Do we live in extra dimensions?

Konstantin Zarembo
(Nordita, Stockholm)

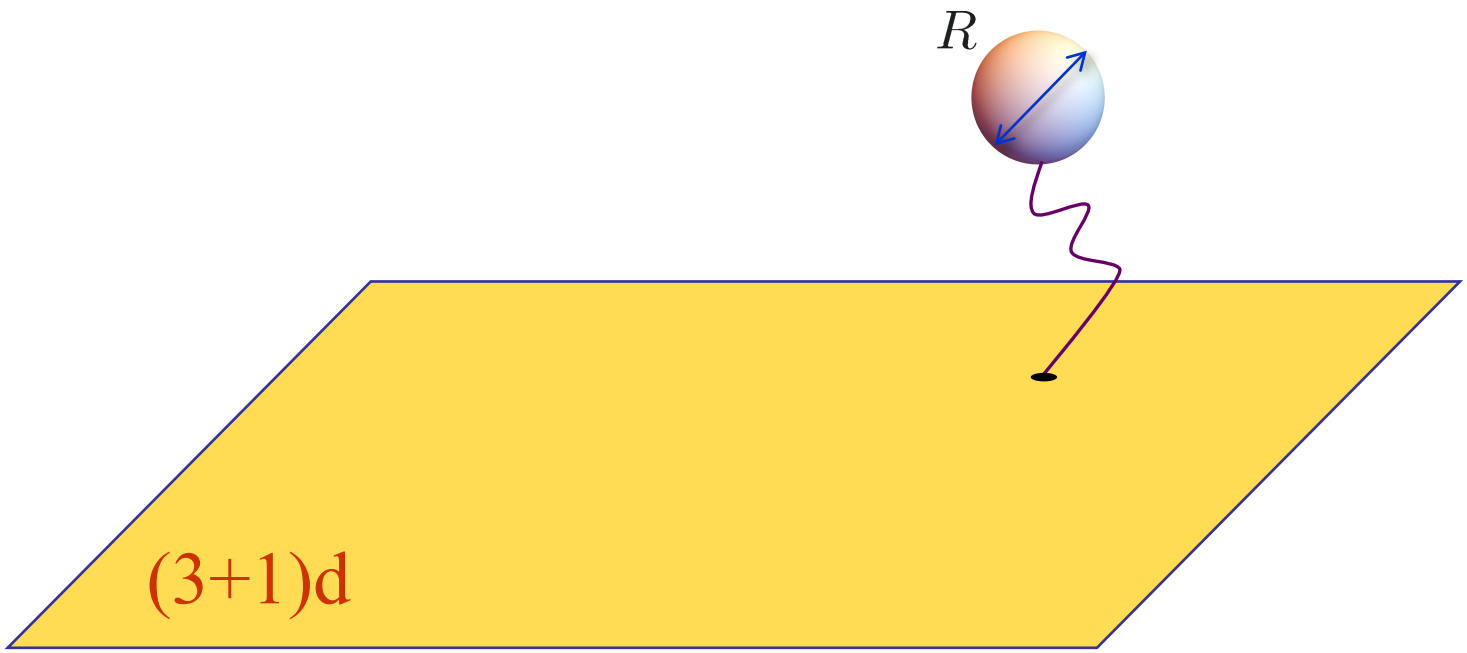
We don’t know …

Kaluza-Klein theory

Gravity in (4+1)d = Gravity in (3+1)d + Electromagentism

Kaluza'21
Klein'26

Kaluza-Klein theory (ctd)



Uncertainty relation: $p \gtrsim \dfrac{\hbar}{R}$

$$E \geqslant pc \gtrsim \frac{\hbar c}{R}$$

Can be very big if R is small

Current exp. bounds:

$R \lesssim$ few $\times$ 10 μm

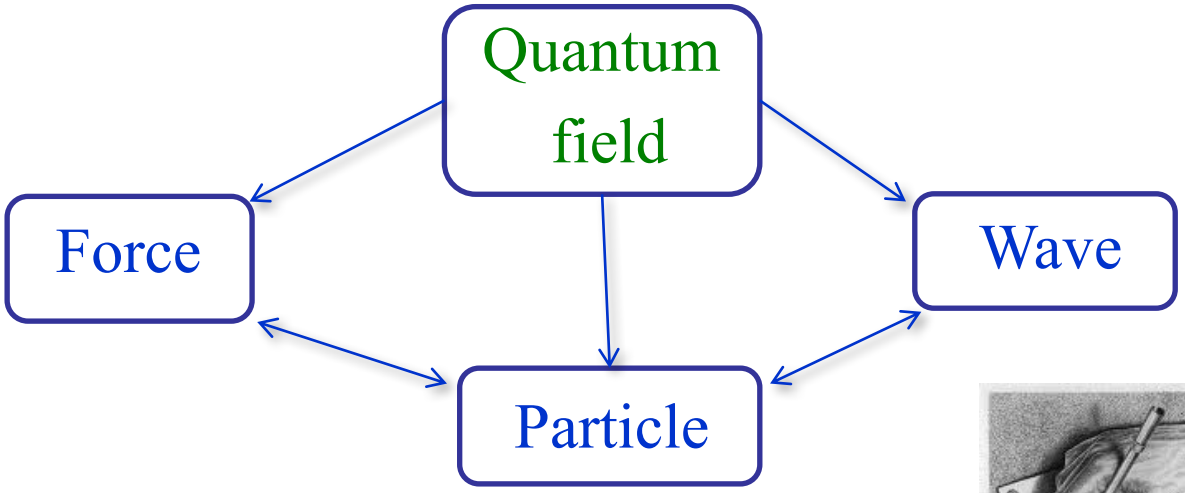
Extra dimensions can arise dynamically,

if interactions among constituents in a system are very strong

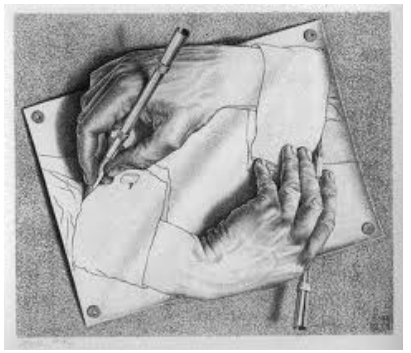
Standard Model of elementary particles

Theoretical basis:

- Quantum Field Theory

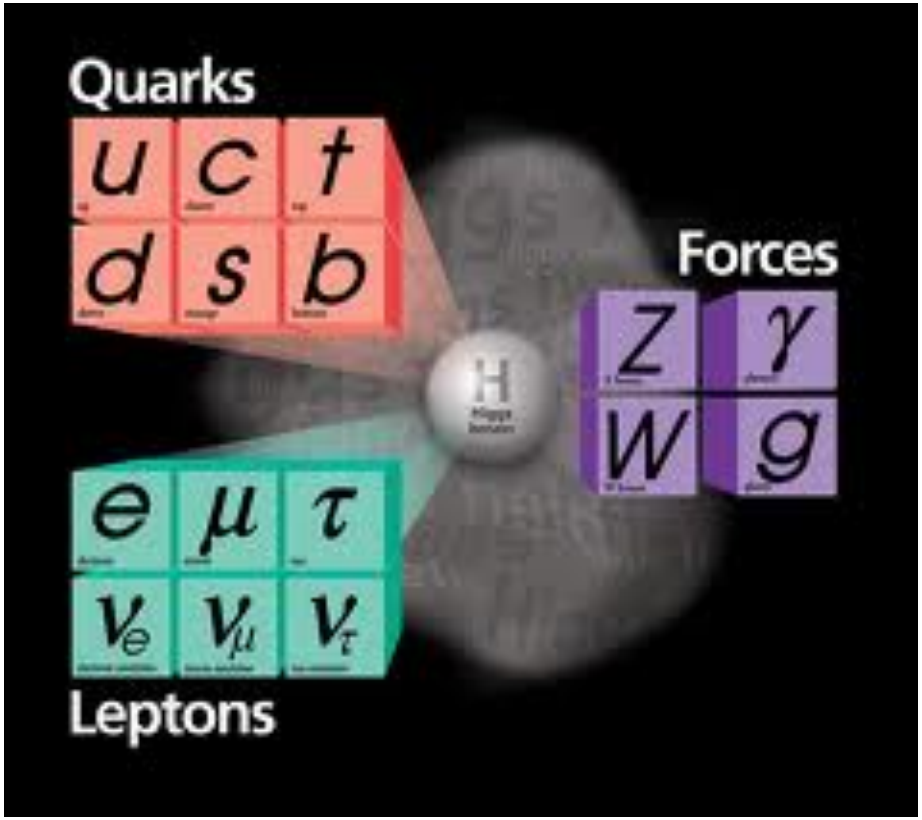


- Gauge Symmetry



- General Relativity

Standard Model (ctd)



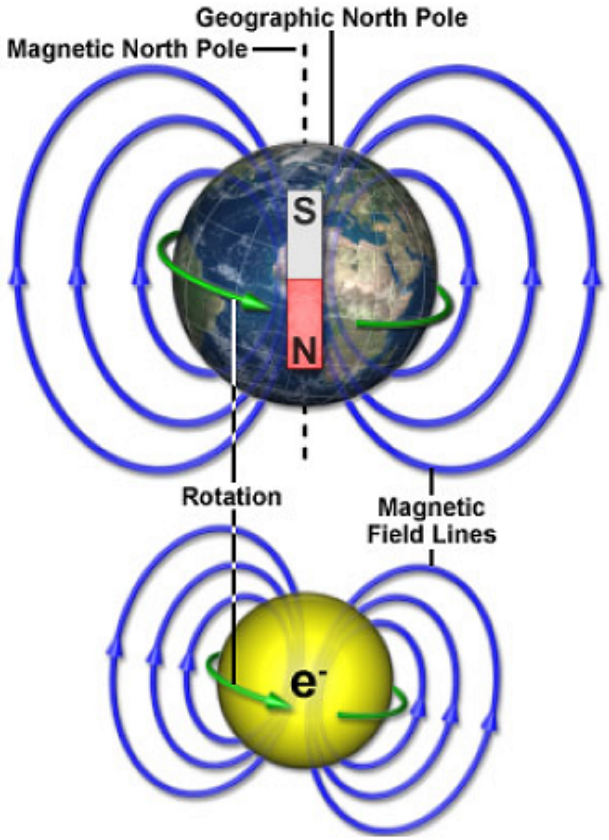
- describes basic constituents of matter and interactions among them
- accuracy reaches 0.1% on average, at weak coupling

Standard Model (ctd)

- Incomplete:
  - ✓ experiment: dark matter, neutrino oscillations, …
  - ✓ theory: hierarchy problem, UV completion, …
  - ✓ esthetic: too many ad hoc parameters, accidental symmetries, …

- Not well understood at strong coupling:
  - ✓ no theory of "strong" strong interactions

Example: Anomalous Magnetic Moment



$$\vec{M} = g \frac{e}{2m} \vec{S}$$

gyromagnetic ratio

Quantum Mechanics:

$$g_e = 2$$

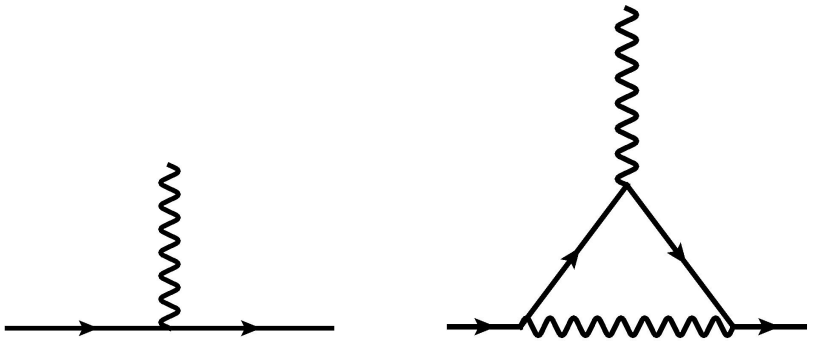
Anomalous Magnetic Moment (ctd)

Vacuum polarization:

$$\left|e^{\mathrm{phys}}\right\rangle = |e\rangle + |e,\gamma\rangle + \ldots$$

Angular momentum: shared between electron and photon

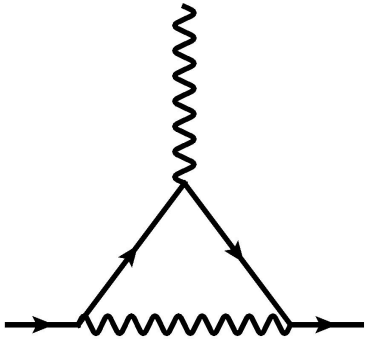
Magnetic moment: carried by electron only



Anomalous Magnetic Moment (ctd)

$$\vec{M} = (1+a)\frac{e}{m}\vec{S}$$

$$a_e \simeq \frac{\alpha}{2\pi} = 0.00116$$



Schwinger'48

Current state of the art:

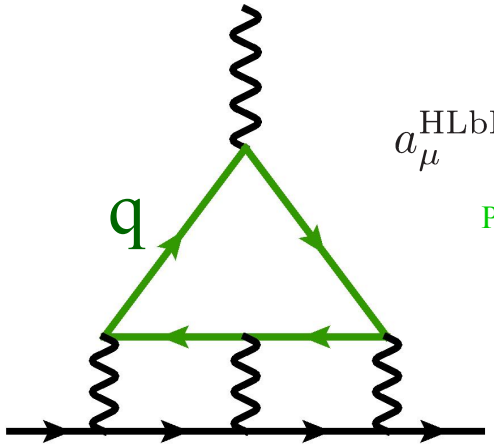
$$a_e = 0.00115965218091(26)$$

most accurately known quantity in physics!

Anomalous Magnetic Moment: Muon

$$a_\mu^{\rm exp} = (116592089 \pm 54) \cdot 10^{-11}$$
$$a_\mu^{\rm th} = (116591803 \pm 42) \cdot 10^{-11}$$

- The 3.4σ difference: is it due to physics beyond the Standard Model?
- Theoretical error cannot be made smaller



$$a_\mu^{\rm HLbL} = (105 \pm 26) \cdot 10^{-11}$$

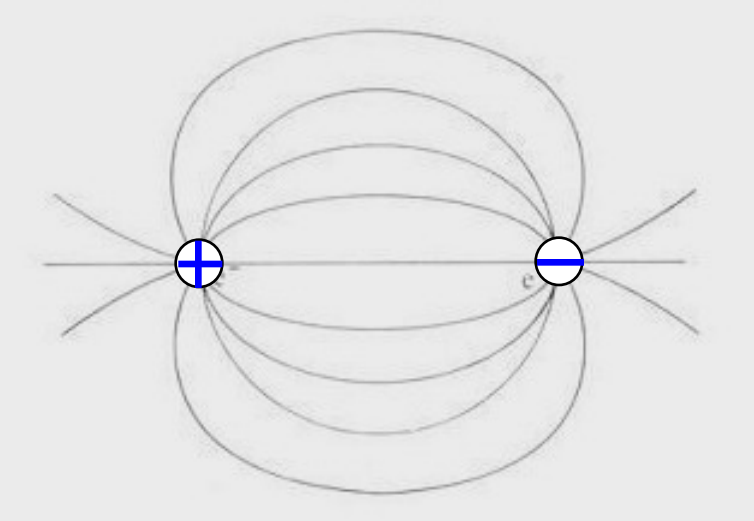
Prades, de Rafael, Vainshtein'09

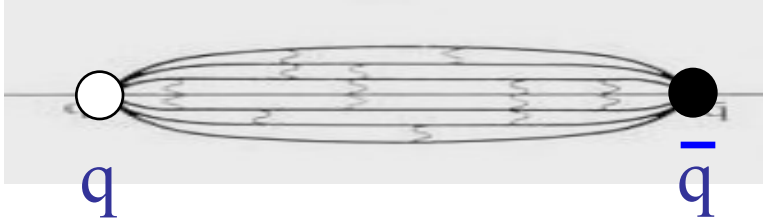
Irreducible uncertaintly due to strong quark interactions

Coulomb field (QED)

vs.

Flux tube (QCD)





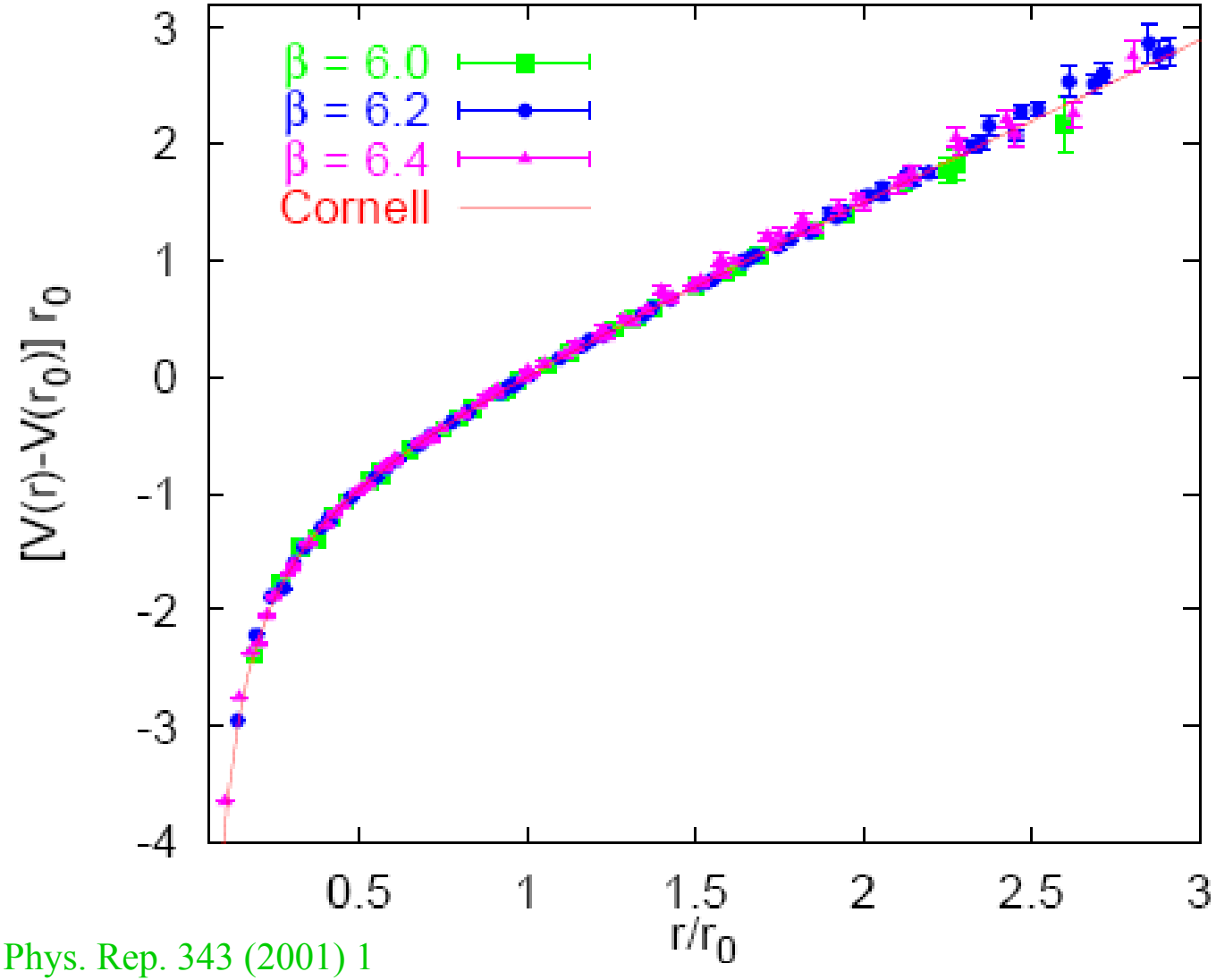
string carries constant energy per unit length

$$V(r) = -\frac{e^2}{4\pi r}$$

$$V(r) \approx Tr$$

$\Downarrow$

confinement



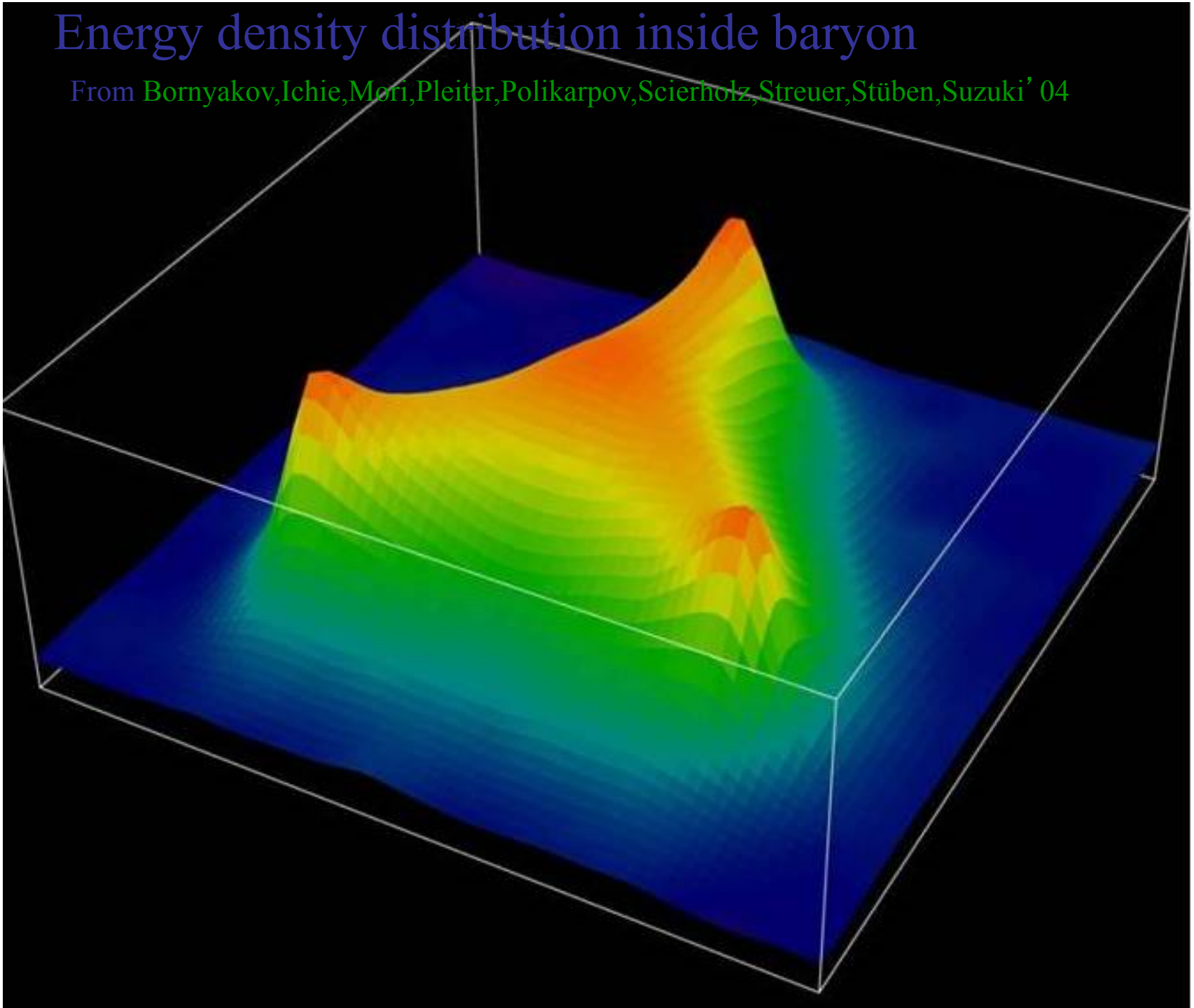
From Bali, Phys. Rep. 343 (2001) 1

More refined lattice simulations clearly indicate that string fluctuates.

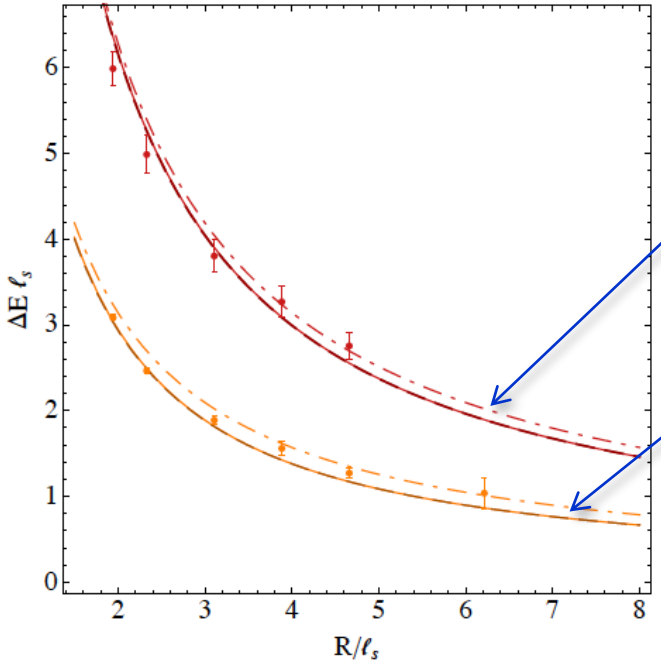
Caselle,Fiore,Gliozzi,Hasenbusch,Provero'97; Caselle,Pepe,Rago'04; …

Energy density distribution inside baryon

From Bornyakov,Ichie,Mori,Pleiter,Polikarpov,Scierholz,Streuer,Stüben,Suzuki' 04



String theory vs. Lattice simulations

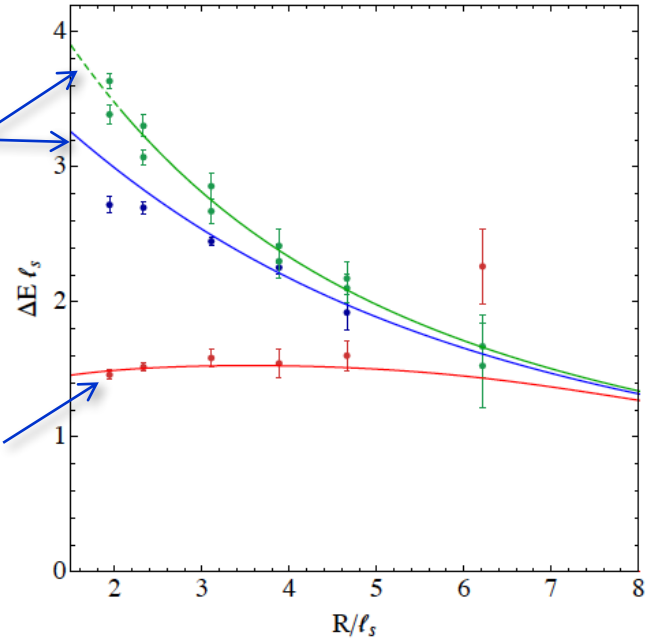


$\alpha^\dagger_{-2}\,|0\rangle$

$\alpha^\dagger_{-1}\,|0\rangle$

$l_s = (2\pi T)^{-1/2}$

$\tilde{\alpha}^\dagger_{-1}\alpha^\dagger_{-1}\,|0\rangle$

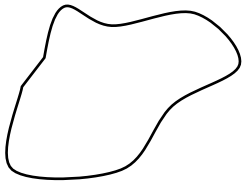


worldsheet axion (new d.o.f. !)

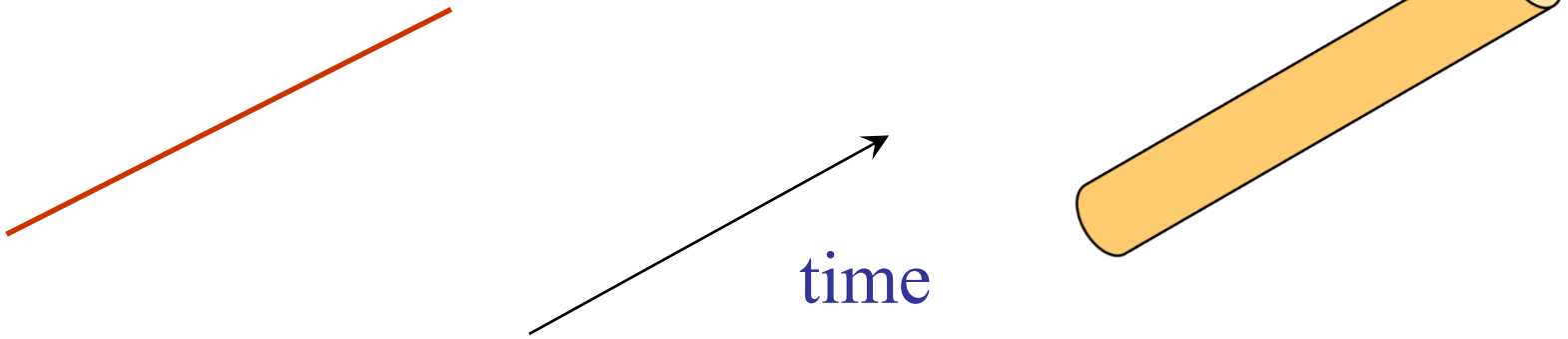
From Dubovsky, Flauger and Gorbenko, JETP 120 (2015) 399

What is String theory?

Particle → String

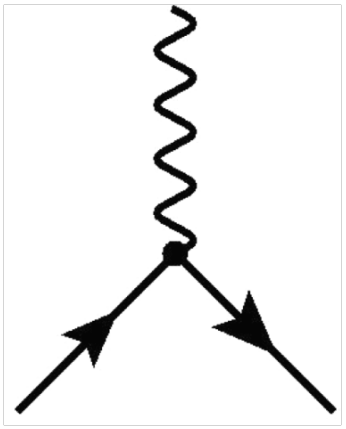


In space-time:

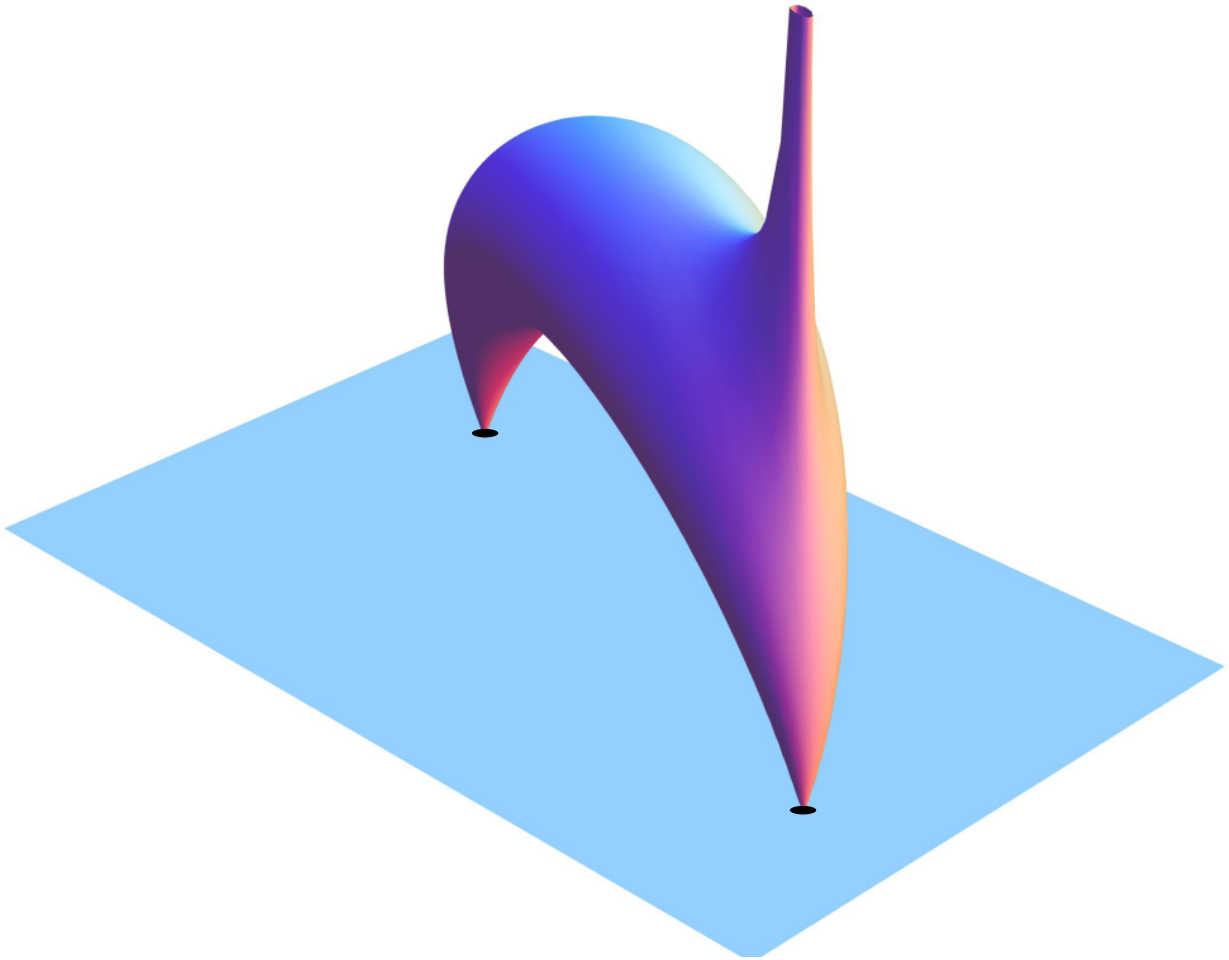


String Interactions

Particles

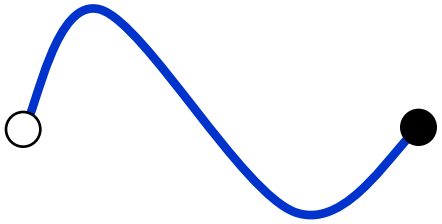


Strings

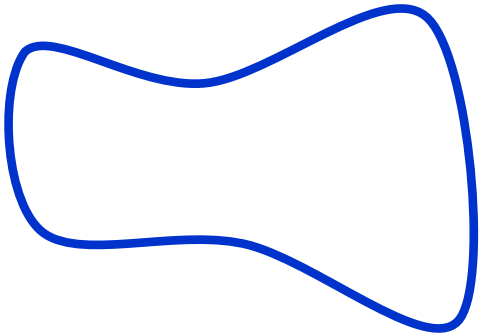


Common lore:

- open strings = mesons



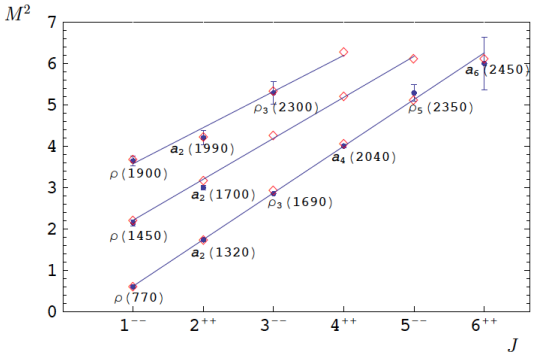
- closed strings = glueballs

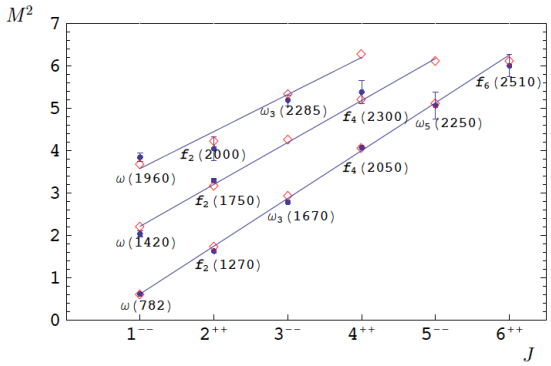


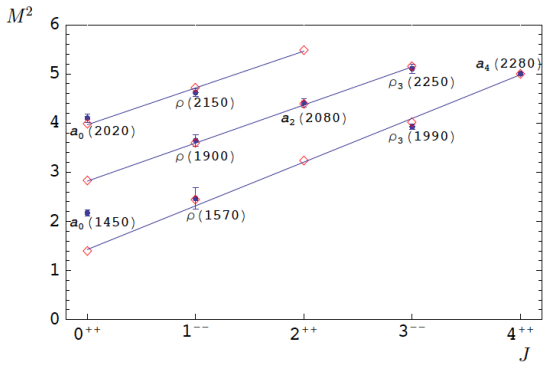
String tension

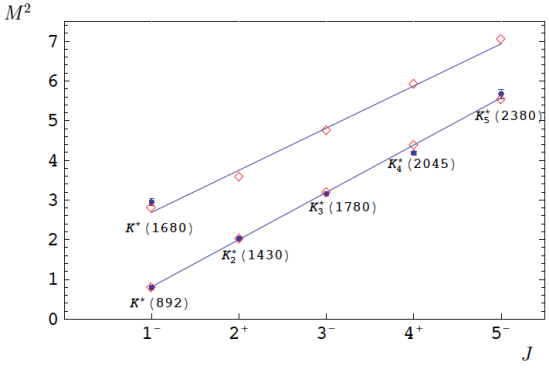
Regge trajectories

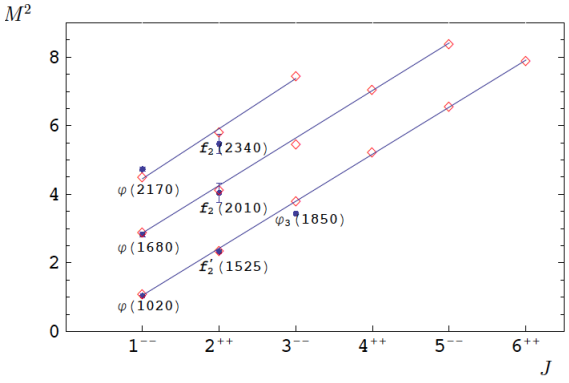
$$M^2 = 2\pi T J + M_0^2$$

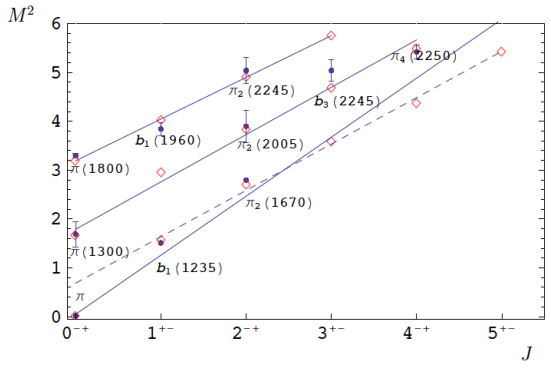












From Ebert, Faustov and Galkin, Phys. Rev. D79 (2009) 114029

Large-N expansion

't Hooft'74

N - the number of colors (N=3)

Consider $\frac{1}{N^2}$ $(= \frac{1}{9})$ as a small parameter

$$\frac{1}{g^2(E')} = \frac{1}{g^2(E)} - \frac{11N}{24\pi^2} \ln \frac{E}{E'}$$

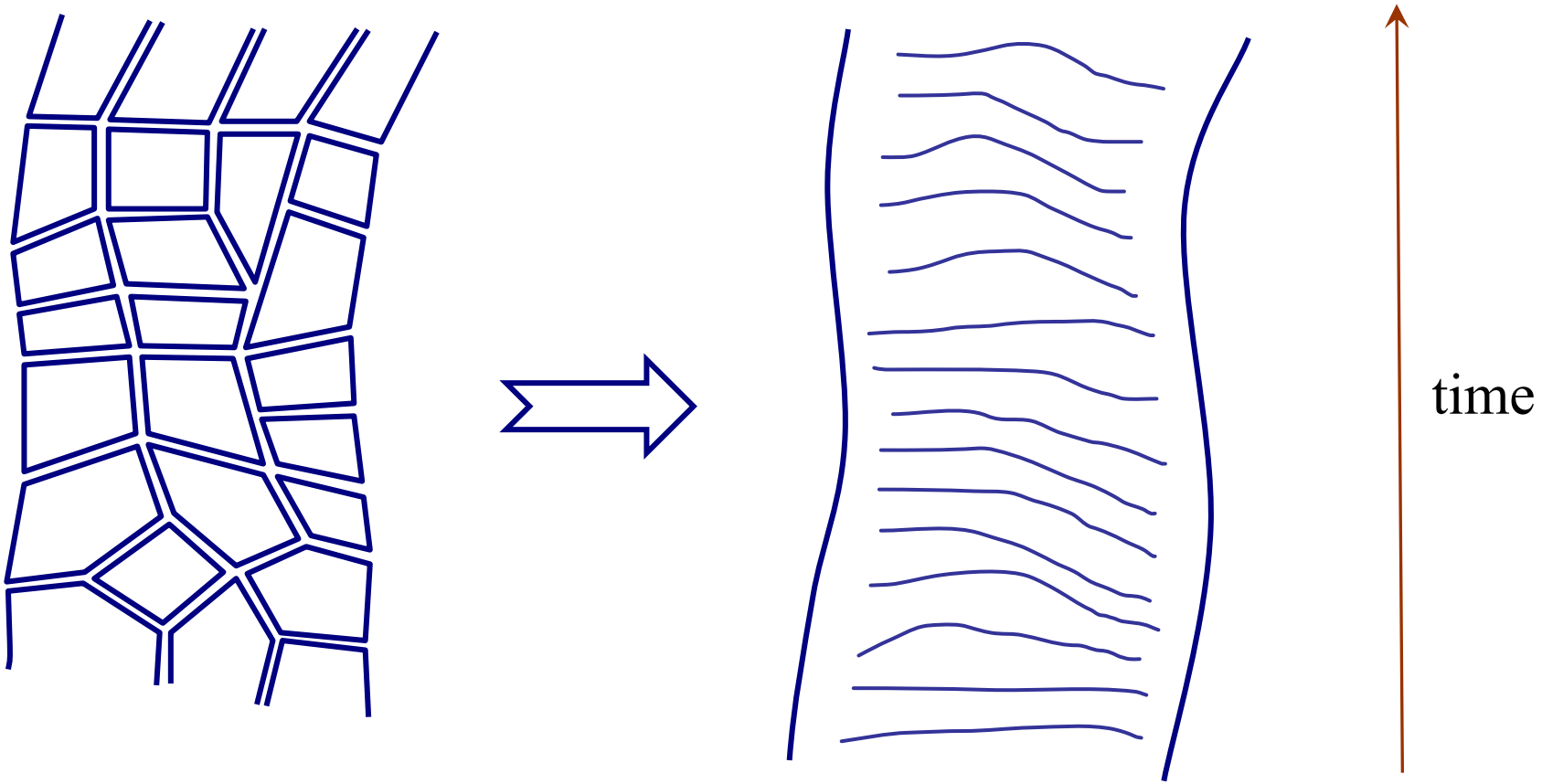
Two coupling constants:

't Hooft coupling: $\lambda = g^2 N$ "large"

string coupling: $\frac{1}{N^2}$ small

Planar diagrams and strings

Large-N limit: $N \to \infty, \qquad \lambda - \text{fixed}$



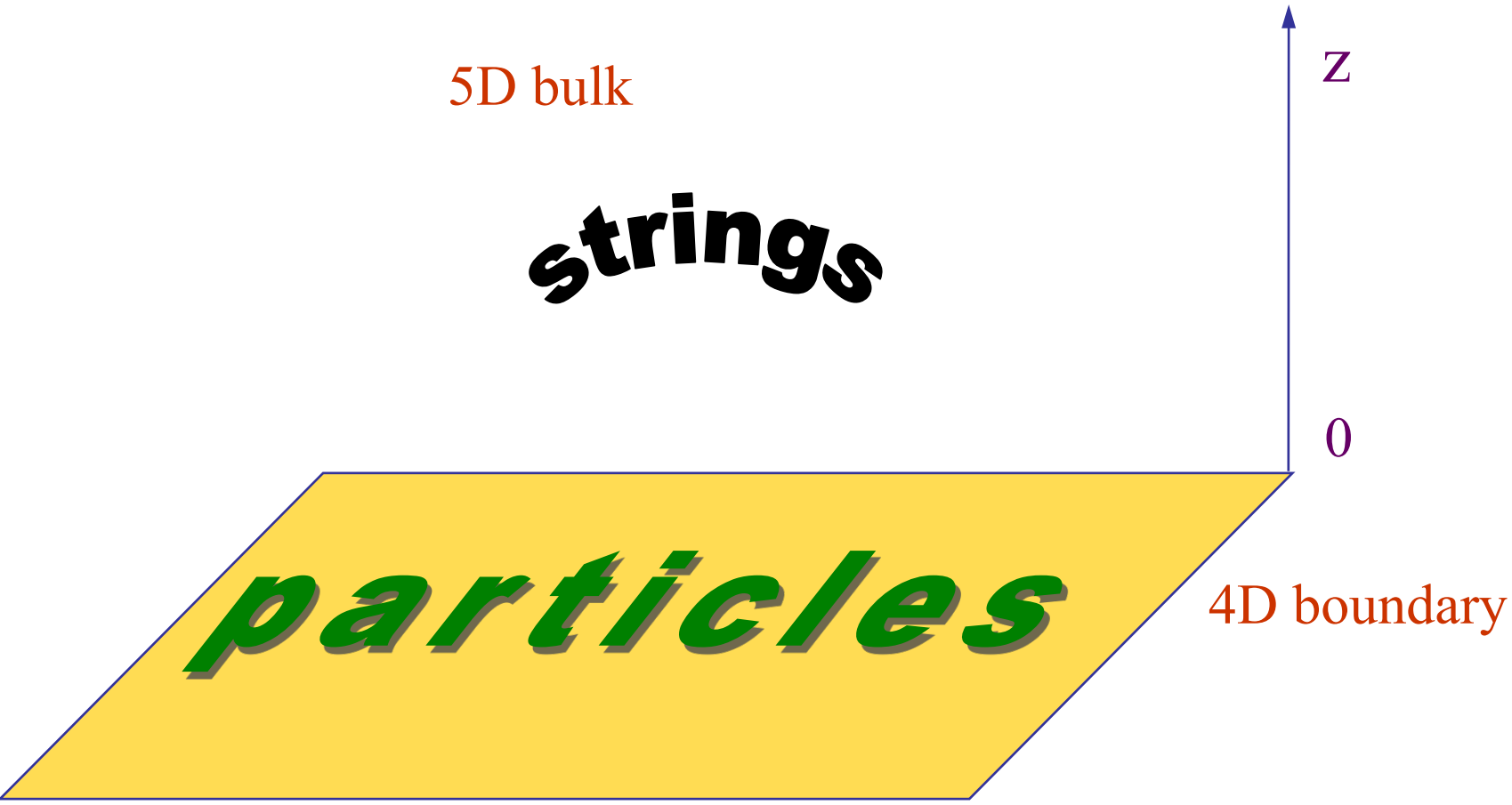
Strings and quantum gravity

- Different vibrational modes of the string = different particles
- The first mode (ground state) describes spin-2, m=0 particle – this is graviton!
- Selfconsistency of string propagation in curved space-time = Einstein's equations

String theory = A theory of quantum gravity

Holography

Maldacena'97



Anti-de-Sitter space (AdS_5)

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

Scale (conformal) invariance:

$$x^\mu \rightarrow ax^\mu$$

$$z \rightarrow az$$

AdS/CFT correspondence

AdS/CFT correspondence

Maldacena'97

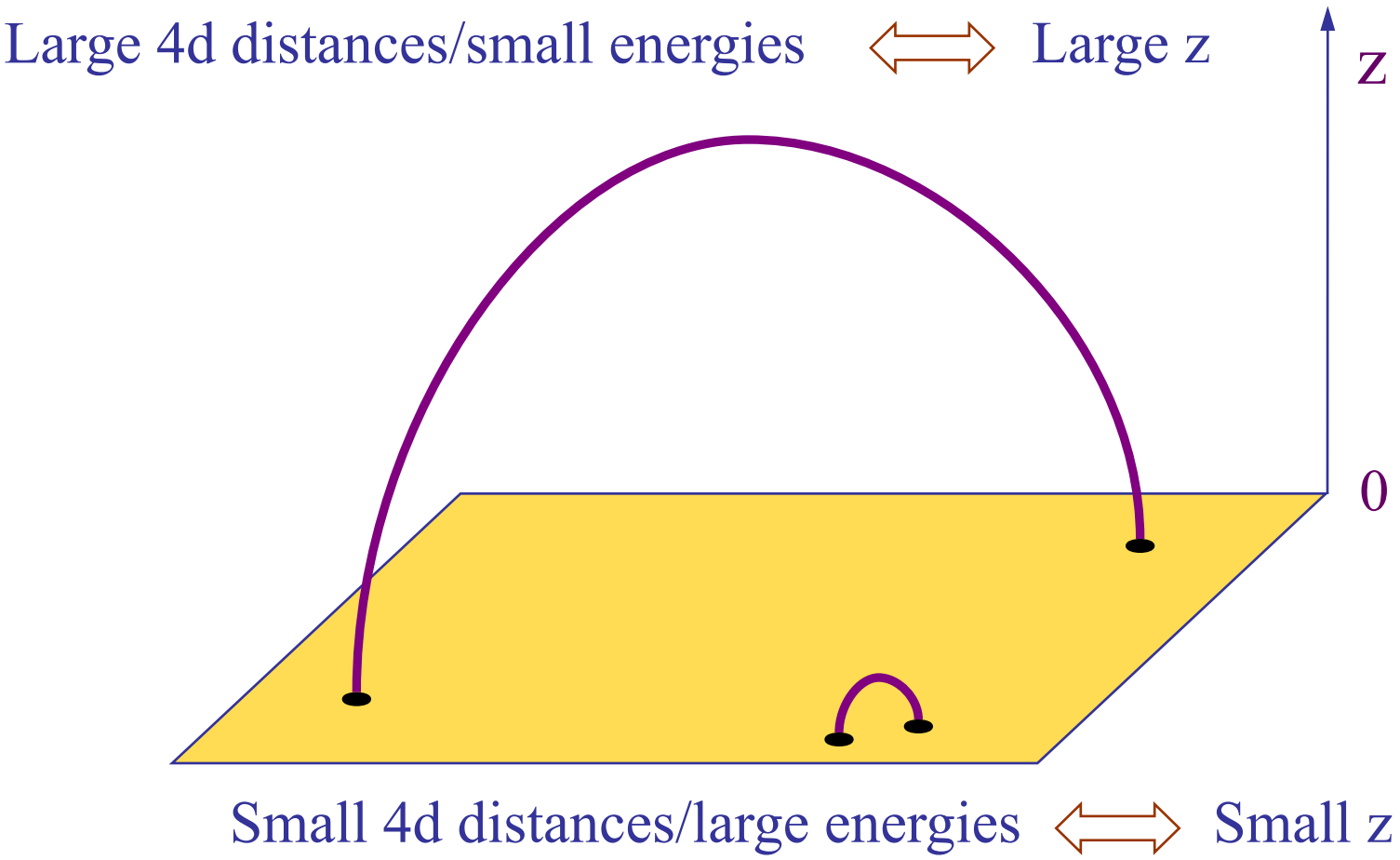
$\mathcal{N}=4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators	String states
Scaling dimension: Δ	Energy: E

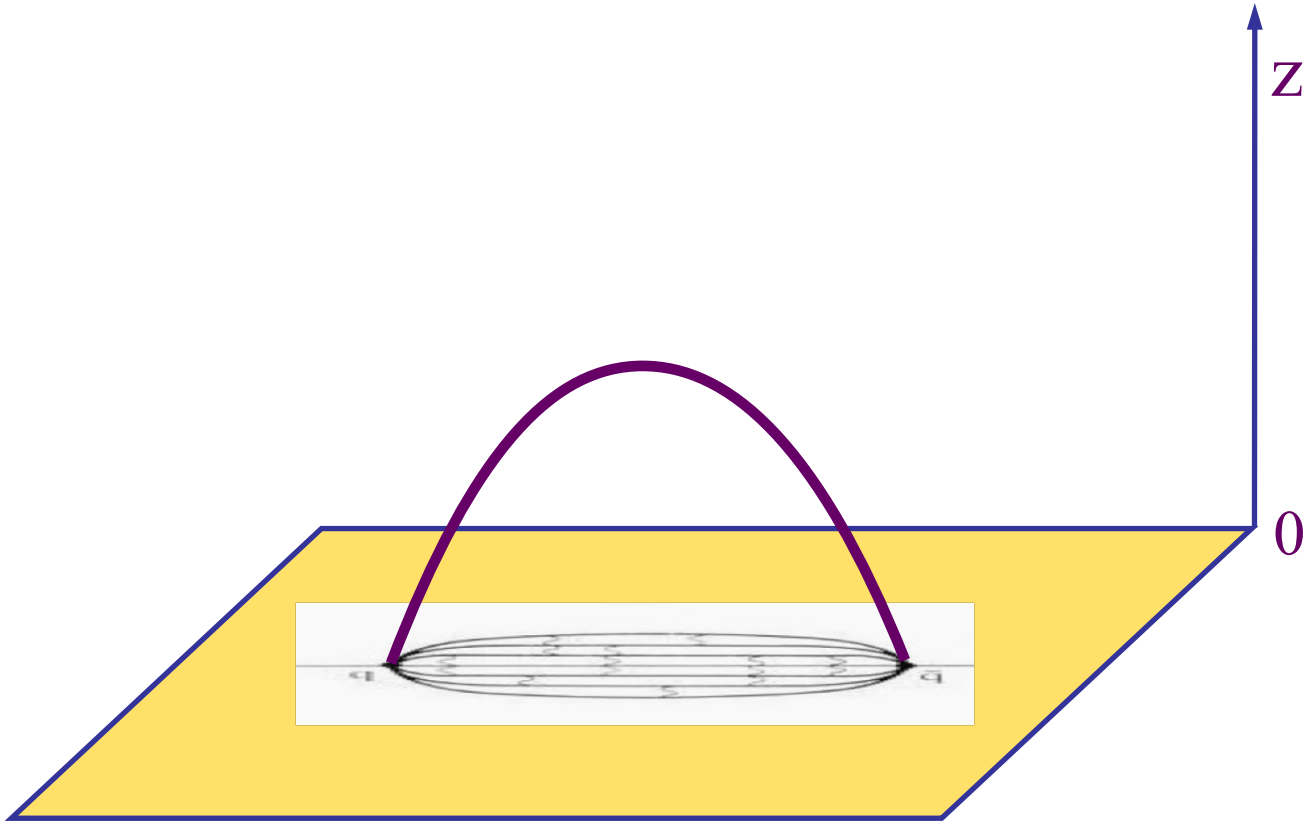
Gubser,Klebanov,Polyakov'98
Witten'98

	$\lambda \ll 1$	$\lambda \gg 1$
Super-Yang-Mills	simple	complicated
Strings	complicated	simple

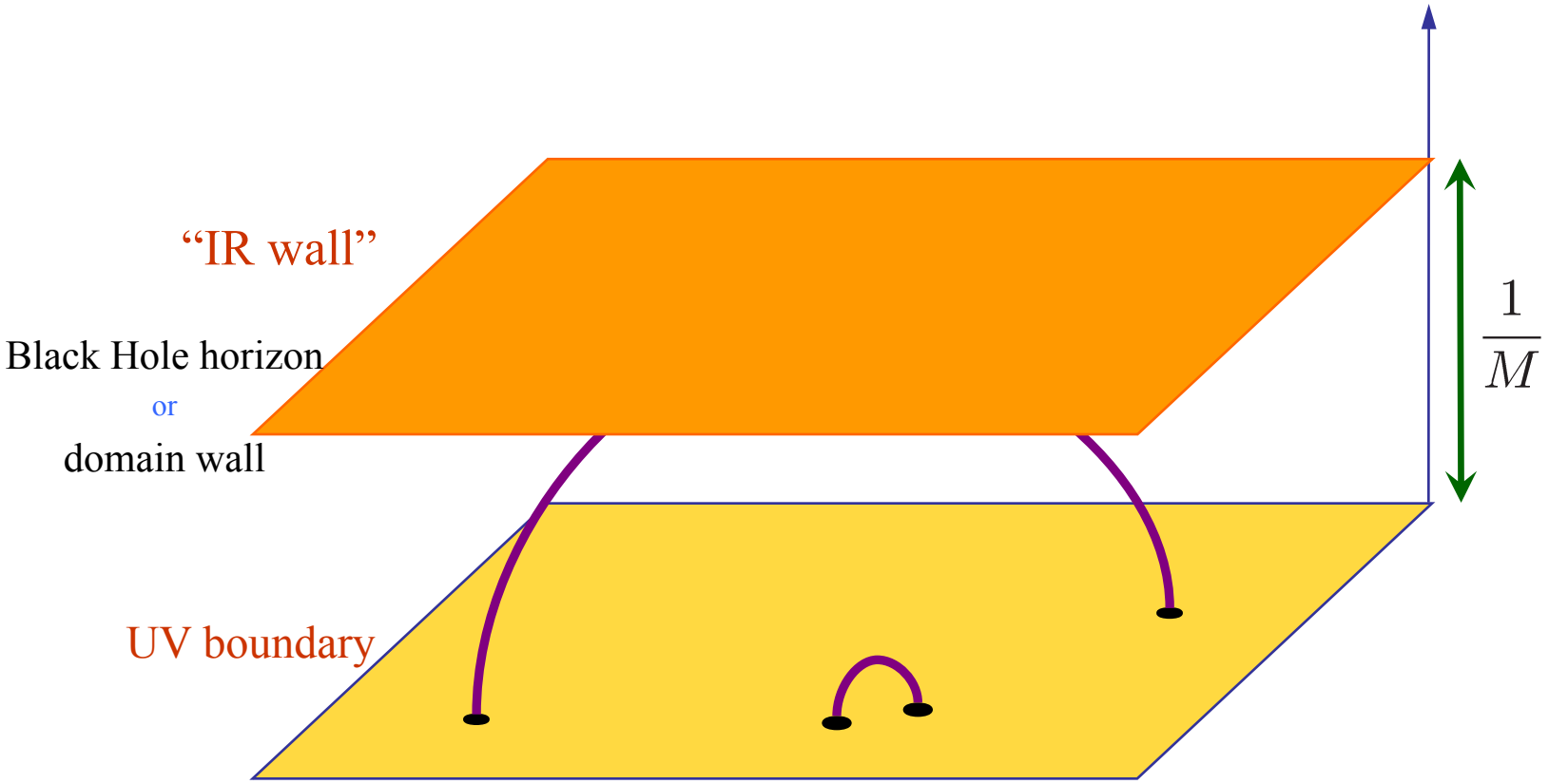
Strong/weak coupling duality

Extra dimension as energy scale

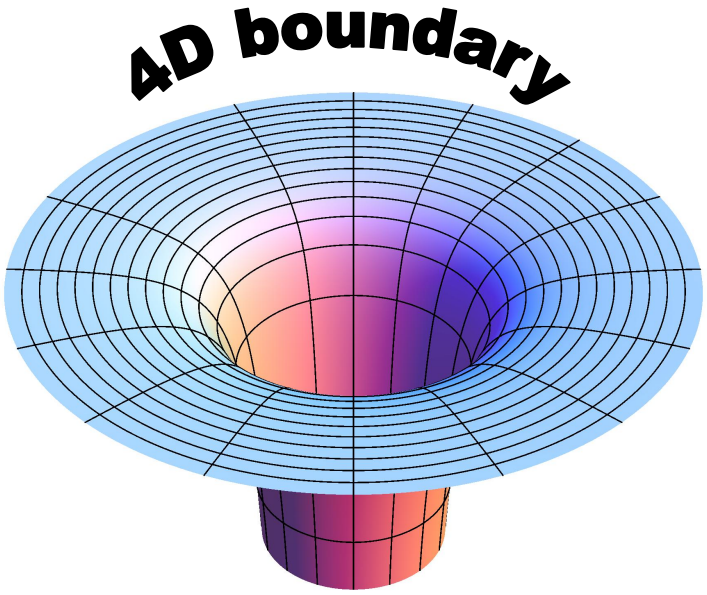



z
0
a
a

Dynamically generated scale



Holography and strongly-coupled plasma



5D black hole

Black Hole inside AdS

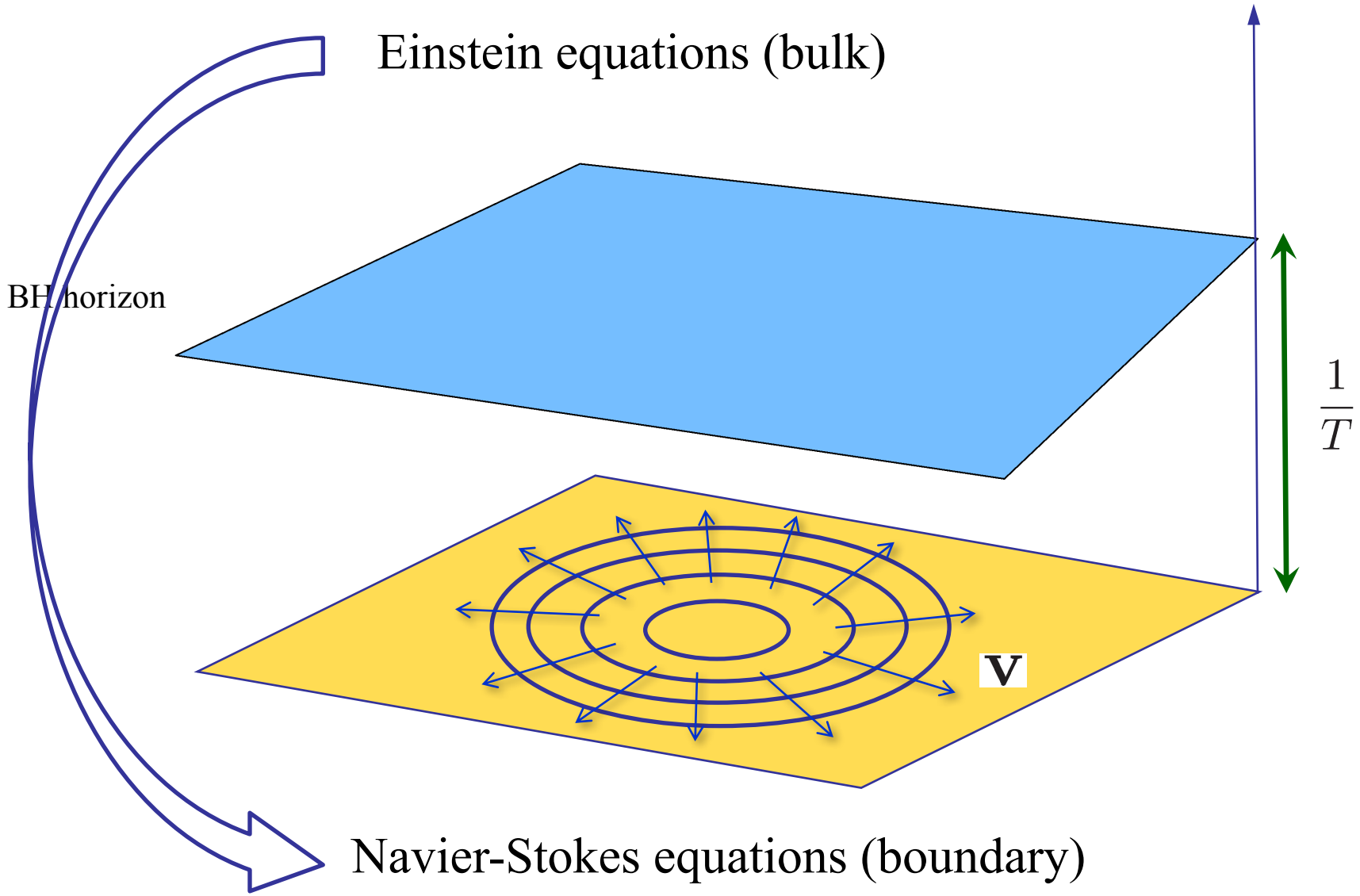
⇕

Gauge-theory plasma

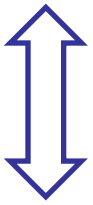
Witten'98

Temperature of the plasma = Hawking temperature of BH

Black holes and hydrodynamics



Oscillation modes of black-hole space-time (quasi-normal modes)



Sound waves in the plasma

viscosity

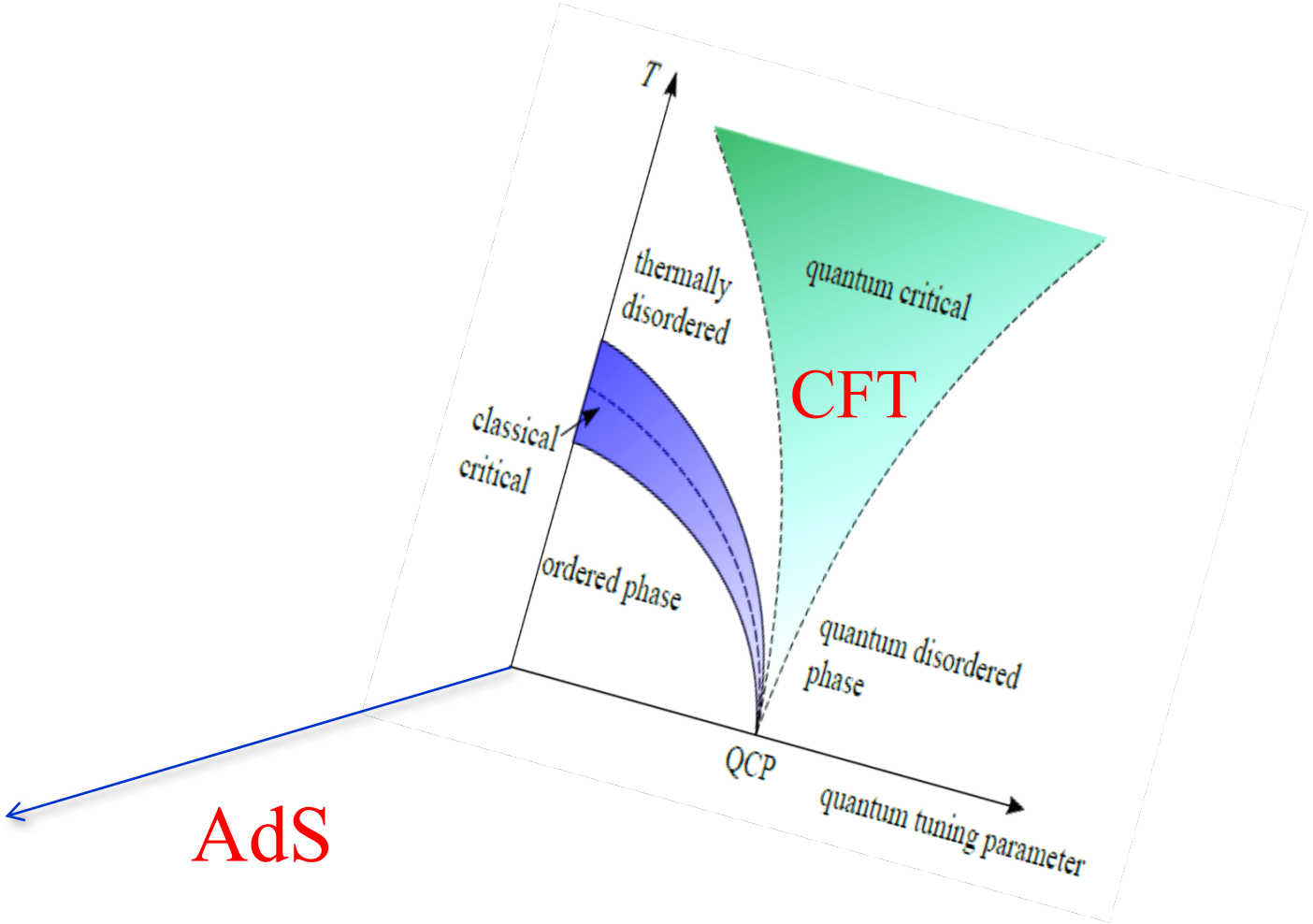
$$\frac{\eta}{\hbar s} = \frac{1}{4\pi}$$

entropy density

Policastro,Son,Starinets'01
Kovtun,Son,Starinets'04

Viscosity of quark-gluon plasma is of the same order of magnitude.

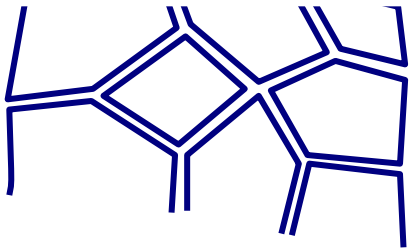
AdS/CMT


T
thermally disordered
quantum critical
CFT
classical critical
ordered phase
quantum disordered phase
QCP
quantum tuning parameter
AdS

Spin chains

Color ordering:

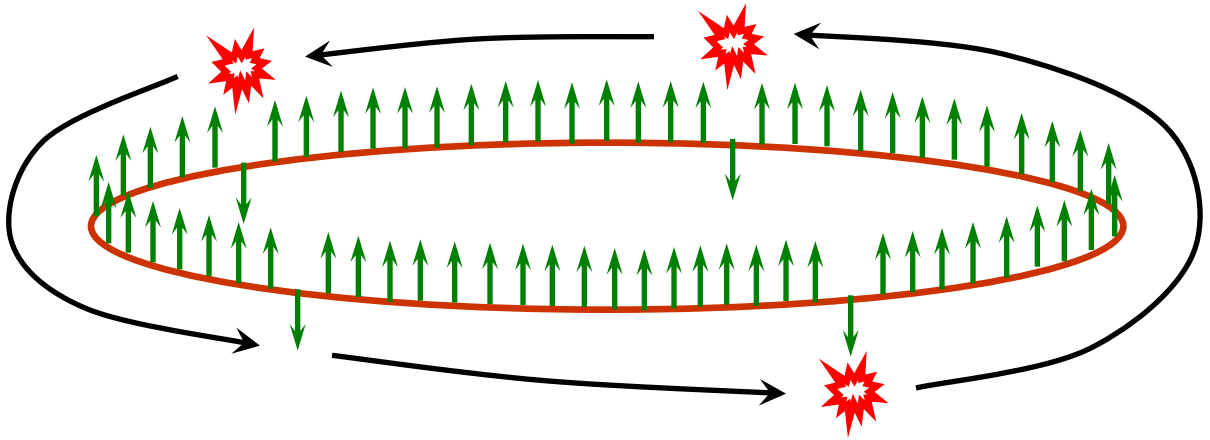
$$|1\rangle \quad |2\rangle \quad |3\rangle$$



spin-chain Hamiltonian

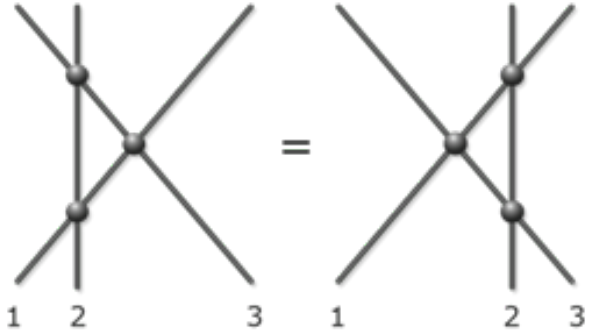
Evolution operator: $\mathrm{e}^{iHt}\,|1\rangle \otimes |2\rangle \otimes |3\rangle$

Constituent gluons $\Longleftrightarrow$ 1d spins



(1+1)d quantum systems are often exactly solvable!

- Consistency (and) symmetry conditions on the S-matrix

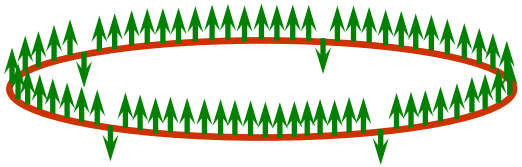


$$S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}$$

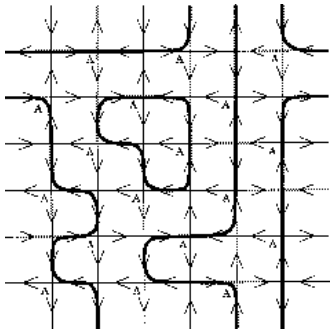
Yang-Baxter equation

Yang'67
Baxter'72
Zamolodchikov'78
Faddeev,Sklyanin,Takhtajan'78

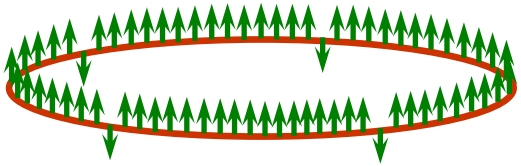
- Map to 2d statistical systems

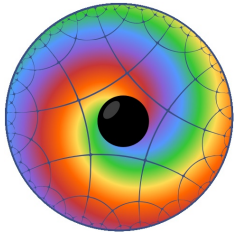


Baxter'72



- Map to 2d Conformal Field Theory





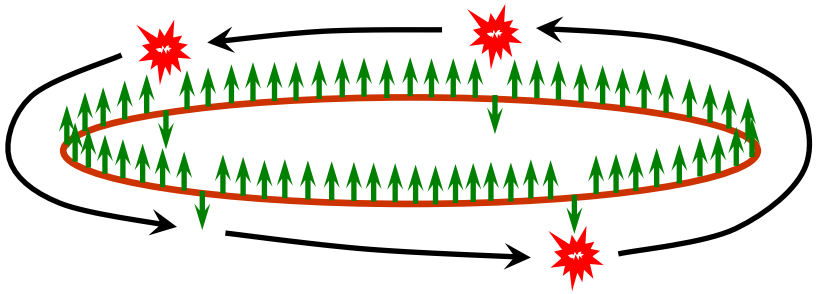
Bazhanov,Lukyanov,Zamolodchikov'94-98
Fioravanti,Rossi'01,03

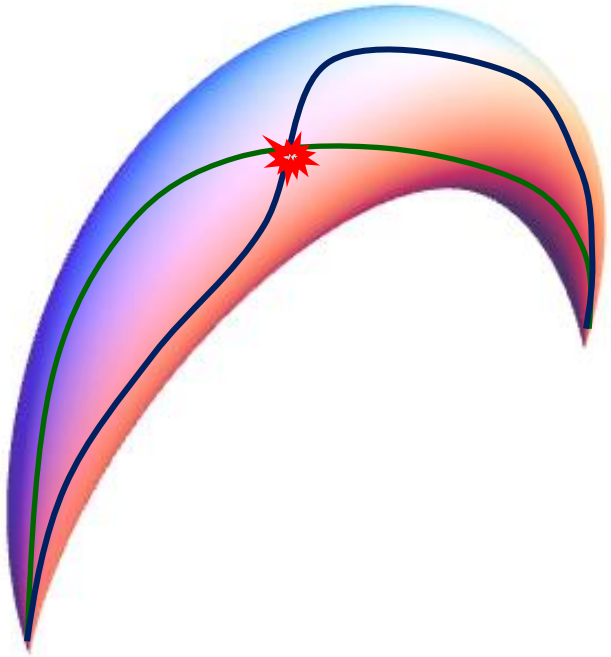
Exact non-perturbative solution
for the AdS/CFT spectrum
was found by methods of integrable systems

Gromov,Kazakov,Vieira'09
Arutyunov,Frolov'09
Bombardelli,Fioravanti,Tateo'09

Spin chain

String





Magnons ⟷ String vibrations

Conclusions

- Extra dimensions can emerge "holographically", due to strong interactions
- (Quantum) gravity is an integral part of the holographic duality
- Many questions in holography can be addressed using integrable systems

Holography is a language

that translates

between

Quantum gravity and "Ordinary", but strong interactions