Trivial and non trivial orders in low dimensional Hubbard-like systems

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Plan of the talk

- Nonlocal orders in spin 1 & extended Bose Hubbard models
- Parity and string orders in 1D Hubbard model
- SPT phases classification of spin-charge decoupled SG models
- Correspondence with group cohomology classification
- beyond 1D case: the 2D Mott insulator

Can order be described just by local observables?

- characterization of long range order: two point correlation function of a local observables non-zero in the thermodynamic limit

$$C(x-y) = <\mathcal{O}^\dagger(x)\mathcal{O}(y)> \xrightarrow[|x-y| \to \infty]{} \text{const}$$

- O(x) orders in the low temperature phase -> SSB: <O(x)> local order parameter goes to zero at phase transition

- true also for quantum 1D systems upon replacing local operators with nonlocal strings

$$\mathcal{O}(x) = \prod_{j<x} \mathrm{e}^{i\alpha S(y)} S(x)$$

Haldane string order in spin 1 models

- Haldane conjecture: the Heisenberg model is gapped for integer spin, gapless otherwise (83)
- Den Nijs and Rommelse (89): in the gapped phase, the non vanishing correlation functions are nonlocal strings, built from SU(2) symmetry generators

$$O^{\alpha}_{\text{string}} = \lim_{|j-k|\to\infty} \omega\left(-S^{\alpha}_{j} \exp\left[i\pi \sum_{l=j+1}^{k-1} S^{\alpha}_{l} \right] S^{\alpha}_{k} \right).$$

- rigorously proved for a similar S=1 model, the AKLT model (87), an integrable bilinear biquadratic Heisenberg model

Some microscopic DOF order in the background of the others

$$|1\ 0\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ -1\ \ldots\rangle$$

$$+|0\ 1\ 0\ -1\ 0\ 0\ 1\ -1\ \ldots\rangle\ldots$$

ignoring 0's
+1 and -1 are
alternated

Order parameter breaks a hidden Z2xZ2 discrete symmetry of the Hamiltonian.

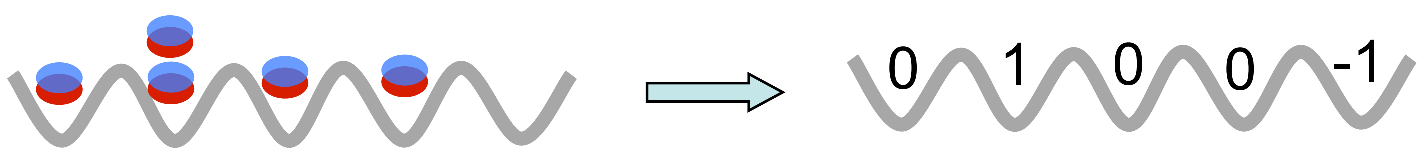
Kennedy Tasaki ('92)

Q: when different spin 1 Hamiltonian (for instance Heisenberg & AKLT) have same gapped Haldane phases?

A: when they can be deformed continuously into each other without breaking the symmetries

Extended Bose Hubbard as spin 1 model

$$H = -t\sum_i (b_i^\dagger b_{i+1} + H.c.) + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_{i,r}\frac{V}{r^3} n_i n_{i+r}$$



$$S_i^z = n_i - \overline{n} = \delta n_i$$

low energy regime-> 3 occupations per site, only particle-hole conserving terms.

$$H_{eff} = J\sum_i (S_i^+ S_{i+1}^- + \mathrm{H.c.}) + \sum_i V S_i^z S_j^z + \frac{U}{2}(S_i^z)^2$$

lambda-D model

Symmetries: $$S^z = \sum_i S_i^z = \sum_i \delta n_i \quad , \quad S_i^z \to -S_i^z$$

string order

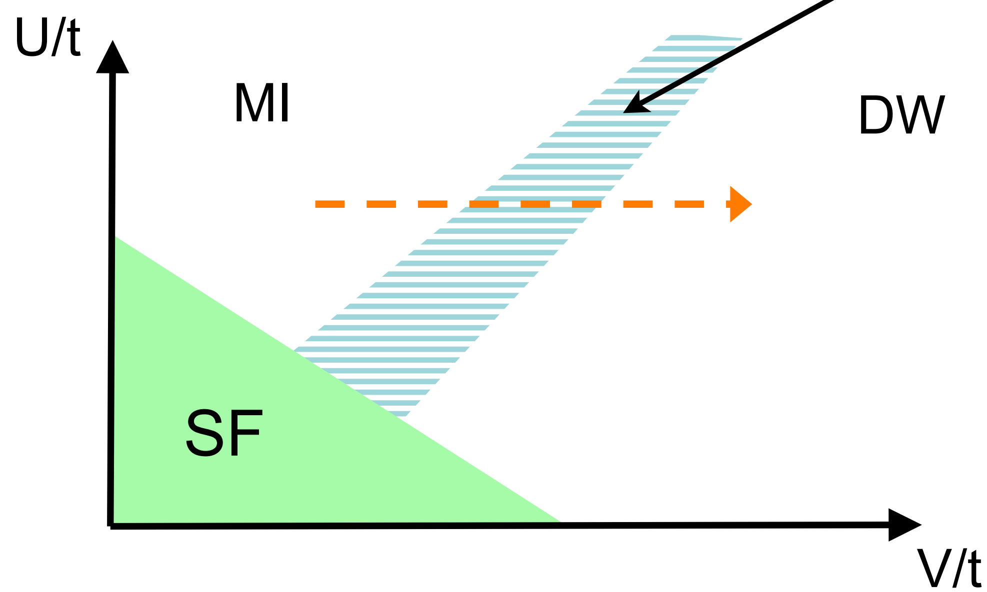
parity order

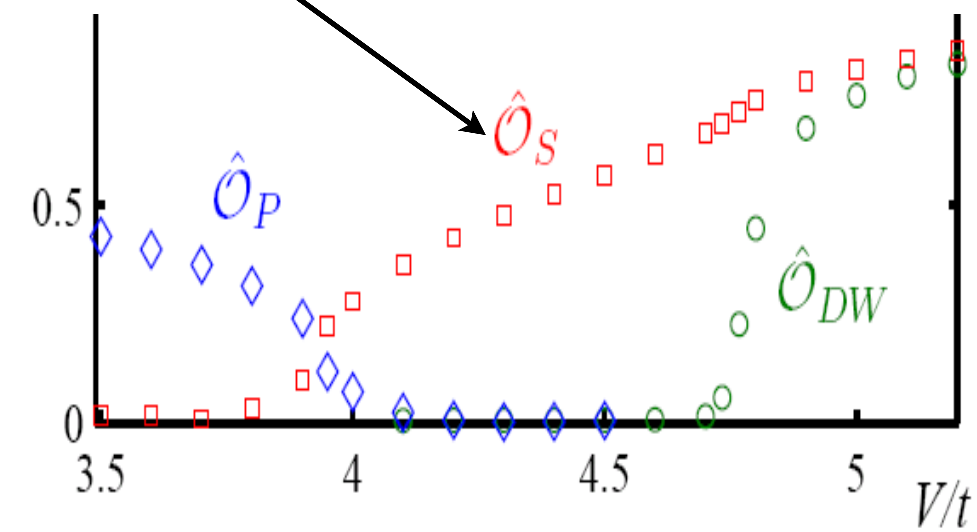
$$C_S = \mathcal{O}_S^2 \equiv \lim_{|i-j|\to\infty} \langle \delta n_i \prod_{i<l<j} (-1)^{\delta n_l} \delta n_j \rangle \neq 0$$

$$C_P = \mathcal{O}_P^2 \equiv \lim_{|i-j|\to\infty} \langle \prod_{i<l<j} (-1)^{\delta n_l} \rangle \neq 0$$

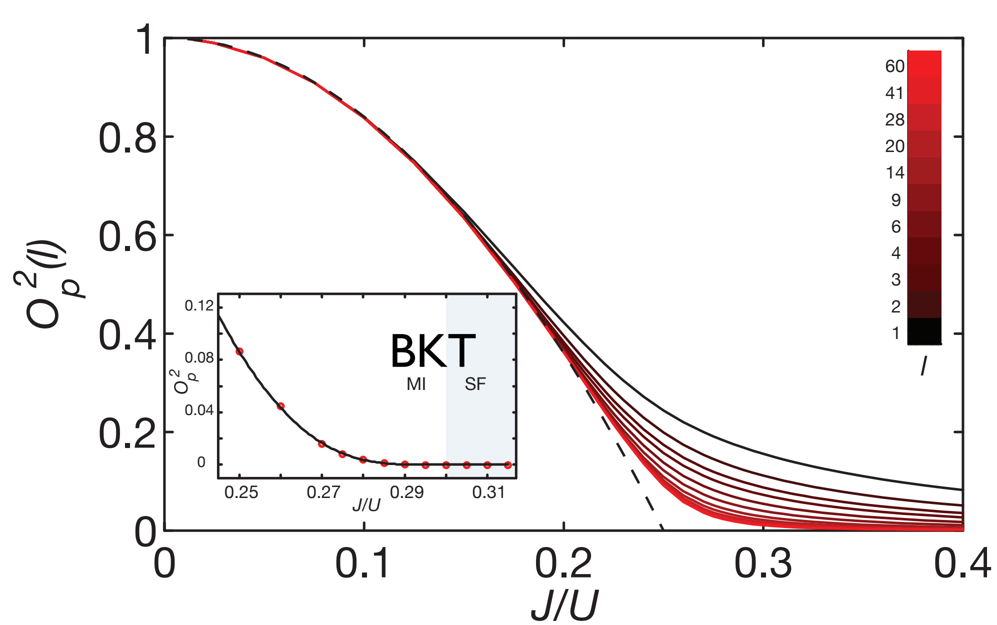
Haldane Insulator

Dalla Torre et al.
PRL 2006



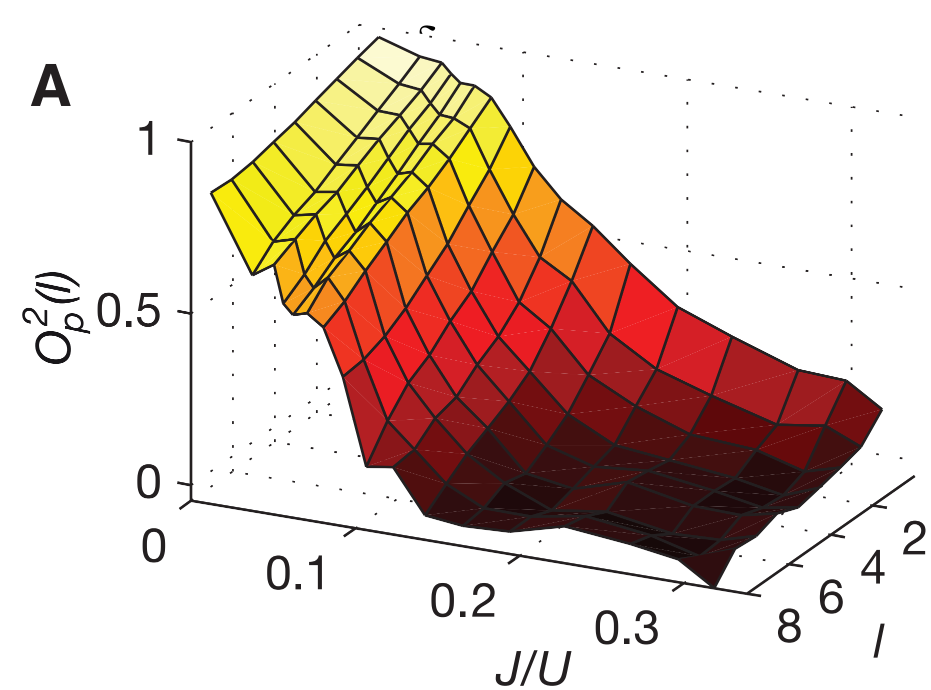


- U and t (J) terms can be tuned independently in optical lattices
- in situ imaging allows to observe on site density fluctuations and to measure experimentally the average value of nonlocal parity operator (pure Bose Hubbard)



Endres et al.
Science 2011

V=0



The Hubbard model: Mott and Luther-Emery Liquid gapped phases

- Is there nonlocal order in the Hubbard model for 1D fermions?

$$\mathcal{H} = -\sum_{i\sigma}(c^\dagger_{i\sigma}c_{i+1,\sigma} + c^\dagger_{i+1,\sigma}c_{i\sigma}) + U\sum_i n_{i\uparrow}n_{i\downarrow}$$

- more symmetries-> more possible nonlocal orders?
- two gapped phases: Mott insulator (U>0, half-filling, open charge gap), Luther Emery liquid (U<0, zero magnetization, open spin gap)
- BKT (-> infinite order) quantum phase transitions

The Hubbard Model: spin and charge parity

- Haldane-like string correlations are vanishing (Anfuso Rosh, PRB 2008)
- What happens to nonlocal parity correlations?
- Introduce nonlocal charge and spin parity operators

$$O_P^{(\nu)}(r) = \prod_{j=1}^{r} e^{2i\pi S_{z,j}^{(\nu)}}$$

- their correlation function reads:

$$C_P^{(\nu)}(r) = \langle \prod_{j=i}^{i+r} \mathrm{e}^{2i\pi S_{z,j}^{(\nu)}} \rangle \qquad \nu = c,\ s.$$

$$S_{z,i}^{(c)} = \frac{1}{2}(n_{i,\uparrow} + n_{i,\downarrow} - 1) \qquad S_{z,i}^{(s)} = \frac{1}{2}(n_{i,\uparrow} - n_{i,\downarrow})$$

->charge parity

->spin parity

The Hubbard model: bosonization

- bosonized Hamiltonian decouples spin and charge dof in two sine-Gordon models. In the charge channel:

$$H_c = \int dx \Big\{ \frac{v_c}{2\pi} \Big[K_c \pi \Pi_c^2 + \frac{1}{K_c} (\partial_x \Phi_c)^2 \Big] - \frac{2U}{(2\pi\alpha)^2} \cos(\sqrt{8}\Phi_c) \Big\}$$

with

$$v_c = v_F \left(1 + \frac{U}{\pi v_F} \right)^{1/2}, \qquad K_c = \left(1 + \frac{U}{\pi v_F} \right)^{-1/2}$$

- due to particle-hole symmetry, substituting U->-U and c->s one obtains also Hs at arbitrary-filling and zero magnetization

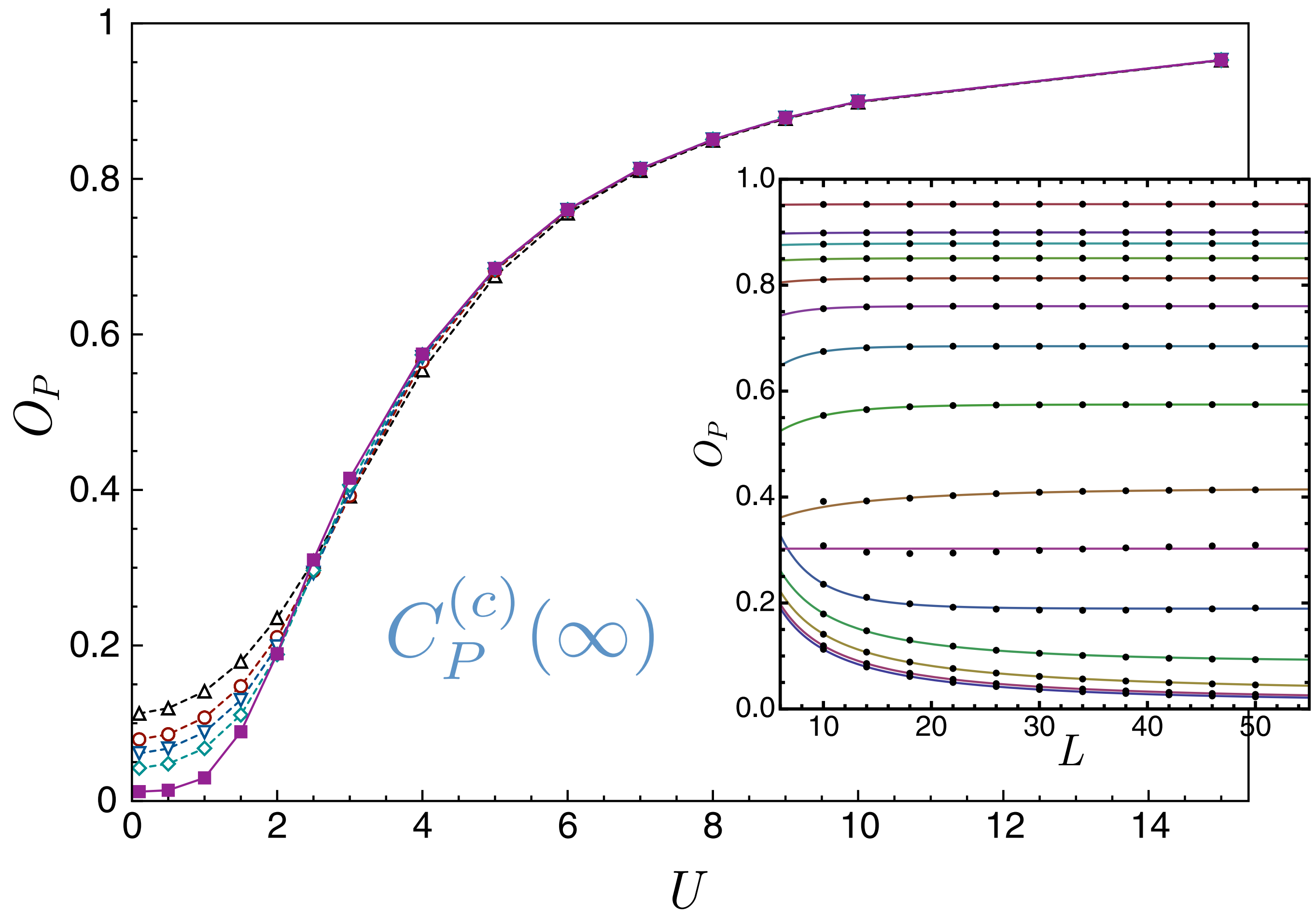
- For U>0 a charge gap opens (MI) when the charge field pins to 0. For U<0 the same holds for the spin gap (LEL) and field.

- charge and spin parity correlations (r continuum) become:

$$C_P^{(\nu)}(r) \approx \langle \cos[\sqrt{2}\Phi_\nu(r)] \cos[\sqrt{2}\Phi_\nu(0)] \rangle.$$

- for locked $\Phi_\nu = 0$ then $O_P^{(\nu)} \neq 0$, 0 in the unlocked case
- the parity correlation functions configure as order parameters for the gapped phases

AM, M Roncaglia (PRL, 2012)



Cartoon: parity orders

- Mott insulator: the sum on each site of fluctuations from half-filling should remain as close as possible to zero-> holon-doublon pairs of finite correlation length

doublon-
holon pairs

$$| \uparrow 0\,2 \uparrow \downarrow \uparrow 2 \downarrow 0 \uparrow \downarrow > + | \uparrow 2\,0 \downarrow \uparrow 2 \downarrow 0 \downarrow \uparrow > + \dots$$

- Luther Emery Liquid: the same holds for fluctuations with respect to zero magnetization-> correlated pairs of single electrons with up and down spin

$$|2\,0\,0 \uparrow 0 \downarrow 2\,0 \downarrow \uparrow > + |2\,0 \downarrow \uparrow 2\,0\,2 \uparrow 0 \downarrow > + \dots$$

up-down pairs

Sine-Gordon model and 1D fermionic systems

- low energy behavior of many 1D interacting quantum systems in the continuum limit described by 2 decoupled spin and charge SG models

$$\mathcal{H} = \sum_{\nu=c,s} \left(H_0^{(\nu)} + \frac{2g_\nu}{(2\pi\alpha)^2} \int dx \cos[q_\nu \sqrt{8}\, \Phi_\nu(x)] \right) \quad = \sum_{\nu=c,s} \mathcal{H}_{SG}^{(\nu)}$$

- interaction may a gap in two ways in each channel (charge/spin), depending on sign of gc/gs (for spin preserving Hamiltonian gs<0)

- to analyze the full phase diagram bosonize also spin and charge string correlators,

$$O_S^{(\nu)}(j) = S_j^{(\nu)} \prod_{l=1}^{j-1} \mathrm{e}^{\mathrm{i}\pi S_l^{(\nu)}} \longrightarrow C_S^{(\nu)}(x) = \langle \sin \sqrt{2}\Phi_\nu(0)\, \sin \sqrt{2}\Phi_\nu(x) \rangle$$

example: phases classification with SU(2) spin symmetry

- parity and/or string non-local order parameters can be non-zero in TDL when the bosonic fields pins to appropriate values

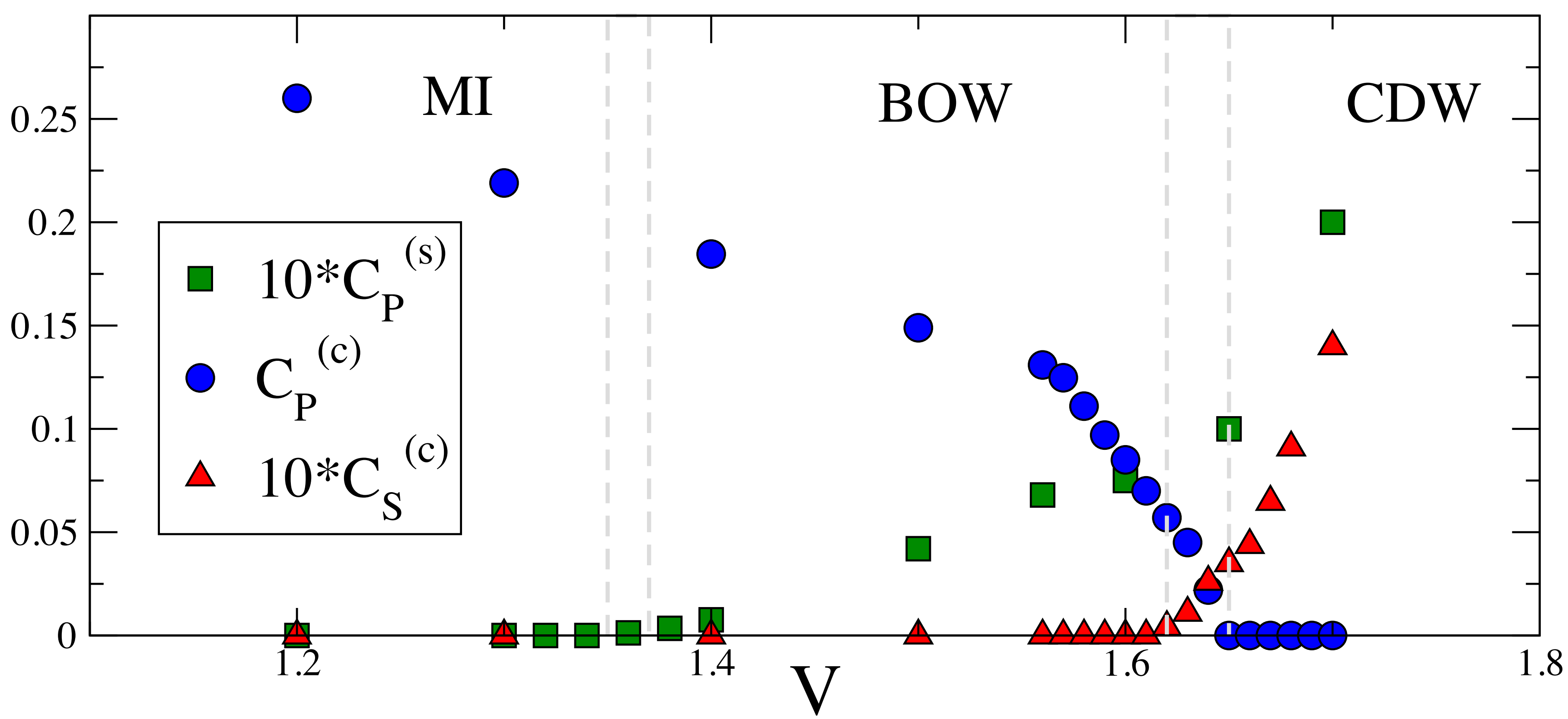
	$q\sqrt{2}\Phi_c$	$\sqrt{2}\Phi_s$	Δ_c	Δ_s	LRO
LL	u	u	0	0	none
LE	u	0	0	open	$O_P^{(s)}$
MI	0	u	open	0	$O_P^{(c)}$
HI	$\pi/2$	u	open	0	$O_S^{(c)}$
BOW	0	0	open	open	$O_P^{(c)}$, $O_P^{(s)}$
CDW	$\pi/2$	0	open	open	$O_S^{(c)}$, $O_P^{(s)}$

charge non-local string order

L. Barbiero, AM, and M. Roncaglia, PRB 2013

Application: DMRG investigation of nonlocal order in extended Hubbard model at half-filling

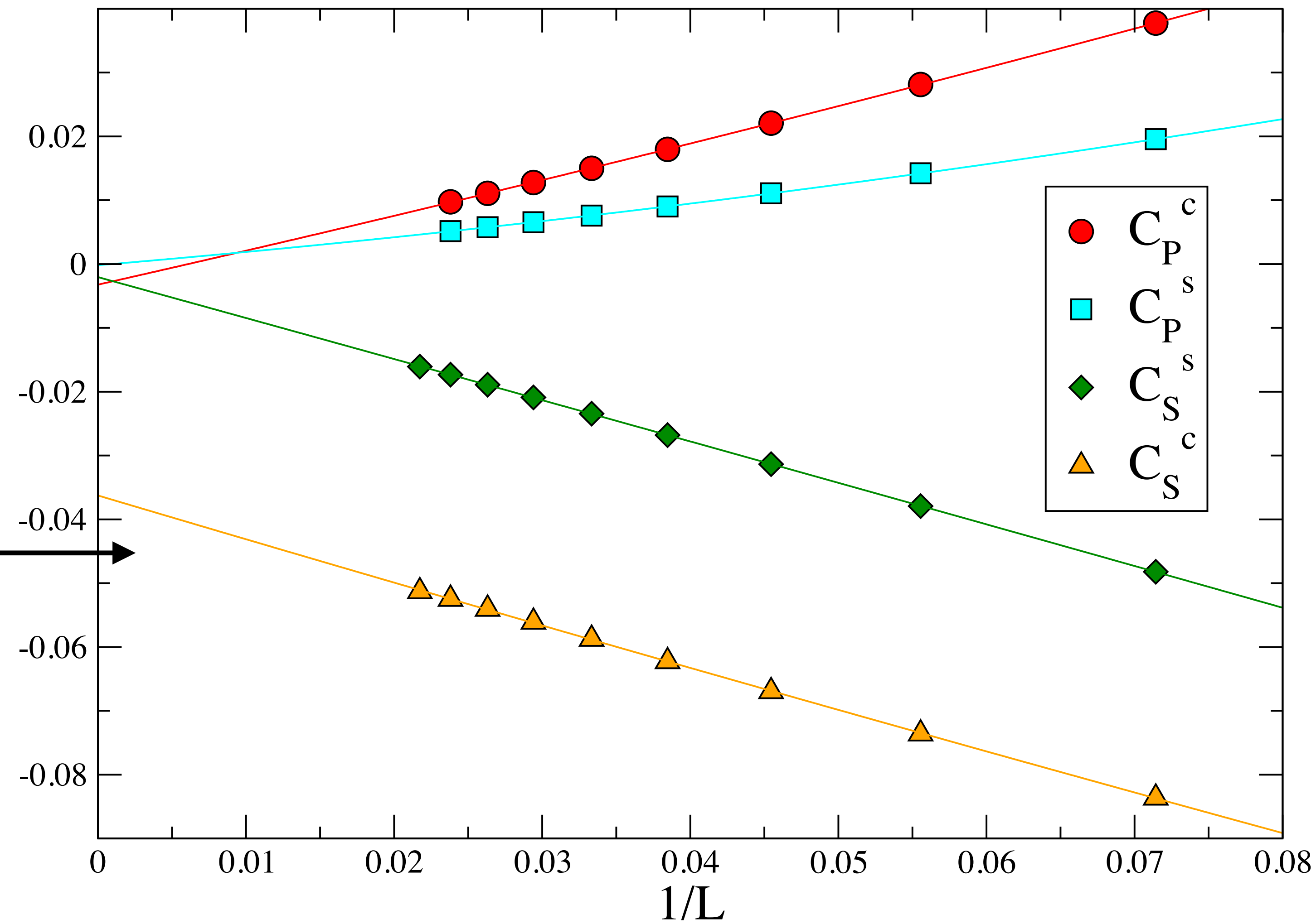
$$H = -t\sum_{j\sigma}(c^\dagger_{j\sigma}c_{j+1,\sigma} + \mathrm{H.c.}) + U\sum_j n_{j\uparrow}n_{j\downarrow} + V\sum_j n_j n_{j+1}$$



example: Haldane charge order

- describes an insulator different from conventional Mott-Hubbard
- at half-filling in extended Hubbard models with correlated hopping, or in presence of 3- and 4-body terms
- charge field pins to nonzero value

HI phase



Symmetry protection of phases of 2 decoupled SG models

- In each channel, the U(1) symmetry implies 2 gapped+1 gapless phases
- Q: when the two gapped phases are distinct?
- A: when symmetry prevents passing from one to the other without closing the gap
- -> avoid terms proportional to $\sin(\alpha\phi_\nu)$
- particle-hole (&time reversal in spin channel) symmetry protects the Haldane phase:

$$\boxed{P\phi_\nu P^{-1} = -\phi_\nu}$$

Fractional edge modes

- $\sqrt{2}\Phi_\nu = \pm\frac{\pi}{2}$ -> non-vanishing string order
- fractionalized edge modes appears also in the SG model, at the interface x with the trivial phase

$$\lim_{a\to 0^+}[S^z_\nu(x) - S^z_\nu(x-a)] \quad = \lim_{a\to 0^+}\frac{1}{\sqrt{2\pi}}[\phi_\nu(x) - \phi_\nu(x-a)] = \pm\frac{1}{4}$$

$$\Phi_\nu = 2\sqrt{\pi}\phi_\nu$$

- kink with half spin/charge of the corresponding dof
- degeneracy in energy at half-filling/zero magnetization

- in each gapped phase ν -parity on the chain of length l can be factorized in its left and right components:

$$\mathcal{P}^{\nu} \doteq \mathcal{O}_P^{\nu}(l) = \mathrm{e}^{i\sqrt{2\pi}[\phi_\nu(l)-\phi_\nu(0)]} \equiv \mathcal{P}_L^{\nu}\mathcal{P}_R^{\nu} \qquad X = L, R$$

- in the Haldane phase $\sqrt{2}\Phi_\nu = \pm\frac{\pi}{2}$ these components have anomalous commutation relation with P:

$$P\mathcal{P}_X^{\nu} = -\mathcal{P}_X^{\nu}P$$

- whereas for $\phi_\nu = 0$

$$P\mathcal{P}_X^{\nu} = \mathcal{P}_X^{\nu}P$$

Group cohomology classification

$$\boxed{P\mathcal{P}_X^\nu = \lambda_\nu \mathcal{P}_X^\nu P \quad , \quad \lambda_\nu = \pm 1}$$

- -1 and +1 denote the non trivial and trivial phases respectively
- anomalous commutation relations at the edge <-> 2 distinct projective representations of the symmetry group G

$$\mathrm{G} \equiv \mathrm{U}(1) \rtimes Z_2$$

- semidirect product denotes the nontrivial commutation relation

$$P\, U_\nu(\beta) = U_\nu(-\beta)\, P.$$

- the two projective representations are distinct elements (under symmetry protection) of the second cohomology group->phases are distinct

SPT phases classification of spin+charge SG, U(1)+U(1) symm

	$\sqrt{8\pi}\phi_c$	$\sqrt{8\pi}\phi_s$	NLO	SP	GCC
LE	u	0	$\mathcal{O}^s_{\mathcal{P}}$	triv	$\lambda_s = 1$
MI	0	u	$\mathcal{O}^c_{\mathcal{P}}$	triv	$\lambda_c = 1$
HLE	u	$\pm\pi$	$\mathcal{O}^c_{\mathcal{S}}$	P,T	$\lambda_s = -1$
HI	$\pm\pi$	u	$\mathcal{O}^c_{\mathcal{S}}$	P	$\lambda_c = -1$
BOW	0	0	$\mathcal{O}^c_{\mathcal{P}}, \mathcal{O}^s_{\mathcal{P}}$	triv	$\lambda_s = 1 = \lambda_c$
CDW	$\pm\pi$	0	$\mathcal{O}^c_{\mathcal{S}}, \mathcal{O}^s_{\mathcal{P}}$	P	$\lambda_s = 1 = -\lambda_c$
SDW	0	$\pm\pi$	$\mathcal{O}^c_{\mathcal{P}}, \mathcal{O}^s_{\mathcal{S}}$	P,T	$\lambda_c = 1 = -\lambda_s$
BSDW	$\pm\pi$	$\pm\pi$	$\mathcal{O}^c_{\mathcal{S}}, \mathcal{O}^s_{\mathcal{S}}$	P	$\lambda_s = -1 = \lambda_c$

spin and charge non-trivial phases

AM et al., PRB 2017

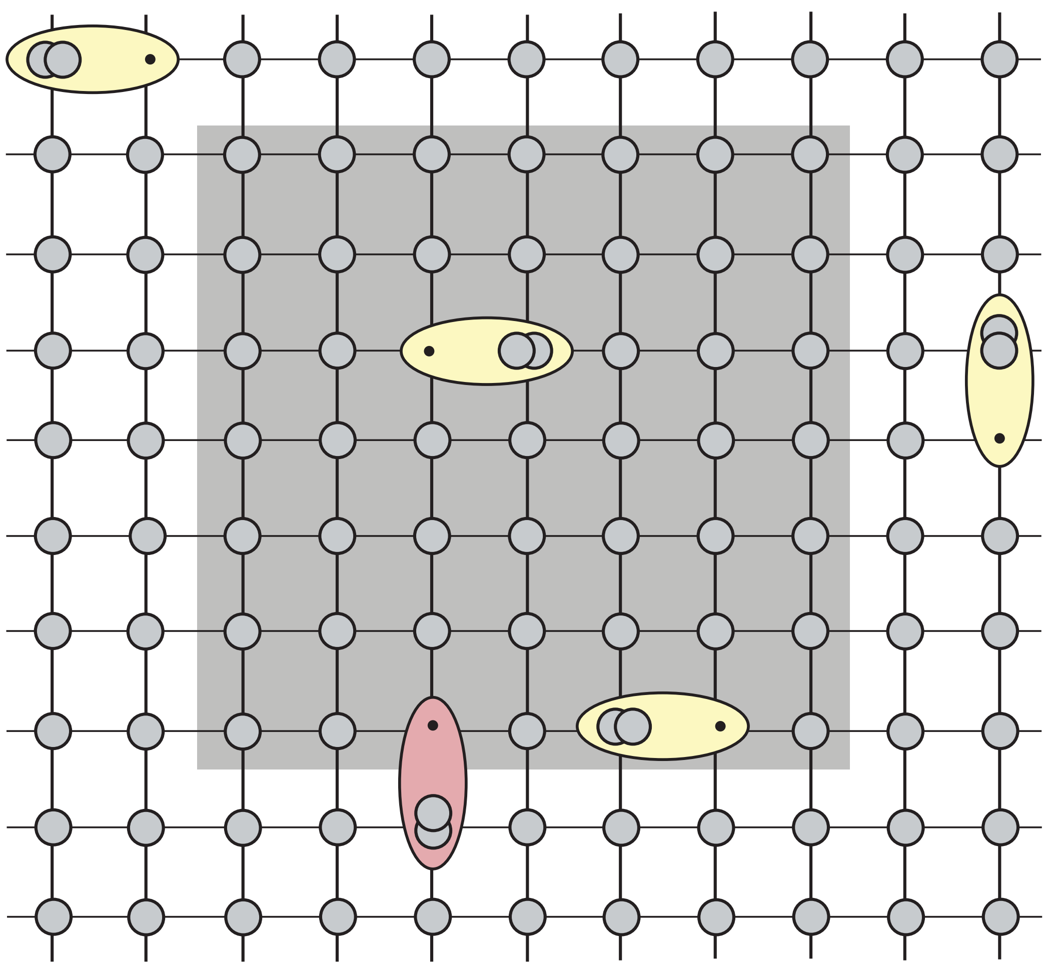
even in absence of p-h symmetry of the microscopic Hamiltonian

Trivial and non-trivial phases

- non trivial phase: particle-hole symmetry implies to flip simultaneously the parity at the two edges -> nonlocal nature of the state
- a trivial state can be connected by symmetry preserving transformations to a separable state; otherwise it is non trivial
- both trivial and non trivial states in 1D can be written as MPS -> unique ground state of parent Hamiltonian
- phases are distinct when they cannot be connected by transformations preserving symmetries of the Hamiltonian
- to which extent previous classification holds at the level of microscopic Hamiltonians? -> classification of phases in the strong coupling limit

Results on 2D case

- Bose-Hubbard model: product of site parities within area D decays exponentially to zero with a perimeter law in MI phase, with super-exponential decay in SF phase

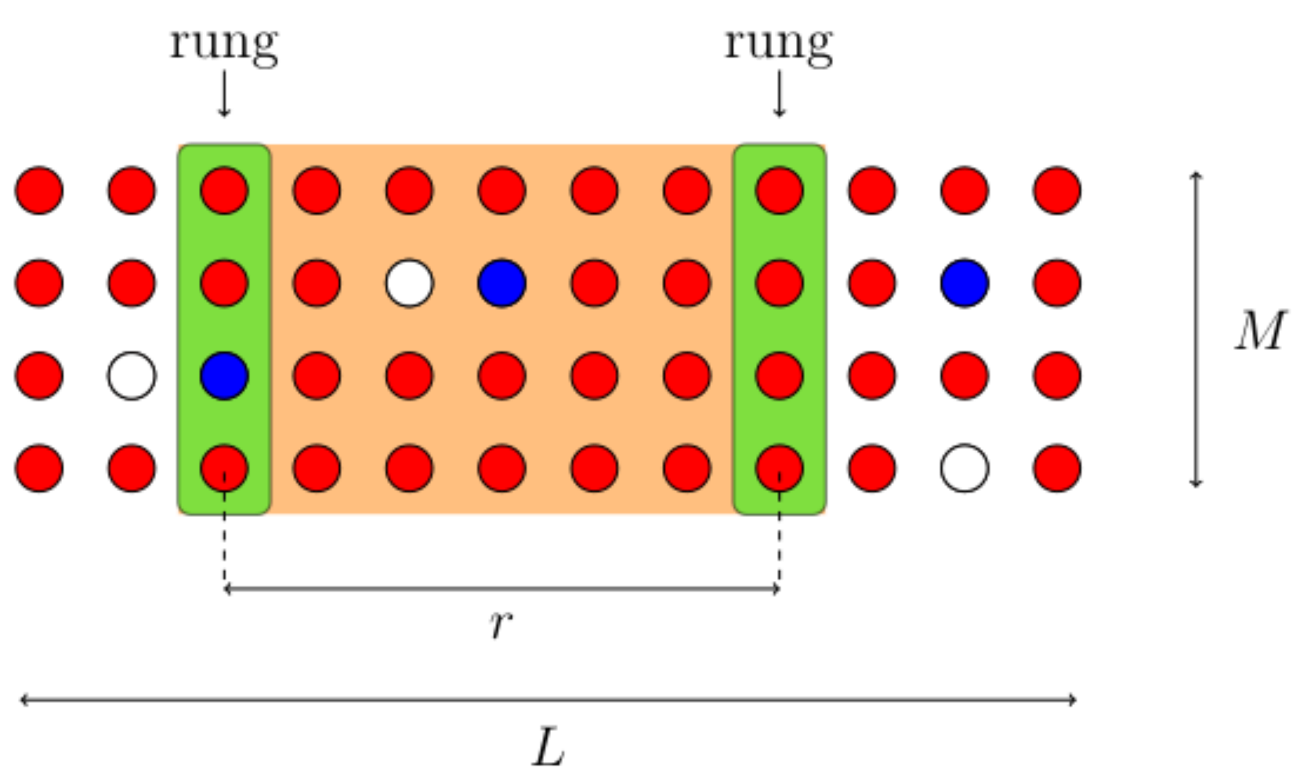


Rath, Simeth, Endres, Zwerger, Ann. Phys. 2013

$$\langle \mathcal{O}^2(\mathcal{D})\rangle = \left\langle e^{i\pi \sum_{i\in\mathcal{D}} \overbrace{(\hat{n}_i - \bar{n})}^{\delta n_i}} \right\rangle$$

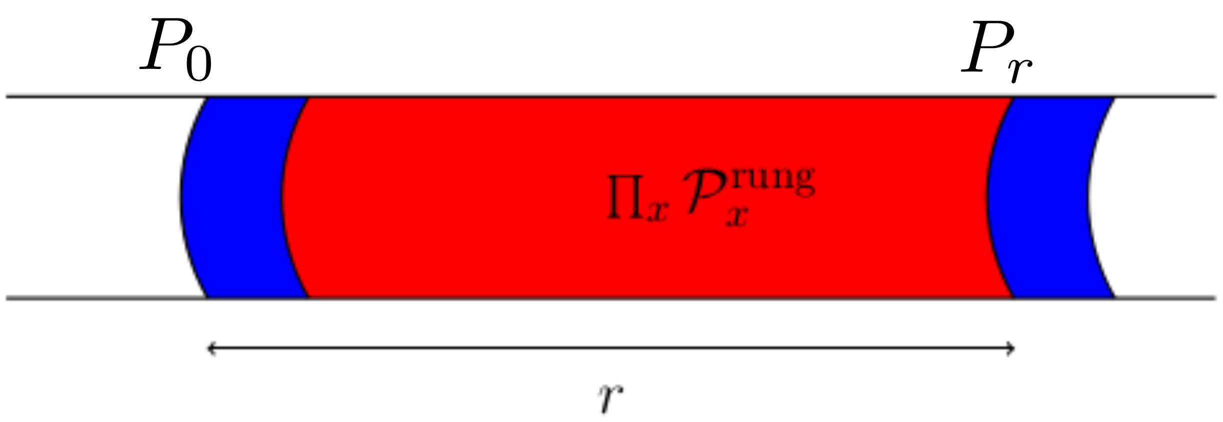
- the physical mechanism is expected to be the same as in 1D: is it possible to characterize MI phase with non-zero order parameter in 2D?

- introduce rung parity $$P_R(M) = \mathrm{e}^{i\pi\delta N_R} \quad , \quad \delta N_R = \sum_{j=1}^{M} \delta n_{R,j}$$



- -> brane parity $$C_P(r, M) = < \prod_{R=1}^{r} P_R(M) >$$

2D parity



$$C_P(M) = \lim_{r\to\infty} C_P(r,M)$$

$$C_P = \lim_{M\to\infty} C_P(M)$$

- leaking from borders: multiple broken holon-doublon pairs change arbitrarily the phase -> decay to zero
- normalize the phase -> fractional brane parity

$$\boxed{C_P^{(\theta)}(M) = \lim_{r\to\infty} \langle \prod_{R=1}^{M} [P_R(M)]^{\frac{\theta}{\pi}} \rangle}$$

S. Fazzini, F. Becca, AM,
PRL '17

- to second order: $C_P^{(\theta)}(r,M) \approx \mathrm{e}^{-\frac{\theta^2}{2}\langle[\delta N(r)]^2\rangle}$, $\delta N(r) = \sum_{R=1}^{r} \delta N_R$

$$C_P^{(\theta)}(M) \approx \begin{cases} \lim_{r\to\infty} r^{-aM\theta^2} & \text{SF}, \\ e^{-bM\theta^2} & \text{MI}, \end{cases}$$

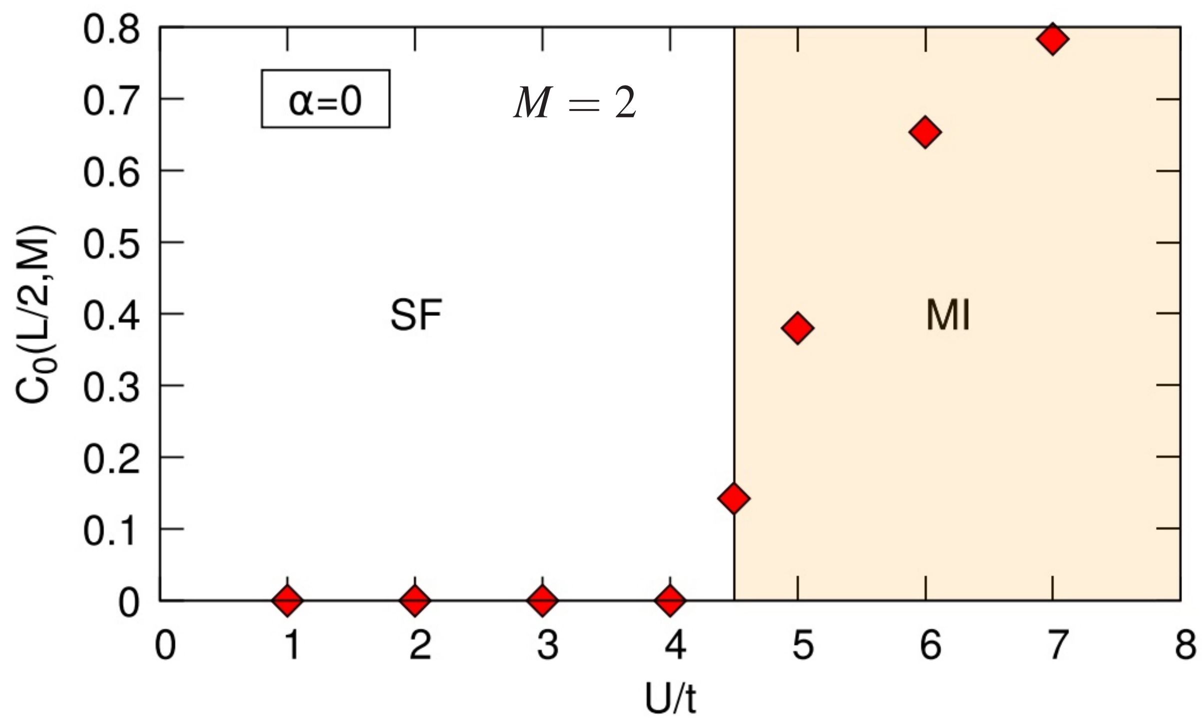
2D

$$C_P^{(\theta)} = \lim_{M\to\infty} C_P^{(\theta)}(M)$$

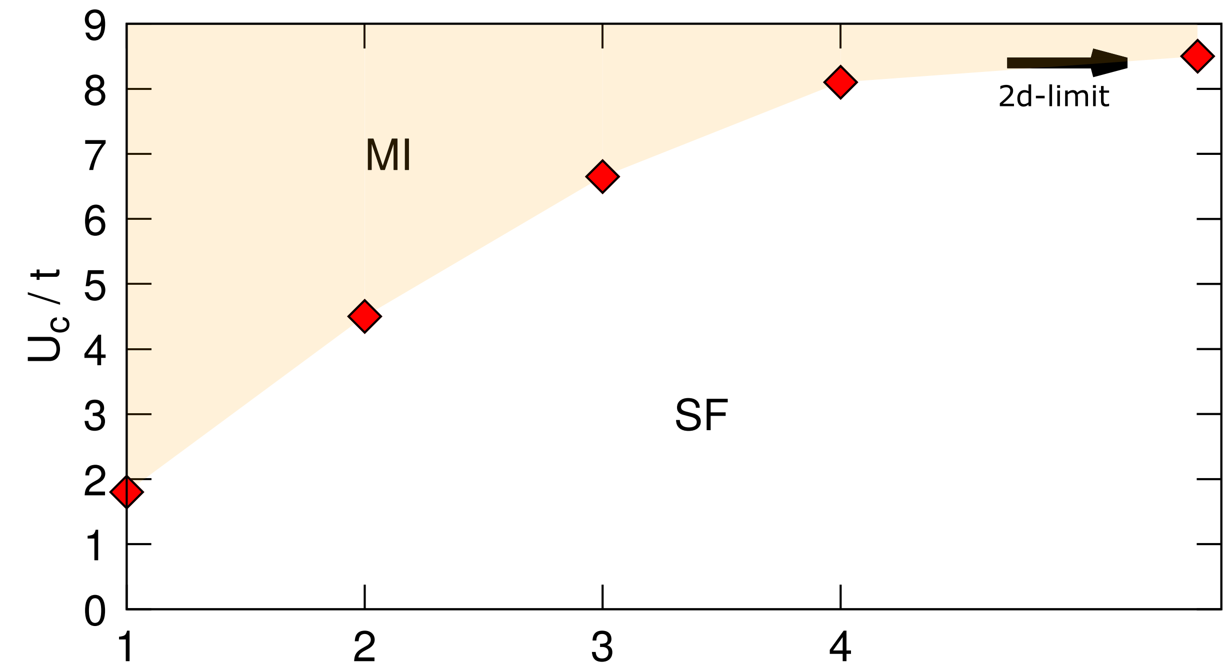
- assuming $\theta \propto M^{-\alpha}$ the 2D fractional parity vanishes in the SF phase
- it is finite in MI phase for $\boxed{\alpha \geq \frac{1}{2}}$
- for $\alpha = 0$ perimeter law decay is recovered

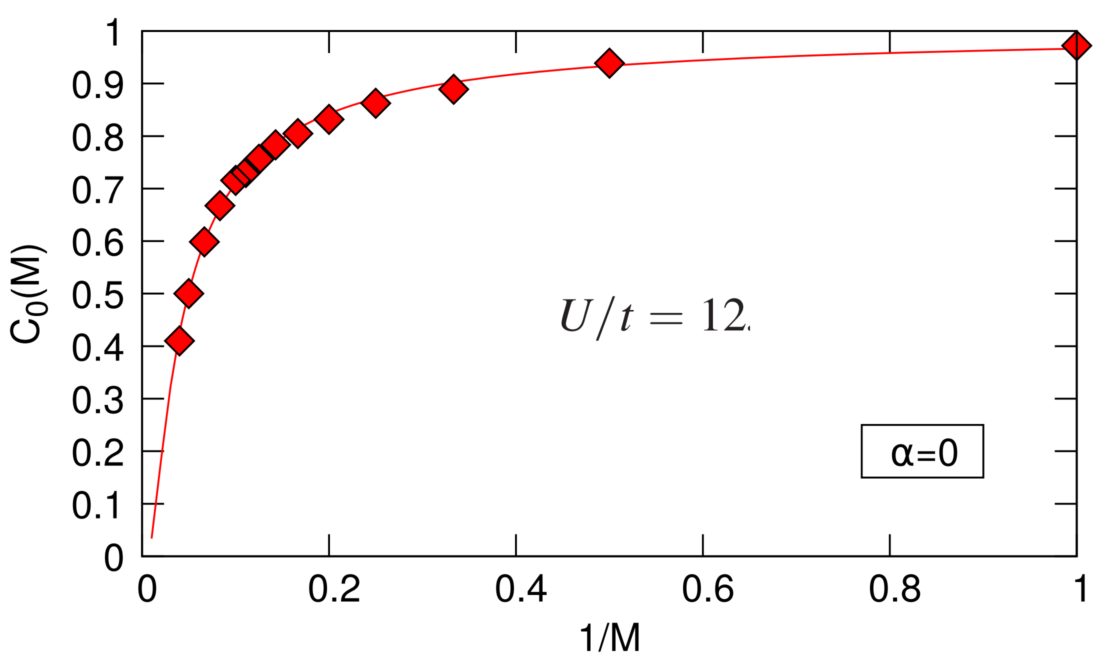
QMC results

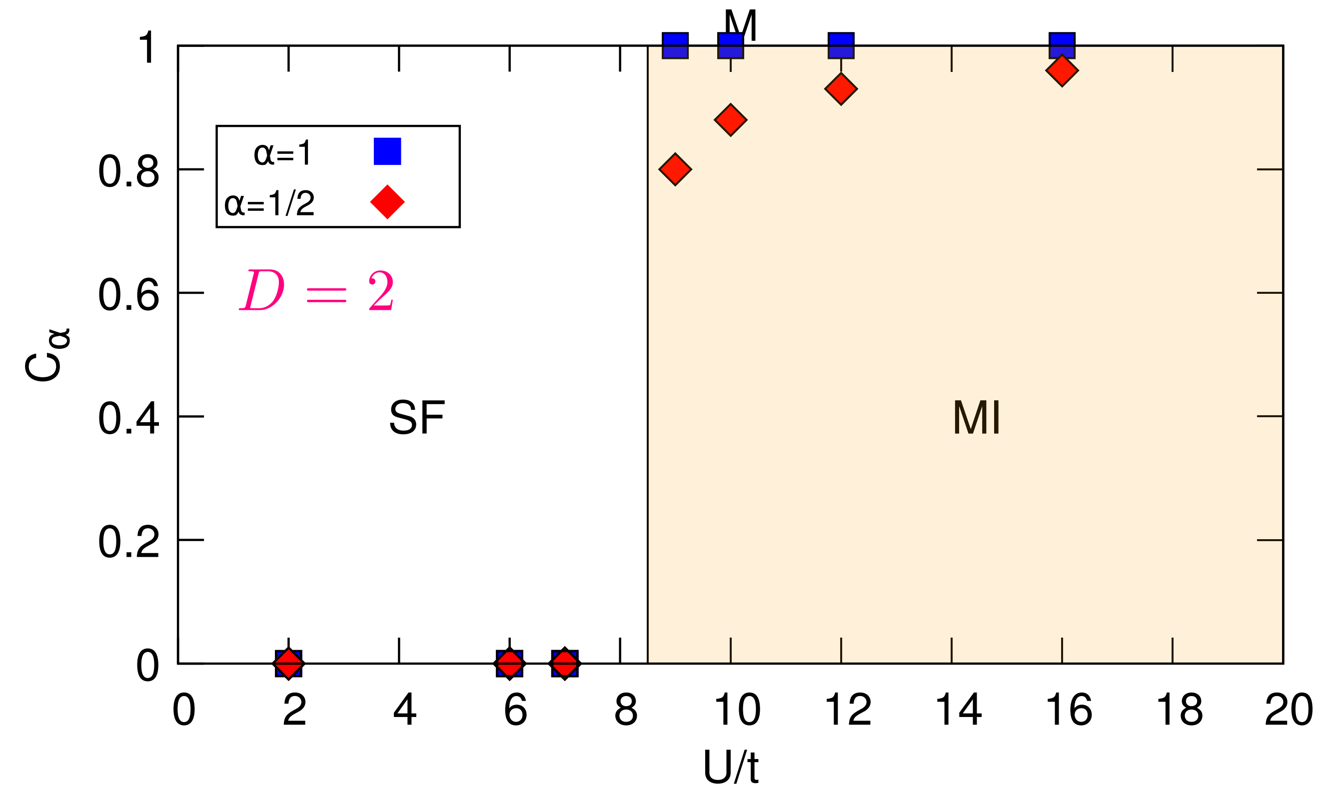
$$C_\alpha \equiv C_P^{\left(\frac{\pi}{M^\alpha}\right)}$$



phase diagram in U-M plane $r \to \infty$







Conclusions and perspectives

- Low temperature phases of 1D quantum systems can be described by the non-vanishing of appropriate non-local orders
- Their number and type depends on the symmetries of the Hamiltonian preserved in the low temperature phase
- Non trivial phases remain distinct from trivial ones under appropriate symmetry protection
- -> higher D: phase diagram and non-local orders built from symmetries of strong coupling Hamiltonian
- -> parity orders in ladders & 2D: Mott and d-wave phases of Hubbard model