Nonextensive Thermodynamics for Hadrons with Finite Chemical Potentials

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Abstract

We derive the nonextensive thermodynamics of an ideal quantum gas composed by bosons and/or fermions with finite chemical potentials. We find agreement with previous works when $\mu \leq m$. However some inconsistencies of previous references are corrected when $\mu > m$ for fermions. This formalism is then used to study the thermodynamical properties of hadronic systems based on a Hadron Resonance Gas (HRG) approach. We show that the nonextensive statistics provides a harder equation of state (EoS) than that provided by the Boltzmann-Gibbs (BG) statistics. We apply this result to study the (proto)neutron star stability under several conditions.

1. Tsallis Statistics

Tsallis statistics constitutes a generalization of BG statistics, under the assumption that the entropy of the system is non additive. For two independent systems A and B

$$S_{A+B} = S_A + S_B + (1-q)S_A S_B \,, \tag{1}$$

where the entropic index q measures the degree of nonextensivity [1]. Let us define the q-exponential $e_q^{(\pm)}(x) = [1 \pm (q-1)x]^{\pm 1/(q-1)}$, with $e_q^{(+)}(x)$ defined for $x \geq 0$ and $e_q^{(-)}(x)$ for $x < 0$, and the q-log function $\log_q^{(\pm)}(x) = \pm(x^{\pm(q-1)} - 1)/(q-1)$. Then the grand-canonical partition function for a nonextensive ideal quantum gas is [2]

$$\log \Xi_q(V,T,\mu) = -\xi V \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \log_q^{(-r)} \left(\frac{e_q^{(r)}(x) - \xi}{e_q^{(r)}(x)} \right), \tag{2}$$

where $x = \beta(E_p - \mu)$, the particle energy is $E_p = \sqrt{p^2 + m^2}$, with m being the mass and μ the chemical potential, $\xi = \pm 1$ for bosons and fermions respectively, and Θ is the step function. Eq. (2) reduces to the Bose-Einstein and Fermi-Dirac partition functions in the limit $q \to 1$. Thermodynamic functions can be derived from the partition function by applying the thermodynamic relations. The average number of particles is

$$\langle N \rangle = \beta^{-1} \frac{\partial}{\partial \mu} \log \Xi_q \Big|_\beta = V \left[C_{N,q}(\mu,\beta,m) + \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \left(\frac{1}{e_q^{(r)}(x) - \xi} \right)^{\tilde{q}} \right], \tag{3}$$

where $\tilde{q}(x \geq 0) = q$ and $\tilde{q}(x < 0) = 2 - q$. The momentum-independent term

$$C_{N,q}(\mu,\beta,m) = \frac{1}{2\pi^2} \frac{\mu\sqrt{\mu^2 - m^2}}{\beta} \frac{2^{q-1} + 2^{1-q} - 2}{q-1} \Theta(\mu - m) \,, \tag{4}$$

appears as a result of a discontinuity at $x = 0$ in the integrand of Eq. (2) [2]. The entropy is

$$S = V \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \Theta(rx) \left[-[\bar{n}_q^{(r)}(x)]^{\tilde{q}} \log_q^{(-r)} \left(\bar{n}_q^{(r)}(x) \right) + \xi [1 + \xi \bar{n}_q^{(r)}(x)]^{\tilde{q}} \log_q^{(-r)} \left(1 + \xi \bar{n}_q^{(r)}(x) \right) \right].$$

We have verified the thermodynamic consistency of these expressions by checking that $(\partial S / \partial E)_{V,N} = \beta$. These results agree with previous works [3, 4] for $x \geq 0$. However, for $x < 0$ this corrects some inconsistencies in the literature related to $\langle N \rangle$ and $\langle E \rangle$ ➞ inclusion of $C_{N,q}$.

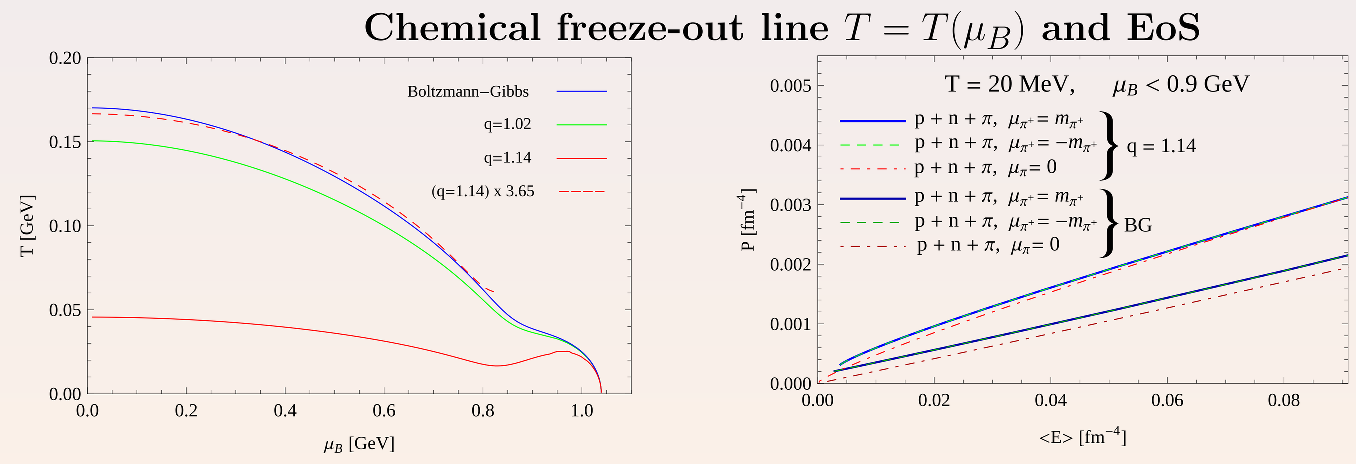
2. Thermodynamical Properties of Hadronic Systems

The thermodynamics of Quantum Chromodynamics (QCD) in the confined phase can be studied within the HRG model. This approach is based on the assumption that physical observables in this phase admit a representation in terms of hadronic states which are treated as non-interacting and point-like particles [5]. These states are taken as the conventional hadrons listed in the review by the Particle Data Group. Within this approach the partition function is then given by

$$\log \Xi_q(V,T,\{\mu\}) = \sum_i \log \Xi_q(V,T,\mu_i) \,, \tag{5}$$

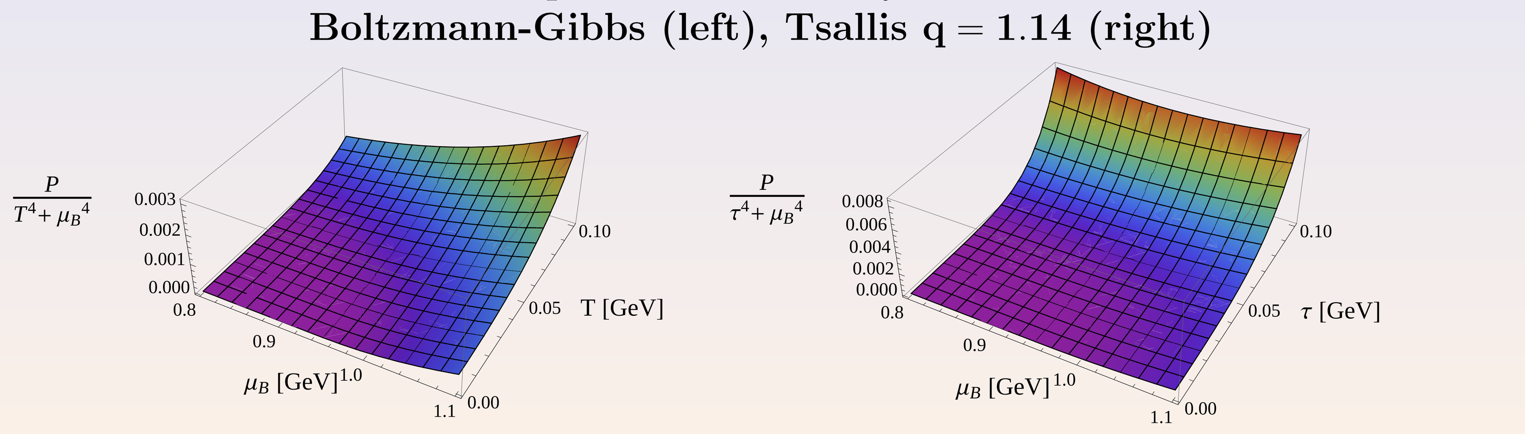
where μ_i refers to the chemical potential for the *i-th* hadron. The total numbers of hadronic states we consider are 808 for mesons and 1168 for baryons (+ anti-baryons), corresponding to masses below 11 GeV and 5.8 GeV respectively. We restrict the summation in Eq. (5) to zero strangeness. Using the arguments of [6], the transition line between the confined and the deconfined regimes can be determined by the condition that $\langle E \rangle / \langle N \rangle = 1$ GeV.

Chemical freeze-out line $T = T(\mu_B)$ and EoS



In Tsallis statistics the temperature τ is an effective parameter linearly related to the physical temperature T [7]. For $\mu_B = 0$ the effective temperature is $\tau_o = 45.6$ MeV for $q = 1.14$, which is in agreement within 25% with the value obtained from the analysis of the p_T-distributions in high energy $p + p$ collisions [8]. The chemical freeze-out lines spam over the region of $0 < \mu_B < 1039.2$ MeV with the maximum value for μ_B corresponding to a null critical temperature.

Pressure vs Temperature and Baryonic Chemical Potential
Boltzmann-Gibbs (left), Tsallis q = 1.14 (right)



EoS at finite μ_B becomes harder ($P(E)$ increases) in Tsallis statistics as compared to BG statistics [2]. The effect of pion chemical potential is to produce an even harder EoS in both statistics.

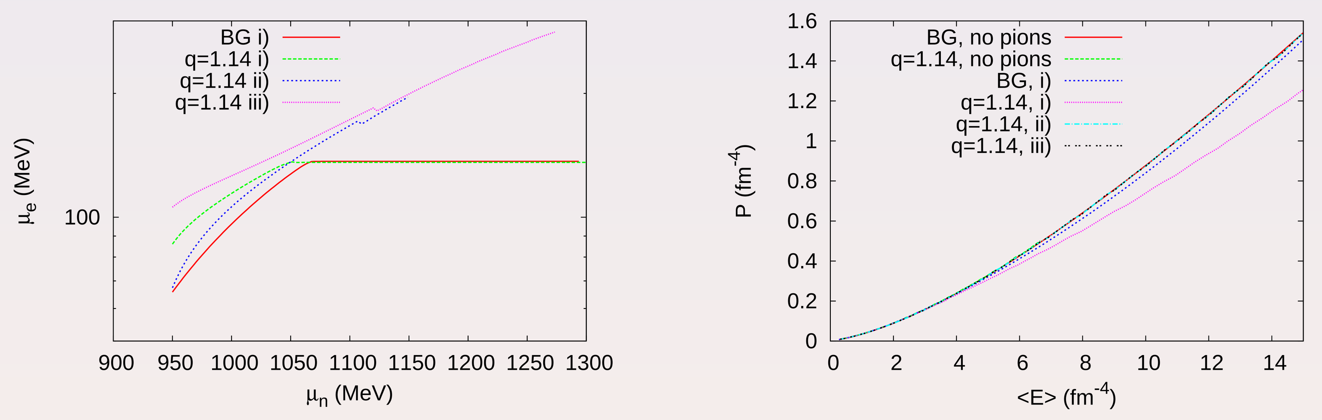
3. Application to (Proto)Neutron Stars

In addition to hadronic matter, leptons are also present in stellar matter and they play a relevant role in the equilibration between gravitational force and the gas pressure due to their small masses. We consider the star composed by neutrons, protons, pions, electrons and muons. The star is a dynamically equilibrated system because protons and neutrons are being converted into one another either through weak decays or scattering with pions. The relevant processes are:

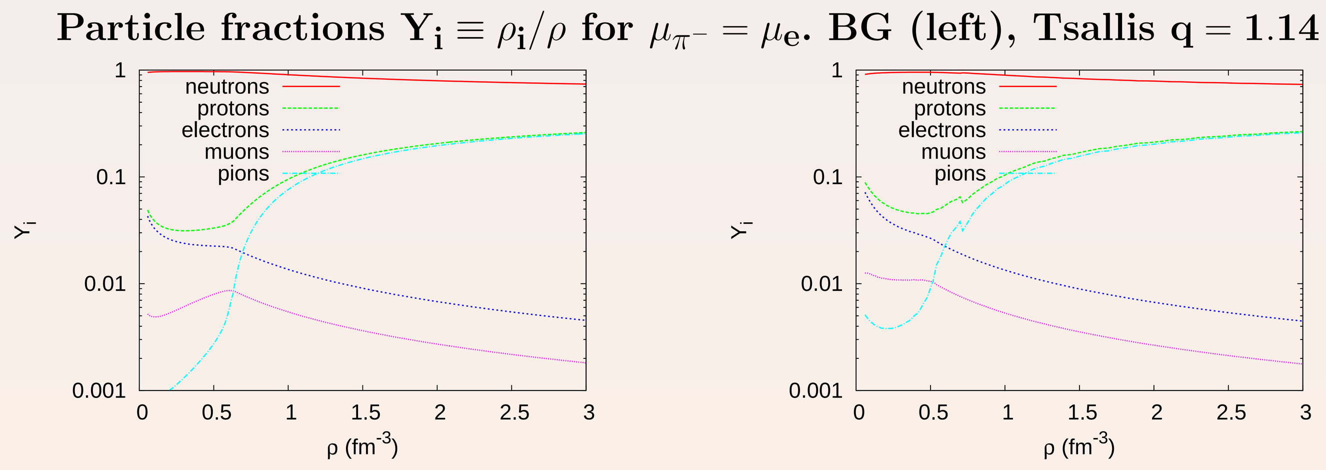
$$\begin{cases} n \to p + e^- + \bar{\nu}_e \\ p \to n + e^+ + \nu_e \end{cases}, \qquad \begin{cases} n + \pi^+ \to p \\ p + \pi^- \to n \end{cases}. \tag{6}$$

The dynamical equilibrium between the relative number of particles leads to the relations $\mu_n = \mu_p + \mu_e$, $\mu_n = \mu_p - \mu_{\pi^+}$ and $\mu_e = \mu_\mu$. As the star is electrically neutral, the condition $\rho_p + \rho_{\pi^+} = \rho_{\pi^-} + \rho_e + \rho_\mu$ is also enforced. We use Tsallis statistics for hadrons and BG statistics for leptons.

μ_e vs μ_n (left) and EoS (right) for: i)$\mu_{\pi^-} = \mu_e$, ii)$\mu_{\pi^-} = m_\pi$ and iii)$\mu_{\pi^-} = -m_\pi$.



Particle fractions $Y_i \equiv \rho_i/\rho$ for $\mu_{\pi^-} = \mu_e$. BG (left), Tsallis q = 1.14 (right).



- Tsallis statistics lead to a discontinuity in the chemical potentials.
- Appearance of pions at low densities increases if Tsallis statistics is used.
- Solving the TOV equations with these EoS, the maximum stellar masses and radii lie around $0.7\,M_\odot$ and 8 Km. This agrees with most of previous models [9] that do not take nuclear interaction into account and, for this reason, it is not enough to explain the maximum values observed ➞ two stars with masses $2\,M_\odot$ have been recently confirmed [10].

4. Conclusions

- We obtained fully thermodynamically consistent expressions for Tsallis statistics of a quantum gas at finite T and μ ➞ Some inconsistencies in previous works have been addressed for $\mu > m$.
- We applied this formalism to study the EoS of hadronic matter and the phase diagram of QCD. The EoS becomes harder in Tsallis statistics as compared to BG statistics.
- The application to (proto)neutron stars shows that the enforcement of charge neutrality and β-equilibrium wash out the hardening effect of the pions on the EoS, and nonextensive statistics only cannot explain the star stability as observed in nature. Nevertheless Tsallis statistics modifies substantially the particle constitution at low densities ➞ this deserves more investigation.

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