γZ Box at Low Energy

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IFAE Institut de Física d'Altes Energies — EXCELENCIA SEVERO OCHOA

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Generalitat de Catalunya

Previous results for the γZ-box

- The polarization asymmetry for the electron proton scatering is

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W^{p,eff} + \triangle_{\square}(s, Q^2) + F(Q^2)\right)$$

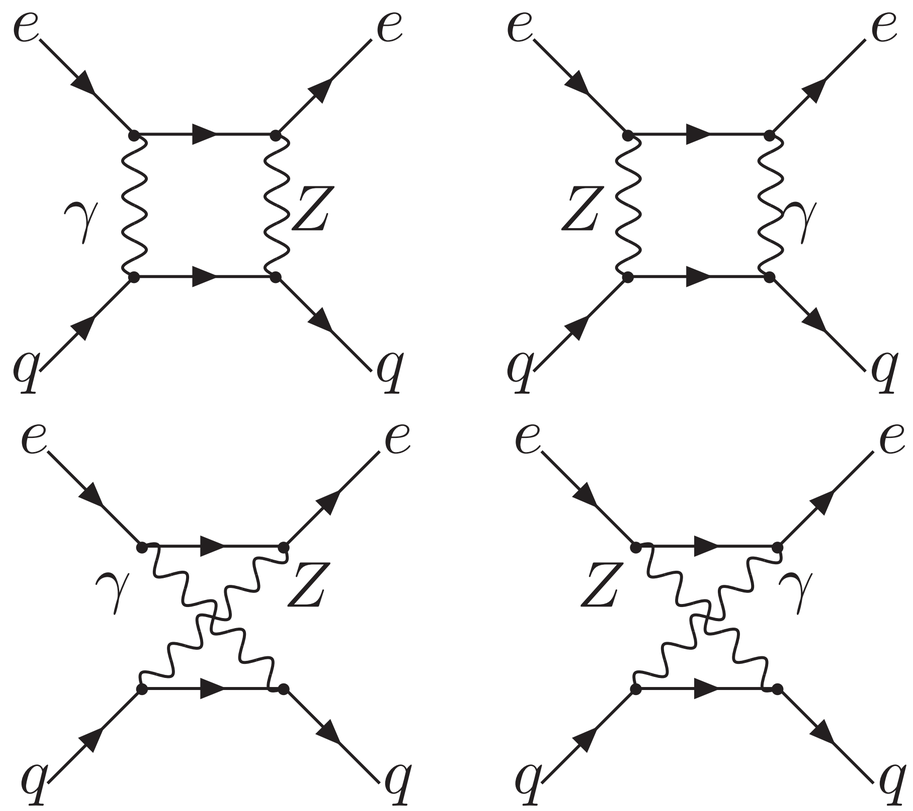
with

$$Q_W^{p,eff} = [\rho_{NC} + \triangle_e]\left[1 - 4\sin^2\hat{\theta}_W(0) + \triangle_e^{'}\right] + \square_{WW} + \square_{ZZ} + \square_{\gamma Z}$$

- In a quark-model calculation

$$\square_{\gamma Z} = -2(2\delta_{box}C_{1u} + \delta_{box}C_{1d})$$

which can be related to the γZ box-graph correction for electron-proton scattering.



Previous results for the γZ-box

- The low-energy effective Lagrangian is

$$\mathscr{L}_{NC}^{eq} = \frac{G_f}{\sqrt{2}} \sum_q [C_{0q}\bar{e}\gamma_\mu e \cdot \bar{q}\gamma^\mu q + C_{1q}\bar{e}\gamma_\mu\gamma_5 e \cdot \bar{q}\gamma^\mu q + C_{2q}\bar{e}\gamma_\mu e \cdot \bar{q}\gamma^\mu\gamma_5 q + C_{3q}\bar{e}\gamma_\mu\gamma_5 e \cdot \bar{q}\gamma^\mu\gamma_5 q]$$

- Corrections to the parity-violating components C_{iq} of $\mathscr{L}_{NC}^{eq}$ coming from the γZ box graphs were given by
  - [Marciano,Sirlin, ’83]: $\delta_{box}C_{2q} = \frac{\alpha}{2\pi} Q_e Q_q g_{AV}^{eq} 3\left(\ln\left(\frac{M_Z^2}{M^2}\right) + \frac{3}{2}\right)$
  - [Erler,Su, ’13]: $\delta_{box}C_{2q} = \frac{\alpha}{2\pi} Q_e Q_q g_{AV}^{eq} 3\left(\ln\left(\frac{M_Z^2}{m_p^2}\right) + \frac{5}{6}\right)$

where in Ref. [Marciano,Sirlin,1983] it is said that they interpret the quark mass as “the scale at which quarks behave as though they were essentially free”.

Previous results for the γZ-box

- In Ref. [Erler et al '03] it is presented

$$\Box_{\gamma Z} = \frac{5\hat{\alpha}}{2\pi}(1-4\hat{s}^2)\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) + C_{\gamma Z}(\Lambda)\right]$$

Hadronic cut-off $\Lambda = m_\rho$

$$C_{\gamma Z}(m_\rho) = 3/2$$

$$\Box_{\gamma Z} = -2(2\delta_{box}C_{1u} + \delta_{box}C_{1d})$$

- Alternative approach ⟶ dispersion relations methods (**neglecting electron mass**).

PROBLEM! **Energy Scale ambiguity**

- Careful analysis of the **partonic one-loop calculation** of the γZ-box graphs is still necessary.
  - This works presents an exact SM calculation with full mass dependence, m_e and m_q.
  - Focus on extraction of C_{iq}, .i.e. forward ($t \to 0$) and threshold limit ($s \to (m_e + m_q)^2$).

Next-to-leading order cross section eq→eq

[Duch et al '26]

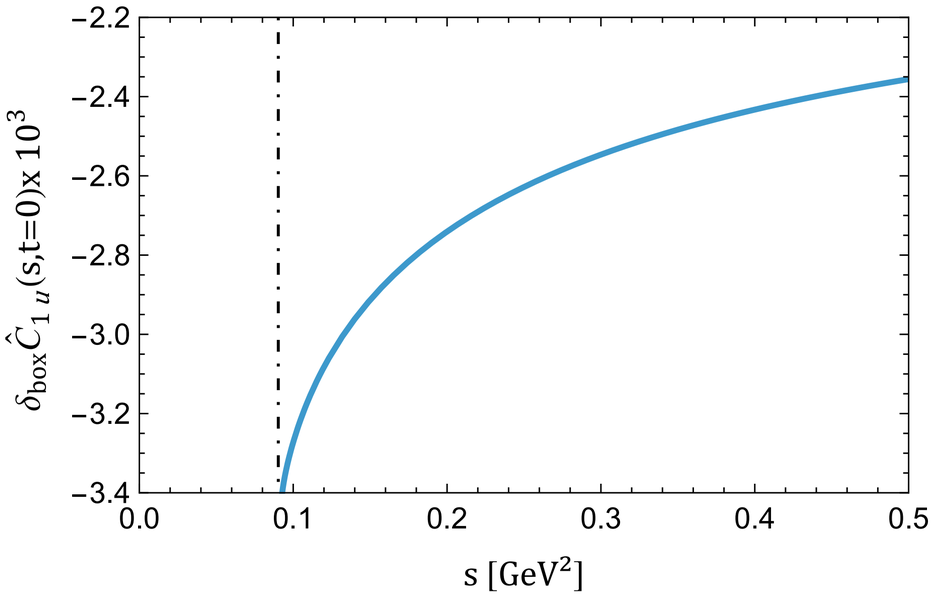


Figure 1: The box-graph corrections $\delta_{box}\hat{C}_{1u}(s,t=0)$ as a function of the Mandelstam variable s, evaluated at fixed $t=0$ for a quark mass of $m_q=0.3$ GeV. The vertical dash-dotted lines indicate the position of the threshold, $s_0=(m_e+m_q)^2$.

- $\delta_{box}C_{iq}$ contains **Sommerfeld enhancement (SE)** at $t=0$, near threshold $s\to s_{th}=(m_e+m_q)^2$.

$$\delta_{box}\hat{C}_{1q}^{SE}=-\frac{\alpha Q_e Q_q g_{AV}^{eq}\pi\sqrt{m_e m_q}}{\sqrt{s-(m_e+m_q)^2}}\,.$$

$$S=\frac{\alpha\pi\sqrt{m_e m_q}}{\sqrt{s-(m_q+m_e)^2}}\to\frac{\alpha\pi}{\beta}$$

- The scattering particles may form a **bound state** when kinetic energy is low.
- The same type of contribution appears in other coefficients $\delta_{box}C_{iq}$ as well
- These are results for **electron-quark scattering**; for **Møller experiment**, **expect similar SE** term, but numerically smaller at high $E_{\rm beam}$.

$\delta_{box}C_{iq}$ at threshold $(s \to (m_e + m_q)^2)$

$$\delta_{box}C_{iq} = \delta_{box}C_{iq}^{eff}(s = (m_e + m_q)^2, t = 0)$$

Next-to-leading order cross section eq→eq

[Duch et al '26]

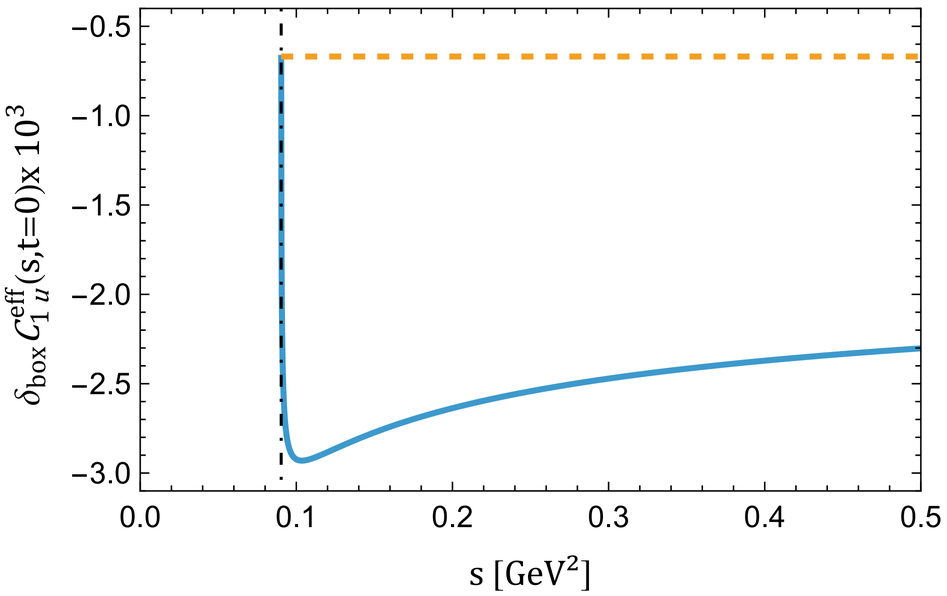


Figure 2: The box-graph corrections $\delta_{box}C_{1u}^{eff}(s, t = 0)$ as a function of the Mandelstam variable s, evaluated at fixed $t = 0$ for a quark mass of $m_q = 0.3$ GeV. The orange dashed lines show the value of $\delta_{box}C_{1u}$ at threshold $(s \to (m_q + m_e)^2)$.

- After subtraction of **SE term**:

$$\delta_{box}C_{iq}^{eff} \equiv \delta_{box}\hat{C}_{iq}(s = (m_e + m_q)^2, t = 0) - \delta_{box}\hat{C}_{iq}^{SE}(s, t = 0)$$

- $\delta_{box}C_{iq}$:

$$\delta_{box}C_{1q} = -\frac{\alpha}{4\pi}Q_eQ_q\left[g_{VA}^{eq}\left(\frac{2m_e^2\left(3m_e^2-5m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_e} - \frac{2m_q^2\left(m_e^2-3m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_q} + \frac{5m_e^2-9m_q^2}{m_e^2-m_q^2}\right)\right.$$
$$\left. +4g_{AV}^{eq}\left(\frac{m_em_q\left(2m_e^2-m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}\ln\left(\frac{m_q^2}{m_e^2}\right) + \frac{m_em_q}{m_e^2-m_q^2}\right)\right]$$

- Taking limits of the electron or quark masses non-zero, we find at threshold:

$$\delta_{box}C_{1q}(m_e = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right) + \frac{3}{2}\right)$$

$$\delta_{box}C_{1q}(m_q = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_e^2}\right) + \frac{5}{6}\right)$$

$$L_{m_i} = \ln\left(\frac{M_Z^2}{m_i^2}\right)$$

$\square_{\gamma Z}^{ep}(s,t)$

$$F_{\gamma Z}^{ep}(s_0,t_0)=0$$

- The ep γZ box-graph correction which contributes to the observable polarization asymmetry

$$\square_{\gamma Z}^{ep}(s,t) = -2\left(2\delta_{box}C_{1u}(s_0,0)+\delta_{box}C_{1d}(s_0,0)\right) + F_{\gamma Z}^{ep}(s,t)$$

Perturbative term (Related to the $Q_W^{p,eff}$)

Non-perturbative term

$m_e = 0$

$$\delta_{box}C_{1q}(m_e=0) = -\frac{\alpha}{4\pi}Q_eQ_q\cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right)+\frac{3}{2}\right)$$

$= 3.39\times10^{-3}$ for $m_q = 0.3$ GeV

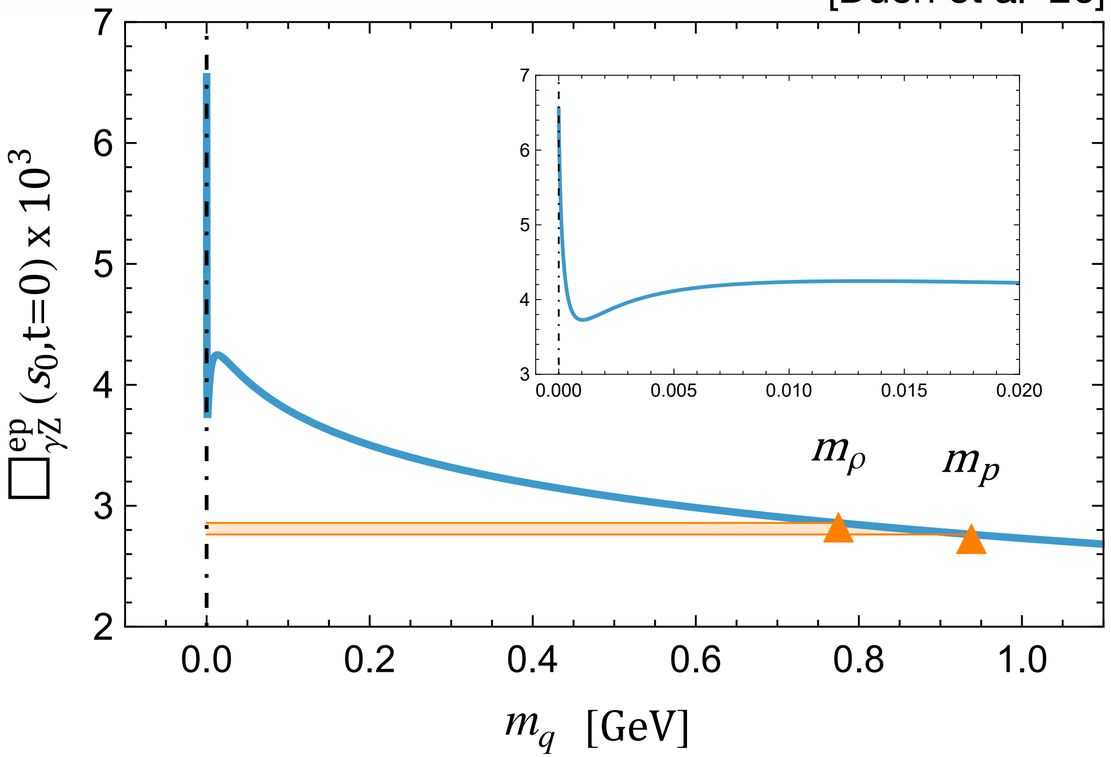
$m_q = 0$

$$\delta_{box}C_{1q}(m_q=0) = -\frac{\alpha}{4\pi}Q_eQ_q\cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_e^2}\right)+\frac{5}{6}\right)$$

$= 6.55\times10^{-3}$

$\square_{\gamma Z}^{ep}(s,t)$

[Duch et al '26]



$$\square_{\gamma Z}^{ep}(s_0,t_0) = -2\left(2\delta_{box}C_{1u}(s_0,0) + \delta_{box} + C_{1d}(s_0,0)\right)$$

Figure 3: The constant term of the box-graph correction $\square_{\gamma Z}^{ep}$ at threshold contributing to the parity-violating asymmetry A_{PV} for scattering of an electron off a proton using the general case, as a function of the quark mass. The horizontal orange band covers the the range for quark masses between the ρ-mass and the proton-mass. The shape at small values of M is shown at an increased scale in the zoom-in plot.

For ^{133}Cs: $\square_{\gamma Z}$ effect

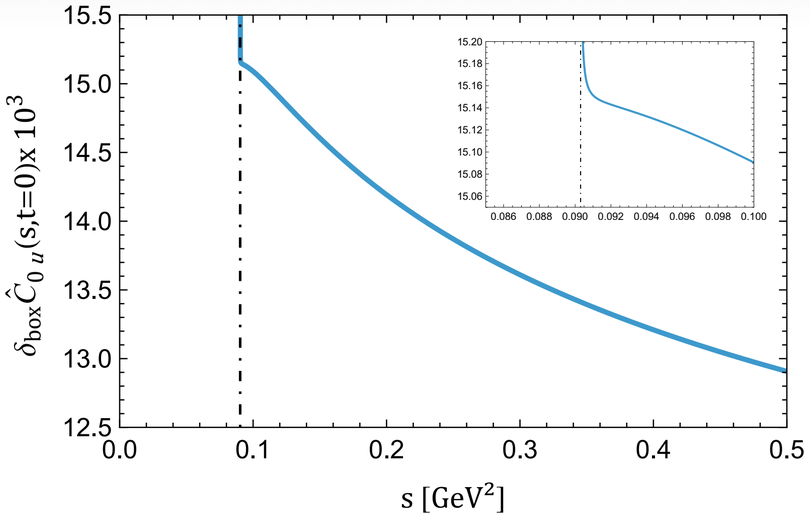
$$\Delta Q_W \approx -0.43 \longrightarrow \Delta \sin^2\theta_W \approx 0.0020 \approx \text{Exp. uncertainty } (\pm 0.0016)$$ **Non-negligible effect!**

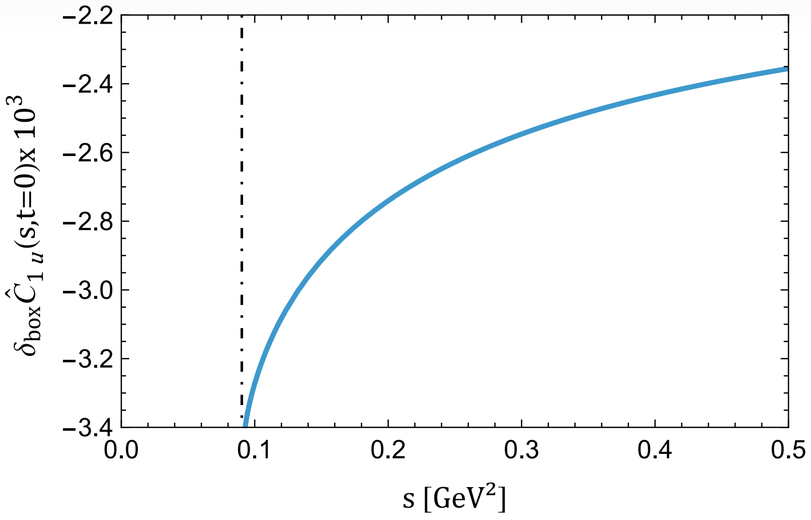
Conclusions

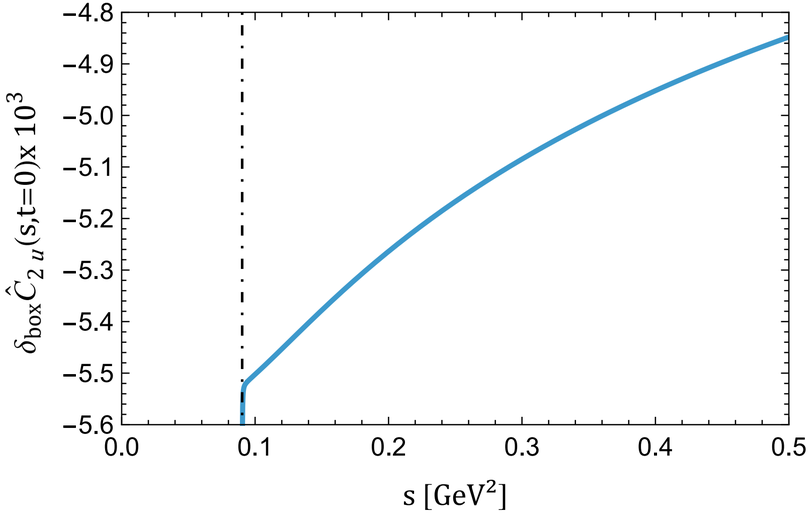
- We have performed an **exact calculation** including the full **mass dependence for both electron and quark.**
- Our results exhibit large logarithmic contributions, and we **explicitly determine the finite constants at $t = 0$ and at threshold.**
- We **identify the leading Coulomb correction**, corresponding to the **Sommerfeld enhancement**.
- Our framework **allows the quark mass to be set to zero**, shifting all nuclear-structure dependence into the non-perturbative contribution.
- These results can be directly used as building blocks for describing electron–quark scattering processes in **deep inelastic scattering**.

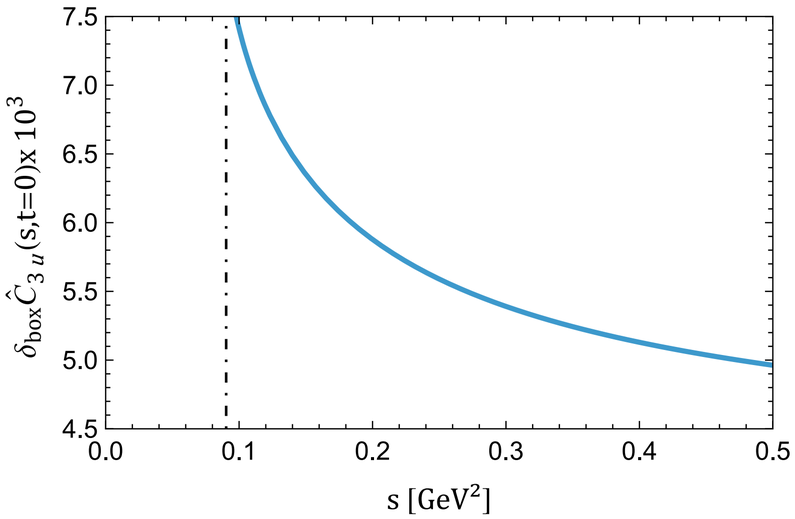
Back up slides

Next-to-leading order cross section eq→eq









[Duch et al '26]

Figure 4: The box-graph corrections $\delta_{box}C_{iu}(s, t = 0)$ as a function of the Mandelstam variable s, evaluated at fixed $t = 0$ for a quark mass of $m_q = 0.3$ GeV. For $\delta_{box}C_{0u}(s, t = 0)$ we inserted a zoom-in at threshold to show that the function is in fact smooth. The vertical dash-dotted lines indicate the position of the threshold, $s_0 = (m_e + m_q)^2$.

Next-to-leading order cross section eq→eq

- Keeping the leading singular terms only, we find

$$\delta_{box}\hat{C}^{SE}_{0q} = \frac{\alpha Q_e Q_q g^{eq}_{VV}\pi\sqrt{m_e m_q}}{\sqrt{s-(m_q+m_e)^2}},\quad \delta_{box}\hat{C}^{SE}_{1q} = -\frac{\alpha Q_e Q_q g^{eq}_{AV}\pi\sqrt{m_e m_q}}{\sqrt{s-(m_e+m_q)^2}},$$

$$\delta_{box}\hat{C}^{SE}_{2q} = -\frac{\alpha Q_e Q_q g^{eq}_{VA}\pi\sqrt{m_e m_q}}{\sqrt{s-(m_q+m_e)^2}},\quad \delta_{box}\hat{C}^{SE}_{3q} = \frac{\alpha Q_e Q_q g^{eq}_{AA}\pi\sqrt{m_e m_q}}{\sqrt{s-(m_e+m_q)^2}}.$$

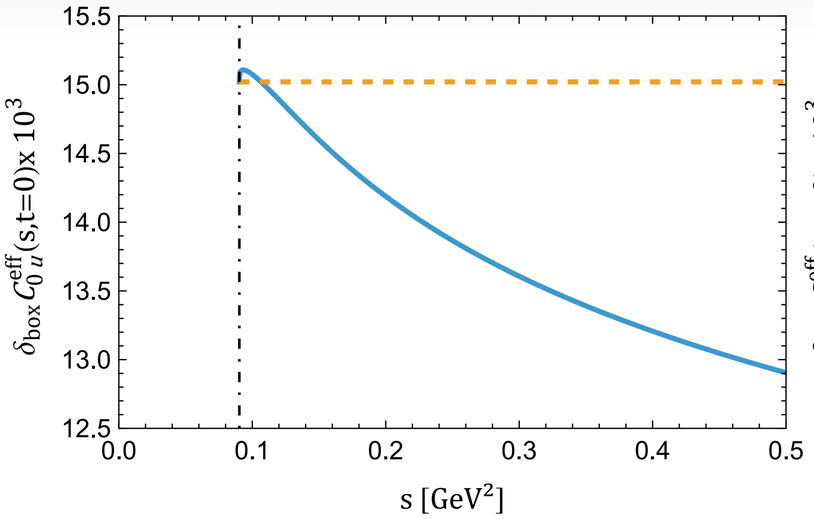
[Duch et al '26]

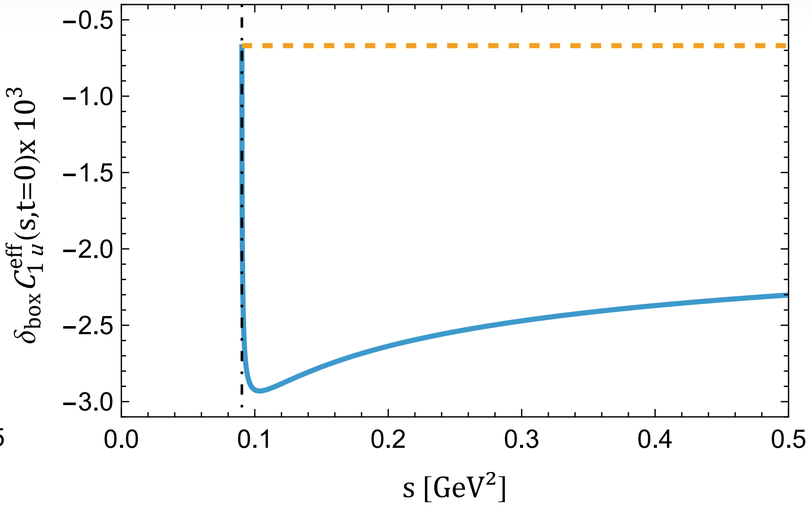
$$S = \frac{\alpha\pi\sqrt{m_e m_q}}{\sqrt{s-(m_e+m_q)^2}} \to \frac{\alpha\pi}{\beta}$$

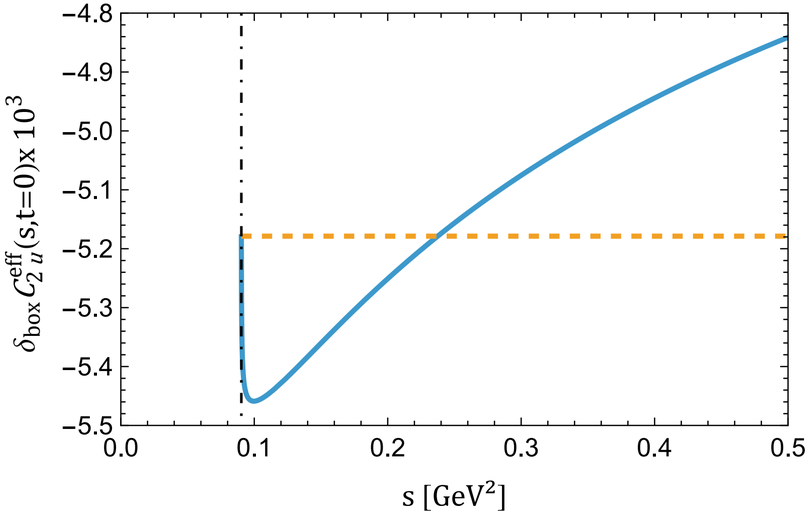
- The effect is known as **Sommerfeld Enhancement (SE)** → The scattering particles may form a **bound state** when kinetic energy is low.
- They are the first order part of **Coulomb interaction**.
- We perform a subtraction,

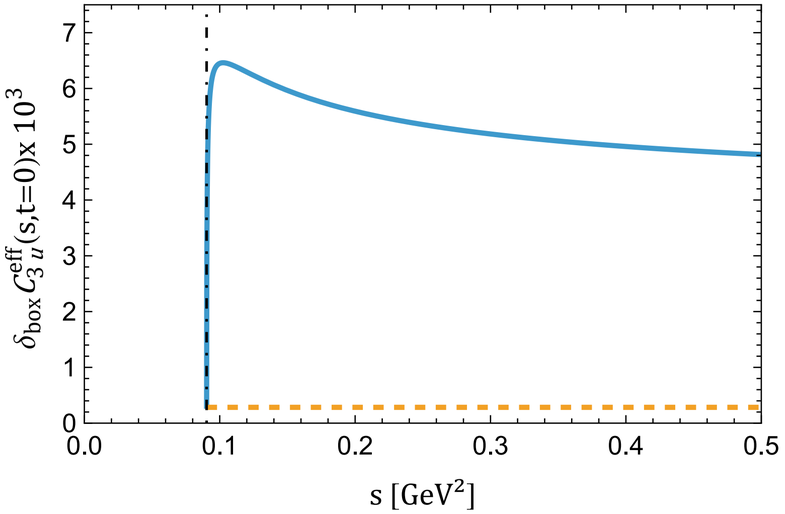
$$\delta_{box}C^{eff}_{iq} \equiv \delta_{box}\hat{C}_{iq}(s=(m_e+m_q)^2, t=0) - \delta_{box}\hat{C}^{SE}_{iq}(s, t=0)$$

Next-to-leading order cross section eq→eq









[Duch et al '26]

$\delta_{box} C_{iq}$ at threshold $(s \to (m_e + m_q)^2)$

$$\delta_{box} C_{iq} = \delta_{box} C^{eff}_{iq}(s = (m_e + m_q)^2, t = 0)$$

Figure 5: The box-graph corrections $\delta_{box} C^{eff}_{iu}(s, t = 0)$ as a function of the Mandelstam variable s, evaluated at fixed $t = 0$ for a quark mass of $m_q = 0.3$ GeV. The orange dashed lines show the value of $\delta_{box} C_{iu}$ at threshold $(s \to (m_q + m_e)^2)$.

Next-to-leading order cross section eq→eq

- Keeping electron and quark masses non-zero, we find at threshold:

$$L_{m_e} = \ln\left(M_Z^2/m_e^2\right)$$
$$L_{m_q} = \ln\left(M_Z^2/m_q^2\right)$$

$$\delta_{box}C_{0q} = -\frac{\alpha}{4\pi}Q_eQ_q\left[6g_{VV}^{eq}\frac{m_em_q}{m_e^2-m_q^2}\ln\left(\frac{m_e^2}{m_q^2}\right)+6g_{AA}^{eq}\left(\frac{-m_e^2L_{m_e}+m_q^2L_{m_q}}{m_e^2-m_q^2}-\frac{3}{2}\right)\right]$$

$$\delta_{box}C_{1q} = -\frac{\alpha}{4\pi}Q_eQ_q\left[g_{VA}^{eq}\left(\frac{2m_e^2\left(3m_e^2-5m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_e}-\frac{2m_q^2\left(m_e^2-3m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_q}+\frac{5m_e^2-9m_q^2}{m_e^2-m_q^2}\right)+4g_{AV}^{eq}\left(\frac{m_em_q\left(2m_e^2-m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}\ln\left(\frac{m_q^2}{m_e^2}\right)+\frac{m_em_q}{m_e^2-m_q^2}\right)\right]$$

$$\delta_{box}C_{2q} = -\frac{\alpha}{4\pi}Q_eQ_q\left[g_{AV}^{eq}\left(\frac{2m_e^2\left(3m_e^2-m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_q}-\frac{2m_q^2\left(5m_e^2-3m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}L_{m_q}+\frac{9m_e^2-5m_q^2}{m_e^2-m_q^2}\right)+4g_{VA}^{eq}\left(\frac{m_em_q\left(m_e^2-2m_q^2\right)}{\left(m_e^2-m_q^2\right)^2}\ln\left(\frac{m_q^2}{m_e^2}\right)-\frac{m_em_q}{m_e^2-m_q^2}\right)\right]$$

$$\delta_{box}C_{3q} = \frac{\alpha}{4\pi}Q_eQ_q\left[8g_{AA}^{eq}\ln\left(\frac{m_e^2}{m_q^2}\right)\frac{m_em_q}{m_q^2-m_e^2}+8g_{VV}^{eq}\left(\frac{m_e^2L_{m_e}-m_q^2L_{m_q}}{m_e^2-m_q^2}+1\right)\right]$$

Next-to-leading order cross section eq→eq

- Taking limits of the electron or quark masses non-zero, we find at threshold:

$m_e = 0$

$$\delta_{box}C_{0q}(m_e = 0) = \frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{AA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right) + \frac{3}{2}\right)$$

$$\delta_{box}C_{1q}(m_e = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right) + \frac{3}{2}\right)$$

$$\delta_{box}C_{2q}(m_e = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{AV}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right) + \frac{5}{6}\right)$$

$$\delta_{box}C_{3q}(m_e = 0) = 1\frac{\alpha}{4\pi}Q_eQ_q \cdot 8g_{VV}^{eq}\left(\ln\left(\frac{M_Z^2}{m_q^2}\right) + 1\right)$$

$m_q = 0$

$$\delta_{box}C_{0q}(m_q = 0) = \frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{AA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_e^2}\right) + \frac{3}{2}\right)$$

$$\delta_{box}C_{1q}(m_q = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 6g_{VA}^{eq}\left(\ln\left(\frac{M_Z^2}{m_e^2}\right) + \frac{5}{6}\right)$$

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$$\delta_{box}C_{3q}(m_q = 0) = -\frac{\alpha}{4\pi}Q_eQ_q \cdot 8g_{VV}^{eq}\left(\ln\left(\frac{M_Z^2}{m_e^2}\right) + 1\right)$$

$\Box^{eq}_{\gamma Z}(s,t)$

Useful for DIS

- The eq γZ box-graph correction which contributes to the observable polarization asymmetry

$$\Box^{eq}_{\gamma Z}(s,t) = \frac{1}{A_0}\frac{d\sigma^{\gamma Z}_{PV}/d\Omega}{d\sigma^{\gamma Z}_{non-PV}/d\Omega + d\sigma^{\gamma}/d\Omega} \simeq \frac{1}{A_0}\frac{d\sigma^{\gamma Z}_{PV}/d\Omega}{d\sigma^{\gamma}/d\Omega}\,.$$

$$p_1 = \frac{\sqrt{\lambda(s,m_e^2,m_q^2)}}{2m_q}$$

$$= 2\delta_{box}C^{eff}_{2q}(s,t)\,\frac{2((m_e^2-m_q^2)^2 - 2(m_e^2+m_q^2)s + s^2)t + (s+m_e^2-m_q^2)t^2}{Q_eQ_q\sqrt{\lambda(s,m_e^2,m_q^2)}\left(2(s-(m_e^2+m_q^2))^2 + 2st + t^2\right)}$$

$$-2\delta_{box}C^{eff}_{1q}(s,t)\,\frac{2\lambda(s,m_e^2,m_q^2)(s-m_e^2-m_q^2) + 2s(s-(m_e^2+m_q^2))t + (s+m_e^2-m_q^2)t^2}{Q_eQ_q\sqrt{\lambda(s,m_e^2,m_q^2)}\left(2(s-(m_e^2+m_q^2))^2 + 2st + t^2\right)}$$

where

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$$

- For zero momentum transfer, $t=0$, this simplifies to

$$\Box^{eq}_{\gamma Z}(s,t=0) = -2\frac{p_1}{E_1}\,\delta_{box}C^{eff}_{1q}(s,0)$$

$$E_1 = \sqrt{p_1^2+m_e^2}$$