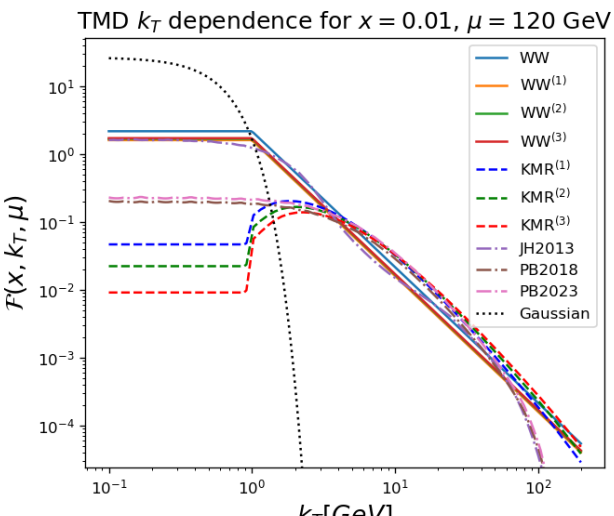
$$\mathcal{F}_{\mathrm{WW}}\left(x,k_T^2\right)=\frac{N_1}{k_0^2}(1-x)^7x^{-\lambda}\times\begin{cases}1 & k_T^2<k_0^2\\ \dfrac{k_0^2}{k_T^2} & k_T^2\geq k_0^2\end{cases}.$$

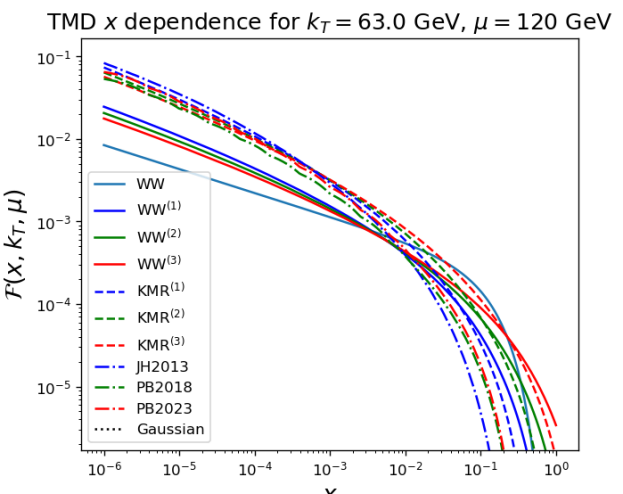
Modified "Weizsäcker–Williams" model

$$\mathcal{F}'_{\mathrm{WW}}\left(x,k_T^2,\mu^2\right)=\frac{xf_g\left(x,\mu^2\right)}{k_0^2\left[1+\log\left(\frac{\mu^2}{k_0^2}\right)\right]}\times\begin{cases}1 & k_T^2<k_0^2\\ \dfrac{k_0^2}{k_T^2} & k_T^2\geq k_0^2\end{cases}.$$

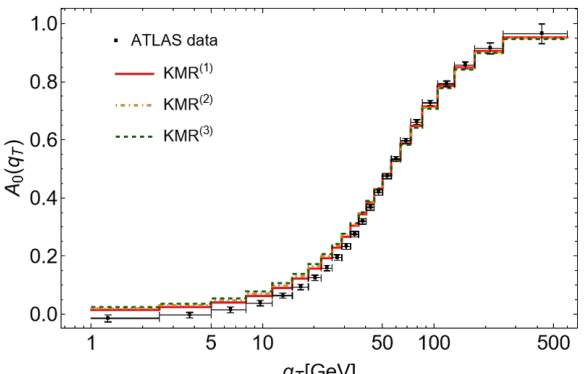
where $f_g\left(x,\mu^2\right)$ is gluon PDF and the parameters have values $N_1=0.611$, $k_0=1$ GeV and $\lambda=0.29$.

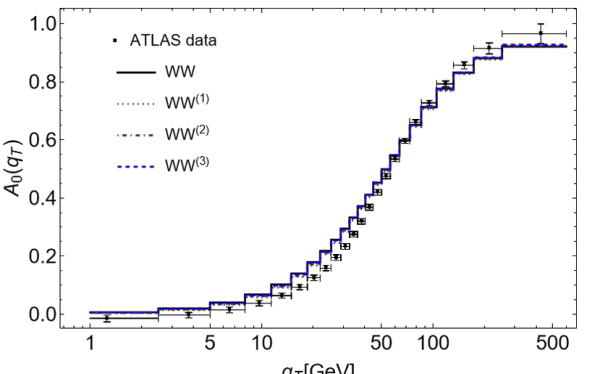
PLOTS OF ALL CONSIDERED GLUON TMD MODELS

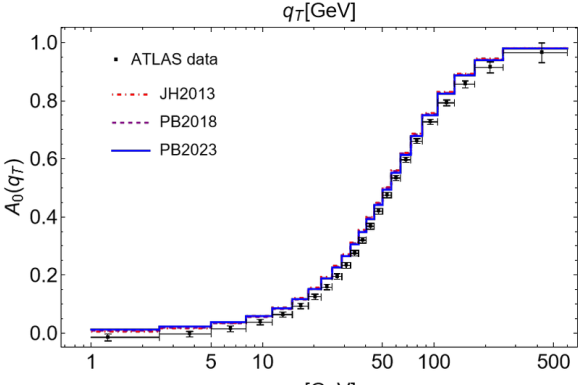


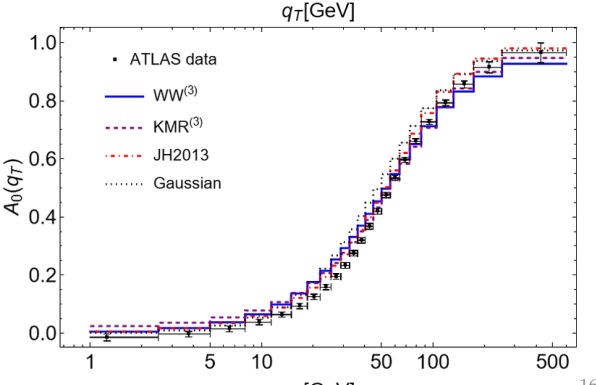


Structure Function A_0

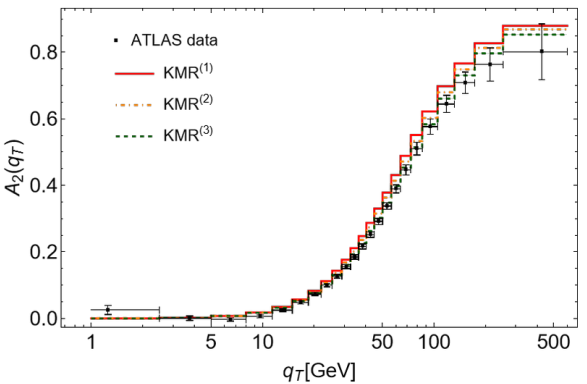


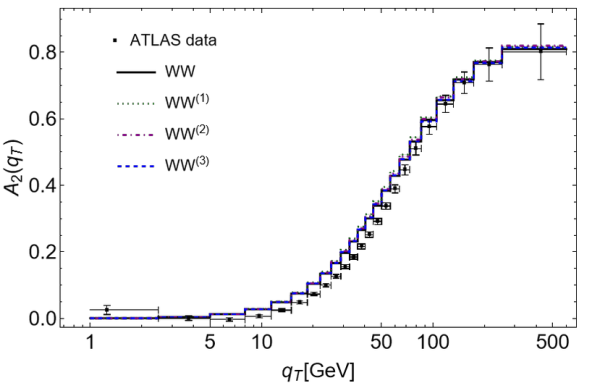


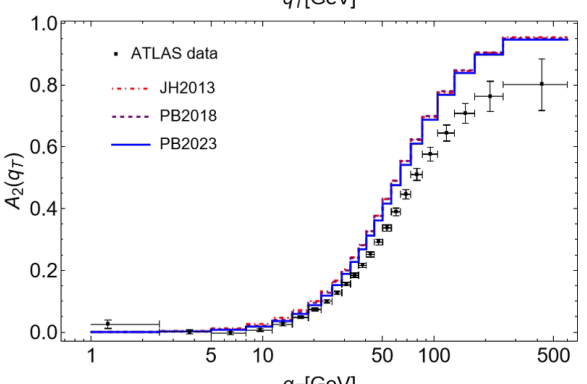


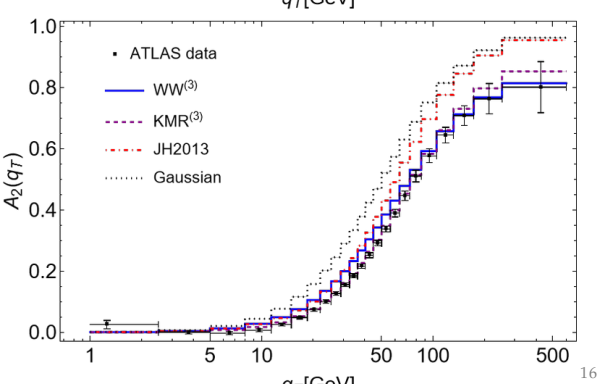


Structure Function A_2

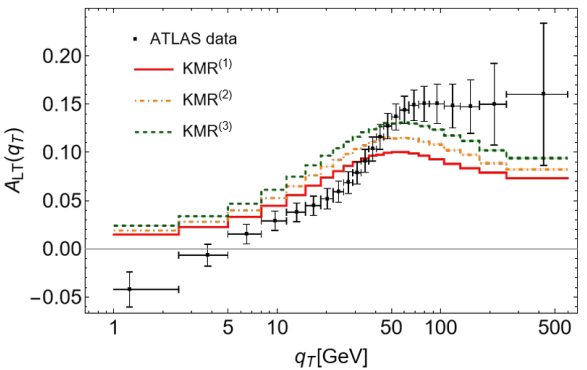


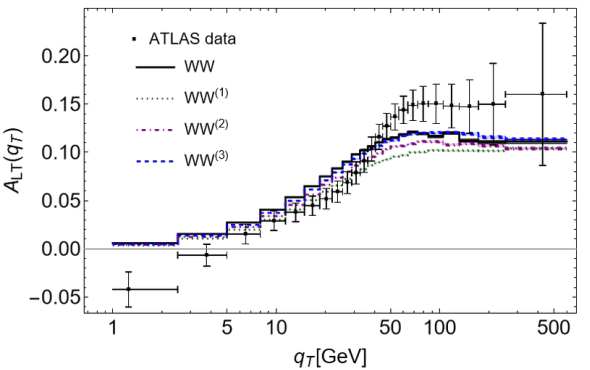


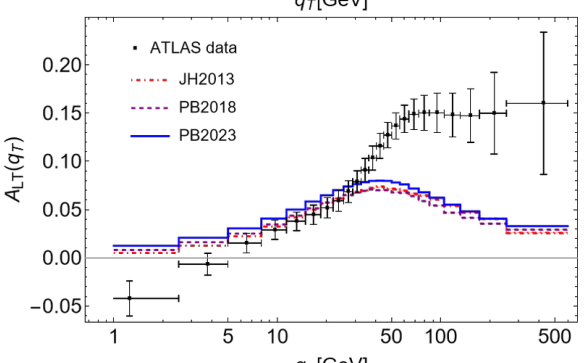


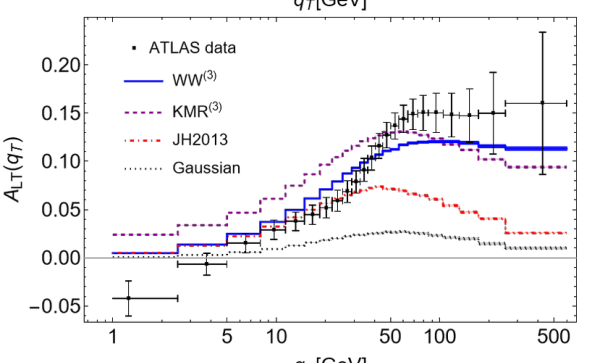


Lam–Tung combination A_{LT}

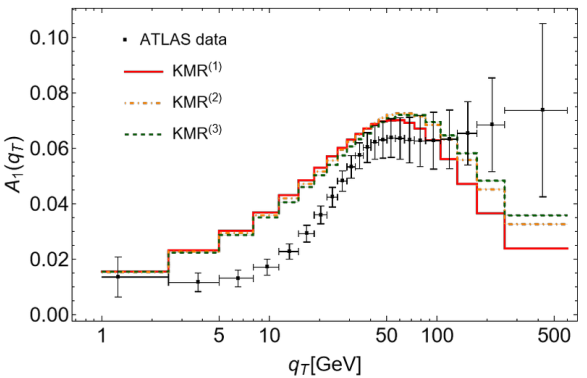


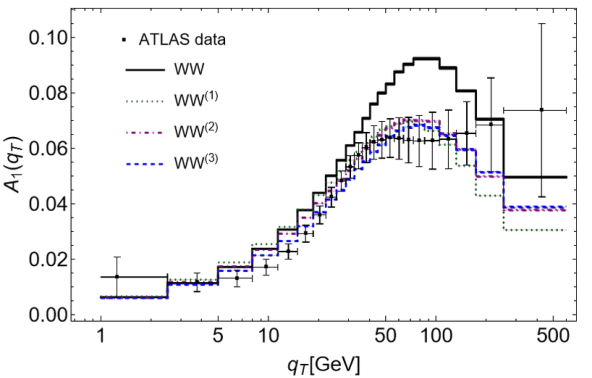


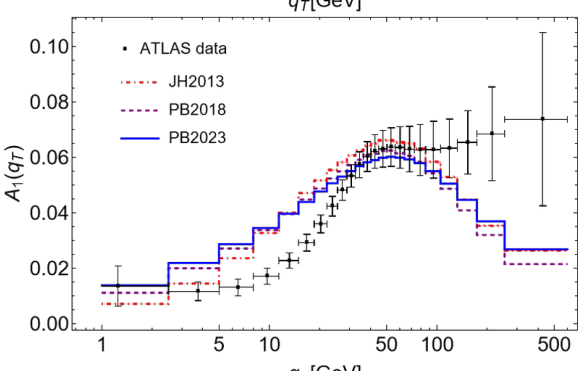


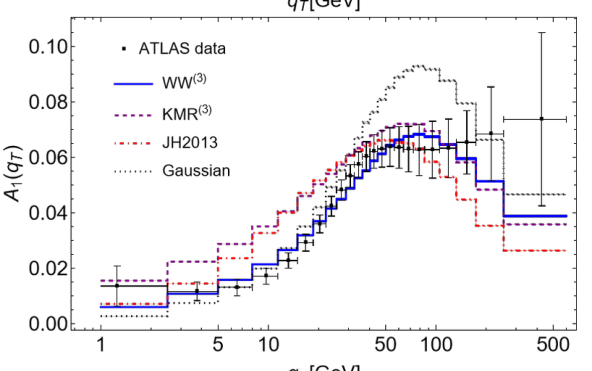


Structure Function A_1

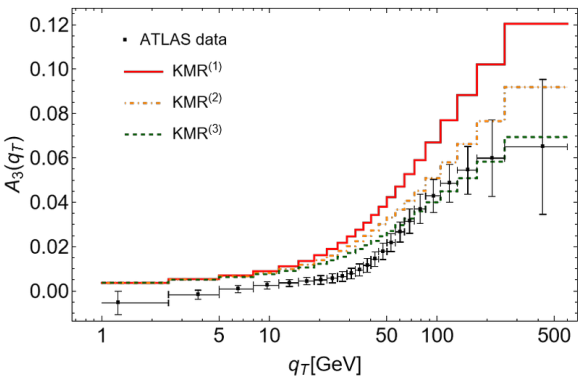


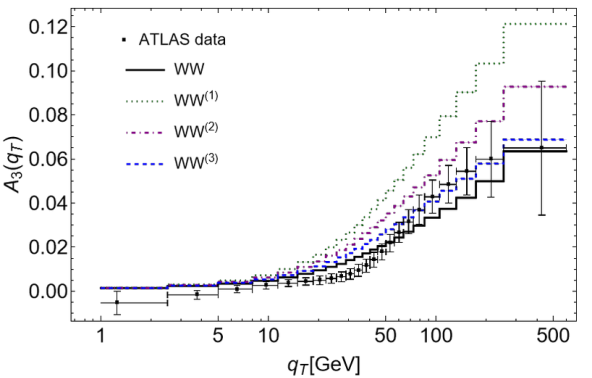


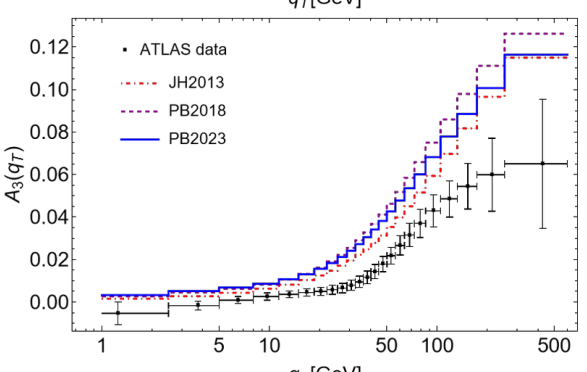


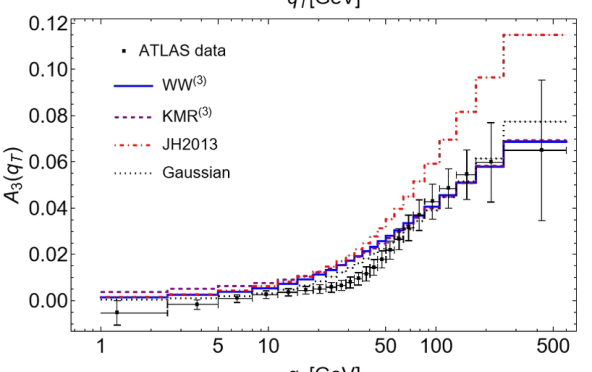


Structure Function A_3

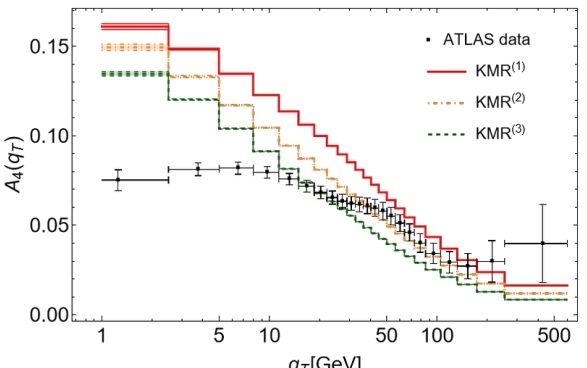


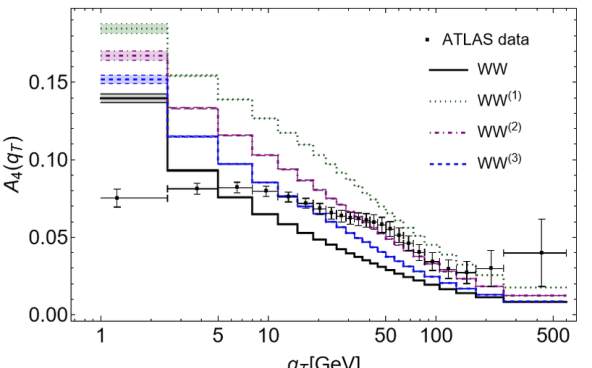


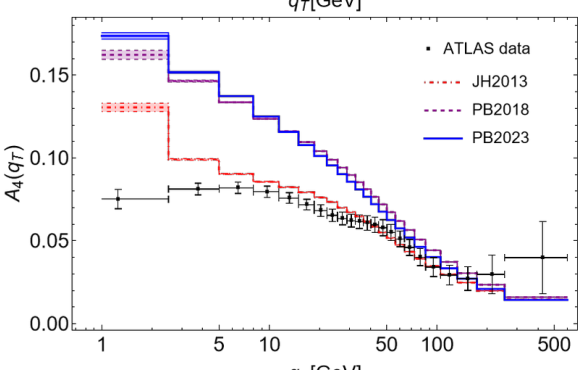


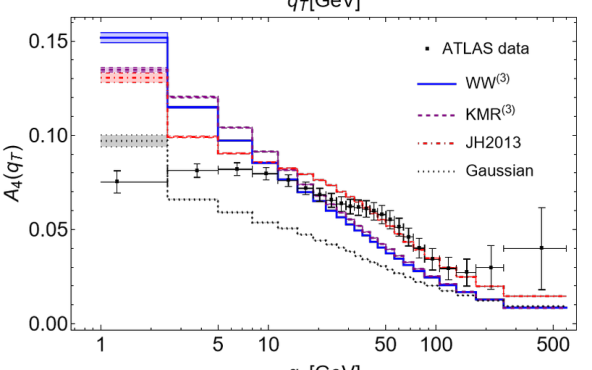


Structure Function A_4









Reduced χ^2 for the resulting structure functions

$$\chi^2_n/\text{d.o.f} = \frac{1}{N_{\text{points}}} \sum_{i=1}^{N_{\text{points}}} \frac{\left(A^{\text{th}}_{n,i} - A^{\text{data}}_{n,i}\right)^2}{\sigma^2_{i,\text{data}} + \sigma^2_{i,\text{th}}} \qquad \text{where} \qquad \sigma_{i,\text{th}} = 0.12 \cdot A^{\text{th}}_{n,i}.$$

$$\chi^2_{\text{min}}/\text{d.o.f} = \min\left\{\chi^2/\text{d.o.f}\right\} \approx 2.50 \qquad \text{where} \qquad \chi^2/\text{d.o.f} = \frac{1}{5}\sum_{n=0}^{4} \chi^2_n/\text{d.o.f.}$$

Model	WW$^{(3)}$	KMR$^{(3)}$	JH2013	Gauss.
$\chi^2_0/\chi^2_{\text{min}}$	0.84	1.26	0.52	0.82
$\chi^2_1/\chi^2_{\text{min}}$	0.15	1.10	0.98	0.70
$\chi^2_2/\chi^2_{\text{min}}$	1.29	0.27	1.56	4.35
$\chi^2_3/\chi^2_{\text{min}}$	1.34	1.78	2.40	0.55
$\chi^2_4/\chi^2_{\text{min}}$	1.39	1.15	0.35	4.70

The ratios of χ^2 per degree of freedom of each structure function (χ^2_n) for each TMD model to the minimal global reduced χ^2 — χ^2_{min}.