

# Femtoscscopy for $D_0^*$ (2300) and $D_{s0}^*$ (2317) states



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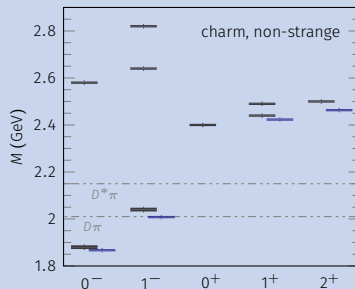
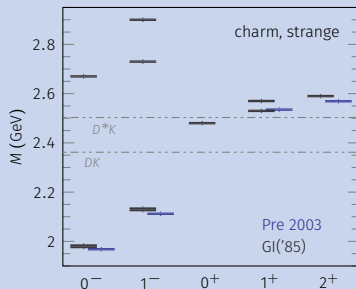


PID2020-112777GB-I00

# Quark model in the open-charm sector

- Quark model  $c\bar{n}$  is still our baseline:

«In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons—from the pion to the epsilon—can be described in a unified framework.» [Godfrey, Isgur, PR,D32,189('85)]





# Theoretical interpretations

## $c\bar{q}$ states

- Colangelo, De Fazio, Phys. Lett. B **570**, 180 (2003)  
Dai *et al.* Phys. Rev. D **68**, 114011 (2003)  
Narison, Phys. Lett. B **605**, 319 (2005)  
Bardeen *et al.*, Phys. Rev. D **68**, 054024 (2003)  
Lee *et al.*, Eur. Phys. J. C **49**, 737 (2007)  
Wang, Wan, Phys. Rev. D **73**, 094020 (2006)

## Pure tetraquarks

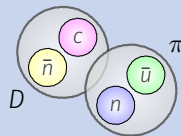
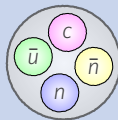
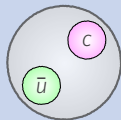
- Cheng, Hou, Phys. Lett. B **566**, 193 (2003)  
Terasaki, Phys. Rev. D **68**, 011501 (2003)  
Chen, Li, Phys. Rev. Lett. **93**, 232001 (2004)  
Maiani *et al.*, Phys. Rev. D **71**, 014028 (2005)  
Bracco *et al.*, Phys. Lett. B **624**, 217 (2005)  
Wang, Wan, Nucl. Phys. A **778**, 22 (2006)

## $c\bar{q}+$ tetraquarks or meson-meson

- Browder *et al.*, Phys. Lett. B **578**, 365 (2004)  
van Beveren, Rupp, Phys. Rev. Lett. **91**, 012003 (2003)

## Heavy-light meson-meson molecules

- Barnes *et al.*, Phys. Rev. D **68**, 054006 (2003)  
Szczepaniak, Phys. Lett. B **567**, 23 (2003)  
Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)  
Hofmann, Lutz, Nucl. Phys. A **733**, 142 (2004)  
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)  
Gamermann *et al.*, Phys. Rev. D **76**, 074016 (2007)  
Faessler *et al.*, Phys. Rev. D **76**, 014005 (2007)  
Flynn, Nieves, Phys. Rev. D **75**, 074024 (2007)



## Meanwhile, in the lattice...

- Masses larger than the physical ones if using  $c\bar{s}$  interpolators only.

Bali, Phys. Rev. D **68**, 071501 (2003)  
UKQCD Collab., Phys. Lett. B **569**, 41 (2003)

- Masses consistent with  $D_0^*(2300)$  and  $D_{s0}^*(2317)$  obtained when “meson-meson” interpolators are employed.

Mohler, Prelovsek, Woloshyn, Phys. Rev. D **87**, 034501 (2013)  
Mohler *et al.*, Phys. Rev. Lett. **111**, 222001 (2013)

- Close to the physical point:

RQCD Collab., Phys. Rev. D **96**, 074501 (2017)

- More complete studies from the HadSpec collaboration:

- ▶  $D\pi$ ,  $D\eta$  and  $D_s\bar{K}$  coupled-channel scattering. A bound state with large coupling to  $D\pi$  is identified with  $D_0^*(2300)$ .

HadSpec Collab., JHEP **1610**, 011 (2016)

- ▶  $D_{s0}^*(2317)$ : A bound state is found in the  $DK$  channel, with:

- $\Delta E = 25(3)$  MeV ( $m_\pi = 391$  MeV)
- $\Delta E = 57(3)$  MeV ( $m_\pi = 239$  MeV)
- Compare with experimental,  $\Delta E \simeq 45$  MeV (the dependence on  $m_\pi$  does not need to be monotonic!

[HadSpec Collab., JHEP 02 (2021) 100; JHEP 07 (2021) 123]

# Lightest $0^+$ open-charm situation and puzzles

- $D_{s0}^*(2317)$  ( $S, l$ ) = (1, 0)  $M_{D_{s0}^*(2317)} = 2317.8 \pm 0.5$  MeV (PDG)
- $D_0^*(2300)$  ( $S, l$ ) = (0, 1/2) Not so well established:

	Collab.	$M$ (MeV)	$\Gamma/2$ (MeV)	Ref.
Neutral	Belle	$2308 \pm 36$	$138 \pm 33$	Phys. Rev. D 69, 112002 (2004)
	BaBar	$2297 \pm 22$	$137 \pm 25$	Phys. Rev. D 79, 112004 (2009)
	FOCUS [?]	$2407 \pm 41$	$120 \pm 40$	Phys. Lett. B 586, 11 (2004)
Charged	LHCb	$2360 \pm 34$	$128 \pm 29$	Phys. Rev. D 92, 032012 (2015) ( $B^0 \rightarrow \bar{D}^0 K^+ \pi^-$ )
	LHCb	$2349 \pm 7$	$109 \pm 9$	Phys. Rev. D 92, 012012 (2015) ( $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ )
	LHCb	$2354 \pm 13$	$128 \pm 13$	Phys. Rev. D 92, 012012 (2015) ( $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ )
	FOCUS [?]	$2403 \pm 38$	$142 \pm 21$	Phys. Lett. B 586, 11 (2004)
	PDG	$2343 \pm 10$	$115 \pm 8$	Neutral and charged

## Three puzzles

1. **Mass problem:** Why are  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  masses much lower than the CQM expectations?
2. **Splittings:** Why  $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D$  (within a few MeV)?
3. **Hierarchy:** Why  $M_{D_0^*(2300)} > M_{D_{s0}^*(2317)}$ , *i.e.*, why  $c\bar{u}$ ,  $c\bar{d}$  heavier than  $c\bar{s}$ ?

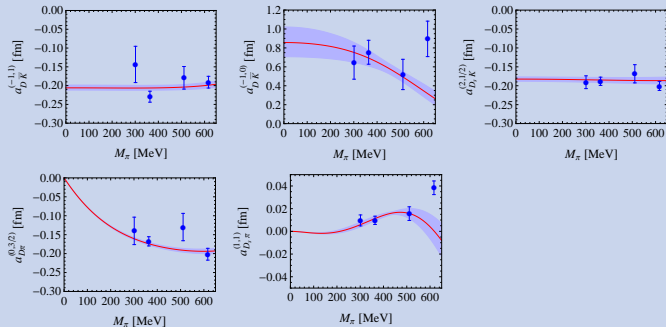
# $D\pi, D\eta, D_s\bar{K}$ scattering amplitudes

- Coupled channel  $T$ -matrix:  $D\pi, D\eta, D_s\bar{K}$  scattering [ $J^P = 0^+, (S, l) = (0, \frac{1}{2})$ ].
- Unitarity:  $T^{-1}(s) = V^{-1}(s) - \mathcal{G}(s)$
- Chiral symmetry used to compute the  $\mathcal{O}(p^2)$  potential:

$$f^2 V_{ij}(s, t, u) = C_{\text{LO}}^{ij} \frac{s-u}{4} + \sum_{a=0}^5 h_a C_a^{ij}(s, t, u)$$

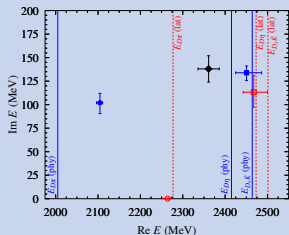
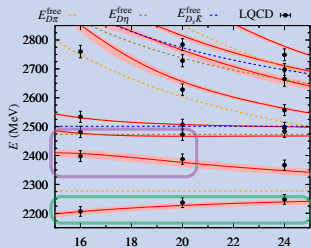
Guo et al., Phys. Lett. B 666, 251 (2008)  
Liu et al., Phys. Rev. D 87, 014508 (2013)

- $h_a$  (LECs), subtraction constants: free parameters **previously fixed**.
- Fitted to reproduce scattering lengths obtained in a LQCD simulation:



# $D\pi, D\eta, D_s\bar{K}$ and $D^*(2300)$ : Comparison with LQCD

MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, Phys. Lett. B 767, 465 (2017)



- $E_n(\mathbf{L})$  are provided for  $D\pi, D\eta, D_s\bar{K}$  in a recent LQCD simulation. [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- Red Bands:** Our amplitude in a finite volume. [MA *et al.*, Phys. Lett. B 767, 465 (2017)]
- No fit** is performed (LECs previously determined)
- Level **below threshold**, associated with a **bound state**.
- Second level** has large shifts w. r. t. thresholds, non-interacting energy levels.
- For lattice masses, we find a **bound state** and a **resonance**.
- For physical masses, both evolve into **resonances**.

	$M$ (MeV)	$\Gamma/2$ (MeV)
Low pole	$2105^{+6}_{-8}$	$102^{+10}_{-12}$
High pole	$2451^{+36}_{-26}$	$134^{+7}_{-8}$

- We also study  $DK, D_s\eta, (S, I) = (1, 0), D_{s0}^*(2317)$  bound state:  $M = 2315^{+18}_{-28}$  MeV.

The  $D_0^*(2300)$  structure actually consists of **two different states** (with complicated interferences with the thresholds)

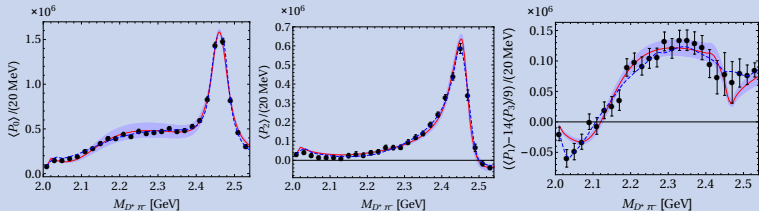
Previously reported in:

Kolomeitsev, Lutz, Phys. Lett. B 582, 39 (2004)  
 Guo *et al.*, Phys. Lett. B 641, 278 (2006)  
 Guo *et al.*, Eur. Phys. J. A 40, 171 (2009)

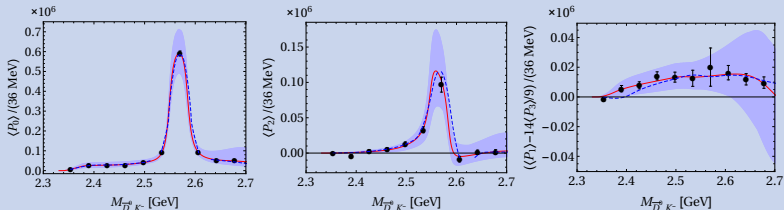
# $D\pi, D\eta, D_S\bar{K}$ and $D^*(2300)$ : Comparison with LHCb data

M.-L. Du, MA, P. Fernández-Soler, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. Nieves, D.-L. Yao, PR,D98,094018('18)

- $B^- \rightarrow D^+ \pi^- \pi^-$  [LHCb Collab., PR,D94,072001('16)]

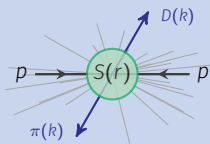


- $B_S^0 \rightarrow \bar{D}^0 K^- \pi^+$  [LHCb Collab., PR,D90,072003('14)]



- Rapid movement in  $\langle P_{13} \rangle$  [no  $D_2(2460)$ ] at 2.4-2.5 GeV [ $D\eta$  and  $D_S\bar{K}$ ].
- Recall: these are the amplitudes with **two states** in the  $D_0^*(2300)$  region, and no fit of the  $T$ -matrix parameters is done (production parameters are fitted).

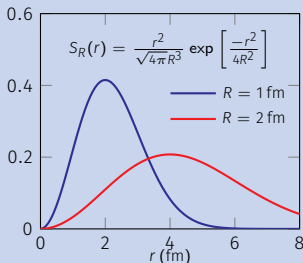
# (Very basic introduction to) femtoscopy



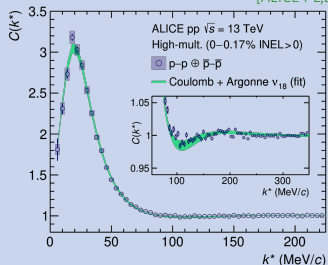
$$C_{\text{exp}}(k) = \xi(k) \frac{N_{\text{same}}(k)}{N_{\text{mixed}}(k)}$$

$$C_{\text{th}}(k) = \sum_{\ell=0} (2\ell + 1) \int dr S_R(r) |\psi_{\ell}(r, k)|^2$$

$$\psi_{\ell}^{\text{[asy,nr]}}(k, r) = j_{\ell}(kr) + f_{\ell}(k) \frac{e^{i(kr - \pi\ell/2)}}{r}$$



[ALICE PL,B805,135419('20)]



- Known the **source  $S(r)$** , explore **interactions** (encoded in the wave function)
- A new method to explore **hadron interactions**
- Lot of attraction. In this conference:

- |                                |                             |
|--------------------------------|-----------------------------|
| ▶ M. Janik [Mon. 11:00]        | ▶ W. Rzesza [Wed. 14:24]    |
| ▶ V. Mantovani [Mon. 14:30]    | ▶ L. Serksnyte [Wed. 15:12] |
| ▶ D. Mihaylov [Mon. 17:40]     | ▶ E. Oset [Thu. 14:30]      |
| ▶ This talk [Now]              | ▶ M. Lesch [Thu. 15:12]     |
| ▶ L. Graczykowski [Wed. 14:00] | ▶ R. Lea [Thu. 15:42]       |

[Fabbietti, Mantovani, Vázquez-Doce, ARNPS,71,377('21)]

Check those talks for more references!

Channels (S-wave only):

- $D_0^*$  ( $S = 0, l = 1/2$ ):  $D\pi, D_S\bar{K}, D\eta$ .
- $D_{S0}^*$  ( $S = 1, l = 0, 1$ ):  $D_S^+\pi^0, D^0K^+, D^+K^0, D_S^+\eta$ .

$$C_i(s) = 1 + \int_0^\infty dr S_R(r) \left[ \sum_j \left| \psi_i^j(s, r) \right|^2 - j_0(p_i r)^2 \right]$$

$$\psi_i^j(s, r) = j_0(p_i r) \delta_{ij} + \tilde{G}_i(s, r) T_{ji}(s)$$

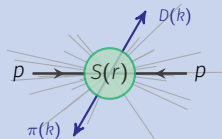
$$T^{-1}(s) = V^{-1}(s) - G(s) \quad (\text{previous slides})$$

$$\tilde{G}_i(s, r) = \frac{1}{\pi} \int_{s_{\text{th}}}^\infty ds' \frac{p_j(s')}{8\pi\sqrt{s'}} \frac{j_0(p_j(s')r)}{s - s' + i\epsilon} \theta(\Lambda - p_j(s'))$$

Goal of our paper [MA, Nieves, Ruiz-Arriola, 2304.03107]

To use our previously fixed  $T$ -matrices in the open-charm sector to **predict** correlation functions to be measured

- $C(k)$  in terms of on-shell  $T$ -matrix:
  - ▶ Feijoo, Vidaña, MA, Nieves, Oset, 2303.06079
- Coupled channels based on:
  - ▶ Lednicky, Lyuboshitz ( $\times 2$ ), PAN,61,2950('98)
  - ▶ Haidenbauer, NPA,981,1('19)
- Cut-off  $\Lambda$ :
  - ▶ Only used in  $\tilde{G}(s, r)$ , not in  $G(s)$
  - ▶ Very soft effect, because  $\tilde{G}$  would be convergent (extra  $1/p$  from  $j_0$ )
  - ▶ Take  $\Lambda \in [0.6, 0.9]$  GeV



- We are considering  $I_z = +\frac{1}{2}$  channels:  $D^+\pi^0/D^0\pi^+$ ,  $D^+\eta$ ,  $D_s^+\bar{K}^0$ .
- We avoid Coulomb interaction, which shows up in  $I_z = -\frac{1}{2}$ , through  $D^+\pi^-$  (not in  $D^0\pi^0$ ) and  $D_s^+K^-$ .
- Q: Does it make sense to use isospin channels, when physical ones are measured?
  - ▶ Physical channels will have combination of  $I = \frac{1}{2}$  and  $I = \frac{3}{2}$ .
  - ▶ Potentially, you can also have Coulomb interaction.

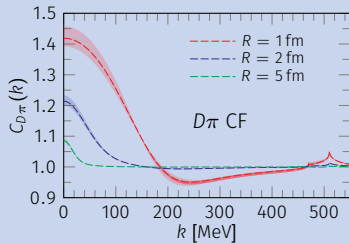
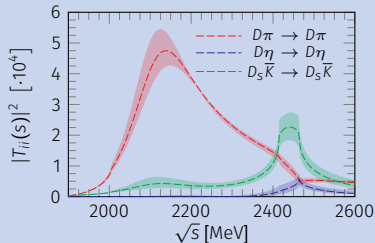
A: Let us see...

- Inserting isospin decompositions in the amplitudes, we get **interesting relations**:
  - ▶  $I = \frac{1}{2}$ :  $C_{D\pi}^{(1/2)} = 2C_{D^0\pi^+} - C_{D^+\pi^0} = \frac{3C_{D^0\pi^+} - C_{D^0\pi^-}}{2}$
  - ▶  $I = \frac{3}{2}$ :  $C_{D\pi}^{(3/2)} = 2C_{D^+\pi^0} - C_{D^0\pi^+} = C_{D^0\pi^-}$

# Open charm femtoscopy: results $S = 0, I = 1/2 [D\pi, D\eta, D_s\bar{K}]$

[MA, Nieves, Ruiz-Arriola, 2304.03107]

- Lower pole, peak at 2135 MeV would correspond to  $k_{D\pi} = 215$  MeV. However, we find a minimum at  $C_{D\pi}(k)$ .

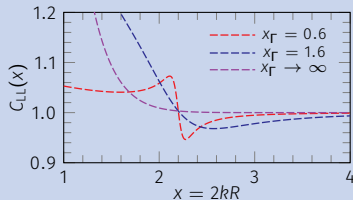


- Take the LL approximation to  $C(k)$ , and  $f(k)$  in terms of  $\delta(k)$ :

$$C_{LL}(k) = 1 + \frac{2 \sin^2 \delta(k)}{x^2} \left( e^{-x^2} + \frac{2xF_1(x)}{\sqrt{\pi}} \cot \delta(k) \right)$$

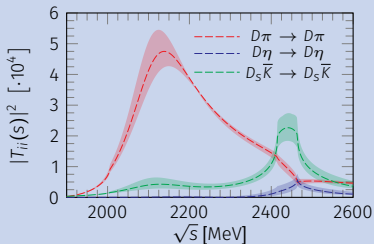
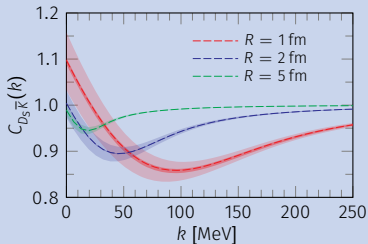
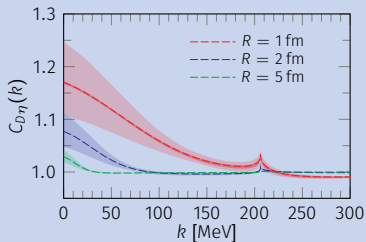
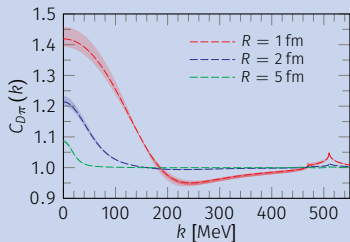
- For a simple BW:  $C_{LL}(k_R) = 1$ ,  $C'_{LL}(k_R) < 0$

Conclusion: the minimum at  $k_{D\pi} = 215$  MeV is a clear signature of the lowest pole.



# Open charm femtoscopy: results $S = 0, I = 1/2 [D\pi, D\eta, D_S\bar{K}]$

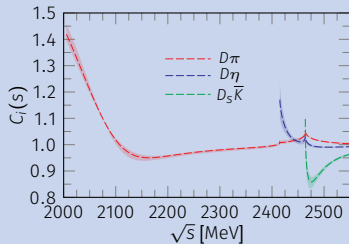
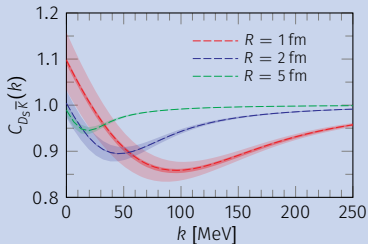
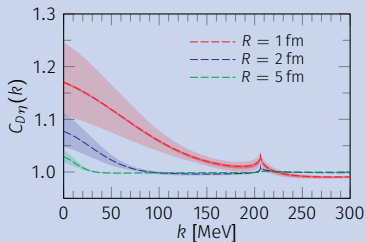
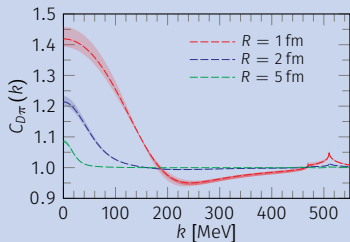
[MA, Nieves, Ruiz-Arriola, 2304.03107]



- Two different minima at  $\sqrt{s} \simeq 2135$  MeV ( $D\pi$  CF) and  $2475$  MeV ( $D_S\bar{K}$  CF), produced by the two different  $D_0^*$  states, can be observed
- Their observation would constitute a strong additional support of the two-state pattern.

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[MA, Nieves, Ruiz-Arriola, 2304.03107]

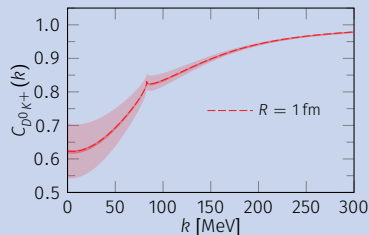


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- Their observation would constitute a strong additional support of the two-state pattern.

# Open charm femtoscopy: results $S = 1, I = 0$ and $1 [D_S^+ \pi^0, D^0 K^+, D^+ K^0, D_S^+ \eta]$

[MA, Nieves, Ruiz-Arriola, 2304.03107]

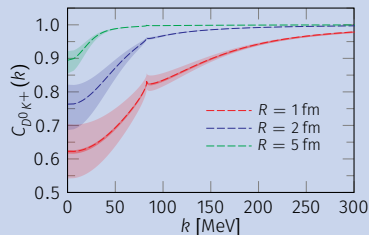
- $D_{s0}^*$  (2317) is a (mostly)  $DK$   $I = 0$  bound state, with  $B = 45$  MeV,  $p_B = 190i$  MeV
- Why use physical channels?
  - ▶  $E_{D^+K^0}^{\text{th}} - E_{D^0K^+}^{\text{th}} \simeq 9$  MeV [ $k_{D^0K^+} \simeq 83$  MeV]
  - ▶ Also,  $C_{D^0K^+} = C_{D^+K^0} = \frac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of  $D_{s0}^*$  (2317)
- Similar trends in recent works, only small differences in the values:
  - ▶ [Liu, Lu, Geng, 2302.01046]
  - ▶ [Ikeno, Toledo, Oset, 2305.16431]



# Open charm femtoscopy: results $S = 1, I = 0$ and $1 [D_S^+ \pi^0, D^0 K^+, D^+ K^0, D_S^+ \eta]$

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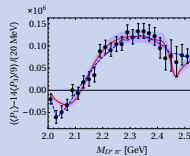
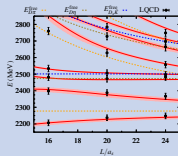
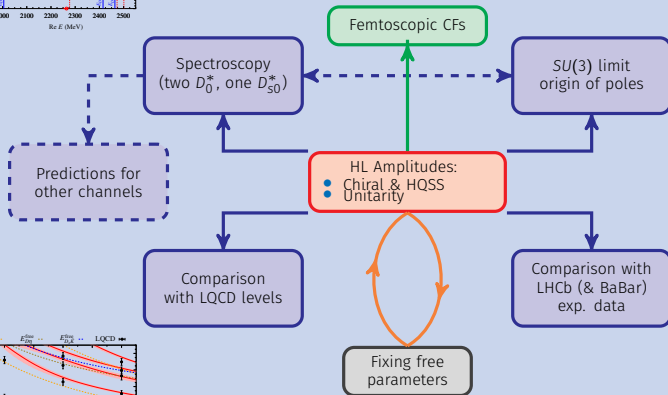
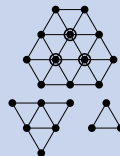
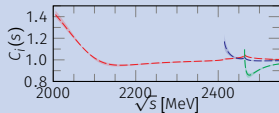
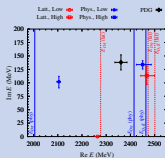
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  - ▶  $E_{D^+K^0}^{\text{th}} - E_{D^0K^+}^{\text{th}} \simeq 9$  MeV [ $k_{D^0K^+} \simeq 83$  MeV]
  - ▶ Also,  $C_{D^0K^+} = C_{D^+K^0} = \frac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of  $D_{s0}^*(2317)$
- Similar trends in recent works, only small differences in the values:
  - ▶ [Liu, Lu, Geng, 2302.01046]
  - ▶ [Ikeno, Toledo, Oset, 2305.16431]



## Summary and conclusions

- We have employed unitarized NLO chiral amplitudes to study the open-charm sector.
- The LECs are fixed through LQCD calculations of  $m_\pi$ -dependent scattering lengths.
- The  $D_0^*(2300)$  structure is actually produced by **two different states** (poles), together with complicated interferences with thresholds.
- This two-state structure receives **strong support**. **Without any fit**, the amplitudes are **compatible** with:
  - ▶ available LQCD simulations,
  - ▶ and experimental data
- This picture, with the lowest  $D_0^*$  pole and  $D^*(2317)$  being **flavour partners** nicely solves simultaneously all the puzzles.
- We have used these amplitudes to predict **femtoscopia correlation functions** in the open-charm sector.
- In particular, we have highlighted the imprints that the two-state structure leaves on  $C_{D\pi}$ ,  $C_{D\eta}$ , and  $C_{D_s\bar{K}}$ .
- The measurements of these CFs can shed further light on the (very interesting!) open-charm sector.

# Scheme of unitary EFTs relations to LQCD and experiments



# Femtoscscopy for $D_0^*$ (2300) and $D_{s0}^*$ (2317) states



Miguel Albaladejo (IFIC-CSIC)

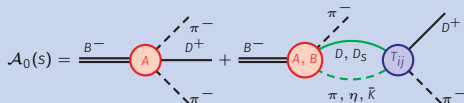
HADRON 2023  
20th International Conference on  
Hadron Spectroscopy and Structure  
Genova Jun. 5-9, 2023



# Comparison with experimental data: $B^- \rightarrow D^+ \pi^- \pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

- $\mathcal{A}(s, z) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z) + \dots$
- $P$ -, $D$ -wave as in LHCb paper:  $D^*$ ,  $D^*(2680)$  in  $P$ -wave,  $D_2^*(2460)$  in  $D$ -wave
- $S$ -wave parameterization:



$$\mathcal{A}_0(s) = A \left\{ E_\pi \left[ 2 + G_1(s) \left( \frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T^{3/2}(s) \right) \right] + \frac{1}{3} E_\eta G_2(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_3(s) T_{31}^{1/2}(s) \right\} + B E_\eta G_2(s) T_{21}^{1/2}(s),$$

- Angular moments:  $\langle P_\ell \rangle(s) = \int dz |\mathcal{A}(s, z)|^2 P_\ell(z)$

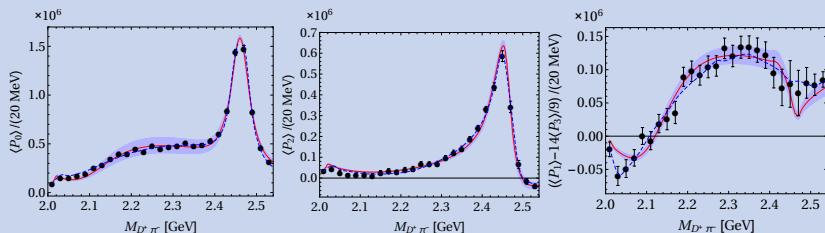
$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2, \quad \langle P_2 \rangle \propto \frac{2}{5} |\mathcal{A}_1|^2 + \frac{2}{7} |\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}} |\mathcal{A}_0| |\mathcal{A}_2| \cos(\delta_0 - \delta_2),$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |\mathcal{A}_0| |\mathcal{A}_1| \cos(\delta_0 - \delta_1).$$

# Comparison with experimental data: $B^- \rightarrow D^+ \pi^- \pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

Data: LHCb Collab., PR,D94,072001('16)

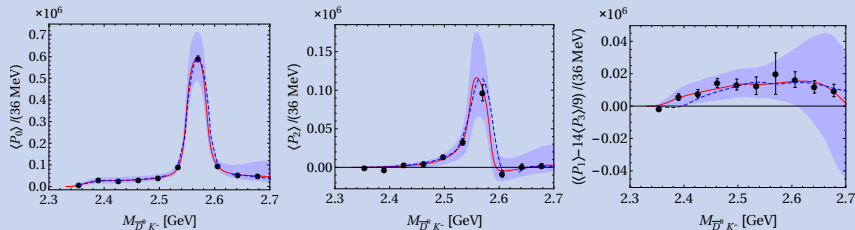


- Parameters:  $B/A = -3.6 \pm 0.1$ ,  $a_A = 1.0 \pm 0.1$ ,  $\chi^2/\text{d.o.f.} = 1.7$
- — This work. - - - LHCb. Bands: fit uncertainty
- **Good agreement** with data & with LHCb fit
- Rapid movement in  $\langle P_{13} \rangle$  [no  $D_2(2460)$ ] between 2.4 and 2.5 GeV. Related to  $D\eta$  and  $D_5\bar{K}$  openings.
- Recall: these are the amplitudes with **two states** in the  $D_0^*(2300)$  region, and no fit of the  $T$ -matrix parameters is done.

# Comparison with experimental data: $B_S^0 \rightarrow \bar{D}^0 K^- \pi^+$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

Data: LHCb Collab., PR,D90,072003('14)



- Exactly the same formalism applied to  $B_S^0 \rightarrow \bar{D}^0 K^- \pi^+$ .

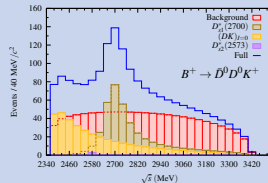
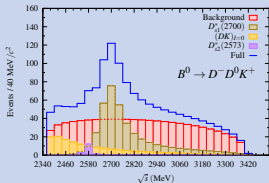
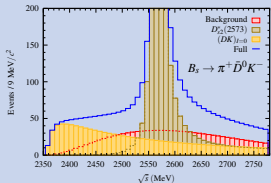
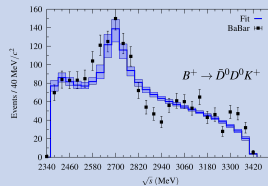
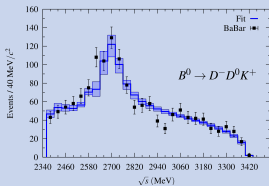
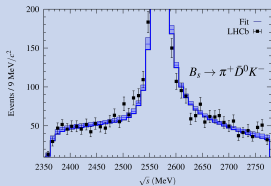
$$\mathcal{A}_0(s) = E_K \left[ C + \frac{C+A}{2} G_1(s) T_{11}^0(s) + \frac{C-A}{2} G_1(s) T_{11}^1(s) \right] - \frac{1}{\sqrt{3}} \left( \frac{3}{2} B - C \right) E_\eta G_2(s) T_{21}^0(s)$$

- Take same value of  $B/A$ ,  $a_A$ , as before:  $C/A = 4.8^{+3.4}_{-1.7}$ ,  $\chi^2/\text{d.o.f.} = 1.6$
- This work. - - - LHCb. Bands: fit uncertainty
- Good agreement with data & with LHCb fit
- See also Du et al., PRL,126,192001('21)

# Old analysis of LHCb data

MA, Nieves, Oset, Jido, EPJ,C76,300('16)  
 Data: LHCb Collab., PR,D90,072003('14)  
 BaBar Collab., PR,D91,052002('15)

- The LHCb (and BaBar) data had already been analyzed with a similar amplitude



# SU(3) light-flavor limit

[M. A. et al., Phys. Lett. B 767, 465 (2017)]

- SU(3) flavor limit:  $m_i \rightarrow m = 0.49$  GeV,  $M_i \rightarrow M = 1.95$  GeV.
- Irrep decomposition:  $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$ .  $T$  and  $V$  can be diagonalized:

$$V_d(s) = D^\dagger V(s) D = \text{diag}(V_{\bar{15}}(s), V_6(s), V_{\bar{3}}(s)) = A(s) \text{diag}(1, -1, -3),$$

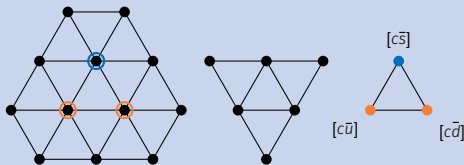
- $\bar{15}$  is repulsive.  $6$  and  $\bar{3}$  are attractive. “Curiously”,  $\bar{3}$  admits a  $c\bar{q}$  interpretation.

$S = 2$

$S = 1$

$S = 0$

$S = -1$



In the SU(3) limit:

- low  $D_0^*$  and  $D_{s0}^*$  (2317) connect with a bound state in  $\bar{3}$
- high  $D_0^*$  connects with a virtual state in  $6$
- See also [Gregory et al., 2106.15391]

State	Channels	$(S, I)$	$\bar{15}$	$6$	$\bar{3}$
$D_0^*$	$D\pi, D\eta, D_s\bar{K}$	$(0, \frac{1}{2})$	✓	✓	✓
$D_{s0}^*$ (2317)	$DK, D_s\eta$	$(1, 0)$	✓	✗	✓

- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[HadSpec Collab., JHEP 02 (2021) 100; JHEP 07 (2021) 123]

## Predictions for other sectors: charm

$(S, l)$	Channels	$\bar{15}$	6	$\bar{3}$	$0^+$		$1^+$	
					M	$\Gamma/2$	M	$\Gamma/2$
$(0, \frac{1}{2})$	$D^{(*)}\pi, D^{(*)}\eta, D_s^{(*)}\bar{K}$	✓	✓	✓	(R) 2105 $^{+6}_{-8}$	102 $^{+10}_{-12}$	(R) 2240 $^{+5}_{-6}$	93 $^{+9}_{-9}$
					(R) 2451 $^{+36}_{-26}$	134 $^{+7}_{-8}$		
$(1, 0)$	$D^{(*)}K, D_s^{(*)}\eta$	✓	✗	✓	(B) 2315 $^{+18}_{-28}$		(B) 2436 $^{+16}_{-22}$	
$(-1, 0)$	$D^{(*)}\bar{K}$	✗	✓	✗	(V) 2342 $^{+13}_{-41}$		-	
$(1, 1)$	$D_s^{(*)}\pi, D^{(*)}K$	✓	✓	✗	-		-	

- HQSS relates  $0^+$  ( $D_{(s)}P$ ) and  $1^+$  ( $D_{(s)}^*P$ ) sectors: **similar resonance pattern**.
- Two pole structure: higher  $D_1$  pole probably affected by  $\rho$  channels.
- $D\bar{K}$  [ $0^+, (-1, 0)$ ]: this virtual state (from 6) has a large impact on the scattering length,  $a_{(-1,0)}^{D\bar{K}} \simeq 0.8$  fm. (Rest of scattering lengths are  $|a| \simeq 0.1$  fm.)

## Predictions for other sectors: bottom

(S, I)	Channels	$\bar{15}$	6	$\bar{3}$	$0^+$		$1^+$	
					M	$\Gamma/2$	M	$\Gamma/2$
$(0, \frac{1}{2})$	$\bar{B}^{(*)}\pi, \bar{B}^{(*)}\eta, \bar{B}_s^{(*)}\bar{K}$	✓	✓	✓	(R) 5537 <sup>+9</sup> <sub>-11</sub>	116 <sup>+14</sup> <sub>-15</sub>	(R) 5581 <sup>+9</sup> <sub>-11</sub>	115 <sup>+13</sup> <sub>-15</sub>
					(R) 5840 <sup>+12</sup> <sub>-13</sub>	25 <sup>+6</sup> <sub>-5</sub>		
(1, 0)	$\bar{B}^{(*)}K, \bar{B}_s^{(*)}\eta$	✓	✗	✓	(B) 5724 <sup>+17</sup> <sub>-24</sub>		(B) 5768 <sup>+17</sup> <sub>-23</sub>	
(-1, 0)	$\bar{B}^{(*)}\bar{K}$	✗	✓	✗	(V-B) thr.		(V-B) thr.	
(1, 1)	$\bar{B}_s^{(*)}\pi, \bar{B}^{(*)}K$	✓	✓	✗	-		-	

- **Heavy flavour symmetry** relates charm (D) and bottom ( $\bar{B}$ ) sectors.
- $(0, \frac{1}{2})$ :  $B_0^*$ , two-pole pattern also observed.
- $(-1, 0)$ : [ $B^{(*)}\bar{K}$ ]: very close to threshold. Relevant prediction. Can be either **bound or virtual** (6) within our errors.
- (1, 1): [ $\bar{B}_s\pi, \bar{B}K, 0^+$ ],  $\chi(5568)$  channel. No state is found:  $\bar{15}$  and 6. If it exists, it is not generated with these  $B_s\pi, \bar{B}K$  interactions.
- (1, 0): Our results for  $B_{s0}^*$  and  $B_{s1}$  agree with **other results** from LQCD:   
M. A. et al., Phys. Lett. B 757, 515 (2016); Guo et al., Commun. Theor. Phys. 65, 593 (2016)
- Comparison of  $0^+, 1^+$  beauty states by Colangelo et al., Phys. Rev. D 86 054024 (2012): agreement in (1, 0) [ $b\bar{s}$ ], but not in  $(0, 1/2)$  [ $b\bar{q}$ ].   
Lang et al., Phys. Lett. B 750, 17 (2015); M. A. et al. Eur. Phys. J. C77, 170 (2017)

# Open questions for the community

- Need of more collaboration between (and simultaneous use of!) different “subcommunities”: LQCD, molecular/tetraquarks/QM models...

- **Spectroscopy, mixing:**

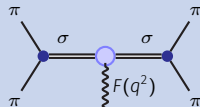
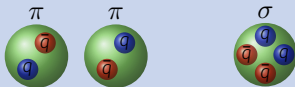
Specific example of  $D_{s0}^*$  (2317), take for granted the presence of a CQM  $c\bar{s}$  state. **Theoretical possibilities:**

- ▶ Genuine  $c\bar{s}$ , (very) renormalized by  $DK$  threshold. Or renormalized by  $DK$  interactions themselves?
- ▶ Or, there is a  $S = 1, L = 0$  state coming from  $DK$  interactions in addition to the  $c\bar{s}$  state. If so, where are those two poles? Which is which?

[Cincioglu *et al.*, EPJ,C76,576('16); MA *et al.*, EPJ,C78,722('19)]

- **Nature/size:**

- ▶ Can we address the question of  $4q, q\bar{q}$ , molecule based on the size of the object?



- ▶ For  $\pi\pi$  scattering,  $\sigma$  meson: MA, Oller, PR,D86,034003('12)

- $\sqrt{\langle r^2 \rangle_\sigma} \simeq 0.44 \text{ fm}$  vs  $\sqrt{\langle r^2 \rangle_\pi} \simeq 0.81 \text{ fm}$

- ▶ Perhaps only theoretical? Future lattice QCD calculations?

Briceño *et al.*, PR,D103,114512('21) [and refs. therein]

# Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$\mathcal{G}_{ii}(s) \rightarrow \mathcal{G}_{ii}(s) + i \frac{p_i(s)}{4\pi\sqrt{s}} \xi_i$$

$SU(3)$  limit:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),$$

$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).$$

- Physical case ( $x = 0$ ): RS specified by  $(\xi_1 \xi_2 \xi_3)$ ,  $\xi_i = 0$  or 1.
- $SU(3)$  symmetric case ( $x = 1$ ): all channels have the same threshold, so there are only two RS (000) and (111).
- To connect the **lower pole** with the  $T_6$  virtual state,

$$\xi_3 = x \quad (1, 1, 0) \rightarrow (1, 1, x)$$

- To connect the **lower pole** with the  $T_{\bar{3}}$  bound state,

$$\xi_1 = 1 - x \quad (1, 0, 0) \rightarrow (1 - x, 0, 0)$$

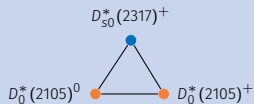
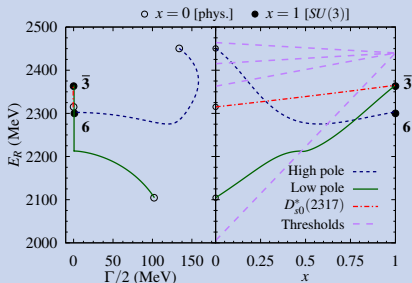
## (II)

Connecting **physical** ( $x = 0$ ) and **flavor  $SU(3)$**  ( $x = 1$ ) limits:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),$$

$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).$$

- The **high  $D_0^*$**  connects with a **6 virtual state** (unph. RS, below threshold).
- The **low  $D_0^*$**  connects with a  $\bar{\mathbf{3}}$  **bound state** (ph. RS, below threshold).
- The  $D_{s_0}^*(2317)$  also connects with the  $\bar{\mathbf{3}}$  **bound state**.



- The low  $D_0^*$  and the  $D_{s_0}^*(2317)$  are  $SU(3)$  **flavor partners**.
- This **solves the “puzzle”** of  $D_{s_0}^*(2317)$  being lighter than  $D_0^*(2300)$ : it is not, the lower  $D_0^*$  pole ( $M = 2105 \text{ MeV}$ ) is lighter.

# Form factors in semileptonic $D \rightarrow \pi \bar{\ell} \nu_{\ell}$

D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, Eur. Phys. J. C **78**, 310 (2018)

- General definitions:

$$\frac{d\Gamma(D \rightarrow \pi \bar{\ell} \nu_{\ell})}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_{\pi}|^3 |V_{cd}|^2 |f_+(q^2)|. \quad [q^2 = 0 : f_+(0) = f_0(0)]$$

$$\langle \pi(p') | \bar{q} \gamma^{\mu} Q | D(p) \rangle = f_+(q^2) \left[ \Sigma^{\mu} - \frac{m_D^2 - m_{\pi}^2}{q^2} q^{\mu} \right] + f_0(q^2) \frac{m_D^2 - m_{\pi}^2}{q^2} q^{\mu},$$

- “Isospin” form factors, related to  $D\pi$ ,  $D\eta$ ,  $D_s\bar{K}$  scattering:

$$\mathcal{F}^{(0,1/2)}(s) \equiv \begin{pmatrix} -\sqrt{\frac{3}{2}} f_0^{D^0 \rightarrow \pi^-}(s) \\ -f_0^{D^+ \rightarrow \eta}(s) \\ -f_0^{D_s^+ \rightarrow K^0}(s) \end{pmatrix}, \quad \text{Im}\mathcal{F}(s) = T^*(s)\Sigma(s)\mathcal{F}(s)$$

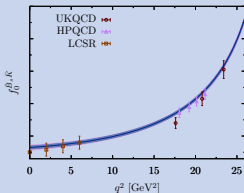
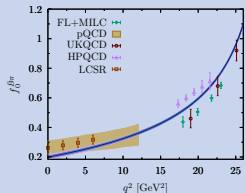
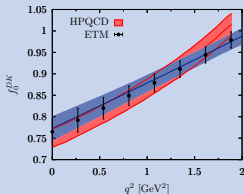
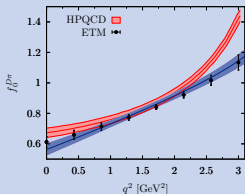
- Write form factors as Omnés matrix times polynomials

$$\mathcal{F}(s) = \Omega(s) \cdot \mathcal{P}(s)$$

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:

$$\mathcal{L}_0 = f_{\mathcal{P}} (\dot{m} \mathcal{P}_{\mu}^* - \partial_{\mu} \mathcal{P}) u^{\dagger} J^{\mu},$$

$$\mathcal{L}_0 = \beta_1 \mathcal{P} u (\partial_{\mu} U^{\dagger}) J^{\mu} + \beta_2 (\partial_{\mu} \partial_{\nu} \mathcal{P}) u (\partial^{\nu} U^{\dagger}) J^{\mu}.$$



- Points mostly from LQCD
- Also LCSR for  $q^2 \rightarrow 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

	This work	Exp.
$10^3  V_{ub} $	4.3(7)	4.49(24) [Incl.] 3.72(19) [Excl.]
$ V_{cd} $	0.244(22)	0.220(5)
$ V_{cs} $	0.945(41)	0.995(16)

## Why is $D_0^*(2300)$ interesting?

- Lightest systems to test **ChPT with heavy mesons**, besides  $D^* \rightarrow D\pi$ .
- $D\pi$  interactions (where it shows up) are relevant, since  $D\pi$  appears as a final state in many reactions that are being considered now (*i.e.*,  $Z_c(3900)$  and  $\bar{D}^* D\pi$ )
- $D_0^*(2300)$  is important in **weak interactions and CKM** parameters:

Flynn, Nieves, *Phys. Rev. D* **76**, 031302 (2007)

D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, *Eur. Phys. J. C* **78**, 310 (2018)

- ▶ It determines the shape of the scalar form factor  $f_0(q^2)$  in semileptonic  $D \rightarrow \pi$  decays.
- ▶ Relation to  $|V_{cd}|$ :  $f_+(0) = f_0(0)$  and  $d\Gamma \propto |V_{cd}f_+(q^2)|^2$ .
- ▶ Even more interesting: the bottom analogue  $|V_{ub}|$ .

## $D\pi$ , $D\eta$ , $D_s\bar{K}$ energy levels in a finite volume

- Periodic boundary conditions imposes **momentum quantization**
- Lüscher formalism:

Commun. Math. Phys. **105**, 153 (1986)

Nucl. Phys. B **354**, 531 (1991)

infinite volume	finite volume
$\vec{q} \in \mathbb{R}^3$	$\vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$
$\int_{\mathbb{R}^3} \frac{d^3q}{(2\pi)^3}$	$\frac{1}{L^3} \sum_{\vec{n} \in \mathbb{Z}^3}$

- In practice, changes in the  $T$ -matrix:  $T(s) \rightarrow \tilde{T}(s, L)$ :

Döring et al., Eur. Phys. J. A **47**, 139 (2011)

$$\mathcal{G}_{ii}(s) \rightarrow \tilde{\mathcal{G}}_{ii}(s, L) = \mathcal{G}_{ii}(s) + \lim_{\Lambda \rightarrow \infty} \left( \frac{1}{L^3} \sum_{|\vec{q}| < \Lambda} l_i(\vec{q}) - \int_0^\Lambda \frac{q^2 dq}{2\pi^2} l_i(\vec{q}) \right),$$

$$V(s) \rightarrow \tilde{V}(s, L) = V(s),$$

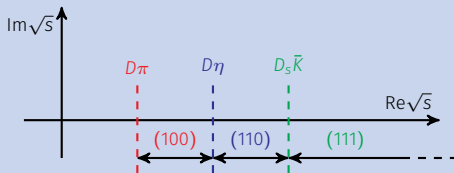
$$T^{-1}(s) \rightarrow \tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{\mathcal{G}}(s, L),$$

- **Free** energy levels:  $E_{n, \text{free}}^{(i)}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$
- **Interacting** energy levels  $E_n(L)$ :  $\tilde{T}^{-1}(E_n^2(L), L) = 0$  (poles of the  $\tilde{T}$ -matrix).

# T-matrix and analytical continuations

- Normalization:  $-ip_{ii}(s)T_{ii}(s) = 4\pi\sqrt{s} \left( \eta_i(s)e^{2i\delta_i(s)} - 1 \right)$ .
- $\mathcal{G}_{ii}(s) = G(s, m_i, M_i)$ , regularized with a subtraction constant  $a(\mu)$  ( $\mu = 1 \text{ GeV}$ ).
- Riemann sheets (RS) denoted as  $(\xi_1\xi_2\xi_3)$ :

$$\mathcal{G}_{ii}(s) \rightarrow \mathcal{G}_{ii}(s) + i \frac{p_i(s)}{4\pi\sqrt{s}} \xi_i$$



# Chiral dynamics and two-state structure(s)

- Other famous two-poles structures rooted in **chiral dynamics**:

$\Lambda(1405) [\Sigma\pi, N\bar{K}]$

Oller, Meißner, Phys. Lett. B **500**, 263 (2001)

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- Recently, [Clymton, Kim, 2305.14812](#) claim a two-pole structure for  $b_1(1235)$ .
- **Chiral dynamics**:
  - ▶ Incorporates the  $SU(3)$  light-flavor structure,
  - ▶ Determines the strength of the interaction,
  - ▶ Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.