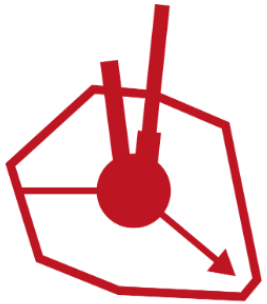
Nucleon Tomography with 0-jettiness



Shuo Lin
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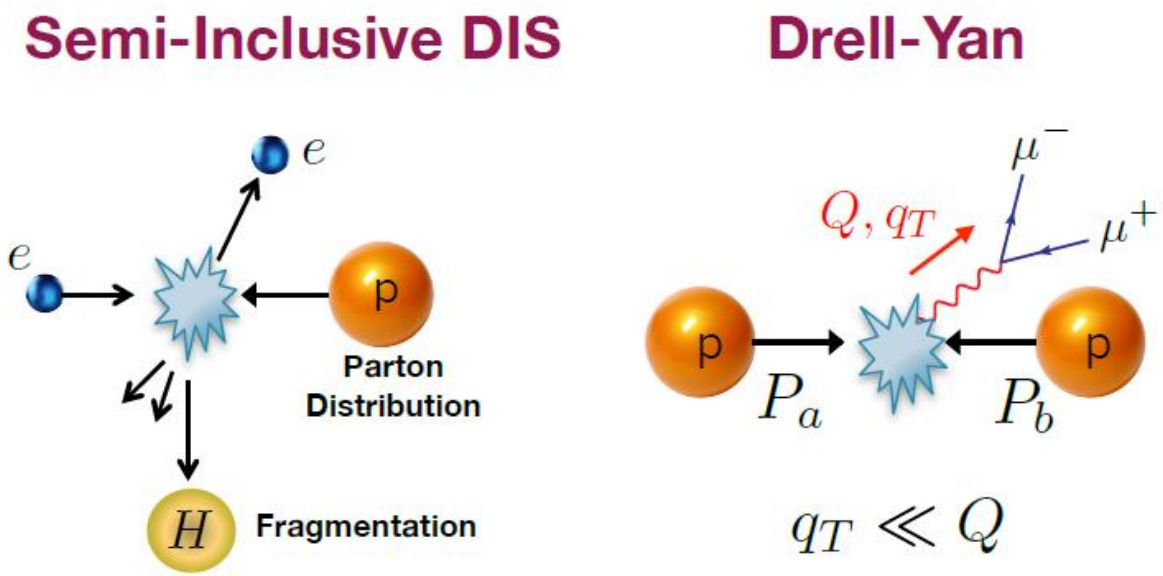
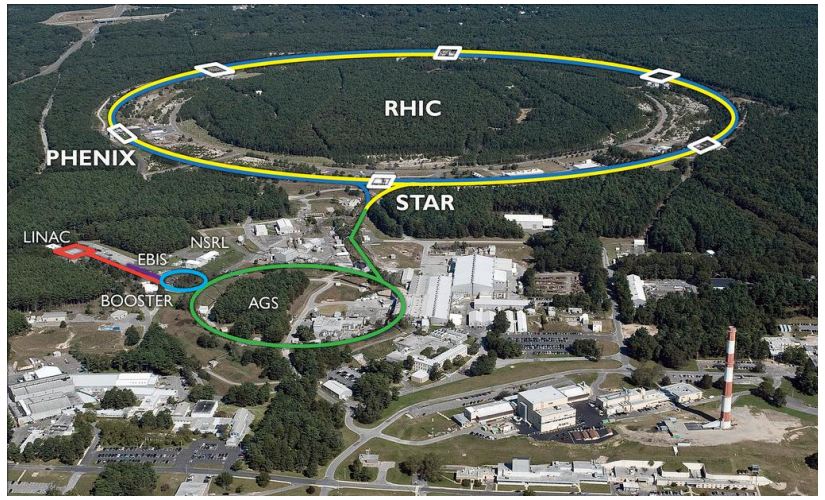
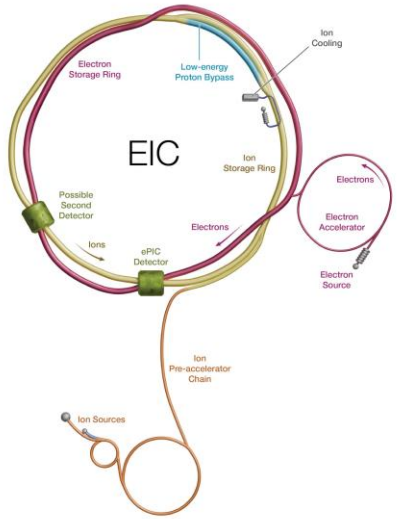
Phys.Rev.Lett. 136 (2026) 2, 021901
In collaboration with Shen Fang,Ding Yu Shao,and Jian Zhou

Nucleon Tomography : a fundamental quest





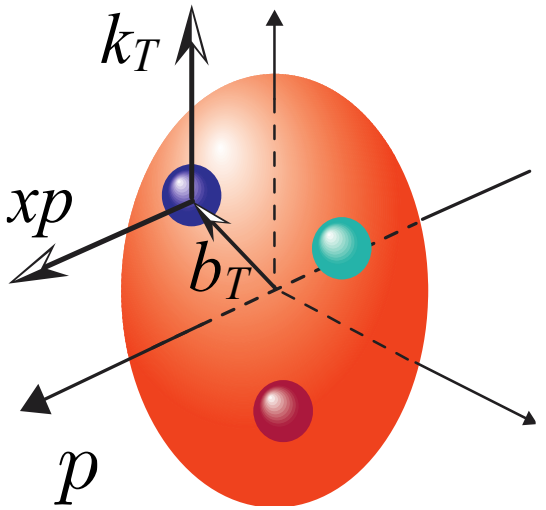


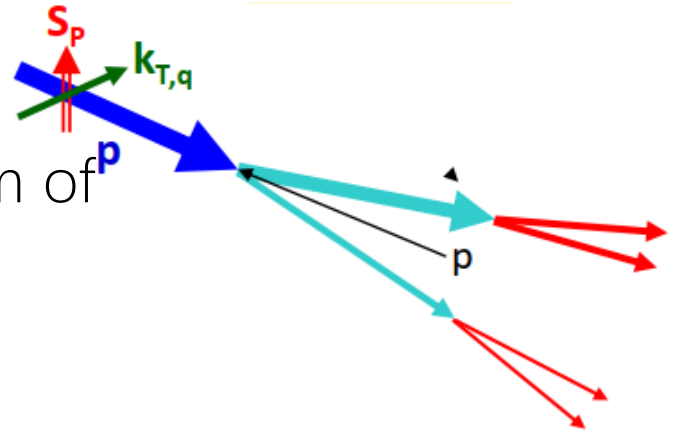
Proton structure: encoded in PDFs

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 \, \hat{\sigma}_{ab}(Q, x_1, x_2, \mu_f) \, f_a(x_1, \mu_f) \, f_b(x_2, \mu_f) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$$

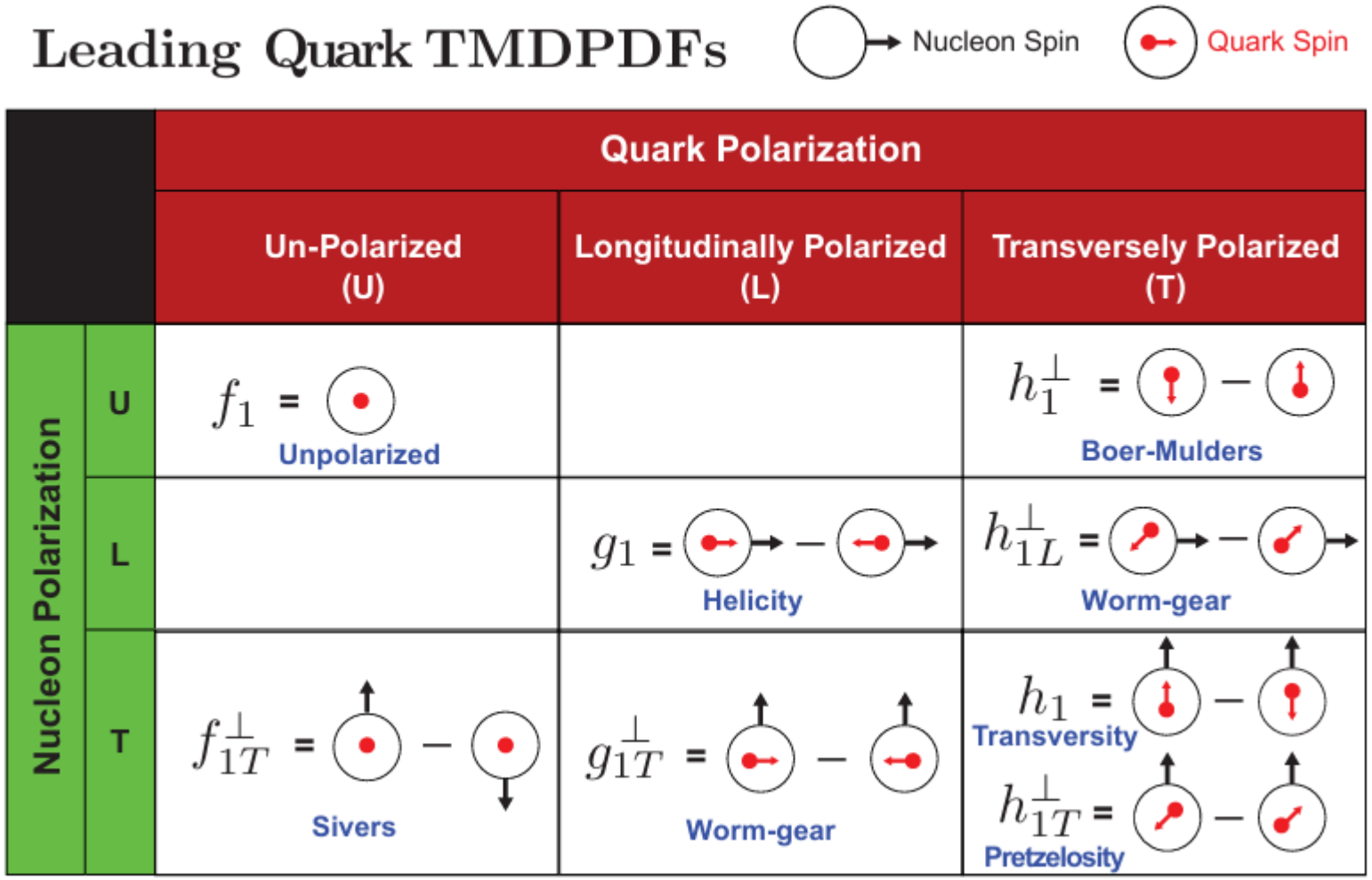
TMDs: center piece of nucleon structure

- Phenomenogical needs
- Both longitudinal and transverse motion of partons inside proton
- Quantum correlation: spin-spin, spin-momentum (orbit) correlations
- Orbital motion
  - Most TMDs would vanish in the absence of parton orbital angular momentum
- Information on the color glass condensate —a unique form of nuclear matter at small x
- QCD factorization

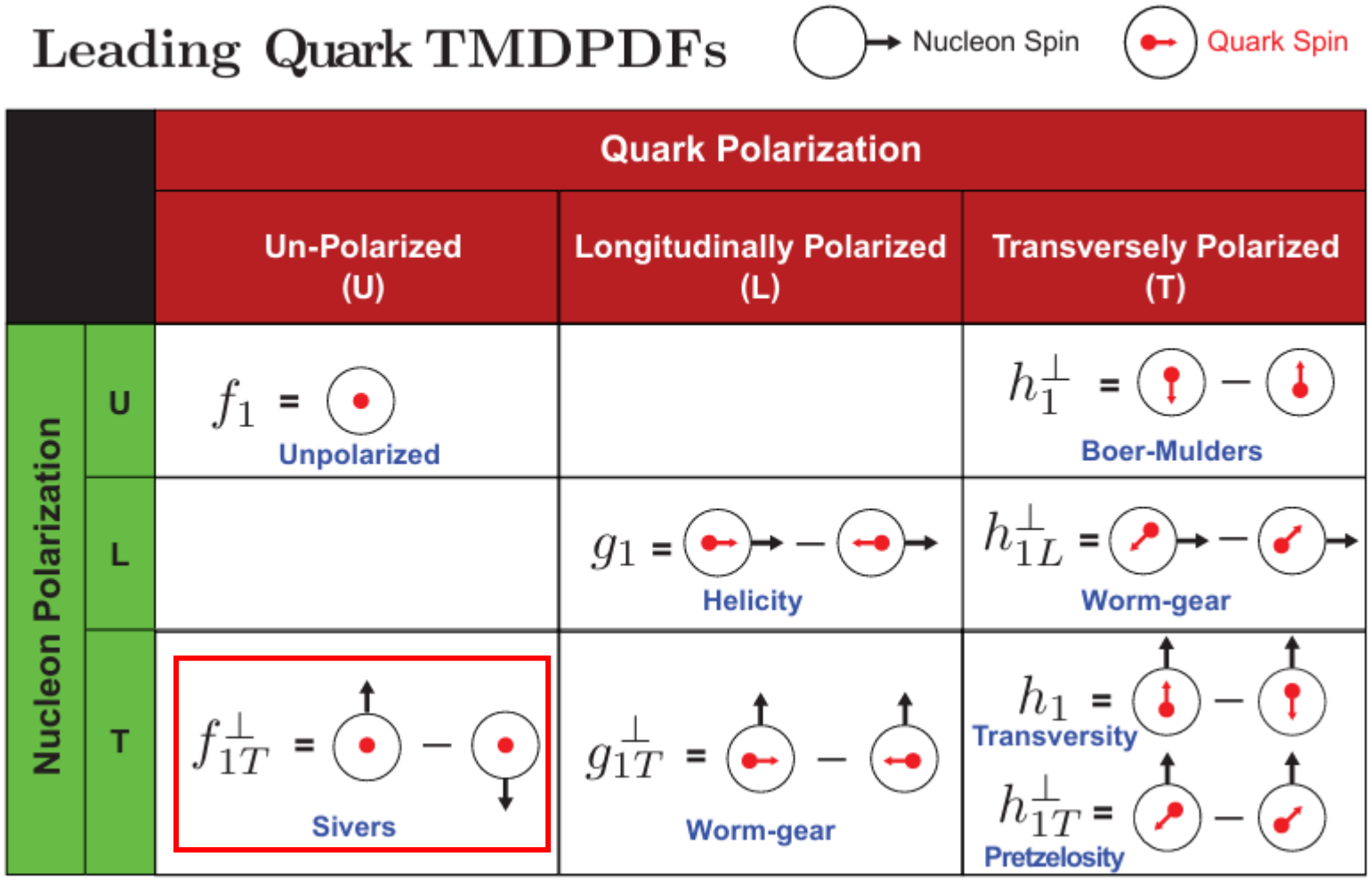




TMDs: center piece of nucleon structure



TMDs: center piece of nucleon structure



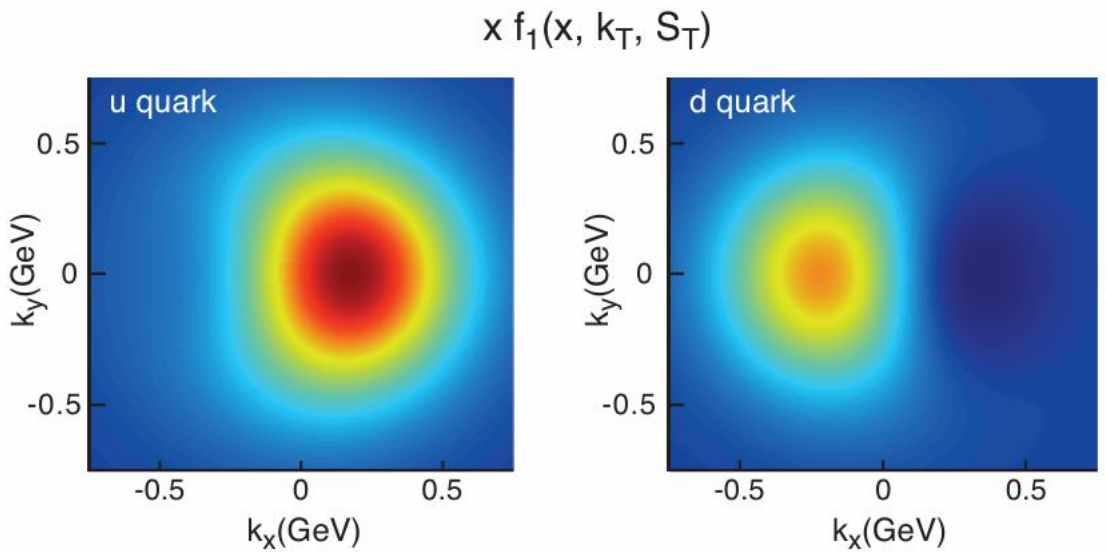
One example: Sivers function

- Sivers function describes the transverse momentum distribution correlated with the transverse polarization vector of the nucleon.

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) - \frac{1}{M} f_{1T}^{\perp q}(x, k_\perp)\, \vec{S} \cdot (\hat{p} \times \mathbf{k}_\perp)$$

Spin-independent

Spin-dependent



A.Accardi et al.Eur.Phys.J.A 52 (2016) 9, 268

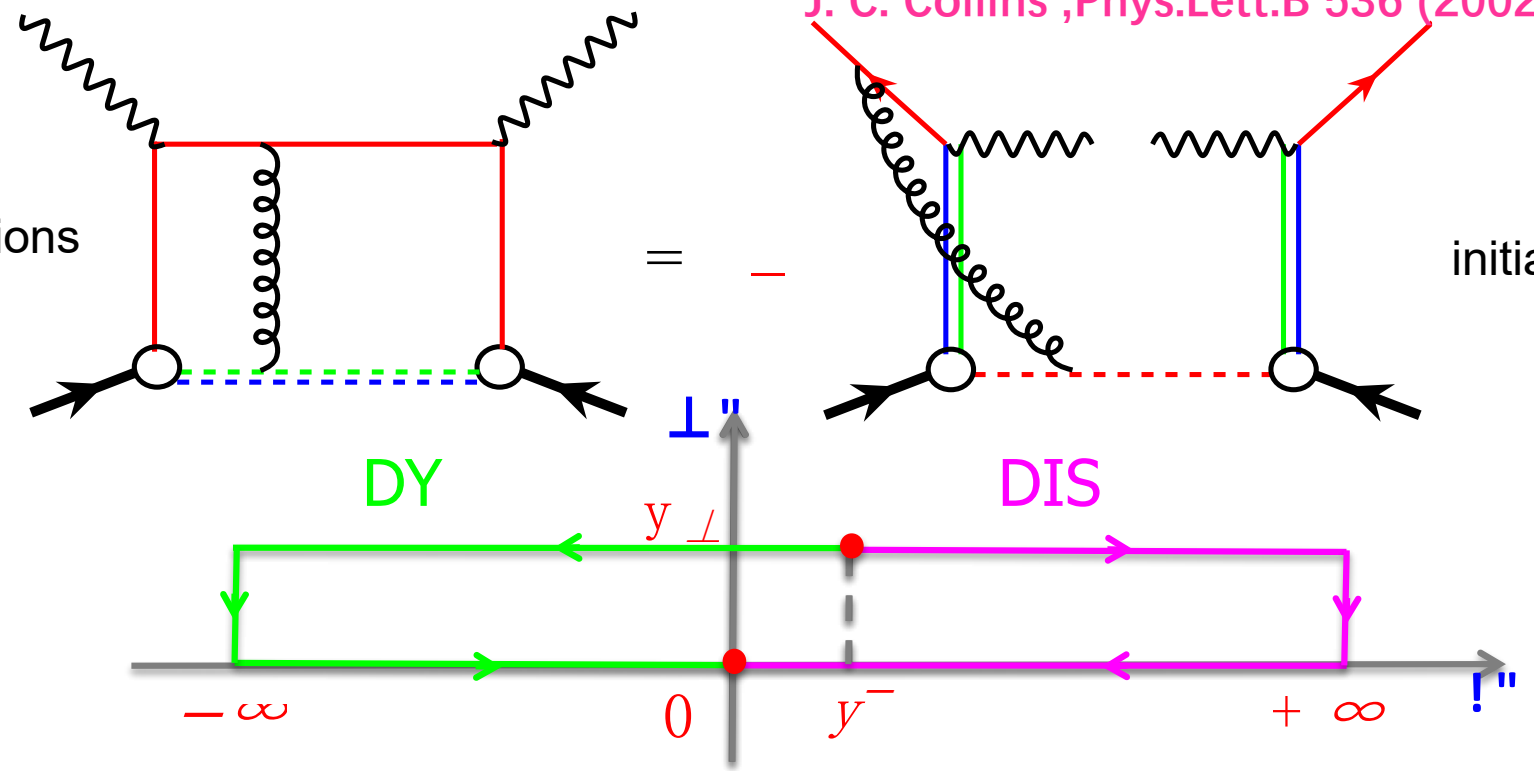
- The quark distribution will be azimuthally asymmetric in the transverse momentum space in a transversely polarized nucleon.

One example: Sivers function

- Naïve time-reversal-odd, and its existence requires a phase (generate through interactions)

S. J. Brodsky, D. S. Hwang, and I. Schmidt, Phys.Lett.B 530 (2002) 99-107
J. C. Collins ,Phys.Lett.B 536 (2002) 43-48

final state interactions



initial state interactions

$$f_{1T}^{\perp,\mathrm{DIS}}(x,k_\perp) = -f_{1T}^{\perp,\mathrm{DY}}(x,k_\perp)$$

Sivers Function and Qiu-Sterman Function

- Qiu-Sterman Function

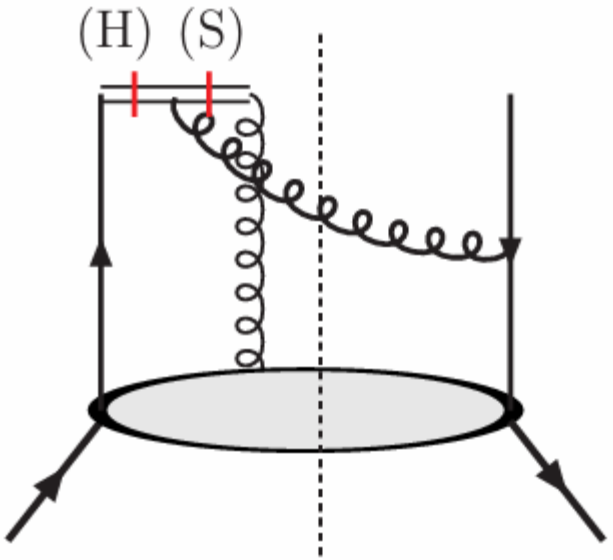
$$T_F(x_2,x_2') \equiv \int \frac{d\zeta^- d\eta^-}{4\pi} e^{i(x_2 P^+ \eta^- + (x_2'-x_2)P_B^+ \zeta^-)} \epsilon_\perp^{\beta\alpha} S_{\perp\beta}$$
$$\times \left\langle PS|\overline{\psi}(0)\mathcal{L}(0,\zeta^-)\gamma^+ gF_\alpha{}^+(\zeta^-)\mathcal{L}(\zeta^-,\eta^-)\psi(\eta^-)|PS\right\rangle$$

- $T_F(x,x) = \int \frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2} f_{1T}^\perp|_{\rm DY}(x,k_\perp)$

D. Boer, P. J. Mulders and F. Pijlman, Nucl. Phys. B 667, 201 (2003

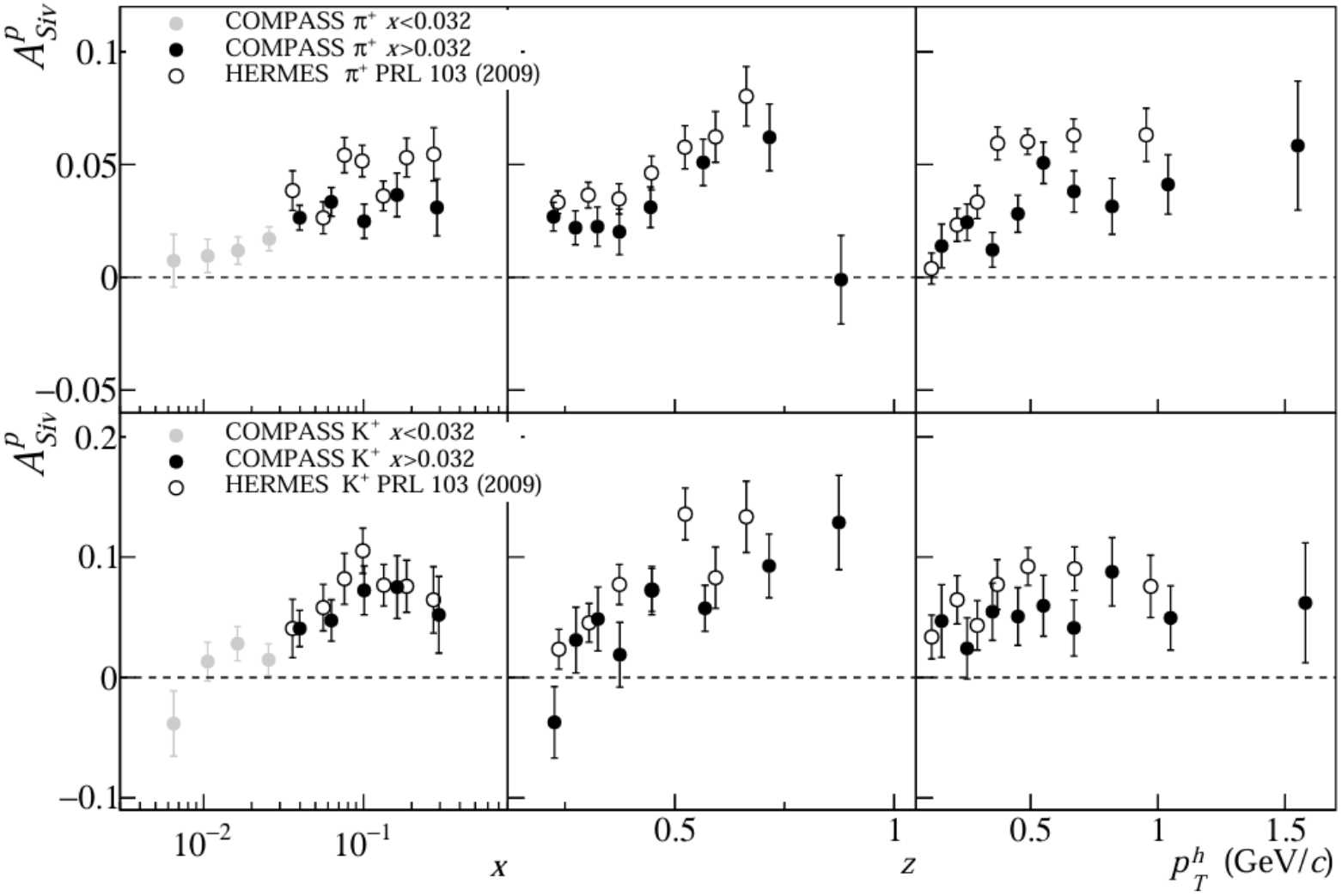
- Sivers function at large kt

$$f_{1T}^\perp(z,k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{M}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{\frac{C_A}{2}T_F(x,z)\frac{1+\xi}{(1-\xi)_+} + T_F(x,x)\frac{-1}{2N_c}\frac{D-2}{2}(1-\xi)\right.$$
$$+\frac{1}{2N_C}\left[\left(x\frac{\partial}{\partial x}T_F(x,x)\right)(1+\xi^2) + T_F(x,x)\frac{(1-\xi)^2(2\xi+1)-2}{(1-\xi)_+}\right]$$
$$\left.+T_F(x,x)\delta(1-\xi)C_F\left(\ln\frac{x^2\zeta^2}{k_\perp^2}-2\right)\right\}\ .$$



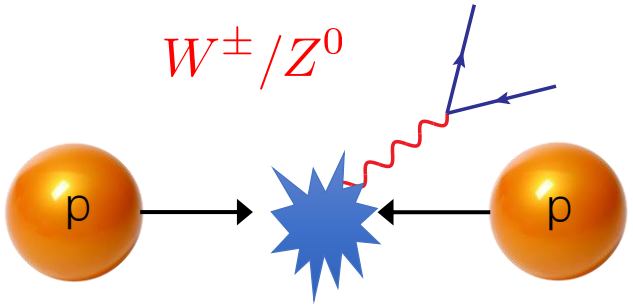
X. Ji, J.W. Qiu, W. Vogelsang, F. Yuan, Phys.Rev.Lett. 97 (2006) 082002 , Phys.Rev.D 73 (2006) 094017
J. Zhou F. Yuan Z. T. Liang,Phys.Rev.D78:114008,2008
Peng Sun, Feng Yuan ,Phys.Rev.D 88 (2013) 11, 114012

Transverse single spin asymmetries in SIDIS

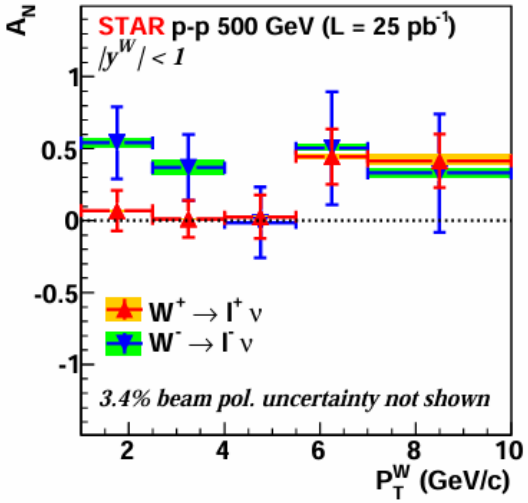


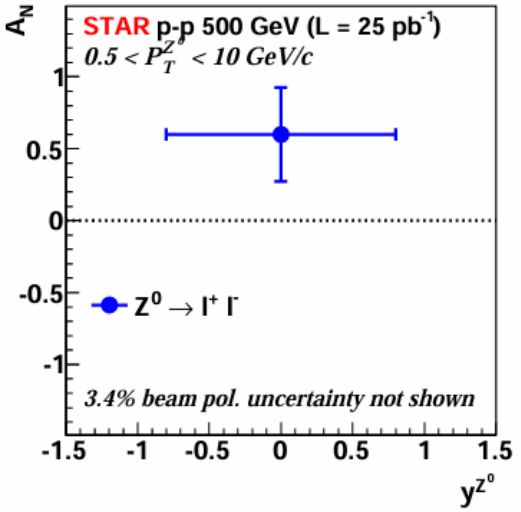
COMPASS
Phys. Lett. B744 (2015) 250
HERMES
Phys. Rev. Lett. 103 (2009) 152002

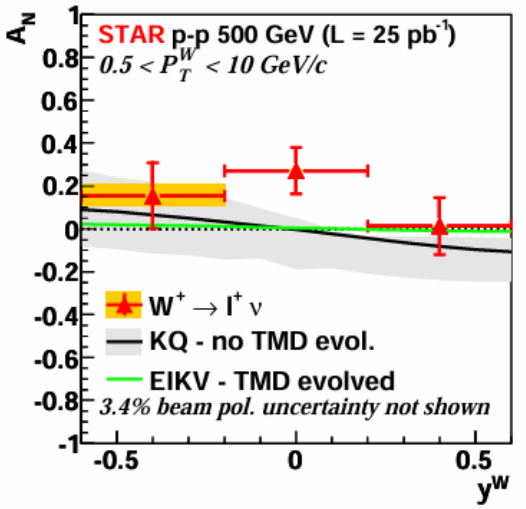
Transverse single spin asymmetries in weak boson production

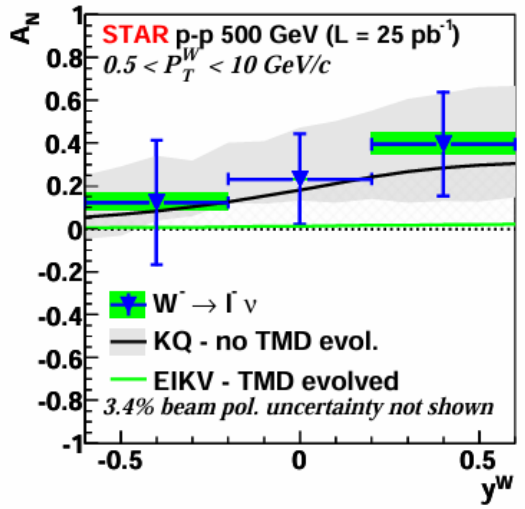


STAR Collaboration,
Phys. Rev. Lett. 116, 132301 (2016)









Resummation

Soft gluon radiation leads to Sudakov Logarithms

$$\ln\left(Q^2 b_\perp^2\right) \sim \ln \frac{Q^2}{q_\perp^2}$$

The large logs will be resummed into the exponential factor

$$W(Q,b) = e^{-\int_{1/b}^{Q} \frac{d\mu}{\mu}\left(\ln \frac{Q}{\mu} A + B\right)} C \otimes f_1 C \otimes f_2$$

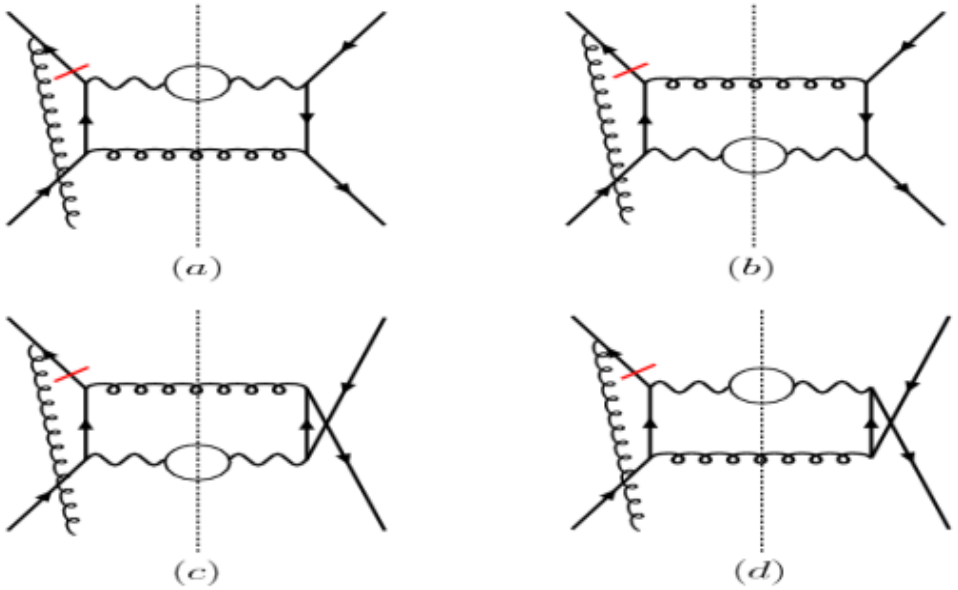
Phenomenogical applications of the QCD resummation to the transverse momentum spectrum have been very successful

Resummation

Z.-B. Kang, B.-W. Xiao, and F. Yuan, Phys.Rev.Lett. 107 (2011) 152002

$$\frac{d\Delta\sigma(S_\perp)}{dydQ^2d^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_\perp^\alpha W_{UT}^\beta(Q;q_\perp)$$

$$\begin{aligned}\widetilde{W}_{UT}^\alpha(Q;b) &= e^{-\mathcal{S}_{UT}(Q^2,b)}\widetilde{W}_{UT}^\alpha(C_1/b,b) \\ &= (-ib_\perp^\alpha/2)\, e^{-\mathcal{S}_{UT}(Q^2,b)}\Sigma_{i,j} \\ &\quad \times\Delta C_{qi}^T \otimes f_{i/A}^{(3)}(z_1)C_{\bar{q}j}\otimes f_{j/B}(z_2)\end{aligned}$$

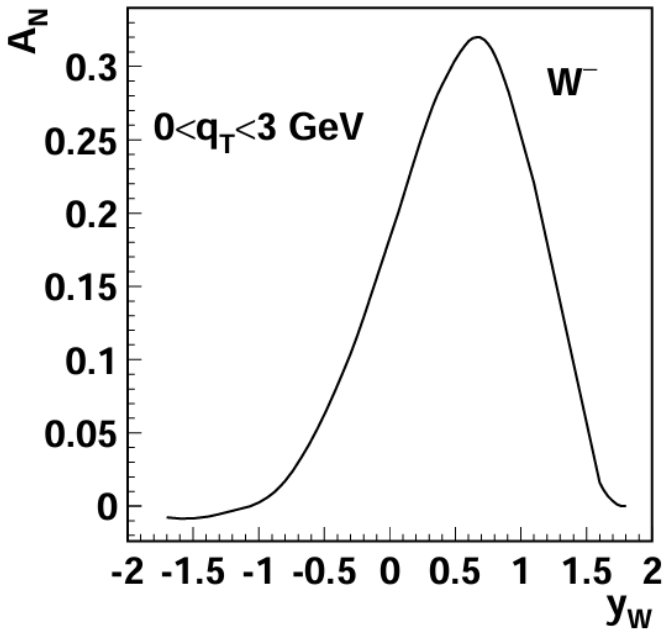


$$\mathcal{S}_{UT}(Q^2,b) = \int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{C_2^2Q^2}{\mu^2}\right)A_{UT}(C_1;g(\mu)) + B_{UT}(C_1,C_2;g(\mu))\right],$$

Sudakov factor S_{UT} have the same form as that for the spin-average case

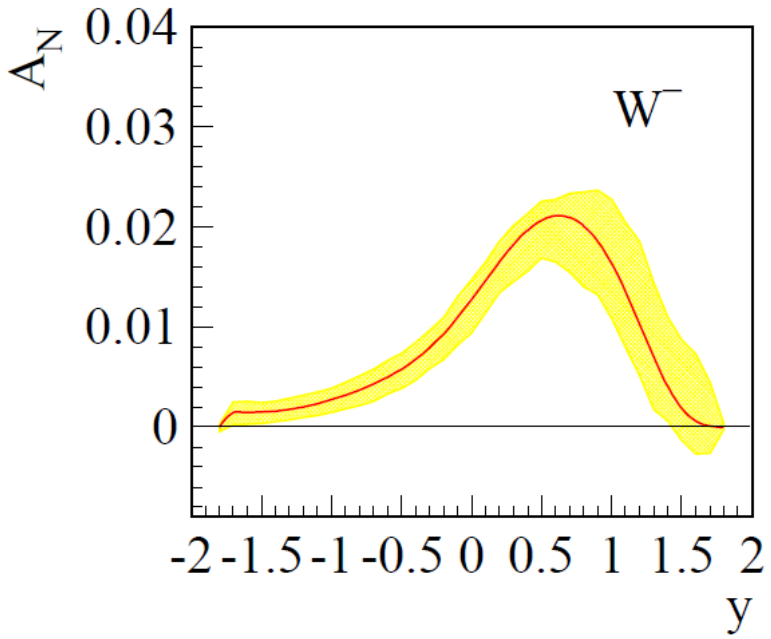
TMD evolution effect

Without TMD evolution



Z.B Kang, J.W Qiu,
Phys.Rev.Lett. 103 (2009) 172001

With TMD evolution



M.G. Echevarria,A. Idilbi,, Z.B Kang, I. Vitev, Phys.Rev.D 89 (2014) 074013

TMD evolution reduces the asymmetry.

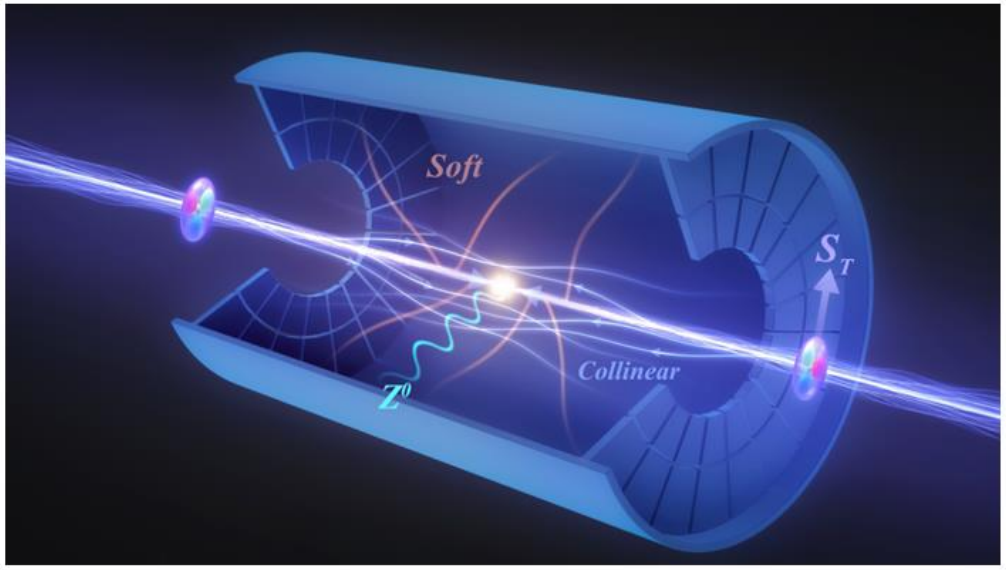
How to enhance the asymmetry?

Our solution: 0-jettiness veto method

$$\tau \equiv \frac{2}{Q^2}\sum_i \min\{p_a \cdot l_i, p_b \cdot l_i\} = \sum_i \frac{|\vec{l}_{\perp,i}|}{Q} e^{-|y_i - y|}$$

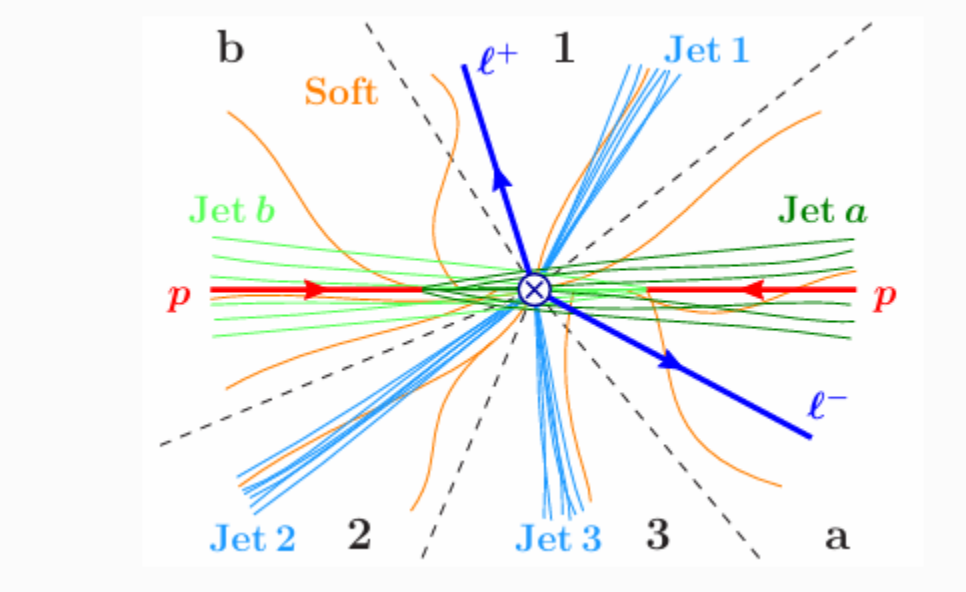
$$\tau < \tau_0$$

S. Fang, S. Lin, D. Y. Shao, and J. Zhou, Phys.Rev.Lett. 136 (2026) 2, 021901



N-Jettiness

N-jettiness is a global event shape defined in terms of the beam $q_{a,b}$ and jet-directions q_j



$$\tau_N = \frac{2}{Q^2} \sum_k \min\{q_a \cdot p_k,\ q_b \cdot p_k,\ q_1 \cdot p_k,\ \ldots,\ q_N \cdot p_k\}$$

I.W. Stewart, F.J. Tackmann, W.J. Waalewijn,Phys.Rev.Lett.105:092002,2010

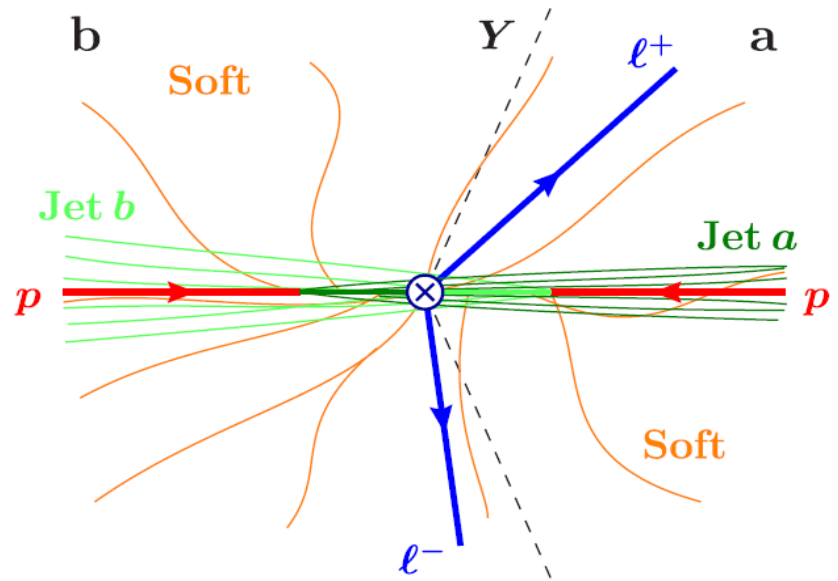
TMDs with 0-jettiness

- For electroweak Drell-Yan processes, the 0-jettiness variable is defined as

$$\tau \equiv \frac{2}{Q^2}\sum_i \min\{p_a \cdot l_i, p_b \cdot l_i\} = \sum_i \frac{|\vec{l}_{\perp,i}|}{Q} e^{-|y_i - y|}$$

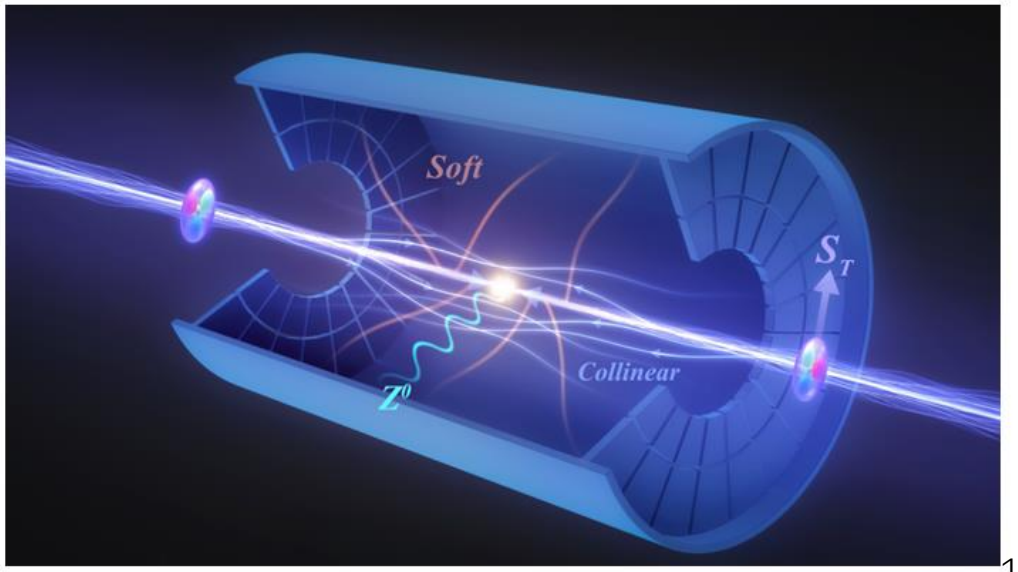
The sum runs over all particles i (excluding the gauge boson) with momentum l_i

$$\tau < \tau_0$$



(b) Isolated Drell-Yan.

- strongly suppresses central gluon radiation and effectively constrains initial state radiation.
- enhance the sensitivity to the intrinsic non-perturbative structure of TMDs



TMDs with 0-jettiness

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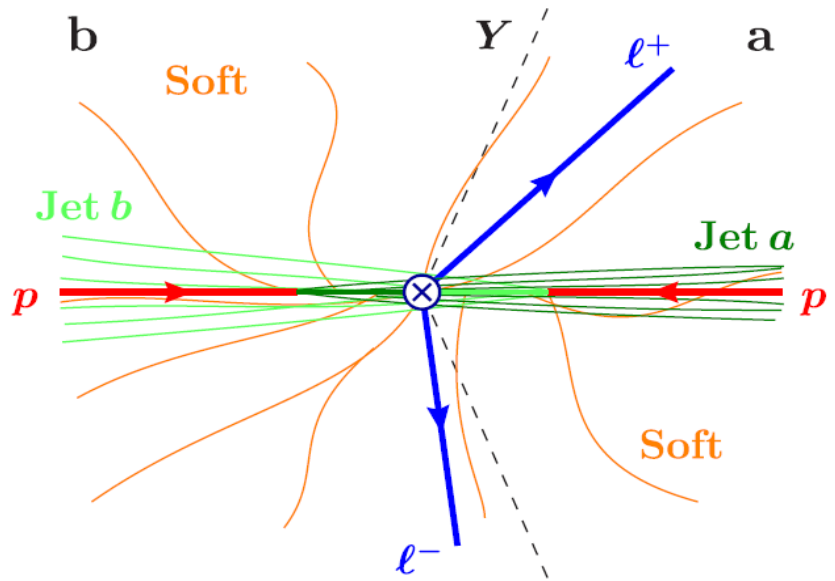
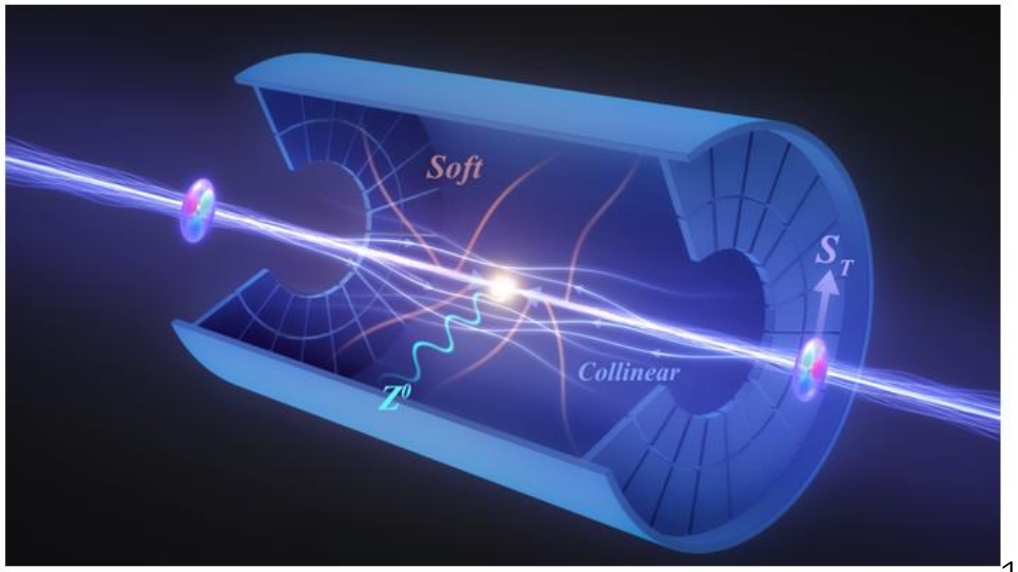
The sum runs over all particles i (excluding the gauge boson) with momentum l_i

$$\tau < \tau_0$$



(b) Isolated Drell-Yan.

- The restricted phase space has a significant impact on the resummation procedure



Joint resummation of TMDs with 0-jettiness

Two types of large logs: $\ln \frac{Q^2}{q_\perp^2}$ $\ln \frac{1}{\tau_0}$

- The modified Sudakov factor with vetoes:

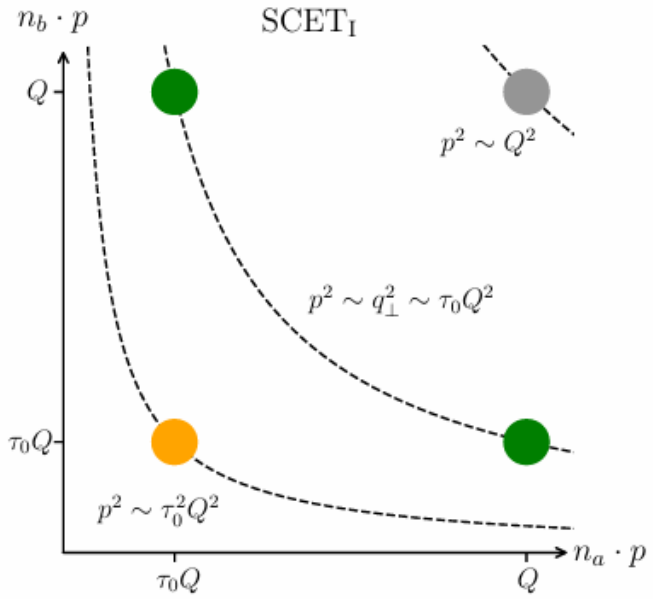
$$\frac{C_F}{\pi}\left[\int_{\mu_b^2}^{t}\frac{d\mu^2}{\mu^2}\left(2\ln\frac{t}{\mu^2}-\frac{3}{2}\right)-\int_{\mu_b^2}^{\tau_0 t}\frac{d\mu^2}{\mu^2}\ln\frac{\tau_0 t}{\mu^2}+\int_{t}^{Q^2}\frac{d\mu^2}{\mu^2}\left(\ln\frac{Q^2}{\mu^2}-\frac{3}{2}\right)+\int_{\tau_0 t}^{t}\frac{d\mu^2}{\mu^2}\ln\frac{\mu^2}{\tau_0 t}\right]\alpha_s(\mu)$$

- The standard Sudakov factor:

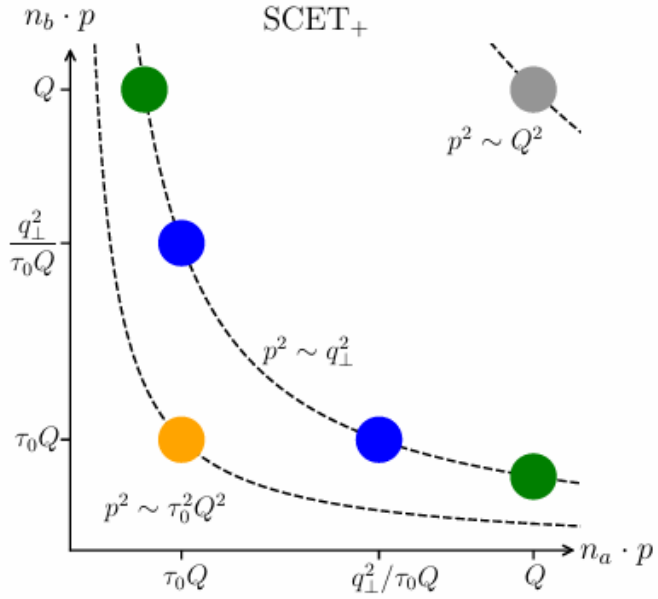
$$\frac{C_F}{\pi}\int_{\mu_b^2}^{Q}\frac{d\mu^2}{\mu^2}\left(\ln\frac{Q}{\mu^2}-\frac{3}{2}\right)\alpha_s(\mu)$$

Formulation in SCET

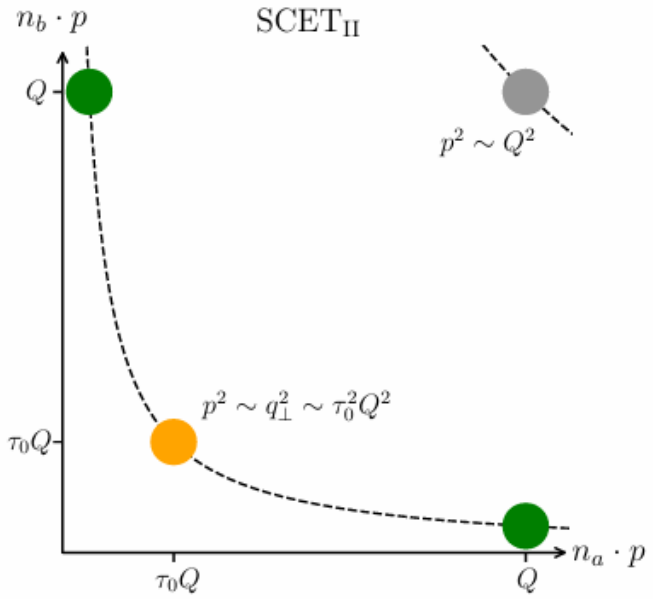
G. Lustermans, J. K. L. Michel, F. J. Tackmann, and W. J. Waalewijn, JHEP 03, 124 (2019), 1901.03331.
M. Procura, W. J. Waalewijn, and L. Zeune, JHEP 02, 117 (2015), 1410.6483.



$$\tau_0^2 Q^2 \ll q_\perp^2 \sim \tau_0 Q^2 \ll Q^2$$



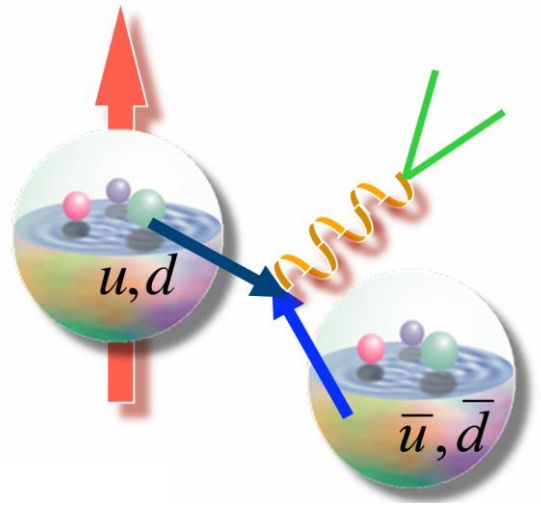
$$\tau_0^2 Q^2 \ll q_\perp^2 \ll \tau_0 Q^2 \ll Q^2$$



$$\tau_0^2 Q^2 \sim q_\perp^2 \ll \tau_0 Q^2 \ll Q^2;$$

Impact of 0-jettiness on spin asymmetries

$Z \to \ell^+\ell^-$



- Unpolarized:

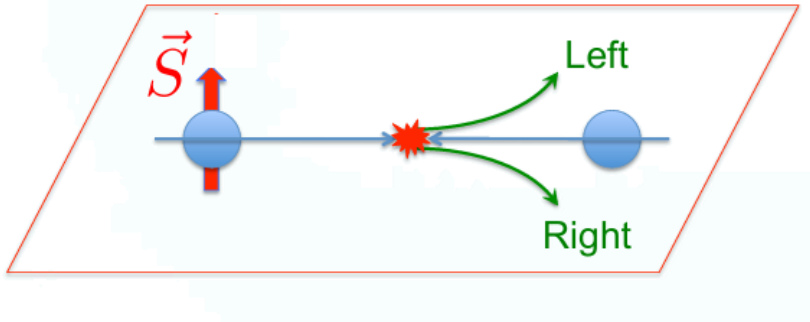
$$\frac{\mathrm{d}\sigma_{UU}}{\mathrm{d}y\,\mathrm{d}^2\vec{q}_\perp} = \sigma_0 \sum_{q,q'} |V_{qq'}|^2 \int_0^\infty \frac{b\,\mathrm{d}b}{2\pi} J_0(b\,q_\perp) \times f_q(x_q,\mu_b) f_{q'}(x_{q'},\mu_b) e^{-S_{\mathrm{P}}(b)} ,$$

- Transversely-polarized:

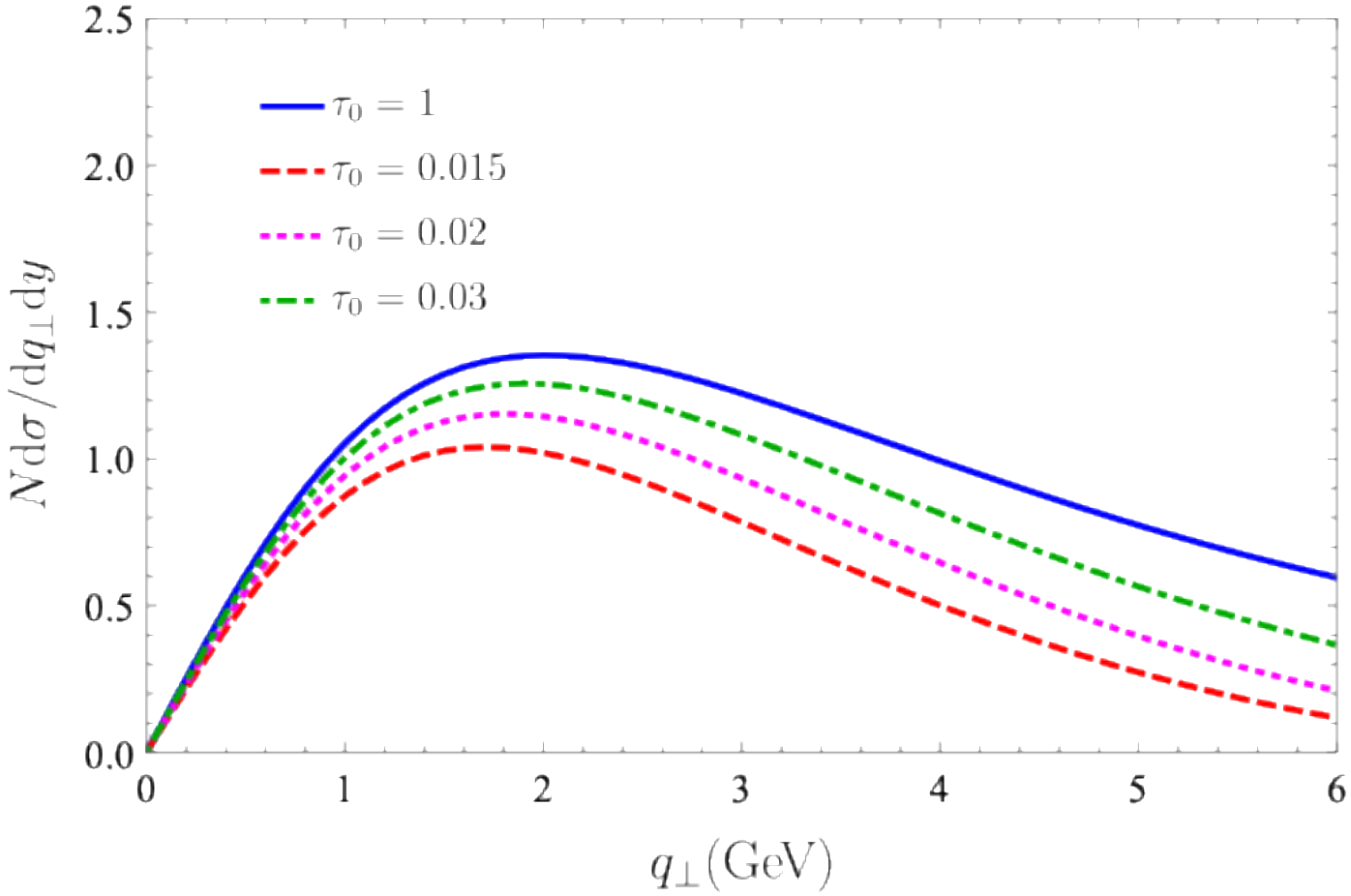
$$\frac{\mathrm{d}\sigma_{UT}(S_\perp)}{\mathrm{d}y\,\mathrm{d}^2\vec{q}_\perp} = -\sin(\phi_q - \phi_S)\,\sigma_0 \int_0^\infty \frac{b^2\,\mathrm{d}b}{4\pi} J_1(b\,q_\perp) \times \sum_{q,q'} |V_{qq'}|^2 T_{F,q}(x_a,x_a,\mu_b) f_{q'}(x_b,\mu_b) e^{-S_{\mathrm{P}}(b)} .$$

- SSA:

$$A_N = \frac{\int_0^{2\pi} \mathrm{d}\phi_q\, 2\sin(\phi_q - \phi_S)\,\mathrm{d}\sigma_{UT}}{\int_0^{2\pi} \mathrm{d}\phi_q\,\mathrm{d}\sigma_{UU}} .$$



Numerical results

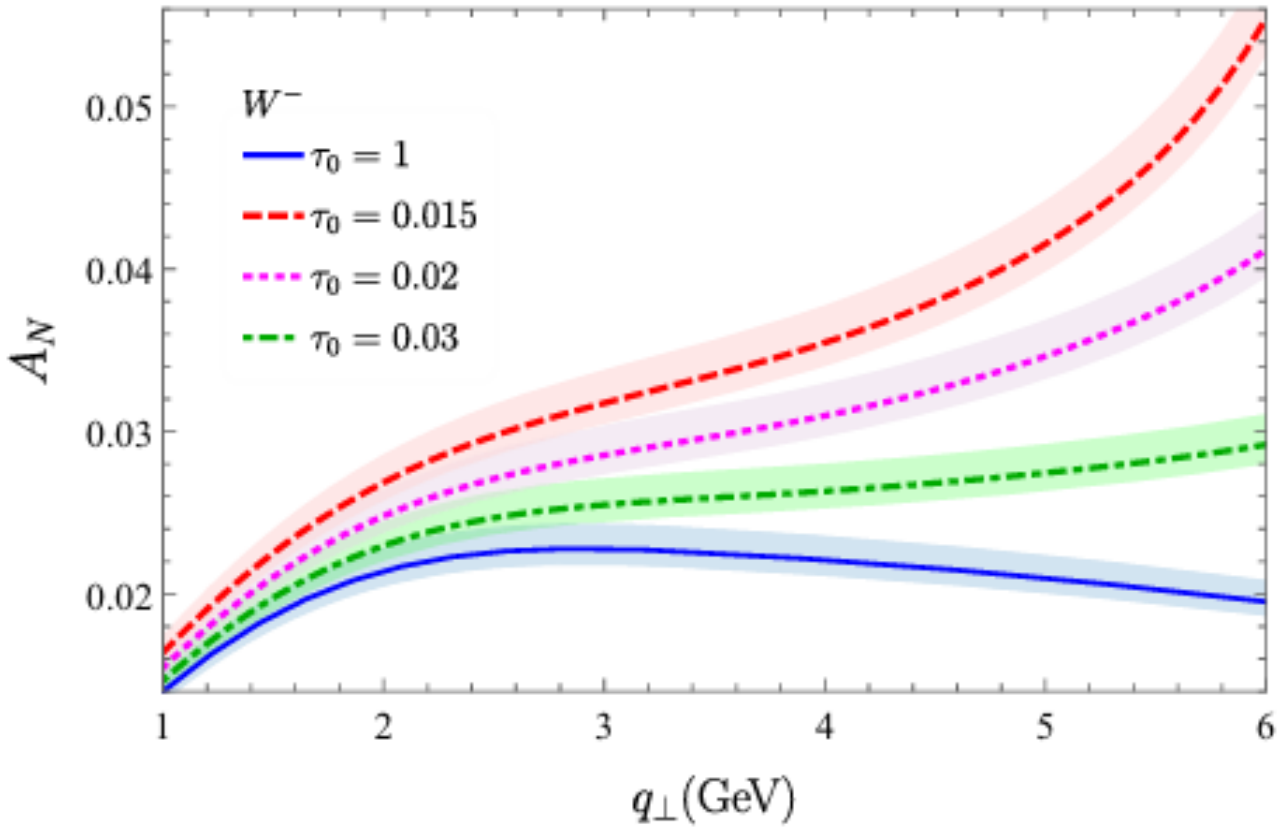


The normalized unpolarized cross section for W^- production at RHIC energy

The enhanced asymmetries

0-jettiness $\tau \equiv \frac{2}{Q^2}\sum_i \min\{p_a \cdot l_i, p_b \cdot l_i\} = \sum_i \frac{|\vec{l}_{\perp,i}|}{Q} e^{-|y_i - y|}$ with veto $\tau < \tau_0$

The SSAs for W^- production at RHIC energy

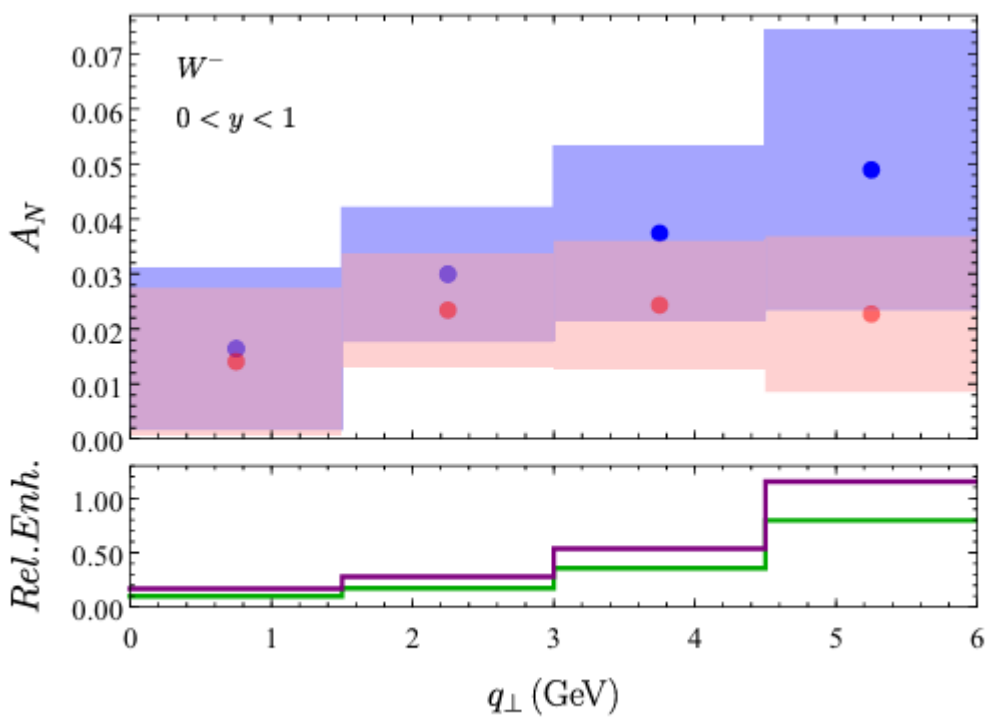


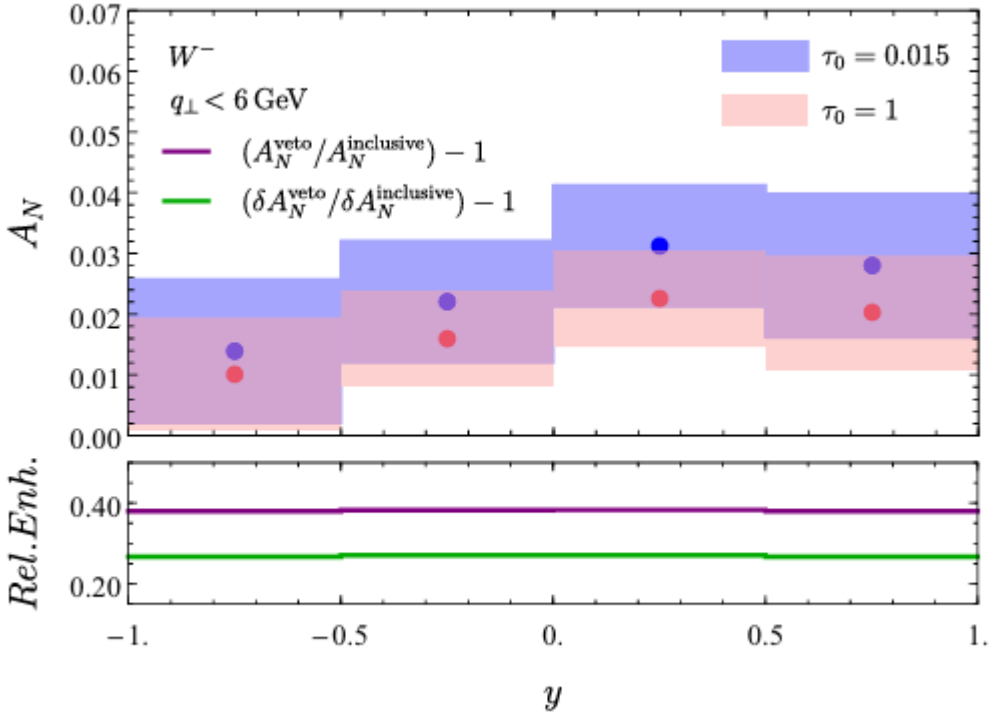
Statistical uncertainties

- Statistical uncertainty: $\delta A_N = \frac{1}{P}\sqrt{\frac{1-(A_N)^2}{\sigma \cdot \mathcal{L}}} \simeq \frac{1}{P}\frac{1}{\sqrt{\sigma \cdot \mathcal{L}}}$

$\mathcal{L} = 780\text{pb}^{-1}$

$P = 53\%$



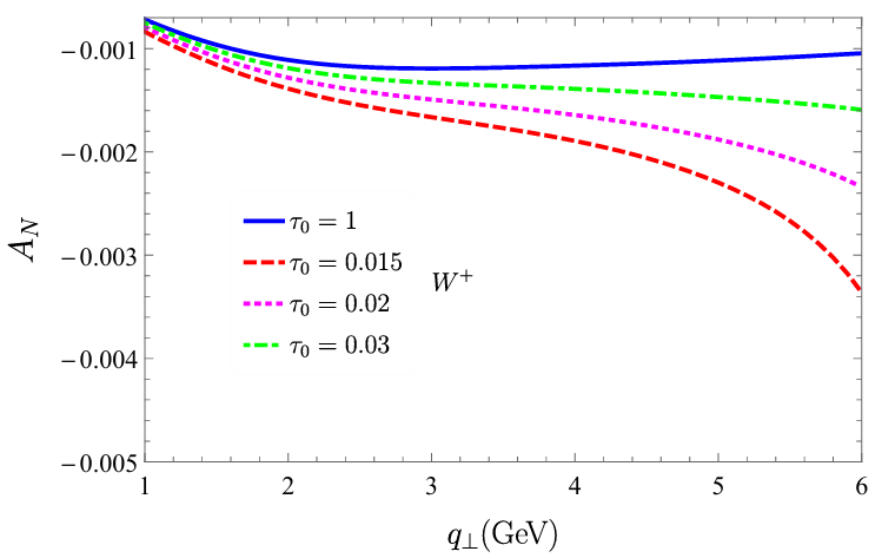


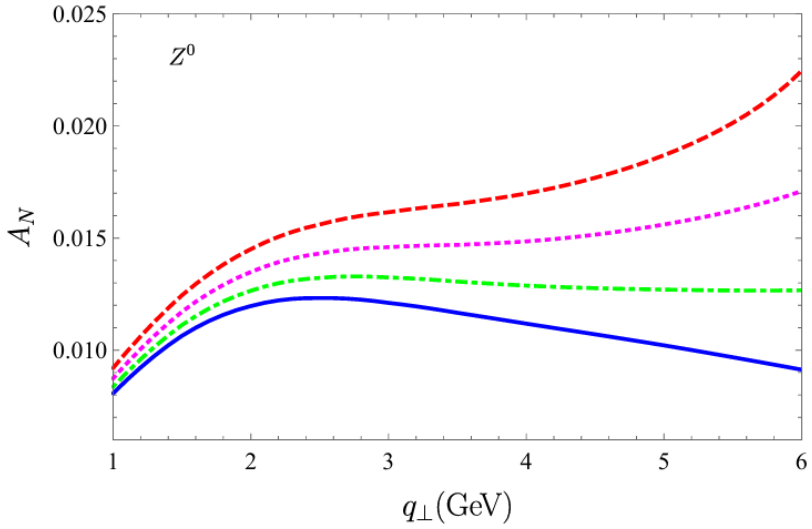
Implementing a 0-jettiness veto can significantly enhance the sensitivity of SSA measurements to the predicted sign flip of the Sivers function, thereby offering a more robust avenue for testing fundamental TMD dynamics in polarized collisions.

The enhanced asymmetries

$$\tau \equiv \frac{2}{Q^2}\sum_i \min\{p_a \cdot l_i, p_b \cdot l_i\} = \sum_i \frac{|\vec{l}_{\perp,i}|}{Q} e^{-|y_i - y|}$$

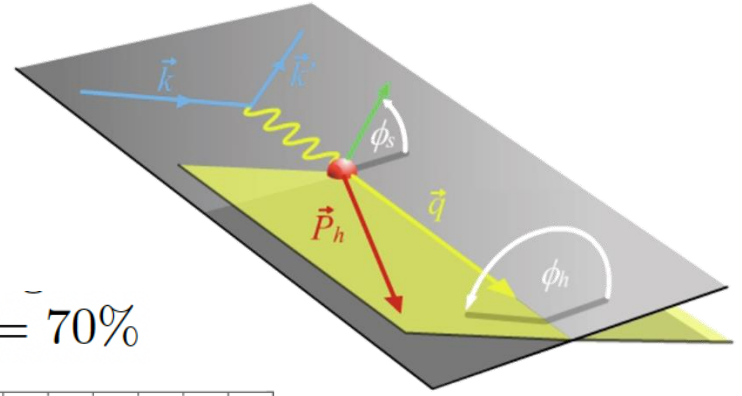
$$\tau < \tau_0$$





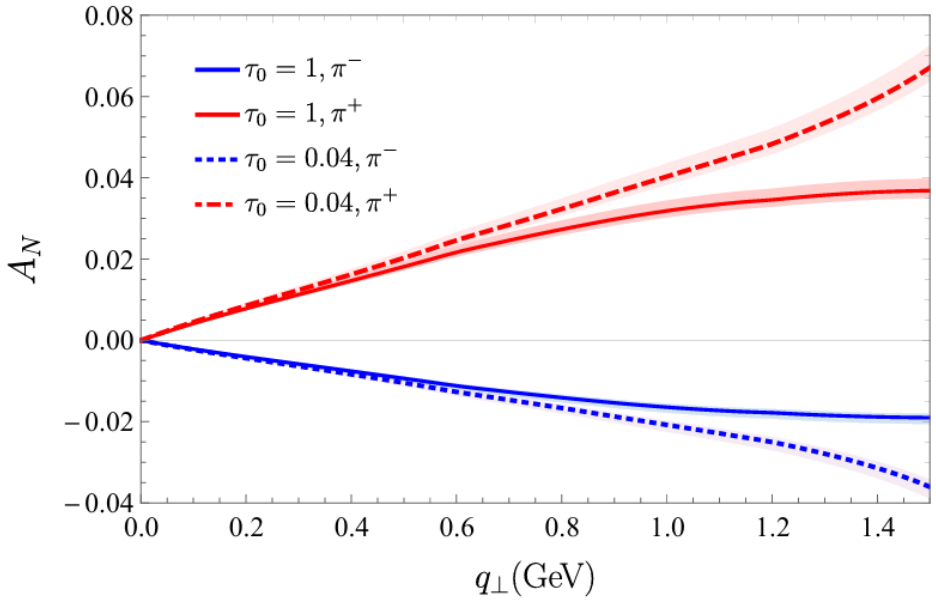
Implementing a 0-jettiness veto can significantly enhance the sensitivity of SSA measurements to the predicted sign flip of the Sivers function, thereby offering a more robust avenue for testing fundamental TMD dynamics in polarized collisions.

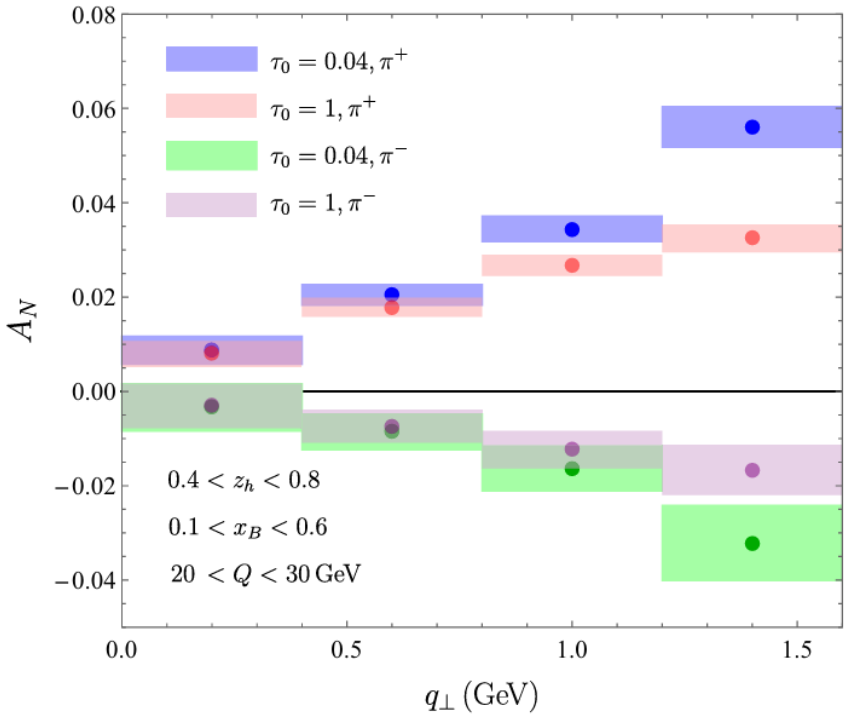
Single spin asymmetry in SIDIS



- 1-jettiness $\ell + p^{\uparrow} \to \ell' + \pi(p_T) + X$

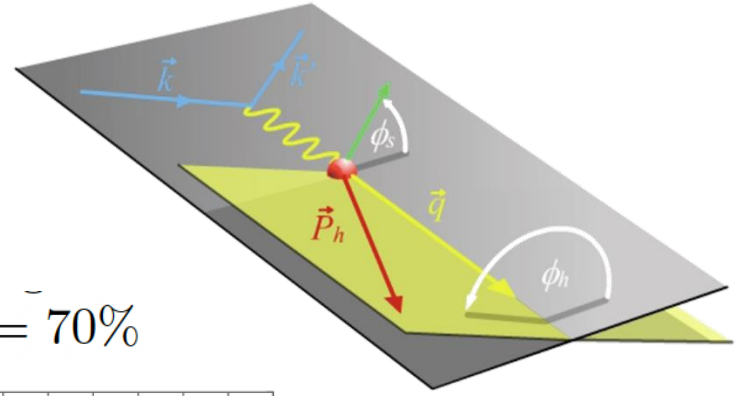
$\mathcal{L} = 100\,\mathrm{fb}^{-1} \quad P = 70\%$





The results show that the asymmetries are enhanced at moderately large pion transverse momentum when the veto is applied

Single spin asymmetry in SIDIS



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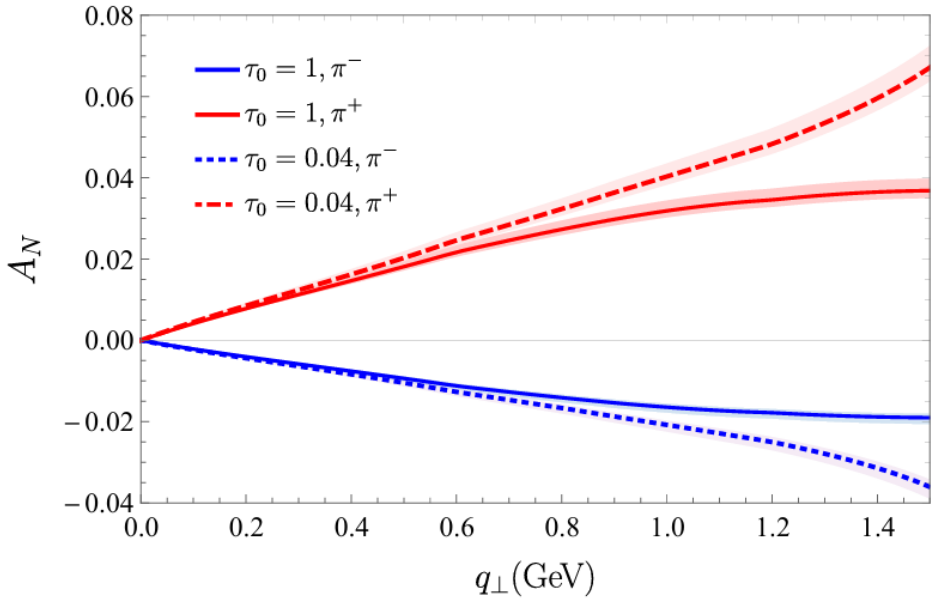
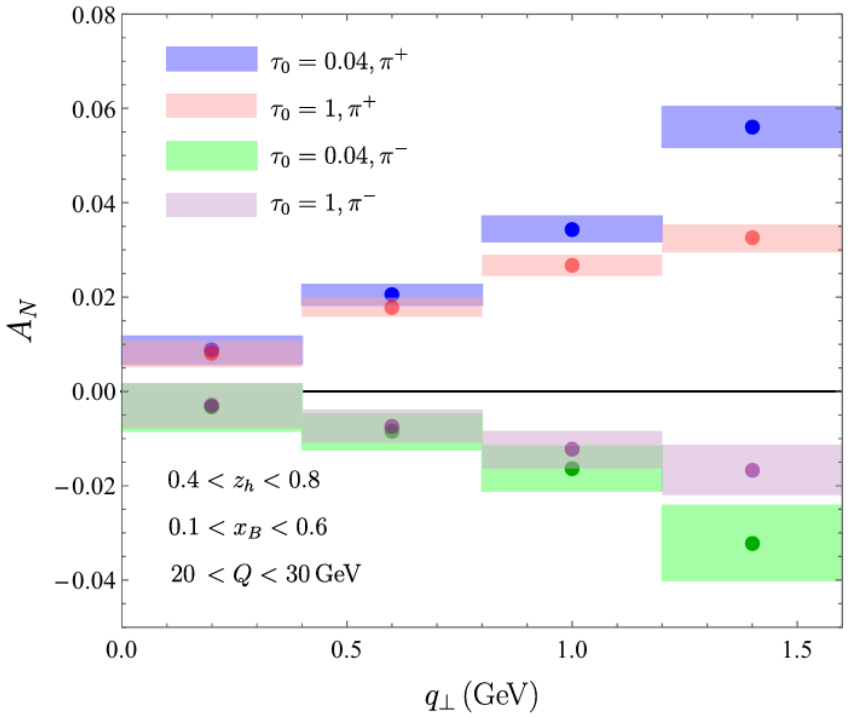




The error bands for the π^+ vetoed and inclusive cases show no overlap for $q_\perp \gtrsim 1$ GeV, confirming a measurable improvement in sensitivity.

Summary

- We introduced a 0-jettiness veto method to suppress TMD evolution effects and probe the nucleon spin structure.
- Single-spin asymmetries (SSAs) in both $W^{\pm}/Z^{0}$ production at RHIC and $\pi^{\pm}$ production at EIC are significantly enhanced with the veto.
- The 0-jettiness veto provides a promising new tool to study the spin dynamics of the nucleon in polarized collisions.

Thanks for your attention!

Back up

SCET_I regime

$$\tau_0^2 Q^2 \ll q_\perp^2 \sim \tau_0 Q^2 \ll Q^2$$

- **Factorization formula:**

Stewart, Tackmann, Waalewijn '09
Jain, Procura, Waalewijn, Zeune '11

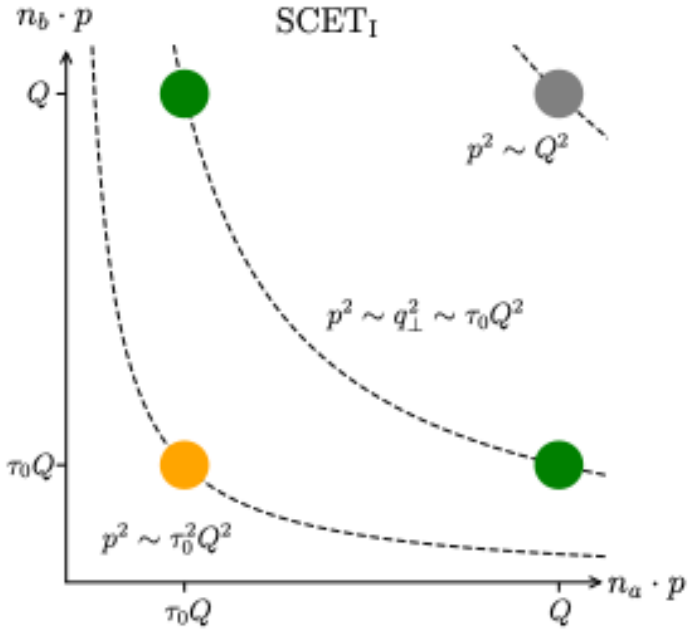
$$\frac{\mathrm{d}\sigma_\mathrm{I}(\tau_0)}{\mathrm{d}y\,\mathrm{d}^2\vec{q}_\perp} = \sigma_0 H(Q,\mu) \sum_{q,q'} C_{qq'} \int_0^{\tau_0 Q} \mathrm{d}\mathcal{T} \int \frac{e^{s\mathcal{T}}\,\mathrm{d}s}{2\pi i} \int_0^\infty \frac{b\,\mathrm{d}b}{2\pi} J_0(b\,q_\perp) \mathcal{B}_{q/p}(s/Q, x_q, b, \mu)\, \mathcal{B}_{q'/p}(s/Q, x_{q'}, b, \mu)\, \mathcal{S}(s,\mu),$$

- **RGEs:**

$$\gamma_\mathcal{B}^q(s/Q,\mu) = -2\Gamma^q_\mathrm{cusp}(\alpha_s) \ln\frac{Q}{\mu^2 s e^{\gamma_E}} + \gamma_\mathcal{B}^q(\alpha_s)\,,$$

$$\gamma_\mathcal{S}^q(s,\mu) = 2\Gamma^q_\mathrm{cusp}(\alpha_s) \ln\frac{1}{\mu^2 s^2 e^{2\gamma_E}} + \gamma_\mathcal{S}^q(\alpha_s)\,,$$

$$\gamma_H^q(Q,\mu) = 2\Gamma^q_\mathrm{cusp}(\alpha_s) \ln\frac{Q^2}{\mu^2} + 2\gamma_V^q(\alpha_s)\,.$$



- **Scale choice:** $\mu_H^\mathrm{I} \sim Q\,, \qquad \mu_\mathcal{B}^\mathrm{I} \sim \sqrt{\tau_0} Q\,, \qquad \mu_\mathcal{S}^\mathrm{I} \sim \tau_0 Q\,.$

SCET$_+$ regime $\tau_0^2 Q^2 \ll q_\perp^2 \ll \tau_0 Q^2 \ll Q^2$

- **Factorization formula:** Procura, Waalewijn, Zeune '14

$$\frac{\mathrm{d}\sigma_+(\tau_0)}{\mathrm{d}y\,\mathrm{d}^2\vec{q}_\perp} = \sigma_0 H(Q,\mu) \sum_{q,q'} C_{qq'} \int_0^{\tau_0 Q} \mathrm{d}\mathcal{T} \int \frac{e^{s\mathcal{T}}\,\mathrm{d}s}{2\pi i} \int_0^\infty \frac{b\,\mathrm{d}b}{2\pi} J_0(b\,q_\perp) B_{q/p}(x_q, b, \mu, \nu/\omega_q) B_{q'/p}(x_{q'}, b, \mu, \nu/\omega_{q'})$$
$$\times \tilde{S}_q(s, b, \mu, \nu) \tilde{S}_{q'}(s, b, \mu, \nu) \mathcal{S}(s, \mu) .$$

- **RGEs:**

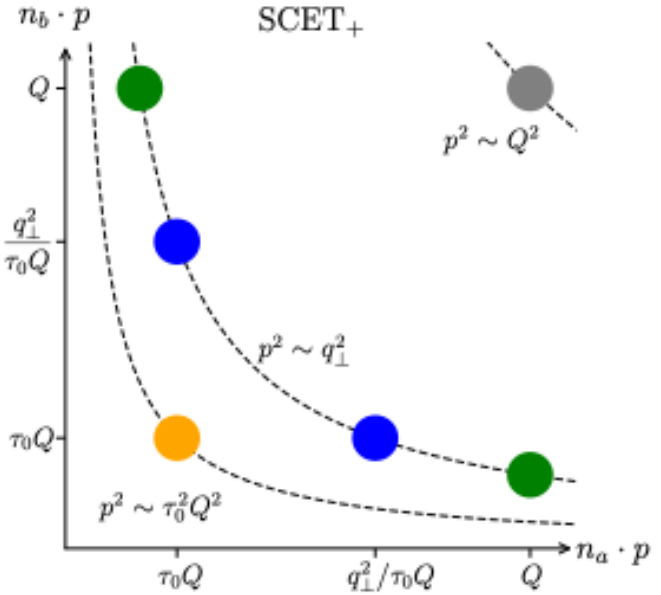
$$\gamma_B^q(\mu, \nu/\omega_q) = \Gamma^q_{\mathrm{cusp}}(\alpha_s) \ln \frac{\nu^2}{\omega_q^2} + \gamma_B^q(\alpha_s) ,$$

$$\gamma_\nu^q(b, \mu) = 2 \left[\int_{\mu^2}^{\mu_b^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \Gamma^q_{\mathrm{cusp}}(\alpha_s) + \gamma_r^q(\alpha_s) \right] ,$$

$$\tilde{\gamma}_S^q(s, \mu, \nu) = -2\Gamma^q_{\mathrm{cusp}}(\alpha_s) \ln \frac{\nu}{\mu^2 s e^{\gamma_E}} + \tilde{\gamma}_S^q(\alpha_s) ,$$

$$\gamma_S^q(s, \mu) = 2\Gamma^q_{\mathrm{cusp}}(\alpha_s) \ln \frac{1}{\mu^2 s^2 e^{2\gamma_E}} + \gamma_S^q(\alpha_s) ,$$

$$\gamma_H^q(Q, \mu) = 2\Gamma^q_{\mathrm{cusp}}(\alpha_s) \ln \frac{Q^2}{\mu^2} + 2\gamma_V^q(\alpha_s) .$$



- **Scale choice:** $\mu_H^+ \sim Q\,, \qquad \mu_B^+ \sim \mu_b\,, \qquad \mu_{\tilde{S}}^+ \sim \mu_b\,, \qquad \mu_S^+ \sim \tau_0 Q\,,$

$$\nu_B^+ \sim Q\,, \qquad \nu_{\tilde{S}}^+ \sim \frac{\mu_b^2}{\tau_0 Q}\,.$$

$\mathbf{SCET_{II}}$ regime

$$\tau_0^2 Q^2 \sim q_\perp^2 \ll \tau_0 Q^2 \ll Q^2;$$

- **Factorization formula:**

Procura, Waalewijn, Zeune '14

$$\frac{\mathrm{d}\sigma_{\rm II}(\tau_0)}{\mathrm{d}y\,\mathrm{d}^2\vec{q}_\perp} = \sigma_0 H(Q,\mu) \sum_{q,q'} C_{qq'} \int_0^{\tau_0 Q} \mathrm{d}\mathcal{T} \int \frac{e^{s\mathcal{T}}\,\mathrm{d}s}{2\pi i} \int_0^\infty \frac{b\,\mathrm{d}b}{2\pi} J_0(b\,q_\perp) B_{q/p}(x_q,b,\mu,\nu/\omega_q) B_{q'/p}(x_{q'},b,\mu,\nu/\omega_{q'}) S(s,b,\mu,\nu),$$

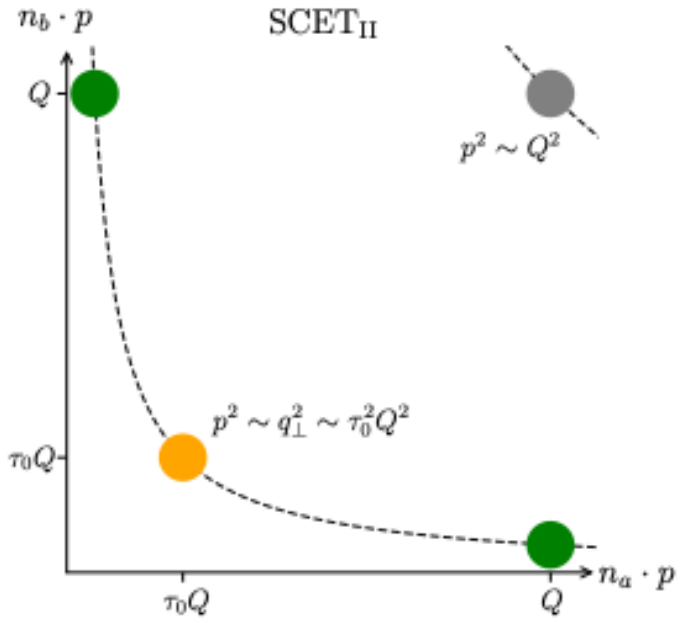
- **RGEs:**

$$\gamma_B^q(\mu,\nu/\omega_q) = \Gamma^q_{\rm cusp}(\alpha_s) \ln\frac{\nu^2}{\omega_q^2} + \gamma_B^q(\alpha_s)\,,$$

$$\gamma_\nu^q(b,\mu) = 2\left[\int_{\mu^2}^{\mu_b^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \Gamma^q_{\rm cusp}(\alpha_s) + \gamma_r^q(\alpha_s)\right],$$

$$\gamma_S^q(\mu,\nu) = 2\Gamma^q_{\rm cusp}(\alpha_s) \ln\frac{\mu^2}{\nu^2} + \gamma_S^q(\alpha_s)\,,$$

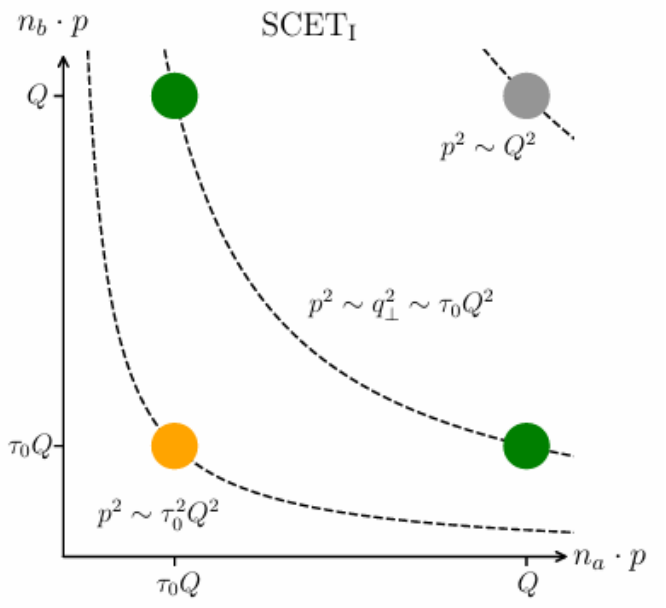
$$\gamma_H^q(Q,\mu) = 2\Gamma^q_{\rm cusp}(\alpha_s) \ln\frac{Q^2}{\mu^2} + 2\gamma_V^q(\alpha_s)\,.$$

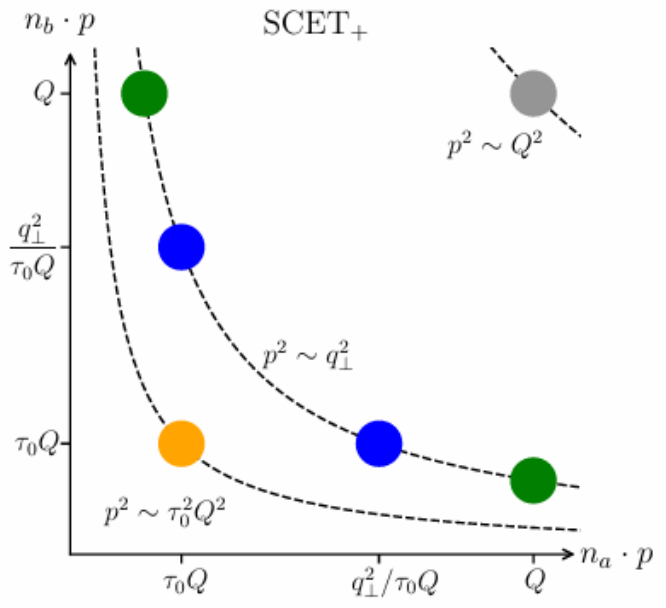


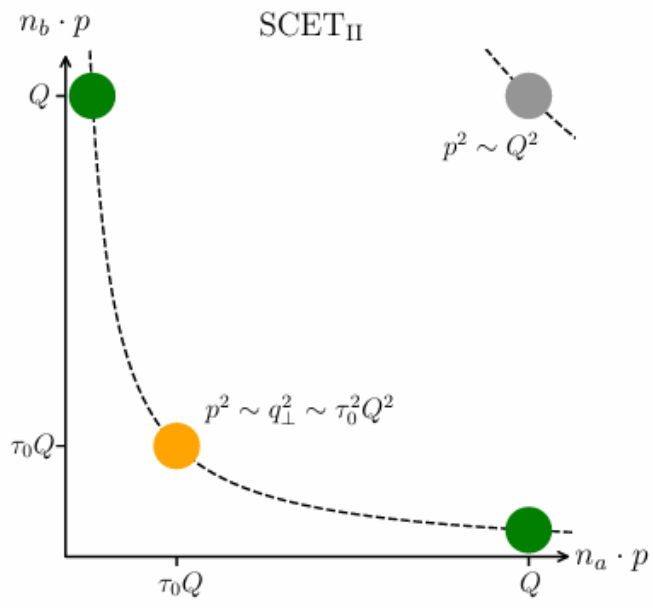
- **Scale choice:**

$$\mu_H^{\rm II} \sim Q\,, \qquad \mu_B^{\rm II} \sim \mu_b\,, \qquad \mu_S^{\rm II} \sim \mu_b\,,$$

$$\nu_B^{\rm II} \sim Q\,, \qquad \nu_S^{\rm II} \sim \mu_b\,.$$







$\tau_0^2 Q^2 \ll q_\perp^2 \sim \tau_0 Q^2 \ll Q^2$ $\tau_0^2 Q^2 \ll q_\perp^2 \ll \tau_0 Q^2 \ll Q^2$ $\tau_0^2 Q^2 \sim q_\perp^2 \ll \tau_0 Q^2 \ll Q^2$;

We implement the transitions between these regimes using Heaviside θ functions, resulting in the final combined perturbative Sudakov factor:

$$S_{\rm P}(b) = \frac{C_F}{\pi}\left[\int_{\tau_0 Q^2}^{\mu_b^2}\frac{{\rm d}\mu^2}{\mu^2}\left(2\ln\frac{\tau_0 Q^2}{\mu^2}-\frac{3}{2}\right)\theta(\mu_b^2-\tau_0 Q^2)-\int_{\tau_0^2 Q^2}^{\mu_b^2}\frac{{\rm d}\mu^2}{\mu^2}\ln\frac{\tau_0^2 Q^2}{\mu^2}\,\theta(\mu_b^2-\tau_0^2 Q^2)\right.$$
$$\left.+\int_{\mu_b^2}^{Q^2}\frac{{\rm d}\mu^2}{\mu^2}\left(\ln\frac{Q^2}{\mu^2}-\frac{3}{2}\right)\right]\alpha_s(\mu)\,.$$

Sivers Function and Qiu-Sterman Function

- Qiu-Sterman Function

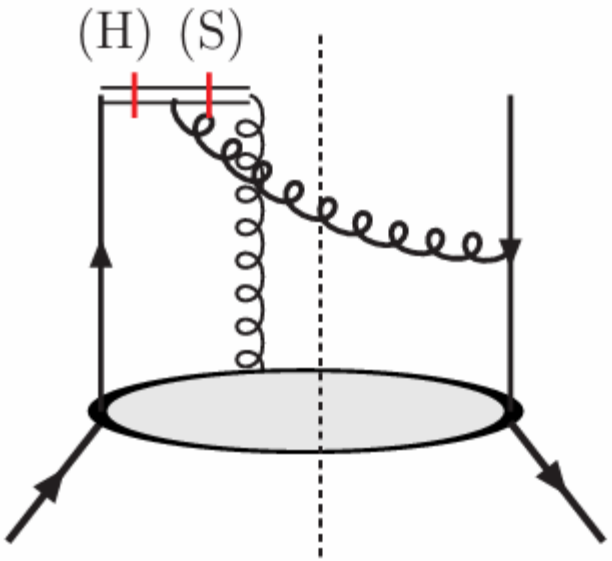
$$T_F(x_2,x_2') \equiv \int \frac{d\zeta^- d\eta^-}{4\pi} e^{i(x_2 P^+\eta^- + (x_2'-x_2)P_B^+\zeta^-)} \epsilon_\perp^{\beta\alpha} S_{\perp\beta}$$
$$\times \left\langle PS|\overline{\psi}(0)\mathcal{L}(0,\zeta^-)\gamma^+ gF_\alpha{}^+(\zeta^-)\mathcal{L}(\zeta^-,\eta^-)\psi(\eta^-)|PS\right\rangle$$

- $T_F(x,x) = \int \frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2} f_{1T}^\perp|_{\rm DY}(x,k_\perp)$

D. Boer, P. J. Mulders and F. Pijlman, Nucl. Phys. B 667, 201 (2003

- Sivers function in small b limit

$$\tilde{f}_{1T}^\alpha(z,b) = \frac{\alpha_s}{2\pi}\left(\frac{-ib_\perp^\alpha}{2}\right)\left\{\left(-\frac{1}{\epsilon}+\ln\frac{c_0^2}{b^2\mu^2}\right)\mathcal{P}_{qg\to qg}^T\otimes T_F(z)\right.$$
$$-\delta(1-\xi)T_F(x,x)C_F\ln\frac{c_0^2}{b^2\mu^2} - \frac{1}{2N_c}T_F(x,x)(1-\xi)$$
$$\left.+\delta(1-\xi)T_F(x,x)C_F\left[\frac{3}{2}\ln\frac{b^2\mu^2}{c_0^2}+\ln\frac{z^2\zeta^2}{\mu^2}-\frac{1}{2}\left(\ln\frac{z^2\zeta^2 b_\perp^2}{c_0^2}\right)^2-2-\frac{\pi^2}{2}\right]\right\}$$



$$\mathcal{P}_{qg\to qg}^T\otimes T_F(z) = \int\frac{dx}{x}\left\{T_F(x,x)\left[C_F\left(\frac{1+\xi^2}{1-\xi}\right)_+ - C_A\delta(1-\xi)\right]\right.$$
$$\left.+\frac{C_A}{2}\left(T_F(x,z)\frac{1+\xi}{1-\xi}-T_F(x,x)\frac{1+\xi^2}{1-\xi}\right)\right\}\ ,$$

Theoretical uncertainty

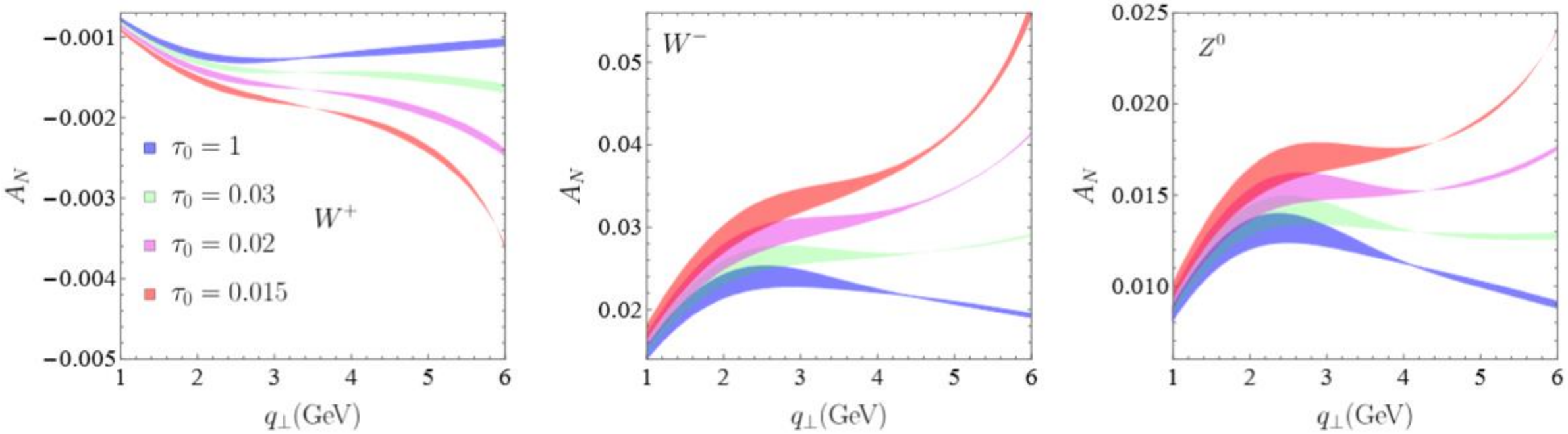


FIG.c: The SSAs for W^+, W^-, and Z^0 production in polarized pp collisions at the RHIC energy $\sqrt{s} = 500$ GeV and rapidity $y = 0$, shown as functions of $q_\perp$ for various values of τ_0. The upper and lower edges of the band correspond to the results obtained using the non-perturbative inputs in the main text and those from [1308.5003], respectively.

Theoretical uncertainty

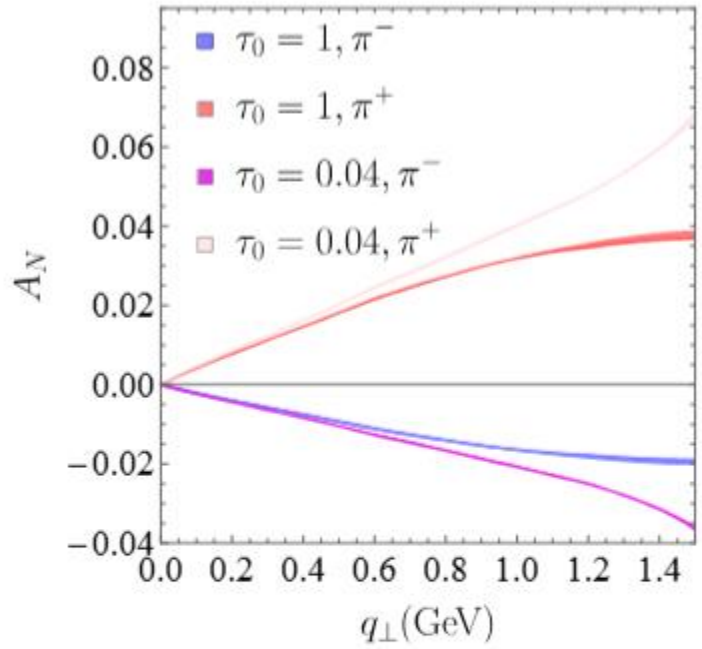


FIG.d: The SSAs for π^+ and π^- production in SIDIS process at $z_h = 0.5$, $x_B = 0.2$, $Q = 25\,\mathrm{GeV}$ and $\sqrt{s} = 100\,\mathrm{GeV}$ are plotted as a function of pions' transverse momentum $q_\perp$. The upper and lower edges of the band correspond to the results obtained using the non-perturbative input in the main text and those from [1401.5078], respectively.

Resummation

A. Idilbi, X. Ji, J.-P. Ma, F. Yuan, Phys.Rev.D 70 (2004) 074021

The scale evolution of the Sivers function at one-loop order,

$$\zeta \frac{\partial}{\partial \zeta} \partial_b^i q_T(x, b, \mu, x\zeta, \rho) = \left(K(b, \mu, \rho) + G(x\zeta, \mu, \rho)\right) \partial_b^i q_T(x, b, \mu, x\zeta, \rho)$$

K and G are the same as those for the unpolarized distribution.

The scale evolution of polarization dependent TMDs is governed by the standard Collins–Soper equation, since the light-cone divergence have the same structure in polarized and unpolarized cases.