

**QCD@Work 2014, 7th International Workshop on QCD - Theory  
and Experiment**

**Giovinazzo(Bari) Italy, 16 -19 June, 2014**

**New Unitary Relations between the QCD sum rules**

**Altug Ozpineci**

METU (Middle East Technical University), Ankara, Turkey

**Valeri Zamiralov**

Skobeltsyn Institute of Nuclear Physics, MSU, Moscow, Russia

## **1 Introduction**

QCD sum rules have become one of the most important tools while studying baryon or meson couplings and the corresponding form factors with photons, mesons or baryons. At the same time the complexity of the formules used in calculations increases in obviously non-linear manner. So the control of one's own and other authors calculations become the plausible problem. Recently we have proposed simple relations between the quantities involving  $\Sigma$  hyperons and those involving  $\Lambda$  hyperons. They are generalizied easily to any  $\Sigma$ -like and  $\Lambda$  -like baryons containing heavy quarks.

Our formules seem to be useful and for example were inserted into the codes by Frank Lee and while studying magnetic moments of the octet baryons.

It is interesting that also the transition  $\Sigma-\Lambda$  quantities could be obtained from the corresponding quantities of the  $\Sigma$  or  $\Lambda$  baryons.

We want here to propose one more relation to control the complex calculations of the transition  $\Sigma - \Lambda$  quantities which has the merit that it does not involve other baryon quantities, neither of  $\Sigma$  nor of  $\Lambda$  baryons.

It is rather trivial for nonrelativistic quark model or simple unitary symmetry model, But it shows us the way to overcome many difficulties in the framework of QCD sum rules as we shall see later.

To show its applicability we consider as an example SR's for magnetic moments of octet baryons.

Our relations for the polarization operators  $\Pi$ 's could be written then as follows:

(Ozpineci, Yakovlev, V.Z., At.Nucl.Phys.Journ.77,(2002))[ozyaz] (Also cited in [Frank X. Lee, Lai Wang PR D78] )

$$2[\tilde{\Pi}^{\Sigma^0(ds)} + \tilde{\Pi}^{\Sigma^0(us)}] - \Pi^{\Sigma^0} = 3\Pi^\Lambda, \quad (1)$$

and similarly for Lambda quantities

$$2[\tilde{\Pi}^{\Lambda(ds)} + \tilde{\Pi}^{\Lambda(us)}] - \Pi^\Lambda = 3\Pi^{\Sigma^0}. \quad (2)$$

Moreover these relations are not constrained by the octet representation of SU(3) but similarly can be written for  $\Sigma_Q$  and  $\Lambda_Q$  and pairs of heavy cascade hyperons  $\Xi_Q$  and  $\Xi'_Q$  with  $Q = c, b$  with the appropriate changes.

In this report we would be more interested with the Sigma-Lambda transitions. First, they can be related also to  $\Sigma$  and  $\Lambda$  quantities as

$$[\tilde{\Pi}^{\Sigma^0(ds)} - \tilde{\Pi}^{\Sigma^0(us)}] = \sqrt{3}\Pi^{\Lambda\Sigma^0}, \quad (3)$$

$$\tilde{\Pi}^{\Lambda(ds)} - \tilde{\Pi}^{\Lambda(us)} = -\sqrt{3}\Pi^{\Lambda\Sigma^0}. \quad (4)$$

Second, and this is the main point of our report, it can be stated in some "autosufficient" way. Our new relation between the corresponding correlation functions reads as

$$\Pi^{\Lambda\Sigma^0(u\leftrightarrow s)} + \Pi^{\Lambda\Sigma^0(d\leftrightarrow s)} = \Pi^{\Lambda\Sigma^0} \quad (5)$$

## 2 Master formules for the octet

As the main ingredient of the polarization operator is the matrix element of the T-production of the interpolating currents and we now put our attention to it. The corresponding formules are often named as "master formules".

We begin with the master formule for the  $\Sigma^0$  [Aliev, Ozpineci, Savci ,PR D66,016002 (2002); Lai Wang, Frank X. Lee PR D78,013003 (2008)]:

$$\begin{aligned} \Sigma^0 &\equiv \langle 0|T\{\eta^{\Sigma^0}\bar{\eta}^{\Sigma^0}\}|0\rangle = \quad (6) \\ &= -\varepsilon_{abc}\varepsilon_{def}\{S_u^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_d^{cf} + S_d^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_u^{cf} + \\ &S_u^{ad}\text{Tr}[\gamma_5 C S_s^{be} C \gamma_5 S_d^{cf}] + S_d^{ad}\text{Tr}[\gamma_5 C S_s^{be} C \gamma_5 S_u^{cf}] + \dots \end{aligned}$$

One can see that every line as to the quark flavors is of the form

$$\Sigma^0 = u s d + d s u$$

Now we construct Lambda quantity using the relation [ozyaz]

$$\begin{aligned} \Lambda &\equiv \langle 0|T\{\eta^\Lambda\bar{\eta}^\Lambda\}|0\rangle = \quad (7) \\ &\varepsilon_{abc}\varepsilon_{def}\{2S_u^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_s^{cf} + 2S_s^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_u^{cf} + \\ &2S_s^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_d^{cf} + 2S_d^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf} \\ &- [S_u^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_d^{cf} + S_d^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_u^{cf}] + \dots \end{aligned}$$

and we retain for a moment only these terms to exhibit the group-theoretical structure. In fact one can see that every three lines as to the quark flavors are of the form

$$\Lambda = 2 u d s + 2 s d u + 2 s u d + 2 d u s - u s d - d s u$$

in accord with [ozyaz].

Transition magnetic moment "master formule" could be read from [ozyaz] as

$$\sqrt{3}\Sigma^0\Lambda \equiv \sqrt{3}\langle 0|T\{\eta^{\Sigma^0}\bar{\eta}^\Lambda\}|0\rangle = \quad (8)$$

$$\begin{aligned} & \varepsilon_{abc}\varepsilon_{def}\{[S_u^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_s^{cf} + S_s^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_u^{cf}] - \\ & [S_s^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_d^{cf} + S_d^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf}] + \\ & [S_u^{ad}\text{Tr}[\gamma_5 C S_d^{be} C \gamma_5 S_s^{cf}] + S_s^{ad}\text{Tr}[\gamma_5 C S_d^{be} C \gamma_5 S_u^{cf}]] - \\ & [S_s^{ad}\text{Tr}[\gamma_5 C S_u^{be} C \gamma_5 S_d^{cf}] + S_d^{ad}\text{Tr}[\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf}]] + \dots \end{aligned}$$

One can see that every two lines as to the quark flavors are of the form

$$u d s + s d u - s u d - d u s$$

Cancelling identical terms in the full expression we obtain

$$\begin{aligned} \sqrt{3}\Sigma^0\Lambda = & \varepsilon_{abc}\varepsilon_{def}\{[S_u^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_s^{cf} + S_s^{ad}\gamma_5 C S_d^{be} C \gamma_5 S_u^{cf}] - \quad (9) \\ & [S_s^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_d^{cf} + S_d^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf}] + \\ & [S_u^{ad}\text{Tr}[\gamma_5 C S_d^{be} C \gamma_5 S_s^{cf}] - S_d^{ad}\text{Tr}[\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf}]] + \dots \end{aligned}$$

New relations (we restrict ourselves with the first terms) yield

$$\Sigma^0\Lambda|_{ds} + \Sigma^0\Lambda|_{us} = \quad (10)$$

$$\begin{aligned} & \varepsilon_{def}\{[S_u^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_d^{cf} + S_d^{ad}\gamma_5 C S_s^{be} C \gamma_5 S_u^{cf}] - \\ & [S_d^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_s^{cf} + S_s^{ad}\gamma_5 C S_u^{be} C \gamma_5 S_d^{cf}]\} + \end{aligned}$$

$$\begin{aligned}
& \varepsilon_{abc}\varepsilon_{def}\{[S_s^{ad}\gamma_5CS_d^{be}C\gamma_5S_u^{cf} + S_u^{ad}\gamma_5CS_d^{be}C\gamma_5S_s^{cf}] - \\
& \quad [S_u^{ad}\gamma_5CS_s^{be}C\gamma_5S_d^{cf} + S_d^{ad}\gamma_5CS_s^{be}C\gamma_5S_u^{cf}]\} = \\
& \varepsilon_{abc}\varepsilon_{def}\{[S_u^{ad}\gamma_5CS_d^{be}C\gamma_5S_s^{cf} + S_s^{ad}\gamma_5CS_d^{be}C\gamma_5S_u^{cf}] - \\
& \quad [S_s^{ad}\gamma_5CS_u^{be}C\gamma_5S_d^{cf} + S_d^{ad}\gamma_5CS_u^{be}C\gamma_5S_s^{cf}]\} = \Sigma^0\Lambda
\end{aligned}$$

One can see that every two lines as to the quark flavors are of the form

$$\begin{aligned}
& (u\ s\ d, +\ d\ s\ u -\ d\ u\ s -\ s\ u\ d) + \\
& (s\ d\ u +\ u\ d\ s -\ u\ s\ d -\ d\ s\ u) = \\
& u\ d\ s +\ s\ d\ u -\ s\ u\ d -\ d\ u\ s.
\end{aligned}$$

Full expression for the transition Sigma-Lambda quantities

$$\Sigma^0\Lambda|_{ds} + \Sigma^0\Lambda|_{us} \quad (11)$$

gives just the  $\Sigma^0\Lambda$  quantity.

Let us give some example.

In [ZhuHwangYang PR D57,1527(1998),arXiv 9802322] one has written QCD SR for the magnetic transition moment which reads in that work in terms of various vev (and for a moment we maintain  $a_u, a_d$  different, the same for other vev's, and the mass terms  $m_u, m_d$ ):

$$\begin{aligned}
SR(\Sigma^0\Lambda) &= (e_u - e_d)\frac{M^6}{4L^{4/9}} + \frac{L^{4/9}}{9M^2}[e_s a_s(a_u - a_d) + e_u a_s(a_u - 3a_d) \\
&\quad - e_d a_s(a_d - 3a_u)] + \frac{M^2 b}{24L^{4/9}}(e_u - e_d) \\
&\quad \frac{M^2 b}{144L^{4/9}}\left[\ln\left(\frac{M^2}{\Lambda^2}\right) - 1 - \gamma_{EM}\right] + 4\left[\ln\left(\frac{M^2}{\Lambda^2}\right) - \gamma_{EM} - \frac{M^2}{2\Lambda^2}\right](e_u - e_d) \\
&+ \left[\frac{-\chi}{6L^{4/27}}\left(M^2 - \frac{m_{(s)0}^2}{8L^{4/9}}\right) \times [a_s(e_u\chi_u a_u - e_d\chi_d a_d) + e_s\chi_s a_s(a_u - a_d)]\right] +
\end{aligned}$$

$$\begin{aligned}
& \frac{M^2}{L^{4/9}} \frac{1}{4} [2(e_u a_u m_d - e_d a_d m_u) - 2e_s a_s (m_u - m_d) - \\
& a_s (e_u m_d - e_d m_u) - e_s (a_u m_d - a_d m_u) + 2(e_d a_d m_u - e_u a_u m_d) + \\
& 2m_s (e_d a_d - e_u a_u) + m_s (e_u a_d - e_d a_u) + e_s (a_d m_u - a_u m_d)] \\
& + \frac{L^{4/9}}{36} (2\kappa - \xi) \times [a_s (e_u (2\kappa_u - \xi_u) a_u - e_d (2\kappa_d - \xi_d) a_d) + e_s (2\kappa_s - \xi_s) a_s (a_u - a_d)] \\
& - \frac{M^4}{4L^{28/27}} [(e_u a_u \chi_u - e_d a_d \chi_d) m_s + e_s a_s \chi_s (m_u - m_d)] + \\
& \frac{1}{18} [(e_u a_u (2\kappa_u - \xi_u) - e_d a_d (2\kappa_d - \xi_d)) m_s + e_s a_s (2\kappa_s - \xi_s) (m_u - m_d)] M^2 - \\
& \frac{M^2}{6} [(e_u a_u \kappa_u - e_d a_d \kappa_d) m_s + e_s a_s \kappa_s (m_u - m_d)] \times \\
& \times [\ln(\frac{M^2}{\Lambda^2}) - 1 - \gamma_{EM}] = \beta_\Sigma \beta_\Lambda \sqrt{3} \mu_{\Sigma^0 \Lambda} e^{-\bar{m}^2/M^2} (1 + A_{\Sigma^0 \Lambda} M^2) + \dots
\end{aligned}$$

One can see that every term in this formule satisfies our new relation

$$\langle \Sigma \Lambda \rangle|_{us} + \langle \Sigma \Lambda \rangle|_{ds} = \langle \Sigma \Lambda \rangle.$$

### 3 FrankLee's formules

Now we try to control the formules (16-20) of [FrankLee] by applying our relation. We put for simplicity  $m_s = 0$ . First we put also parameter  $\beta = 0$ . Checking the formules one by one one can see that all of them have the form

$$\kappa(e_u - e_d)$$

which obviously satisfies our relation:

$$\kappa(e_u - e_s) + \kappa(e_s - e_d) = \kappa(e_u - e_d).$$

## 4 Baryonic Distribution Amplitudes

But our group-theoretical relations for the product of interpolating currents do not work if we go to the higher  $Q^2$  (baryon electromagnetic form factors at high momentum transfer, form factors of lepton beta-decay of the heavy baryons etc).

QCD SR's now are formulated for the product of baryon interpolating current and the photon or weak bozon one. And in the ket bracket we have now not photon, meson or weak boson but baryon. Correspondingly one should work not with photon or boson distrubution amplitudes but with the baryon distribution amplitudes.

But the resulting formules are even more cumbersome than in the case of ordinary QCD SR's basing on the product of the baryon inrerpolating currents.

And the final formules of several authors are very different. That is why we decided to construct if possible some group theoretical relations for the case with only one baryon interpolating current.

New group-theoretical relations for the SR's with baryon distribution amplitudes read as

$$(1) \sqrt{3}\langle 0|\eta_\Lambda\eta_\gamma|\Lambda\rangle = 2\langle 0|\eta_\Sigma^{us}\eta_\gamma|\Lambda\rangle + \langle 0|\eta_\Sigma\eta_\gamma|\Lambda\rangle$$

$$(2) \sqrt{3}\langle 0|\eta_\Lambda\eta_\gamma|\Sigma\rangle = 2\langle 0|\eta_\Sigma^{us}\eta_\gamma|\Sigma\rangle + \langle 0|\eta_\Sigma\eta_\gamma|\Sigma\rangle$$

$$(3) \sqrt{3}\langle 0|\eta_\Sigma\eta_\gamma|\Lambda\rangle = -2\langle 0|\eta_\Lambda^{ds}\eta_\gamma|\Lambda\rangle + \langle 0|\eta_\Lambda\eta_\gamma|\Lambda\rangle$$

$$(4) \sqrt{3}\langle 0|\eta_\Sigma\eta_\gamma|\Sigma\rangle = -2\langle 0|\eta_\Lambda^{ds}\eta_\gamma|\Sigma\rangle + \langle 0|\eta_\Lambda\eta_\gamma|\Sigma\rangle$$

We have 4 different matrix elements and try to write the solutions in general form as

$$\langle 0 | \eta_\Lambda \eta_\gamma | \Lambda \rangle = a e_u + b e_d + c e_s$$

wherefrom

$$\begin{aligned} \sqrt{3} \langle 0 | \eta_\Lambda \eta_\gamma | \Sigma \rangle &= -2(a e_u + b e_s + c e_d) + (a e_u + b e_d + c e_s) = \\ &= -a e_u + (-2c + b) e_d + (-2b + c) e_s; \end{aligned}$$

wherefrom

$$\langle 0 | \eta_\Lambda \eta_\gamma | \Lambda \rangle = a(e_u - e_d), \quad \sqrt{3} \langle 0 | \eta_\Sigma \eta_\gamma | \Lambda \rangle = -a(e_u + e_d - 2e_s).$$

Similar calculations can be made for another couple of relations. Upon writing general form

$$\langle 0 | \eta_\Sigma \eta_\gamma | \Sigma \rangle = \tilde{a} e_u + \tilde{b} e_d + \tilde{c} e_s$$

we write

$$\begin{aligned} \sqrt{3} \langle 0 | \eta_\Lambda \eta_\gamma | \Sigma \rangle &= 2(\tilde{a} e_s + \tilde{b} e_d + \tilde{c} e_u) + \tilde{a} e_u + \tilde{b} e_d + \tilde{c} e_s = \\ &= (\tilde{a} + 2\tilde{c}) e_u + 3\tilde{b} e_d + (\tilde{c} 2\tilde{a}) e_s, \end{aligned}$$

and

$$\begin{aligned} 3 \langle 0 | \eta_\Sigma \eta_\gamma | \Sigma \rangle &= 3(\tilde{a} e_u + \tilde{b} e_d + \tilde{c} e_s) = \\ &= (-\tilde{a} - 2\tilde{c}) e_u + (-4\tilde{a} + 3\tilde{b} - 2\tilde{c}) e_d + (2\tilde{a} - 6\tilde{b} + \tilde{c}) e_s, \end{aligned}$$

wherefrom general form of solution would be

$$\langle 0 | \eta_\Sigma \eta_\gamma | \Sigma \rangle = \tilde{a}(e_u + e_d - 2e_s), \quad \sqrt{3} \langle 0 | \eta_\Lambda \eta_\gamma | \Sigma \rangle = 3\tilde{a}(e_d - e_u).$$

Thus solutions for the matrix elements  $\langle 0 | \eta_\Sigma \eta_\gamma | \Sigma \rangle$  and  $\sqrt{3} \langle 0 | \eta_\Sigma \eta_\gamma | \Lambda \rangle$  should be similar up to some overall factor. We have proved them in the NRQM.



Now we try to prove now the validity of these relations for QCD sum rules. As an example we take  $\Sigma\Lambda$  transition from Aliev,Azizi,Savci, arXiv:1303.6897v2[hep-ph]

$$F_1^{\Sigma\Lambda}(Q^2) = \frac{1}{4\sqrt{2}\lambda_{\Sigma^0}} e^{m_{\Sigma^0}^2} \int_{x_0}^1 dx \left( -\frac{\rho_2(x)}{x} + \frac{\rho_4(x)}{M^2 x^2} - \frac{\rho_6(x)}{2M^4 x^3} \right) e^{-\left(\frac{Q^2 x}{M^2} + \frac{m_{\Lambda}^2 x}{M^2}\right)} + \dots$$

where

$$\rho_6(x) = 4m_{\Lambda}^3 x (m_{\Lambda}^2 + Q^2) (1 + \beta) (e_u B_6(x) + e_d \tilde{B}_6(x) + 2e_s \hat{B}_6(x));$$

$$\rho_4(x) = 2m_{\Lambda}^3 x (1 + \beta) \{ [e_u (2B_6(x) - 5B_8(x)) + e_d (2\tilde{B}_6(x) - 5\tilde{B}_8(x)) + 4e_s (2\hat{B}_6(x) - 5\hat{B}_8(x))] -$$

$$2 \int_0^x dx_1 5(e_u T_1^M(x, 1-x-x_1, x_1) + e_d T_1^M(x_1, x, 1-x-x_1) + 2e_s T_1^M(x_1, 1-x-x_1, x)) \}$$

Now if we change all the  $\Lambda$  symbols to  $\Sigma^0$  ones we just arrive at the formules for  $F_1^{\Sigma^0}(Q^2)$  for the  $\Sigma^0$  form factor (see Appendix arXiv:1303.6798).

This solution proves the validity of our relation for baryon DA's.

## 5 Conclusion

1. We have obtained a new group-theoretical relation for the LC QCD SR describing transition Sigma Lambda quantities in terms of the meson or photon DA's and proved it for "master formules" as well as for the QCD SR's for transition magnetic moments.

2. We have obtained new relations for the case of baryon DA's and have shown its validity on the example of the LC QCD SR for the electromagnetic form factors of Sigma baryon and Sigma-Lambda transition.