Some Novel Ways for Neutrino Mass Generation

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OUTLINE

MOTIVATION

- Lorentz- & CPT -Violating Backgrounds in the Early Universe:

  WHY? Novel ways for generating matter antimatter asymmetry at tree-level – non-trivial Geometry as extra source of CP Violation

  HOW? Role of Torsion as a CPT- and Lorentz-Violating Background

PART I

- String-Inspired Space-time Backgrounds with (Kalb-Ramond) Torsion and Leptogenesis → Baryogenesis
- Majorana neutrino mass generation due to torsion quantum fluctuations: **Novel role of axion fields coupled with the torsion**

PART II

- **Lorentz-Violation as UV regulator & Dynamical Neutrino Masses**

  of relevance to Interacting Dark Matter Models

Motivation

Theoretical models for CPT & Lorentz Violation
– Standard Model Extension (SME)

Kostelecky *et al.*

- CPT well-defined quantum mechanical operator Θ, **not commuting** with the (Lorentz Violating) Hamiltonian

$$[\Theta, H] \neq 0$$

- Field Theoretical Formalism: CPTV induced by Lorentz-violating backgrounds → **Standard Model Extension (SME)**

- **Sakharov's conditions in Early Universe modified/surpassed: Early Universe matter/antimmater asymmetry already at thermal equilibrium, Lorentz Violation (LV) → CPT Violation (CPTV) →**
→ different dispersion relations between particles/antiparticles in thermal distributions (for Dirac fermions, not Majorana)
→ **LV Geometry as extra source for tree-level CP violation**
(through decays of Massive right-handed Majorana neutrinos)

De Cesare, Ellis, NEM, Sarkar

STANDARD MODEL EXTENSION

Kostelecky *et al.*

$$\mathcal{L} = \tfrac{1}{2} i \bar{\psi} \Gamma^{\nu} \overleftrightarrow{\partial}_{\nu} \psi - \bar{\psi} M \psi, \qquad M \equiv m + a_{\mu}\gamma^{\mu} + b_{\mu}\gamma_5\gamma^{\mu} + \tfrac{1}{2} H^{\mu\nu}\sigma_{\mu\nu}$$

$$\Gamma^{\nu} \equiv \gamma^{\nu} + c^{\mu\nu}\gamma_{\mu} + d^{\mu\nu}\gamma_5\gamma_{\mu} + e^{\nu} + i f^{\nu}\gamma_5 + \tfrac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

+ Gauge Sectors

$$\boxed{O^{\mathrm{SM}}_{\mu\nu\ldots} C^{\mu\nu\ldots} \to O^{\mathrm{SM}}_{\mu\nu\ldots} \langle C^{\mu\nu\ldots} \rangle}$$

Bolokhov, Pospelov 0703291.

Contributions to Matter & Gauge sectors → Complete classification
Of dimension five Operators (gauge invariance requirement)

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Lorentz & CPT - Violating

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Our Models include

(i) SME

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Lorentz & CPT - Violating

(ii) Extended SME

Alexandre, Leite & NEM, arXiv:1404.7429

$$\mathcal{L}_{SME} = \overline{\psi}\left(i\partial\!\!\!/ - m + Q\right)\psi$$ + FERMION SELF INTERACTIONS

$$Q = A + iB\gamma^5 + C_{\mu}\gamma^{\mu} + D_{\mu}\gamma^{\mu}\gamma^5 + E_{\mu\nu}\sigma^{\mu\nu}$$

$$A = \frac{b}{M}\Delta \ , \quad C_0 = -i\frac{a}{M^2}\Delta\partial_0 \ , \quad \vec{C} = -i\frac{c}{M^2}\Delta\vec{\partial} \ , \quad B = D_{\mu} = E_{\mu\nu} = 0$$

M is the scale of LV,

$$\Delta = \partial_i\partial^i \; i = 1, \ldots 3$$

Alexandre, Leite & NEM, arXiv:1304.7706

AND IN THE GAUGE SECTOR

(iii)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(1 - \frac{\Delta}{M^2})F^{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - \tau A\!\!\!/)\Psi,$$

$$\tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} = \frac{e_1 + e_2}{2}\mathbf{1} + \frac{e_1 - e_2}{2}\sigma_3 + \epsilon\sigma_2$$

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M is the scale of LV, if regulator M → infinity

(i) Discuss briefly the Microscopic origin of such models

(ii) Discuss dynamical mass generation for fermions, in particular neutrinos

PART I

Microscopic Origin of SME coefficients?

Several ``Geometry-induced'' examples:

Non-Commutative Geometries

Axisymmetric Background

Geometries of the Early Universe

Torsionful Geometries (including strings…)

Early Universe T-dependent effects:
large @ high T, low values today
for coefficients of SME

Microscopic Origin of SME coefficients?

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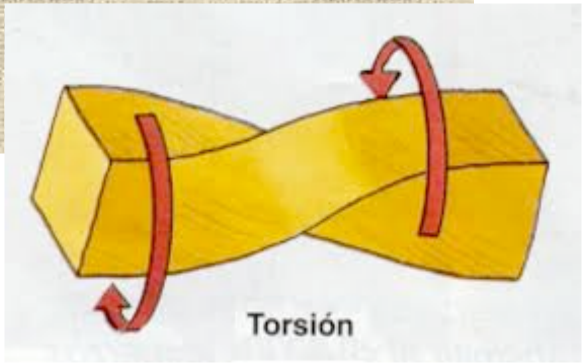

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

$$\mathcal{L} = \sqrt{-g}\left(i\,\bar{\psi}\,\gamma^a D_a \psi - m\,\bar{\psi}\psi\right)$$

$$D_a = \left(\partial_a - \frac{i}{4}\omega_{bca}\sigma^{bc}\right),$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \tfrac{i}{2}\left[\gamma^a, \gamma^b\right]$$

$$\omega_{bca} = e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\gamma\mu} e_c^\gamma e_a^\mu\right).$$

$$e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar{\psi}\left[\left(i\gamma^a\partial_a - m\right) + \gamma^a\gamma^5 B_a\right]\psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\alpha\mu} e_c^\alpha e_a^\mu\right)$$

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B^d may be constant in a given frame
In some (torsionful) background
Geometries → SME

3. Fermions in Gravity with TORSION

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If torsion then $\Gamma_{\mu\nu} \neq \Gamma_{\nu\mu}$
antisymmetric part is the contorsion tensor, contributes

A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

NEM & Sarben Sarkar, arXiv:1211.0968
John Ellis, NEM & Sarkar, arXiv:1304.5433
De Cesare, NEM & Sarkar arXiv:1412.7077

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Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symemtric rank 2 tensor (graviton)
spin 1 asntisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

Effective field theories (low energy scale E << M_s) `` gauge'' invariant

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

Depend only on field strength : $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$

Bianchi identity : $$\boxed{\partial_{[\sigma}H_{\mu\nu\rho]} = 0 \to d \star \mathbf{H} = 0}$$

ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

4-DIM PART

$$S^{(4)} = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{6}H_{\mu\nu\rho}\,H^{\mu\nu\rho}\Big)$$
$$= \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}\overline{R}\ \Big)$$

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT $\boxed{\sim \frac{1}{2}\partial^\mu b\, \partial_\mu b}$

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IN 4-DIM DEFINE DUAL OF H AS :

$$\boxed{-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}}$$

b(x) **= Pseudoscalar (Kalb-Ramond (KR) axion)**

FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2}\int d^4x\sqrt{-g}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}_\mu\psi - (\overline{\mathcal{D}}_\mu\overline{\psi})\gamma^\mu\psi\Big)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\overline{\mathcal{D}}_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\sigma^{bc}$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2}\Big(T_{cab} - T_{abc} - T_{bca}\Big)$$

$$H_{cab}$$

Non-trivial contributions to ***B^μ***

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu}e^\alpha_c e^\mu_a\right)$$

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Non-trivial contributions to $\boldsymbol{B^\mu}$

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In string theory a constant ***B⁰*** background is guaranteed by exact conformal Field theory with linear in FRW time ***b = (const) t***

Antoniadis, Bachas, Ellis, Nanopoulos

Strings in Cosmological backgrounds

$$ds^2 = g^E_{\mu\nu}(x) dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j$$

$$a(t) = t$$

$$\Phi = -\ln a(t) + \phi_0$$

$$H_{\mu\nu\rho} = e^{2\Phi} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b(x)$$

$$b(x) = \sqrt{2} e^{-\phi_0} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t$$

Central charge of uderlying world-sheet conformal field theory

$$n \in Z^+$$

$$c = 4 - 12Q^2 - \frac{6}{n+2} + c_I$$

``internal'' dims central charge

Kac-Moody algebra level

Perturbatively we may also have such a constant ***B^0*** background in the presence of **Lorentz-violating condensates** of fermion axial current temporal component

$<0 \,|\, J^{05} \,|0> \neq 0$

at the high-density, high-temperature Early Universe epochs

De Cesare, NEM & Sarkar arXiv:1412.7077

$$\partial^\mu \Big(\sqrt{-g} \big[\epsilon_{\mu\nu\rho\sigma} (\partial^\sigma \overline{b} - \tilde{c}\, J^{5\,\sigma}) + \mathcal{O}\Big((\partial \overline{b})^3 \Big) \big] \Big) = 0$$

$$\dot{\overline{b}} = \tilde{c}\, \langle J^5_0 \rangle = \tilde{c}\, \langle \psi_i^\dagger \gamma^5 \psi_i \rangle = \text{constant} \neq 0$$

Condensate may be subsequently destroyed at a temperature Tc **$<0 \,|\, J^{05} \,|0> \rightarrow 0$**

by relevant operators so eventually in an expanding FRW Universe **for T < T_c**

$$\dot{\overline{b}} \sim 1/a^3(t) \sim T^3$$

When db/dt = constant → Torsion is constant

Covariant Torsion tensor

$$\overline{\Gamma}^{\lambda}_{\ \mu\nu} = \Gamma^{\lambda}_{\ \mu\nu} + e^{-2\Phi} H^{\lambda}_{\mu\nu} \equiv \Gamma^{\lambda}_{\ \mu\nu} + T^{\lambda}_{\ \mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk}\, \dot{b}$$

Constant

$$\boxed{S_\psi \ni \int d^4x \overline{\psi} \gamma^a \gamma^5 B_a \psi}$$

constant ***B⁰***

Standard Model Extension type with CPT and Lorentz Violating background b^0

De Cesare, NEM, Sarkar arXiv:1412.7077

A background of Kalb-Ramond H-Torsion generates Matter-Antimatter Asymmetry (Leptogenesis) via (Right-handed) neutrino CP Violating (tree-level) decays in the Early Universe

$$\boxed{\mathcal{L} = i\overline{N}\not{\partial}N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.}$$

$$N_I \to H\nu,\ \bar{H}\bar{\nu}$$

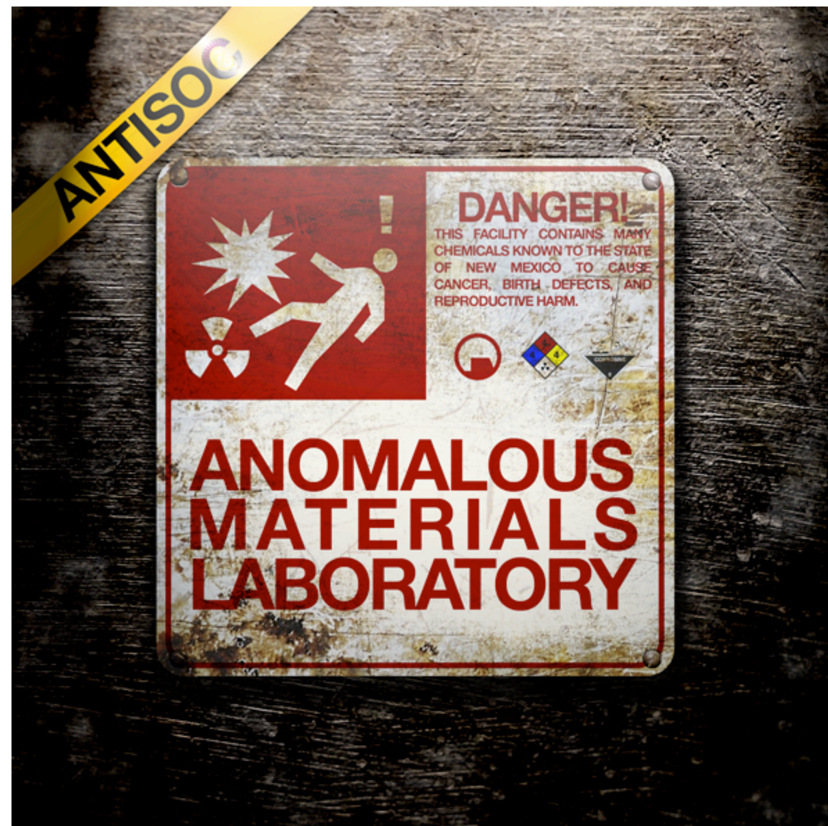
What About the Quantum Fluctuations of the H-torsion ?
Even in the absence of a non-trivial H-background

Physical Effect in Generating Majorana masses for neutrinos
via coupling to ordinary axion fields

ANOMALOUS GENERATION OF RIGHT-HANDED MAJORANA NEUTRINO MASSES THROUGH TORSIONFUL QUANTUM GRAVITY

UV complete string models ?

NEM & Pilaftsis 2012
PRD 86, 124038
arXiv:1209.6387



Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

Fermions: $S_\psi \ni -\frac{3}{4}\int d^4\sqrt{-g}\, S_\mu \bar{\psi}\gamma^\mu\gamma^5\psi = -\frac{3}{4}\int S \wedge {}^\star J^5$

+ standard Dirac terms without torsion

$$\mathbf{S} = {}^\star\mathbf{T}$$

$$S_d = \frac{1}{3!}\epsilon^{abc}{}_d T_{abc}$$

$$T_{abc} \to H_{cab} = \epsilon_{cabd}\,\partial^d b$$

Bianchi identity

$$\mathrm{d}\,{}^\star\mathbf{S} = 0$$

classical

conserved
``torsion " charge

$$Q = \int {}^\star\mathbf{S}$$

Postulate conservation at quantum level by adding counterterms

Implement $\mathrm{d}\,{}^\star\mathrm{S} = 0$ via $\delta(d\,{}^\star\mathrm{S})$ constraint
→ lagrange multiplier in Path integral → b-field

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$$
\int D\mathbf{S}\, Db \exp\Big[i \int \frac{3}{4\kappa^2}\mathbf{S}\wedge {}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge {}^\star\mathbf{J}^5 + \Big(\frac{3}{2\kappa^2}\Big)^{1/2} b\, d^\star\mathbf{S}\Big]
$$
$$
= \int Db \exp\Big[-i \int \frac{1}{2}\mathrm{d}b\wedge {}^\star\mathrm{d}b + \frac{1}{f_b}\mathrm{d}b\wedge {}^\star\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge {}^\star\mathbf{J}^5\Big];
$$

multiplier field $\Phi(x) \equiv (3/\kappa^2)^{1/2} b(x)$.

$$
f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}
$$

$$\int D\mathbf{S}\, Db \exp\left[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge{}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge{}^\star\mathbf{J}^5 + \left(\frac{3}{2\kappa^2}\right)^{1/2} b\, d^\star\mathbf{S}\right]$$
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partial integrate

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partial integrate

Use chiral anomaly equation (one-loop) in curved space-time:

$$\begin{aligned}\nabla_\mu J^{5\mu} &= \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}\\ &\equiv G(\mathbf{A},\omega)\ .\end{aligned}$$

Hence, effective action of torsion-full QED

$$\int Db \exp\left[-i\int \frac{1}{2}\mathbf{d}b\wedge{}^\star\mathbf{d}b - \frac{1}{f_b}bG(\mathbf{A},\omega) + \frac{1}{2f_b^2}\mathbf{J}^5\wedge{}^\star\mathbf{J}^5\right].$$

$$\int D\mathbf{S}\, Db\, \exp\left[i\int \frac{3}{4\kappa^2}\mathbf{S}\wedge {}^\star\mathbf{S} - \frac{3}{4}\mathbf{S}\wedge {}^\star\mathbf{J}^5 + \left(\frac{3}{2\kappa^2}\right)^{1/2} b\, d^\star\mathbf{S}\right]$$

$$= \int Db\, \exp\left[-i\int \frac{1}{2}\mathrm{d}b\wedge {}^\star\mathrm{d}b + \frac{1}{f_b}\mathrm{d}b\wedge {}^\star\mathbf{J}^5 + \frac{1}{2f_b^2}\mathbf{J}^5\wedge {}^\star\mathbf{J}^5\right],$$

partial integrate

Use chiral anomaly equation (one-loop) in curved space-time:

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}$$
$$\equiv G(\mathbf{A},\omega)\ .$$

$$\boxed{bR\widetilde{R} - bF\widetilde{F}}$$

coupling

Hence, effective action of torsion-full QED

$$\int Db\, \exp\left[-i\int \frac{1}{2}\mathrm{d}b\wedge {}^\star\mathrm{d}b - \frac{1}{f_b}bG(\mathbf{A},\omega) + \frac{1}{2f_b^2}\mathbf{J}^5\wedge {}^\star\mathbf{J}^5\right].$$

Fermionic Field Theories with H-Torsion
EFFECTIVE ACTION AFTER INTEGRATING OUT QUANTUM TORSION FLUCTUATIONS

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} +$$

+ Standard Model terms for fermions

SHIFT SYMMETRY b(x) → b(x) + c

$c\,R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma}$ and $c\,F^{\mu\nu}\widetilde{F}_{\mu\nu}$ total derivatives

Mavromatos, Pilaftsis arXiv: 1209.6387

ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

OUR SCENARIO ***Break*** such ***shift symmetry*** by coupling first b(x) to another pseudoscalar field such as QCD axion a(x) (or e.g. other string axions)

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a\left(\overline{\psi}_R^{\ C}\psi_R - \overline{\psi}_R\psi_R^{\ C}\right)\Big]\,,$$

Mavromatos, Pilaftsis arXiv: 1209.6387

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Yukawa

neutrino fields

Field redefinition

$$b(x) \to b'(x) \equiv b(x) + \gamma a(x)$$

so, effective action becomes

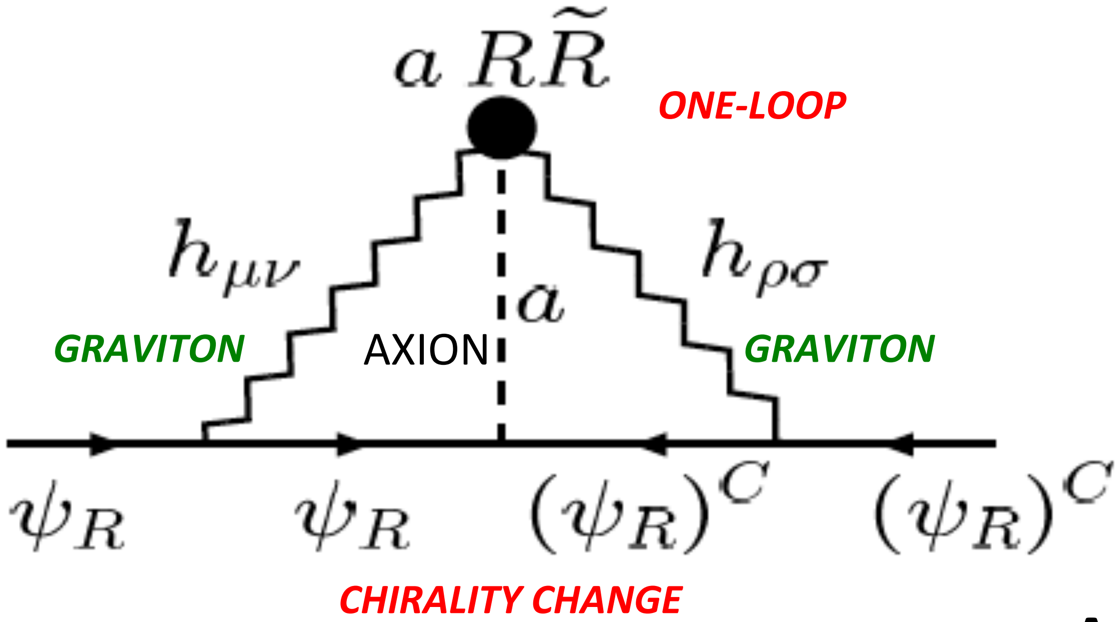
$$\mathcal{S} = \int d^4x \sqrt{-g} \Big[\frac{1}{2}(\partial_\mu b')^2 + \frac{1}{2}\left(1-\gamma^2\right)(\partial_\mu a)^2 + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} - y_a i a \left(\overline{\psi}_R^C \psi_R - \overline{\psi}_R \psi_R^C\right)\Big] .$$

must have $\boxed{|\gamma| < 1}$ otherwise axion field a(x) appears as a ghost → canonically normalised kinetic terms

$$\boxed{\mathcal{S}_a = \int d^4x \sqrt{-g} \Big[\frac{1}{2}(\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1-\gamma^2}} R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} - \frac{i y_a}{\sqrt{1-\gamma^2}} a \left(\overline{\psi}_R^C \psi_R - \overline{\psi}_R \psi_R^C\right) + \frac{1}{2f_b^2} J^5_\mu J^{5\mu}\Big] .}$$

CHIRALITY CHANGE

THREE-LOOP ANOMALOUS FERMION MASS TERMS



Λ = UV cutoff

$$M_R \sim \frac{1}{(16\pi^2)^2}\,\frac{y_a\,\gamma\,\kappa^4\Lambda^6}{192\pi^2 f_b(1-\gamma^2)} = \frac{\sqrt{3}\,y_a\,\gamma\,\kappa^5\Lambda^6}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}$$

SOME NUMBERS

$\Lambda = 10^{17}$ GeV

$\gamma = 0.1$

M_R is at the TeV

for $y_a = 10^{-3}$

$\Lambda = 10^{16}$ GeV

$M_R \sim 16$ keV,

$y_a = \gamma = 10^{-3}$

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$M_R \sim 16$ keV,

$y_a = \gamma = 10^{-3}$

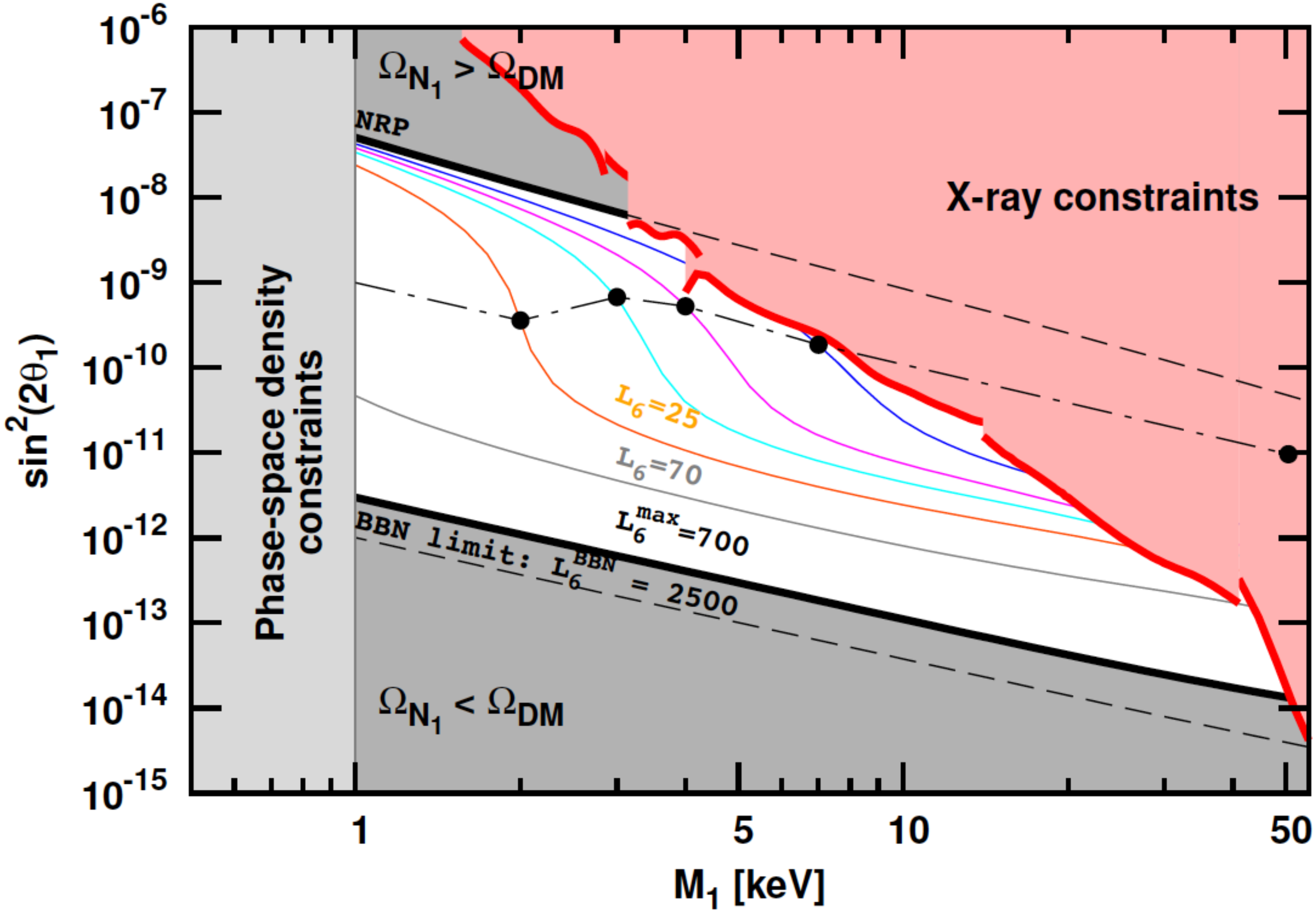
INTERESTING WARM DARK MATTER REGIME

Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter cosntraints can be arranged by choosing Yukawa couplings

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…

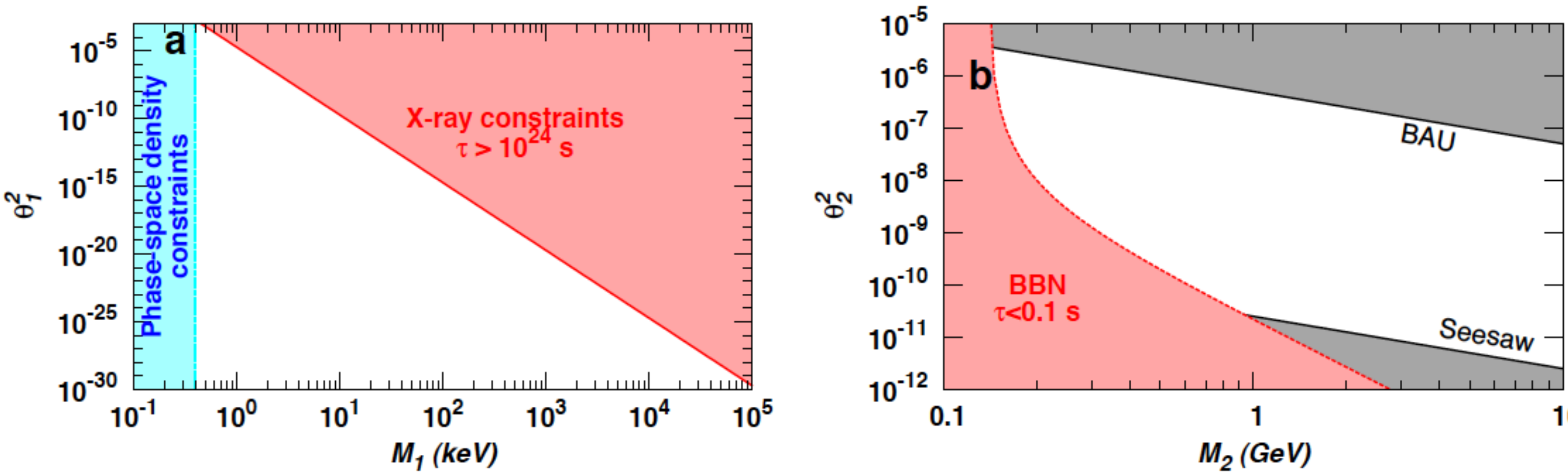
MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS



More than one sterile neutrino needed to reproduce Observed oscillations

νMSM

Boyarski, Ruchayskiy, Shaposhnikov…



Constraints on two heavy degenerate singlet neutrinos

N_1 DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos

FINITENESS OF THE MASS

Arvanitaki, Dimopoulos *et al.*

MULTI-AXION SCENARIOS (e.g. string axiverse)

$$\mathcal{S}_a^{\rm kin} = \int d^4x\,\sqrt{-g}\,\Bigg[\frac{1}{2}\sum_{i=1}^{n}\Big((\partial_\mu a_i)^2 - M_i^2\Big) + \gamma(\partial_\mu b)(\partial^\mu a_1)$$
$$-\frac{1}{2}\sum_{i=1}^{n-1}\delta M_{i,i+1}^2\, a_i a_{i+1}\Bigg]\,;$$

$$\delta M_{i,i+1}^2 \;<\; M_i M_{i+1}$$

positive mass spectrum
for all axions

simplifying all mixing equals

$$M_R \sim \frac{\sqrt{3}\, y_a\,\gamma\,\kappa^5\Lambda^{6-2n}(\delta M_a^2)^n}{49152\sqrt{8}\,\pi^4(1-\gamma^2)} \qquad n\le 3$$

$$M_R \sim \frac{\sqrt{3}\, y_a\,\gamma\,\kappa^5(\delta M_a^2)^3}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}\,\frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \quad n>3$$

FINITENESS OF THE MASS

MULTI-AXION SCENARIOS (e.g. string axiverse)

$$\mathcal{S}_a^{\rm kin} = \int d^4x \sqrt{-g}\left[\frac{1}{2}\sum_{i=1}^{n}\Big((\partial_\mu a_i)^2 - M_i^2\Big) + \gamma(\partial_\mu b)(\partial^\mu a_1) - \frac{1}{2}\sum_{i=1}^{n-1}\delta M^2_{i,i+1}\, a_i a_{i+1}\right];$$

$$\delta M^2_{i,i+1} \;<\; M_i M_{i+1}$$

positive mass spectrum for all axions

simplifying all mixing equals

$$M_R \sim \frac{\sqrt{3}\, y_a\, \gamma\, \kappa^5 \Lambda^{6-2n}(\delta M_a^2)^n}{49152\sqrt{8}\,\pi^4(1-\gamma^2)} \qquad n \le 3$$

$$M_R \sim \frac{\sqrt{3}\, y_a\, \gamma\, \kappa^5 (\delta M_a^2)^3}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}\,\frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \quad n > 3$$

M_R: UV finite for n=3 @ 2-loop independent of axion mass

PART II

DYNAMICAL MASS GENERATION FOR SELF-INTERACTING FERMIONS IN LORENTZ-VIOLATING EXTENDED SME MODELS

(ii) Extended SME

Alexandre, Leite & NEM, arXiv:1404.7429

$$\mathcal{L}_{SME} = \overline{\psi}\left(i\partial\!\!\!/ - m + Q\right)\psi$$

+ FERMION SELF INTERACTIONS

$$Q = -i\partial_0\gamma^0\frac{a}{M^2}\Delta + i\vec{\partial}\cdot\vec{\gamma}\left(i\frac{b}{M}\vec{\partial}\cdot\vec{\gamma} + \frac{c}{M^2}\Delta\right)$$

$$\Delta = \partial_i\partial^i \; i = 1,\ldots 3$$

M is the scale of LV,

$$\mathcal{L}_1 = \bar{\psi}\left[i\partial_0\gamma^0\left(1 - \frac{a}{M^2}\Delta\right) - i\vec{\partial}\cdot\vec{\gamma}\left(1 - i\frac{b}{M}\vec{\partial}\cdot\vec{\gamma} - \frac{c}{M^2}\Delta\right) - m\right]\psi + \frac{g^2}{M^2}(\overline{\psi}\psi)^2$$

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$$\Delta = \partial_i\partial^i \; i = 1,\ldots 3$$

M is the scale of LV,

NB: For a=0 but $c \neq 0$ → z=3 Lifshitz theory

$$\mathcal{L}_1 = \bar{\psi}\left[i\partial_0\gamma^0\left(1 - \frac{a}{M^2}\Delta\right) - i\vec{\partial}\cdot\vec{\gamma}\left(1 - i\frac{b}{M}\vec{\partial}\cdot\vec{\gamma} - \frac{c}{M^2}\Delta\right) - m\right]\psi + \frac{g^2}{M^2}(\overline{\psi}\psi)^2$$

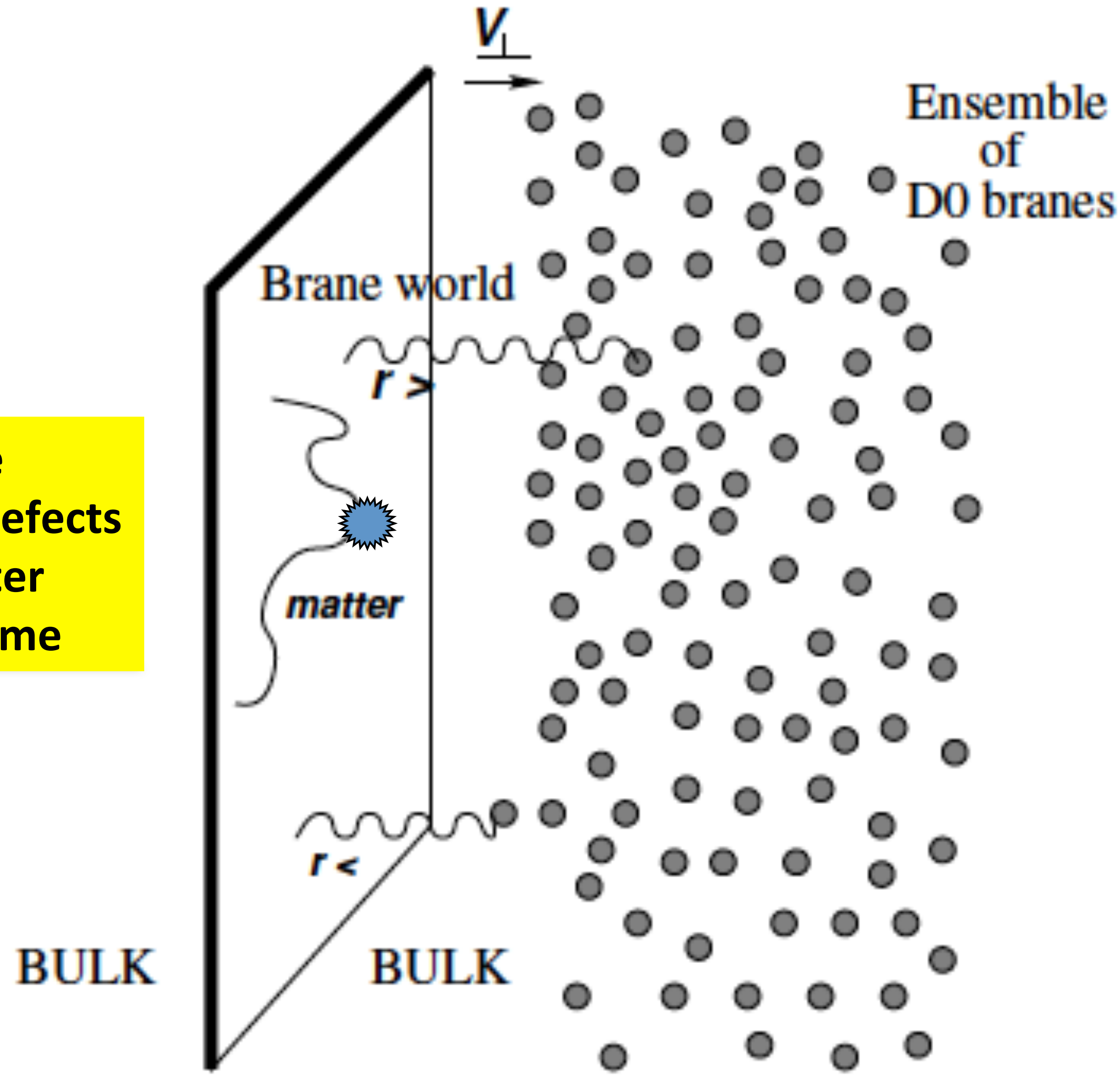
A String-inspired Macroscopic Model for extended SME

NEM arXiv:1011.3528

$$r_< \; : \; r \ll \ell_s$$

$$r_> \; : \; r \gg \ell_s$$

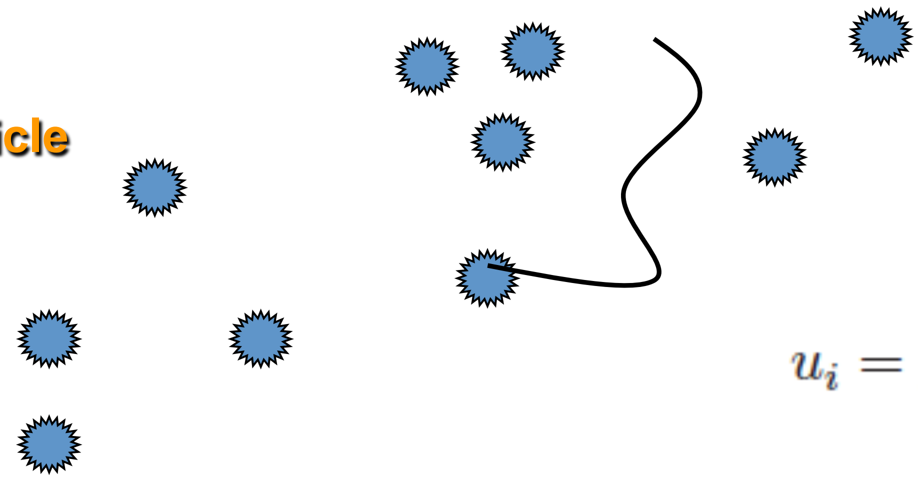
Matter on Brane
Encounters D0 defects
Recoil of the latter
Distorts space-time



Ellis, NEM, Westmuckett

Brane Universe

D-particle



Recoil Momentum transfer

$$u_i = g_s \frac{\Delta p_i}{M_s} = g_s n^\star \frac{r\, p_i}{M_s} \ll 1,$$

$$\begin{aligned} S &= \int d^4x \sqrt{g}\, \overline{\psi}\Big[i g_{\mu\nu}\gamma^\mu \partial^\nu - m \Big]\psi = \\ &= \int d^4x \Big(1 + u_i u_j \delta^{ij}\Big)\overline{\psi}\Big[i\eta_{\mu\nu}\gamma^\mu\partial^\nu + i u_i \gamma^0 \partial^i + i\gamma^i u_i \partial_0 - m\Big]\psi \;; \end{aligned}$$

$$\boxed{M = \frac{M_s}{g_s \sqrt{\ll (n^\star r)^2 \gg}}}$$

$$u_i u_j \delta^{ij} \to (g_s\, r n^\star)^2 \frac{\Delta}{M_s^2}\;, \quad \Delta \equiv \partial_i \partial_j \delta^{ij}$$

A stringy model for Extended SME → fluctuations of D-particles induce non-trivial metric depending on momenta of incident matter strings → Finsler quantum geometry

Dispersion Relation , relativistic when regulator M → ∞

$$\omega^2 = m^2 \left(\frac{1 + bp^2/(Mm)}{1 + ap^2/M^2} \right)^2 + p^2 \left(\frac{1 + cp^2/M^2}{1 + ap^2/M^2} \right)^2 .$$

Focus here on case *a c ≠ 0* → IR (p << M) & UV (p >> M) regimes relativistic ,
Only intermediate regime $p \sim M$ non relativistic

$$v_p v_g = \frac{\omega}{p}\frac{d\omega}{dp} = 1 + \frac{2}{M^2}\frac{m + bp^2/M}{(1 + p^2/M^2)^3}(bM - m)$$

NB: in stringy microscopic model M is proportional to density n of defects in space
Hence taking n → 0 removes the cutoff M → ∞ & restores Lorentz symmetry

....But **dynamical** fermion **masses** remain **finite** in this lint as we shall not show

Massless model and dynamical mass generation

$$\mathcal{L}'_1 = \bar{\psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1-\frac{\Delta}{M^2}\right) + b\frac{\Delta}{M}\right]\psi - \frac{M^2}{4}\phi^2 - g\phi\overline{\psi}\psi$$

Auxiliary

Integration over fermions → (one-loop) effective potential

$$V_1(\phi) = \frac{M^2}{4}\phi^2 + i\,tr\int\frac{d^4p}{(2\pi)^4}\ln\left[(\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1+p^2/M^2) - bp^2/M - g\phi\right]$$

Gap Equation from minimization $\boxed{0 = \delta V_1 / \delta\phi \,|\phi=\phi_1}$

$$\frac{M^2}{2}\phi_1 = \frac{g}{\pi^3}\int p^2dp\int d\omega\left[\frac{(g\phi_1 + bp^2/M)}{(\omega^2+p^2)(1+p^2/M^2)^2 + (g\phi_1+bp^2/M)^2}\right]$$

$$\boxed{m_{dyn} = g\phi_1}$$

$$g^2 << 1,$$

$$\boxed{\begin{aligned} m_{dyn} &\simeq bg^2M\frac{2\ln 2 - 1}{2\pi^2} \quad \text{for } 0 \neq b << 1 \\ m_{dyn} &\simeq g^2M\left(\frac{4-\pi}{2\pi^2} + \mathcal{O}(1/b^2)\right) \quad \text{for } b >> 1\,. \end{aligned}}$$

NO CRITICAL $g > g_c$
COUPLING REQUIRED

Finite as $M \to \infty$ if $g^2 \to 0$ such that g^2M = finite

Two-flavour case and dynamical flavour oscillations

$$\mathcal{L}_2 = \bar{\Psi}\left[i(\partial_0\gamma^0 - \vec{\partial}\cdot\vec{\gamma})\left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M}\right]\Psi + \frac{1}{M^2}(\overline{\Psi}\tau\Psi)^2,$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \text{and} \quad \tau = \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix},$$

Effective potential for auxiliary field linearising 4-ferrmion interactions

$$V_2 = \frac{M^2}{4}\phi^2 + i\, tr \int \frac{d^4p}{(2\pi)^4}(\ln\lambda_+ + \ln\lambda_-)$$

$$\lambda_\pm = (\omega\gamma^0 - \vec{p}\cdot\vec{\gamma})(1 + p^2/M^2) - p^2/M - h_\pm\phi$$

$$h_\pm = \frac{1}{2}(g_1 + g_2) \pm \frac{1}{2}\sqrt{(g_1 - g_2)^2 + 4g_3^2}$$

minimization $(dV_2/d\phi)_{\phi_2} = 0$ leads to

$$\frac{M^2}{2}\phi_2 = \sum_{s=+,-} \frac{h_s}{\pi^3} \int p^2 dp \int d\omega \left[\frac{(h_s\phi_2 + p^2/M)}{(\omega^2 + p^2)(1 + p^2/M^2)^2 + (h_s\phi_2 + p^2/M)^2} \right]$$

$h_\pm << 1$ → **Mass Matrix**

$$\boxed{\mathcal{M} = \alpha(g_1 + g_2)M \begin{pmatrix} g_1 & g_3 \\ g_3 & g_2 \end{pmatrix}}$$

Mass eigenvalues $m_\pm$ and mixing angle θ

$$\begin{aligned} m_\pm &= \frac{\alpha}{2}M\left[(g_1+g_2)^2 \pm \sqrt{(g_1^2 - g_2^2)^2 + 4g_3^2(g_1+g_2)^2}\right] \\ \tan\theta &= \frac{g_1 - g_2}{2g_3} + \sqrt{1 + \left(\frac{g_1 - g_2}{2g_3}\right)^2}. \end{aligned}$$

g_i^2 M finite in the limit $g_i \to 0$, $M \to \infty$

Dynamical Oscillations

$$\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(E_+ - E_-)\,t}{2}\right]$$

$E_\pm \sim p + m_\pm^2/2p + \ldots,$ for $m_\pm \ll p$,

$$\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(m_+^2 - m_-^2)L}{4E}\right]$$

NB: Dispersion relation

$$\omega^2 = m^2\left(\frac{1 + bp^2/(Mm)}{1 + ap^2/M^2}\right)^2 + p^2\left(\frac{1 + cp^2/M^2}{1 + ap^2/M^2}\right)^2 .$$

$m_\pm^2/p^2 \ll 1$

$$(E_+ - E_-)t = \frac{(m_+^2 - m_-^2)L}{2E} + (m_+ - m_-)\frac{EL}{M} + \mathcal{O}(m_\pm^2/M^2)$$

Dynamical Oscillations

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$$\mathcal{P}(\nu_{\beta_1} \to \nu_{\beta_2}) = \sin^2(2\theta)\sin^2\left[\frac{(m_+^2 - m_-^2)L}{4E} + (m_+ - m_-)\frac{EL}{2M} + \ldots\right],$$

$M \to \infty$

$$\simeq \frac{\sin^2[A\ (g_1+g_2)^3\sqrt{(g_2-g_1)^2+4g_3^2}\ ]}{1+(g_2-g_1)^2/(4g_3^2)}, \quad \text{with } A = \frac{\alpha^2 M^2 L}{4E}$$

NB: For large but finite M (microscopic string model) → phenomenological Constraints on M from linear term in EL/M

Majorana Fermions

$$(\nu^M)^c \equiv C(\bar{\nu}^M)^T = \nu^M$$

$$\nu_L = \begin{pmatrix} \nu_{\beta_1 L} \\ \nu_{\beta_2 L} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{kin}^M &= \bar{\nu}_L \left[i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M} \right] \nu_L \\ &= \frac{1}{2} \sum_{j=1,2} \bar{\nu}_j \left[i(\partial_0 \gamma^0 - \vec{\partial} \cdot \vec{\gamma}) \left(1 - \frac{\Delta}{M^2}\right) + \frac{\Delta}{M} \right] \nu_j \end{aligned}$$

Dynamical masses

$$m_i = 2\alpha M(g_1 + g_2) g_i, \quad i = 1, 2$$

Seesaw type Extension

$$\mathcal{L}^{M+D} = -\frac{1}{2}\overline{\nu}_L\, m_L\,(\nu_L)^c - \overline{\nu}_L\, m^D\, N_R - \frac{1}{2}\overline{N}_R\, m_R\,(N_R)^c + \mathrm{h.c.}$$

Mass matrix

$$\mathcal{M} = \alpha M g_2 \begin{pmatrix} 0 & g_3 \\ g_3 & g_2 \end{pmatrix} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

with the following eigenvalues

$$\begin{aligned} m_+ &\simeq \alpha M g_2^2 = m_R \\ m_- &\simeq \alpha M g_3^2 = \frac{m_D^2}{m_R} \ll m_R\ . \end{aligned}$$

NB: m_D , *m_R generated dynamically* ***not*** *through Higgs mechanism*

Alexandre, Leite & NEM, arXiv:1304.7706

(iii) Minimal Lorenz Violation in ``gauge'' sector

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(1 - \frac{\Delta}{M^2})F^{\mu\nu} + \bar{\Psi}(i\partial\!\!\!/ - \tau A\!\!\!/)\Psi,}$$

$$\tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} = \frac{e_1+e_2}{2}\mathbf{1} + \frac{e_1-e_2}{2}\sigma_3 + \epsilon\sigma_2$$

Structure of Dynamical Mass matrix

$$\mathrm{M} = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix} = \frac{m_1+m_2}{2}\mathbf{1} + \frac{m_1-m_2}{2}\sigma_3 + \mu\sigma_1$$

Schwinger-Dyson Gap equations

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu}\ \tau\gamma^\mu\ G\ \tau\gamma^\nu$$

Alexandre, Leite & NEM, arXiv:1304.7706

Gap Equations

$$\begin{aligned}
\frac{m_1}{4+\zeta} &= (e_1^2 m_1 + \epsilon^2 m_2) I_1 + (\mu^2 - m_1 m_2)(e_1^2 m_2 + \epsilon^2 m_1) I_2 \\
\frac{m_2}{4+\zeta} &= (e_2^2 m_2 + \epsilon^2 m_1) I_1 + (\mu^2 - m_1 m_2)(e_2^2 m_1 + \epsilon^2 m_2) I_2 \\
\frac{\mu}{4+\zeta} &= \mu(e_1 e_2 - \epsilon^2)[I_1 - (\mu^2 - m_1 m_2) I_2] \\
0 &= \epsilon(e_1 m_1 + e_2 m_2) I_1 + \epsilon(\mu^2 - m_1 m_2)(e_1 m_2 + e_2 m_1) I_2
\end{aligned}$$

$$\begin{aligned}
I_1 &\simeq \frac{1}{16\pi^2} \frac{1}{A_+^2 - A_-^2} \left[A_-^2 \ln\left(\frac{A_-^2}{M^2}\right) - A_+^2 \ln\left(\frac{A_+^2}{M^2}\right) \right] \\
I_2 &\simeq \frac{1}{16\pi^2} \frac{1}{A_+^2 - A_-^2} \ln\left(\frac{A_-^2}{A_+^2}\right) .
\end{aligned}$$

$$A_\pm^2 = \frac{m_1^2 + m_2^2 + 2\mu^2}{2} \pm \frac{\sqrt{(m_1^2 - m_2^2)^2 + 4\mu^2(m_1 + m_2)^2}}{2}$$

Various cases .....

Dynamical Flavour oscillations $m_1 = m_2 = m \neq 0$

$$\mu^2 = m_1 m_2 = m^2 \qquad\qquad e_1 = e_2; \qquad \epsilon = 0$$

$$\boxed{m = \frac{M}{2}\exp\left(-\frac{8\pi^2}{(4+\zeta)e^2}\right)}$$

NB: non-perturbative in coupling e nature of mass

$$M \to \infty \quad \text{and} \quad e_1,\, e_2,\, \epsilon \to 0$$

No Gauge fixing parameter ζ dependence
Mass finite in this limit → UV regulator gauge fields

Mass eigenvalues

$$\lambda_+ = 2m = M\exp\left(-\frac{8\pi^2}{(4+\zeta)e^2}\right) \quad , \quad \lambda_- = 0$$

MASS EIGENSTATES

$$\psi_\pm = \frac{1}{\sqrt{2}}(\psi_2 \pm \psi_1)$$

Mixing angle

$$\boxed{\theta = \mp\pi/4}$$

Mechanism also extended to Majorana neutrinos

Application to Standard Model with Right-Handed (sterile) Neutrinos

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}(1-\frac{\Delta}{M^2})F^{\mu\nu} + \bar{N}(i\partial\!\!\!/ - e_1 A\!\!\!/)\,\frac{1}{2}\Big(1+\gamma_5\Big)\,N$$
$$+\,\bar{\psi}(i\partial\!\!\!/ - e_2 A\!\!\!/)\,\frac{1}{2}\Big(1-\gamma_5\Big)\,\psi\ ,$$

Gauge field is viewed as a vector field in the case of Majorana neutrinos – there is no Gauge invariance of the Lagrangian

Majorana doublet $\nu^M = \begin{pmatrix} \psi \\ N \end{pmatrix}$

Dynamical mass for right-handed neutrinos only in this case

$$\boxed{\begin{aligned} m_1 &= \lambda_- = 0 \\ m_2 &= \lambda_+ = M\exp\left(\frac{-8\pi^2}{(4+\zeta)e_2^2}\right) \end{aligned}} \Rightarrow \boxed{M^{M+D} = \begin{pmatrix} 0 & 0 \\ 0 & m_2 \end{pmatrix}}$$

CONCLUSIONS

- Reviewed theoretical models for CPT-and Lorentz symmetry- Violation in the Early Universe in connection with dynamical mass generation for fermions/**neutrinos**

- Such models are those entailing (constant) Kalb-Ramond Torsion backgrounds : they play a role in generating matter-antimatter asymmetry in the early Universe
- Torsion flucts → Majorana neutrino mass generation

  Role of axions interacting with neutrinos via **shift-symmetry breaking**

- Lorentz-Violating fluctuating backgrounds as regulators for neutrino mass generation in (weakly) self-interacting neutrino models

- **Of relevance to interacting dark matter – keV mass neutrinos interacting via vector fields may play a crucial role in galactic core and halo structure**

Arguelles, NEM, Rueda, Ruffini (2015)

IS THIS CPTV ROUTE WORTH FOLLOWING? ….

CPT Violation

Construct Microscopic Quantum Gravity models with strong CPT Violation in Early Universe, but weak today… Fit with all available data… Estimate in this way matter-antimatter asymmetry in Universe.

Thank you for your attention !

SPARES

De Cesare, NEM, Sarkar arXiv:1412.7077

CPT VIOLATION IN THE EARLY UNIVERSE

GENERATE Baryon and/or Lepton ASYMMETRY in the Universe via B^0 –Torsion induced CPT Violation

N = right-handed Majorana neutrino

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{M}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Lepton number & CP Violations @ tree-level due to *N* decays in Lorentz/CPTV Background

$$N_I \to H\nu,\ \bar{H}\bar{\nu}$$

Produce Lepton asymmetry

$$\Delta L^{TOT} = \frac{2\Omega B_0}{\Omega^2 + B_0^2} n_N$$

$$\frac{B_0}{m} \simeq 10^{-8} \quad \Leftarrow \quad \frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$n_N = e^{-\beta m}\left(\frac{m}{2\pi\beta}\right)^{\frac{3}{2}}$$

$$T_D \simeq m \sim 100 \text{ TeV}$$

$$m \geq 100\text{TeV} \to$$

$$B^0 \sim 1\text{MeV}$$

$$B^0 \ll T, m \quad T_D \simeq m$$

$Y_k = 10^{-5}$

De Cesare, NEM, Sarkar arXiv:1412.7077

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Equilibrated electroweak B+L violating sphaleron interactions

B-L conserved

$$\boxed{\frac{\Delta L}{n_\gamma} \simeq 10^{-10},}$$

Environmental Conditions Dependent

Observed Baryon Asymmetry In the Universe (BAU)

De Cesare, NEM, Sarkar arXiv:1412.7077

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Consistent with SME bounds

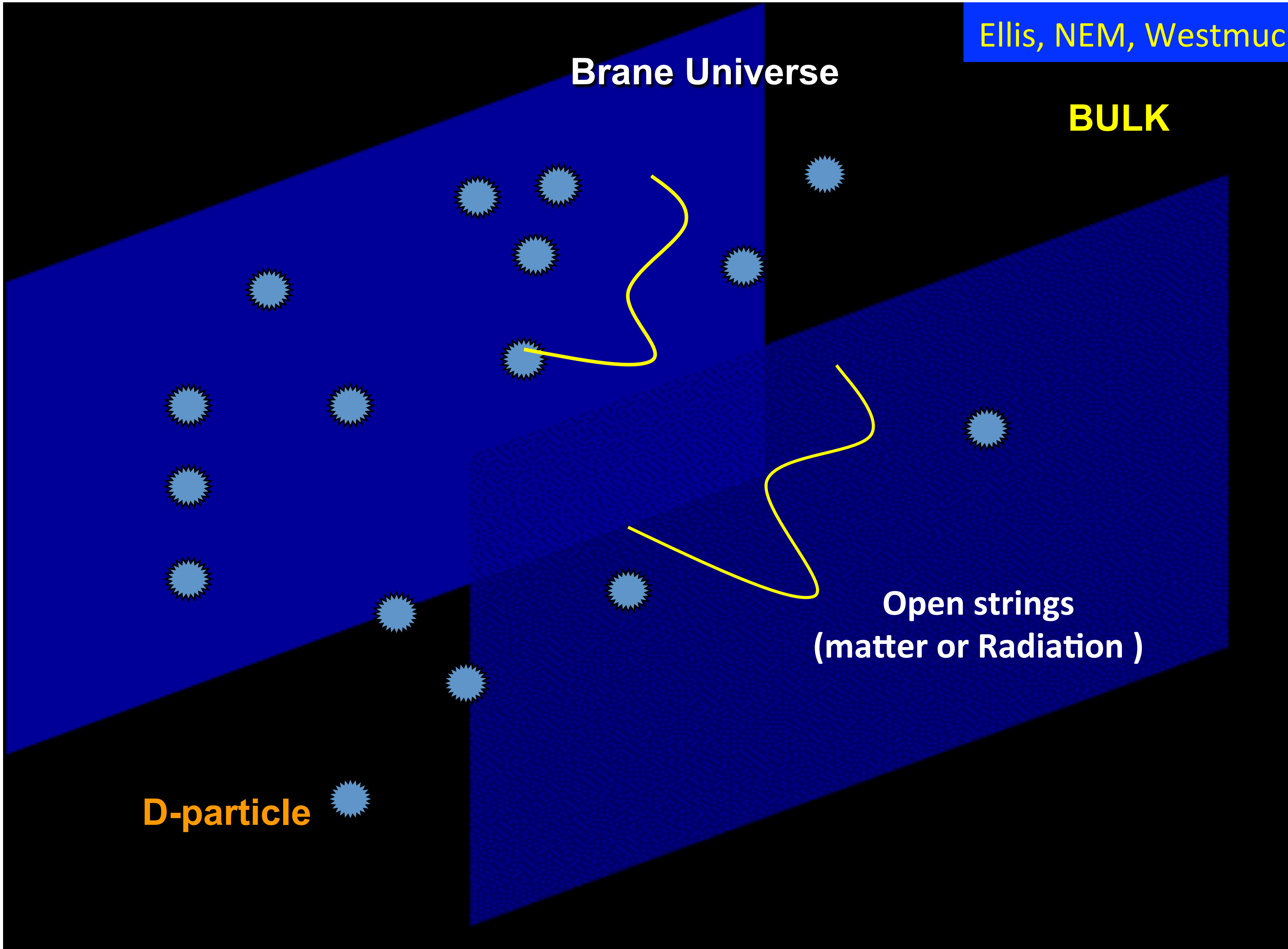
NB: Relaxation law:

$$B_0 = c_0 T^3 \qquad B_{0\,\mathrm{today}} = \mathcal{O}\left(10^{-44}\right)\ \mathrm{meV}$$

$$|B^0| < 10^{-2}\,\mathrm{eV}$$

$$B_i \equiv b_i < 10^{-31}\,\mathrm{GeV}$$

Ellis, NEM, Westmuckett



A stringy model for Extended SME → fluctuations of D-particles induce non-trivial metric depending on momenta of incident matter strings → Finsler quantum geometry