Holographic study of 3d YM-CS theory with defects

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① Introduction

3 dim YM-CS theory (SU(N)$_k$ theory)

$$S = \int_{\rm 3dim} \frac{1}{g^2} \mathrm{Tr}(F \wedge *F) + \frac{k}{4\pi} \int_{\rm 3dim} \omega_3(A)$$

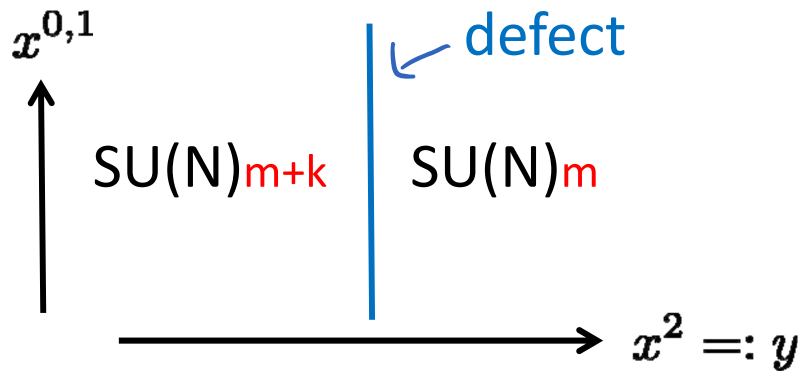
A: SU(N) gauge field

$$\omega_3(A) = \mathrm{Tr}\left(AdA + \frac{2}{3} A^3 \right)$$ CS 3-form

- k is an integer called "level"
- Low energy limit is the CS-theory
- U(1) case is used as an effective theory of FQH effect
- We consider large N with k and g^2N fixed
- level / rank duality SU(N)$_k$ ↔ U(k)$_N$

Level changing defects

- Consider a 1+1 dim defect that changes the CS level.



- Note that the CS-term is *not* gauge invariant in this case:

$$\delta_\lambda S_{\rm CS} = \frac{k}{4\pi}\int_{\rm defect} {\rm Tr}(\lambda dA)$$

- To cancel this anomaly, we put k chiral fermions on the defect:

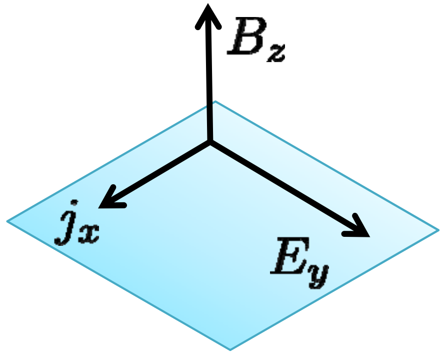
$$S_\psi = \int_{\rm defect} d^2x\, \psi^{\dagger}_{-i}(\partial_+ + A_+)\psi^i_-$$

$\psi^i_- \quad i = 1 \sim k$ negative chirality fermions

$A_+ = A_0 + A_1$

Motivation

* FQH states with edges



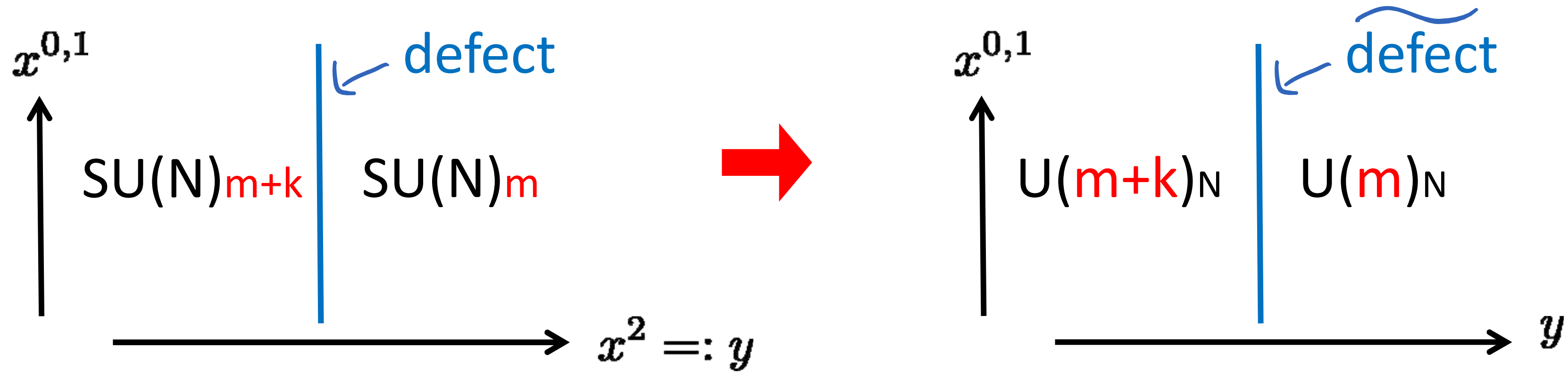
Hall conductance

$$j_x = \sigma_{xy} E_y \qquad \sigma_{xy} = \nu \frac{e^2}{h} \qquad \nu \in \mathbf{Q}$$

$$\nu = \frac{1}{m} \quad \Rightarrow \quad \text{effective theory:} \quad U(1)_m \ \text{CS-theory}$$

(Laughlin)

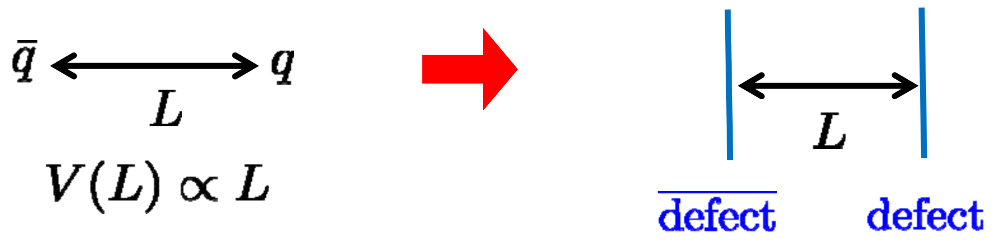
* level/rank duality with defects



How can we understand this?

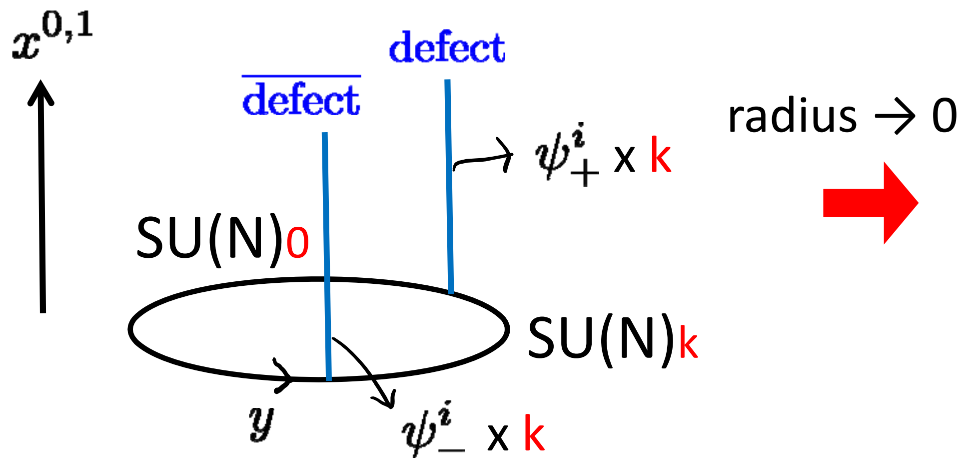
* possible new probe to study confinement ?

3 dim YM is a confining theory.



* Relation to 2 dim QCD

compactify y direction to S^1



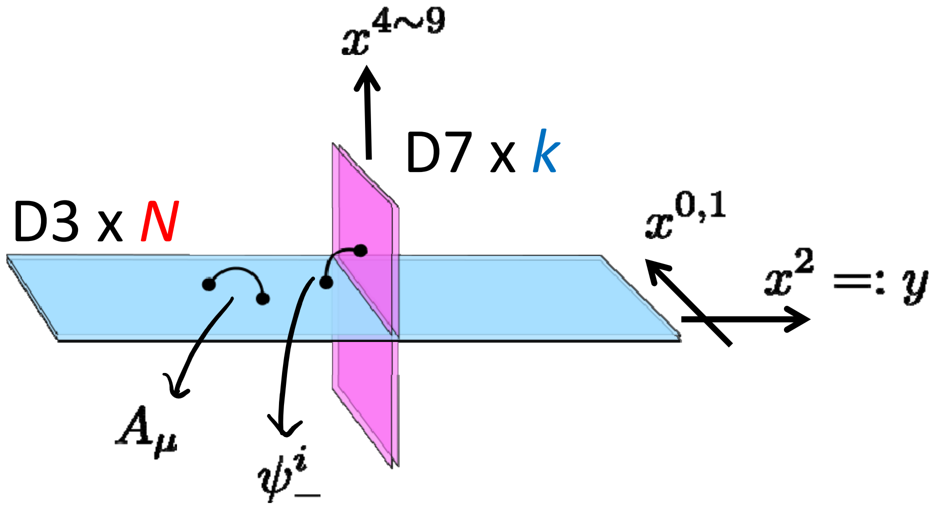
2 dim
SU(N) QCD
with k flavors

Plan

1. Introduction
2. Brane configuration
3. Holographic description
4. Defect operators
5. Free energy & phase transition
6. Summary & Discussion

② Brane configuration

	$\overbrace{0\ 1\ 2}^{\mathbf{R}^{1,2}}$			$\overset{S^1}{3}$	4	5	6	7	8	9
N D3 :	o	o	o	o						
k D7 :	o	o			o	o	o	o	o	o



fermions are anti-periodic along the S^1

⇒ SUSY is completely broken

⇒ 3d YM + 2d chiral fermions on the defect + CS-term
(at low energy)

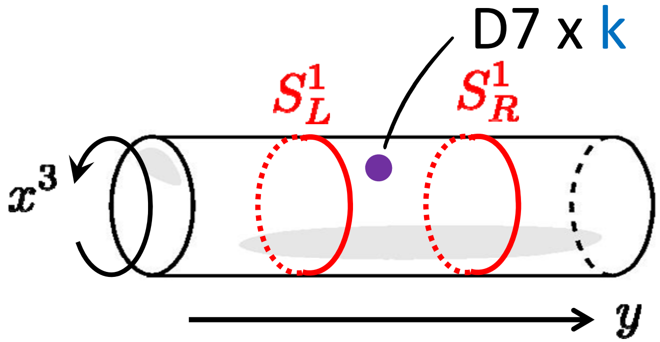
↓ next side

(※ More precisely, we get
4d N=4 SU(N) SYM on this S^1 + 2d fermions + CS-term)

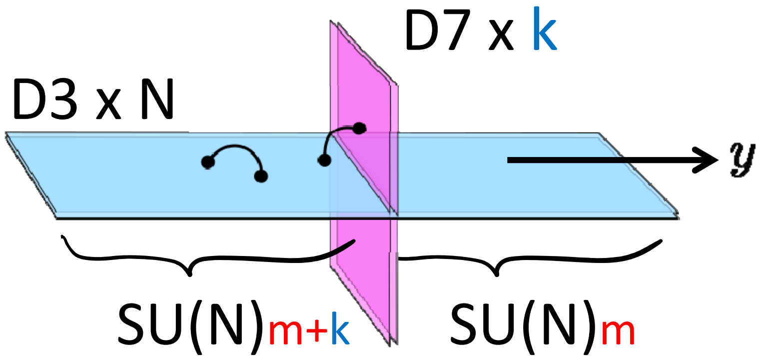
Chern-Simons term

RR 0-form

$$S_{\mathrm{CS}}^{\mathrm{D3}} = \frac{1}{8\pi^2}\int_{\mathrm{4dim}} C_0\,\mathrm{Tr}F^2 \sim \frac{1}{4\pi}\underbrace{\left(\frac{1}{2\pi}\int_{S^1} dC_0\right)}_{\text{CS level}}\int_{\mathrm{3dim}}\omega_3(A)$$



$$k = \underbrace{\frac{1}{2\pi}\int_{S_L^1} dC_0}_{\text{m+k}} - \underbrace{\frac{1}{2\pi}\int_{S_R^1} dC_0}_{\text{m}}$$



③ Holographic description

D3 → "AdS$_5$-soliton" x S^5 [Witten '98]

D7 → probe brane (assuming $k \ll N$) [Karch-Katz '02]

Geometry

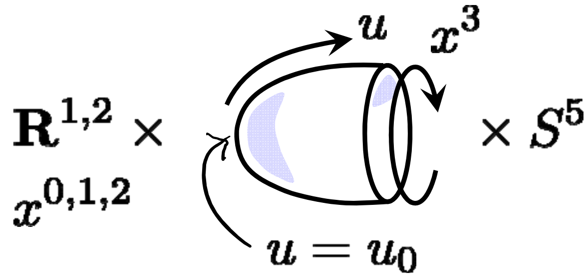
radial

$\mathbf{R}^{1,2}$ S^1 $\mathbf{R}^6 \sim \{u\} \times S^5$

0 1 2 3 4 5 6 7 8 9

N D3 : o o o o

"AdS$_5$-soliton" x S^5

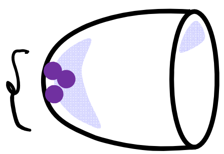


$$ds^2 = u^2(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)(dx^3)^2) + \frac{du^2}{u^2 f(u)} + d\Omega_5^2 \qquad \frac{1}{2\pi}\int_{S^5} dC_4 = N$$

$$f(u) = 1 - \frac{u_0^4}{u^4}$$

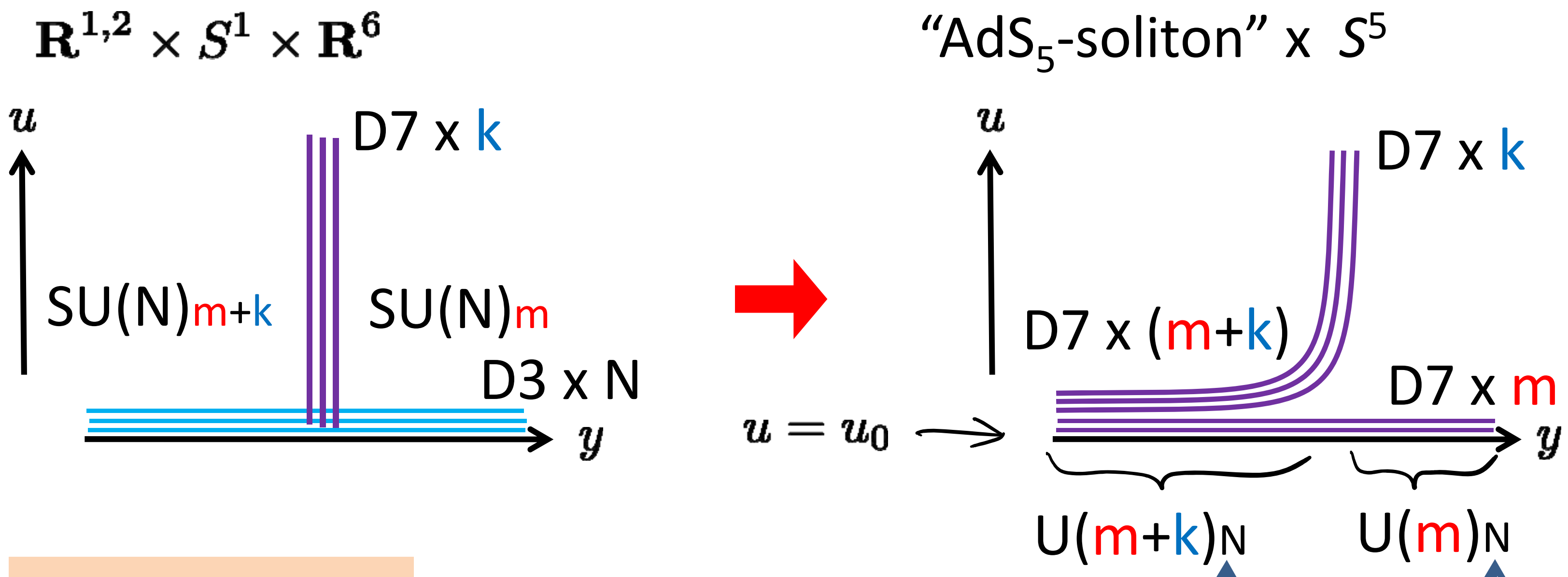
CS level

$$\text{CS level} = \frac{1}{2\pi}\int_{S^1} dC_0 = \#\text{D7}$$

D7 {


[Fujita-Li-Ryu-Takayanagi '11]

D7-brane configuration



CS-term on D7

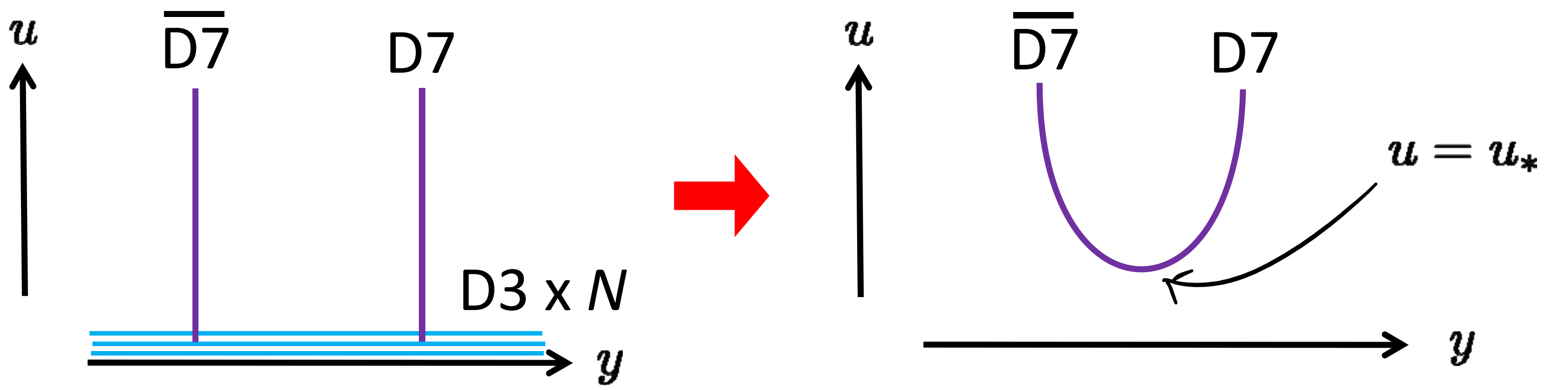
$$S_{\rm CS}^{\rm D7} \simeq \frac{1}{4\pi}\overbrace{\left(\frac{1}{2\pi}\int_{S^5} dC_4\right)}^{\rm N}\int_{\rm 3dim}\omega_3(a)$$

gauge field on D7

$\Rightarrow$ Consistent with the level/rank duality!

[Fujita-Li-Ryu-Takayanagi '11]
(for the case without defect)

In the following, we mainly consider



Solution with $\partial_0 = \partial_1 = 0$

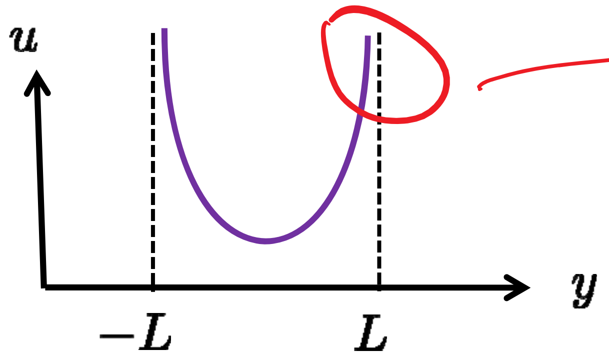
$a_\pm = a_0 \pm a_1$ gauge field on D7

$$y(u) = c_y \int_{u_*}^{u} \frac{du'}{F(u')} \qquad a_\pm(u) = a_\pm^{(0)} \pm \frac{c_\pm}{8\pi\alpha'} \exp\left(\mp 4 \int_{u_*}^{u} du' \frac{u'^4}{F(u')}\right)$$

$$F(u) = \sqrt{(u^4 - u_0^4)(u^6 + u^4 c_+ c_- - c_y^2)}$$

const. (+ pure gauge)

④ Defect operators



near $y \to L\ ,\ \ u \to \infty$

$$y \sim L + \mathcal{O}(u^{-4})$$
$$a_+ \sim \mathcal{A}_+ + \mathcal{O}(u^{-4})$$
$$a_- \sim \mathcal{C}_- u^4 + \mathcal{O}(u^2)$$

$$\mathcal{A}_+ = a_+^{(0)}|_{\mathrm{bdry}}\ ,\quad \mathcal{C}_- = -\frac{N}{4\pi}\frac{c_-}{8\pi\alpha'}u_*^4 e^{-\xi(L)}$$

Dictionary [Harvey-Royston '08]

on-shell action (after holographic renormalization)

$$\frac{\delta S}{\delta L} \sim \langle \mathcal{O}^{(3)} \rangle\ ,\qquad \frac{\delta S}{\delta \mathcal{A}_+} \sim \langle J_- \rangle\ ,\qquad \frac{\delta S}{\delta \mathcal{C}_-} \sim \langle \mathcal{O}_+^{(5)} \rangle$$

dim 3 operator — current — dim 5 operator

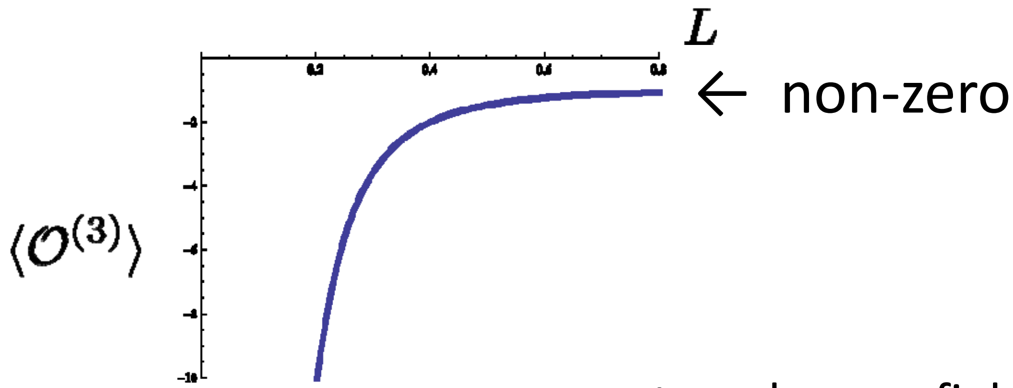
$$\mathcal{O}^{(3)} \sim \psi_-^\dagger F_{+y}\psi_- \qquad J_- \sim \psi_-^\dagger \psi_- \qquad \mathcal{O}^{(5)} \sim \psi_-^\dagger F_{+y}F_{+y}\psi_-$$

Results (consinstency check + prediction)

*** condensation of $\mathcal{O}^{(3)}$**

we can show

$$\langle \mathcal{O}^{(3)} \rangle \neq 0$$



*** anomaly**

external gauge field

$$\langle J_- \rangle \propto a_-^{(0)}|_{\mathrm{bdry}} \quad \Rightarrow \quad \partial_+ \langle J_- \rangle \propto \partial_+ a_-^{(0)}|_{\mathrm{bdry}} = \partial_- \mathcal{A}_+$$

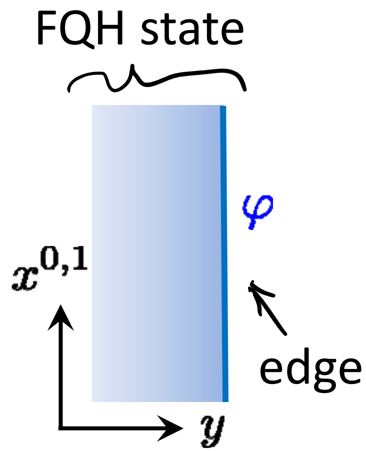
reproduces correct anomaly equation [Jensen '10, Yee-Zahed '11]

*** edge modes**

$a_\pm^{(0)}$ is the pure gauge part $\Rightarrow$ $a_\pm^{(0)}|_{\mathrm{bdry}} =: \partial_\pm \varphi$

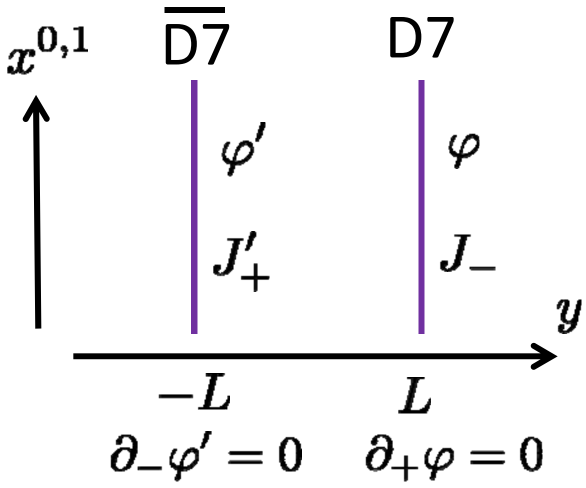
For $\mathcal{A}_+ = 0$, we have $\partial_+ \varphi = 0$ $\Rightarrow$ chiral boson

This is an analog of an edge mode in FQH states.



* chiral symmetry breaking

With two edges, we have two edge modes:



They satisfy (when $\mathcal{A}_\pm = 0$)

$$\langle J_- \rangle \propto \partial_- \varphi \qquad \langle J'_+ \rangle \propto \partial_+ \varphi'$$

U(1) x U(1)' rotation $\leftrightarrow$ shift of φ & φ'

$\varphi + \varphi'$ can be shifted without changing the gauge field.

(Recall: $a_\pm^{(0)} =: \partial_\pm \phi$, $\phi|_{y\to L} = \varphi$, $\phi|_{y\to -L} = \varphi'$)

sym breaking $U(1) \times U(1)' \to U(1)_{\mathbf{diag}}$
→ reproduces chiral sym breaking in large *N* 2d QCD

$\varphi - \varphi'$ is the Nambu-Goldstone mode

* 2 pt function for $\mathcal{O}_{\pm}^{(5)}$

operator at *y* = + *L*

source at *y* = - *L*

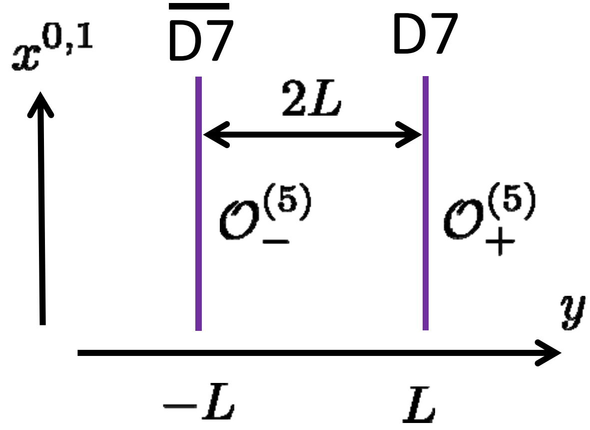
$$\langle \mathcal{O}_{+}^{(5)} \rangle \sim e^{-\alpha L} \mathcal{C}_{+}$$

(for large *L*)

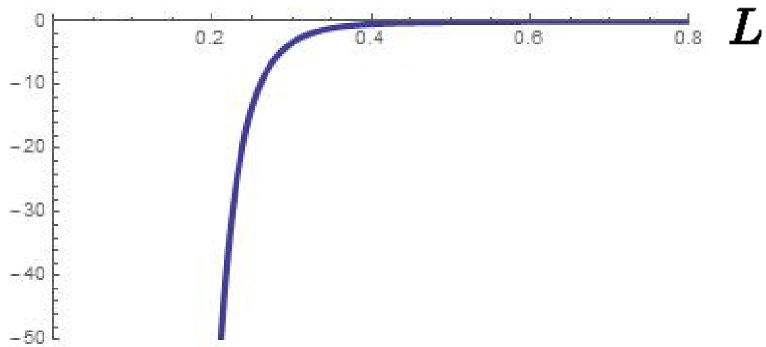
$$\frac{\delta}{\delta \mathcal{C}_{+}}$$

$$\langle \mathcal{O}_{-}^{(5)} \mathcal{O}_{+}^{(5)} \rangle \sim e^{-\alpha L}$$

(averaged over $x^{0,1}$)



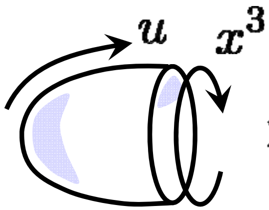
$\langle \mathcal{O}_{-}^{(5)} \mathcal{O}_{+}^{(5)} \rangle$

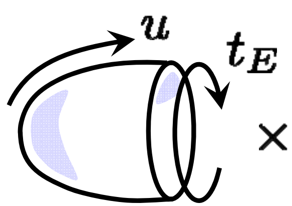


⑤ Free energy & phase transition

Finite temperature

* It is possible to introduce temperature by compactifying the Wick rotated time direction.
* It is known that there is a critical temperature at which the geometry changes. [Witten ’98, Kruczenski-Mateos-Myers-Winter ‘03, …]

$\underset{t_E}{S^1} \times \underset{x^{1,2}}{\mathbf{R}^2} \times$ 

$\times S^5$ $\Rightarrow$ 

$\times \underset{x^{1,2}}{\mathbf{R}^2} \times \underset{x^3}{S^1} \times S^5$

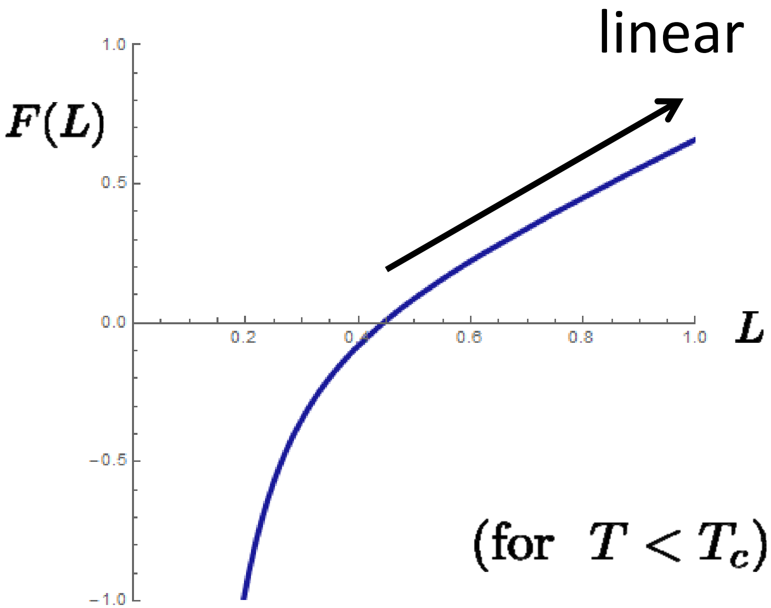
$T < T_c$ confined

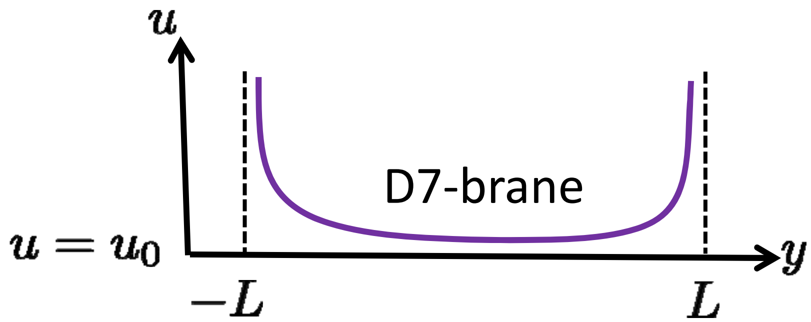
$T > T_c$ deconfined

* We are interested in the L dependence of the free energy.

on-shell action (after holographic renormalization)

$$S \sim \beta F(L)$$

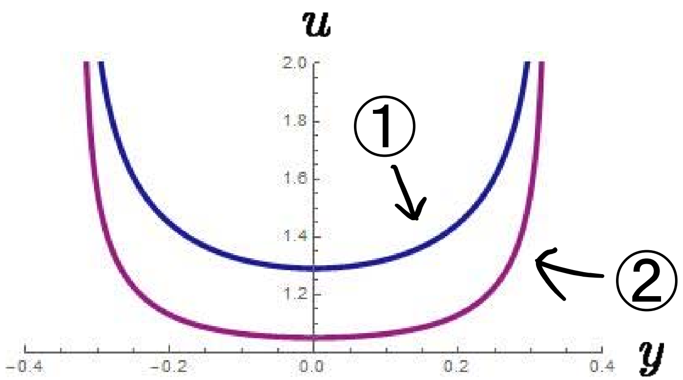


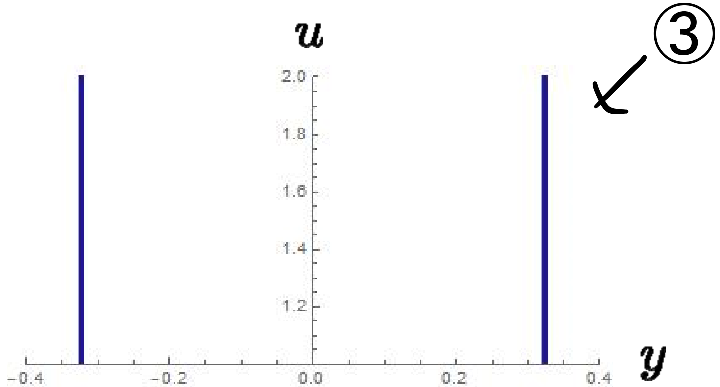


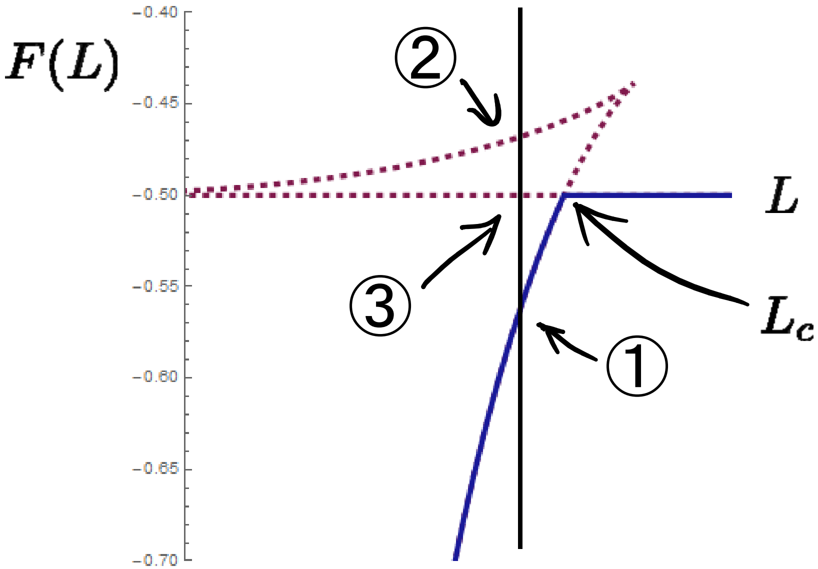
Linear behavior comes from the fact that D7-brane tension is non-zero at $u = u_0$.

Same origin as the linear behavior for the q-$\bar{\text{q}}$ potential.

* For $T > T_c$ there are three solutions.





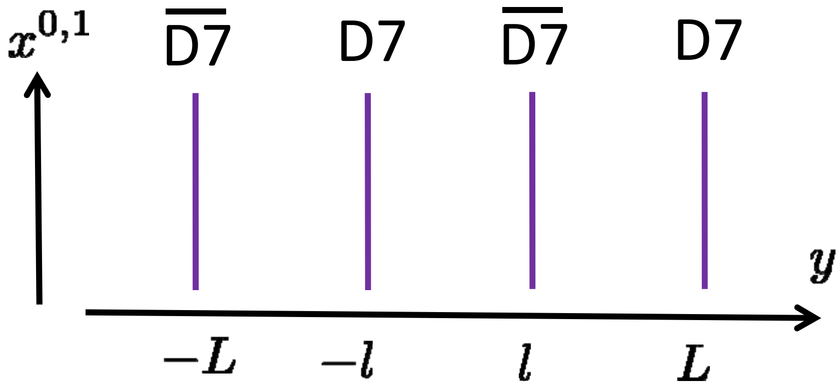


$L < L_c \Rightarrow$ ①

$L > L_c \Rightarrow$ ③

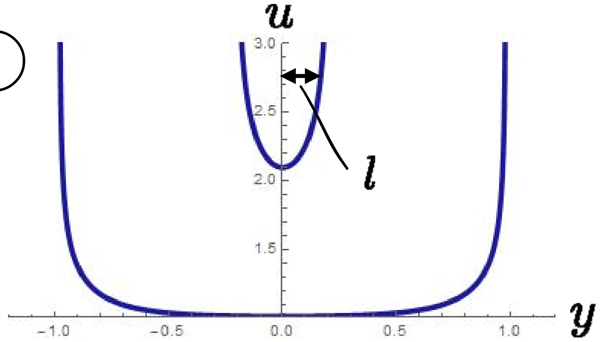
A case with 4 defects

Consider a case with 4 defects:

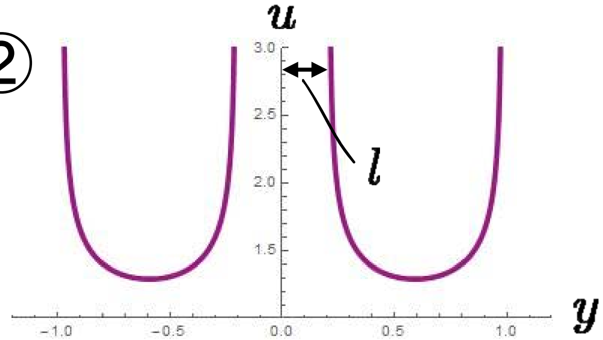


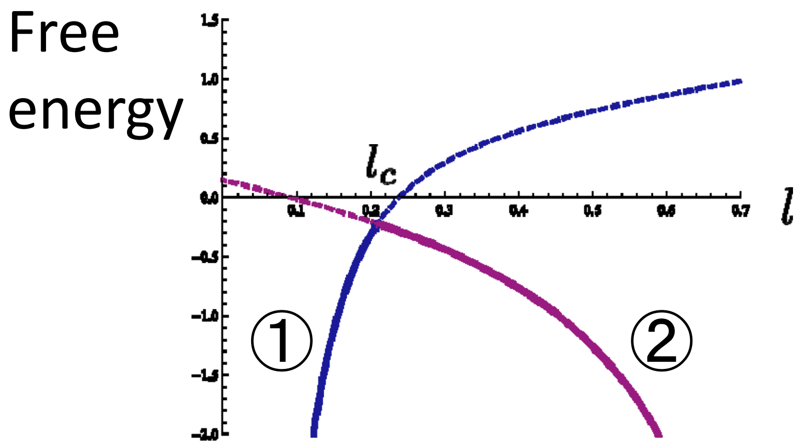
There are two solutions: (T<Tc)

①



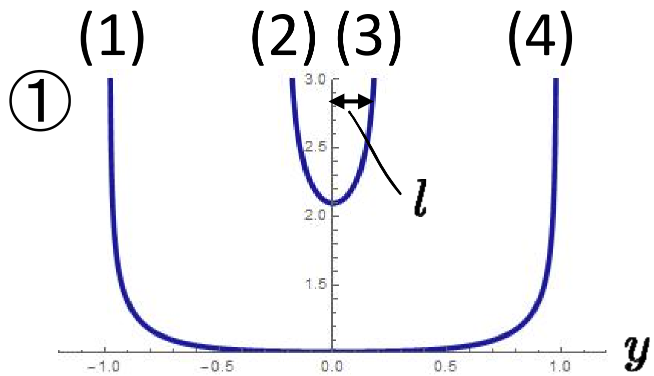
②





$l < l_c \Rightarrow$ ①

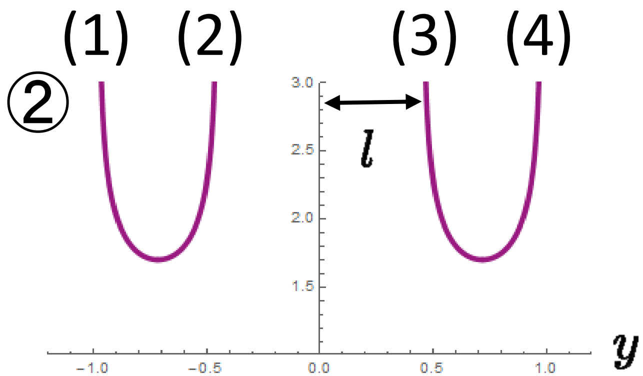
$l > l_c \Rightarrow$ ②



$l < l_c$

$$\langle \mathcal{O}_-^{(5)}(1)\mathcal{O}_+^{(5)}(4)\rangle \neq 0$$

$$\langle \mathcal{O}_+^{(5)}(2)\mathcal{O}_-^{(5)}(3)\rangle \neq 0$$

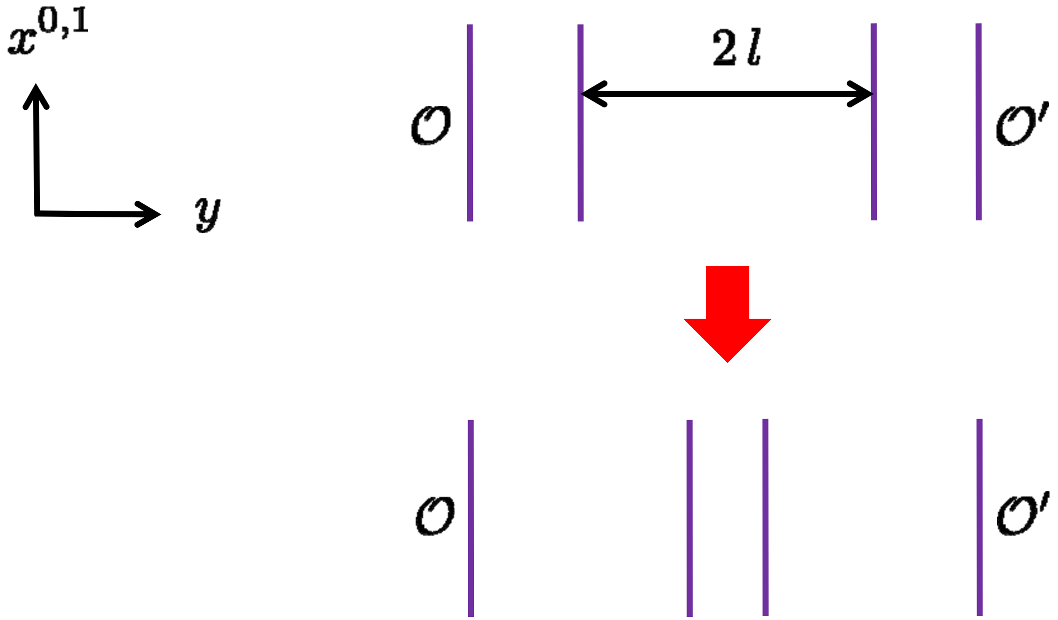


$l > l_c$

$$\langle \mathcal{O}_-^{(5)}(1)\mathcal{O}_+^{(5)}(2)\rangle \neq 0$$

$$\langle \mathcal{O}_-^{(5)}(3)\mathcal{O}_+^{(5)}(4)\rangle \neq 0$$

In particular,



$\langle \mathcal{O}\mathcal{O}' \rangle = 0$

(for $l > l_c$)

$\langle \mathcal{O}\mathcal{O}' \rangle \neq 0$

(for $l < l_c$)

When l is small, the farthest pair starts to communicate!

Q Do you know any experiments that show similar behavior ?

⑥ Summary & discussion

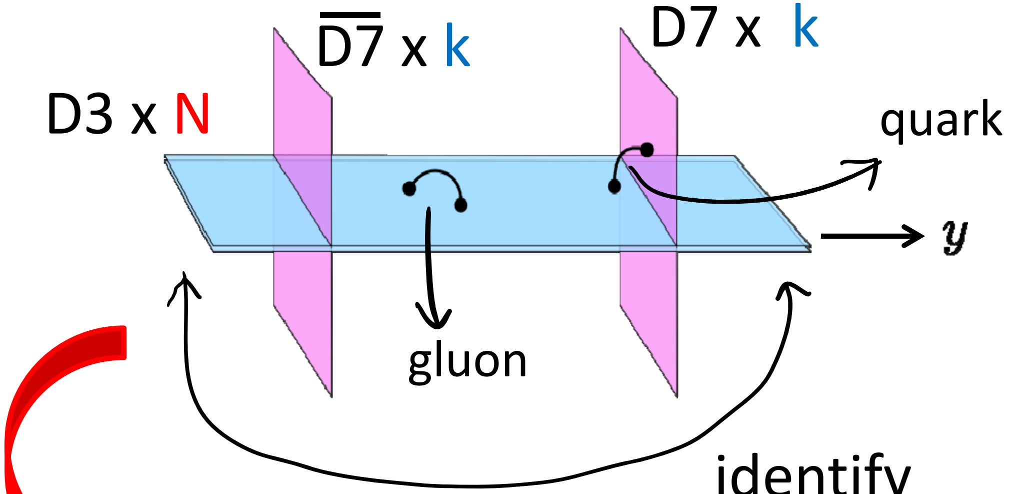
We studied "3d YM-CS theory with defects" using holography.

➡ {
- anomaly
- symmetry breaking
- confinement
- phase transition
- etc.

<u>Q</u> Any application to real world physics ?

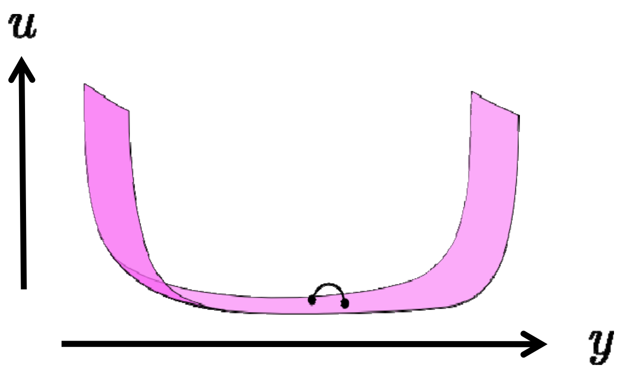
2 dim QCD vs FQH states

compactify y direction to S^1



→ 2 dim
SU(N) QCD
with k flavors
(+ KK modes)

meson sector



→ 3 dim
$U(k)_N$ YM-CS theory

[Yee-Zahed '11]

↔ FQH states:
3 dim $U(1)_N$ CS-theory

Dictionary

FQH states		QCD (2dim, large N)
Electric charge Q	↔	Baryon number charge
Electron	↔	Baryon
Quasiparticle with Q=1/N	↔	Quark
gapless edge state	↔	Nambu-Goldstone mode (Pion)

<u>Q</u> Any implications to 4 dim QCD ?

Thank you!