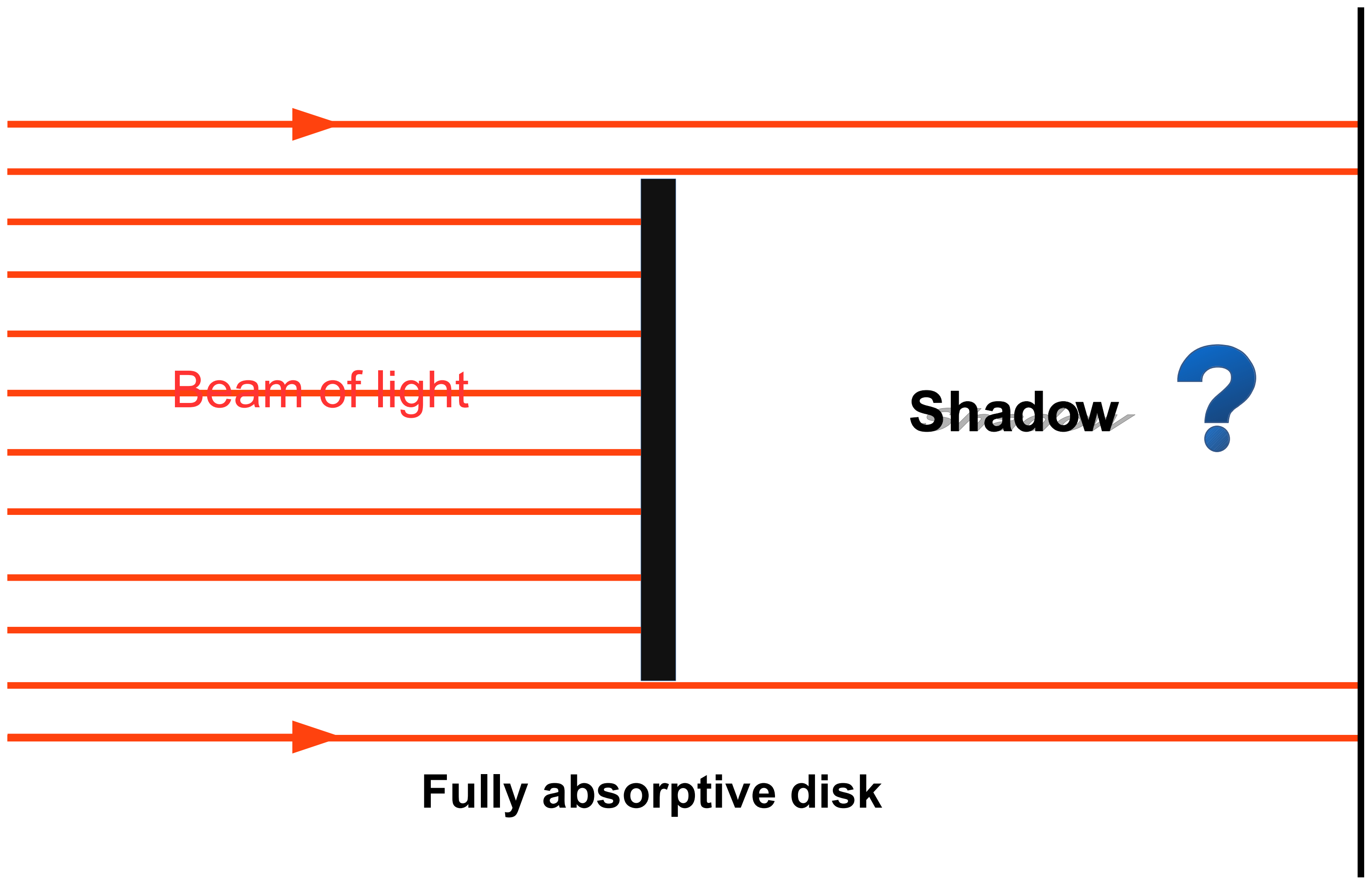
Rapidity gap distribution in diffractive deep-inelastic scattering and parton genealogy

Stéphane Munier

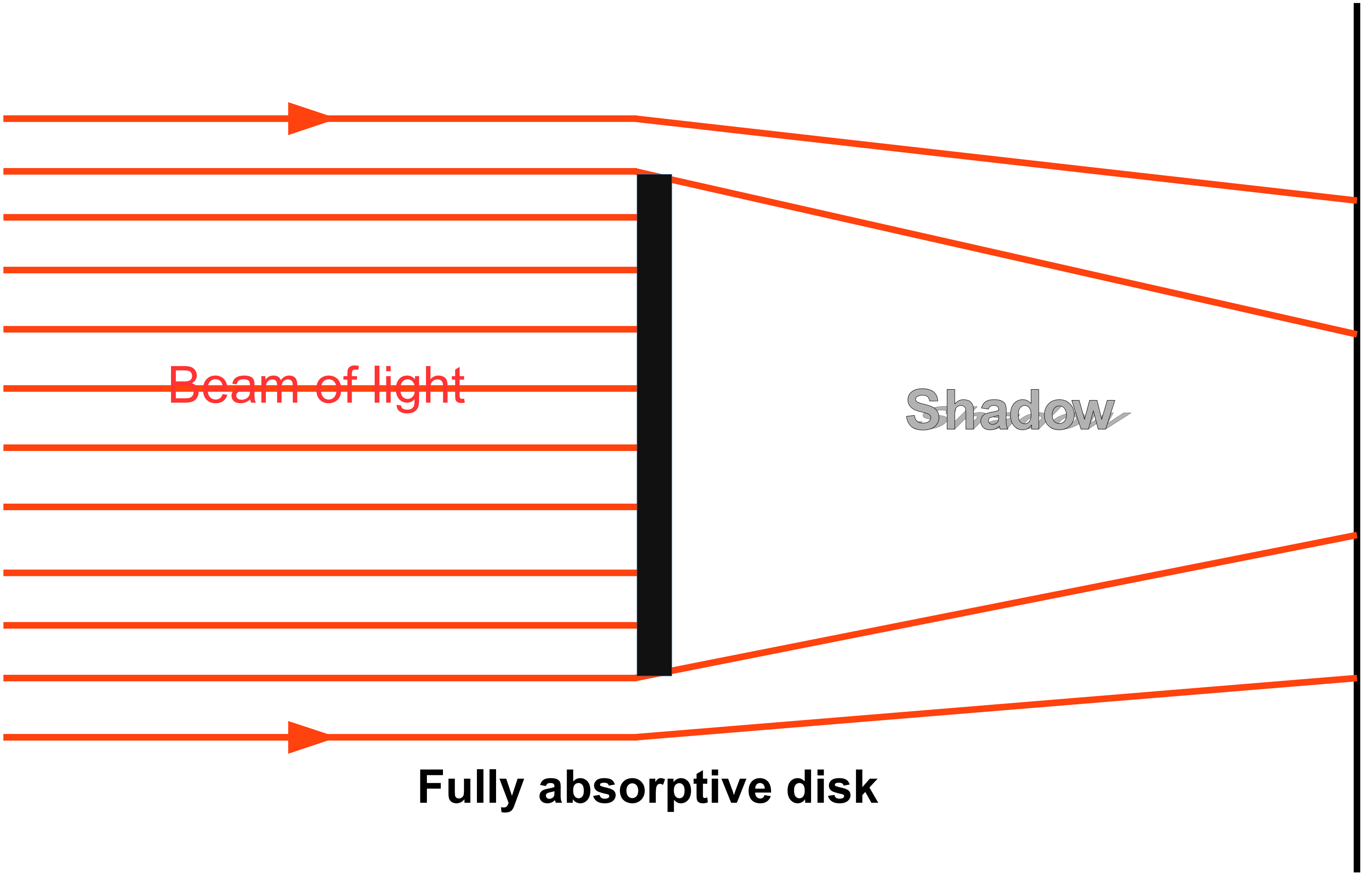
Centre de physique théorique
École polytechnique, CNRS, Université Paris-Saclay

Based on work with A.H. Mueller (arXiv:1805.02847,1805.09417)
and with Dung Le (to appear)

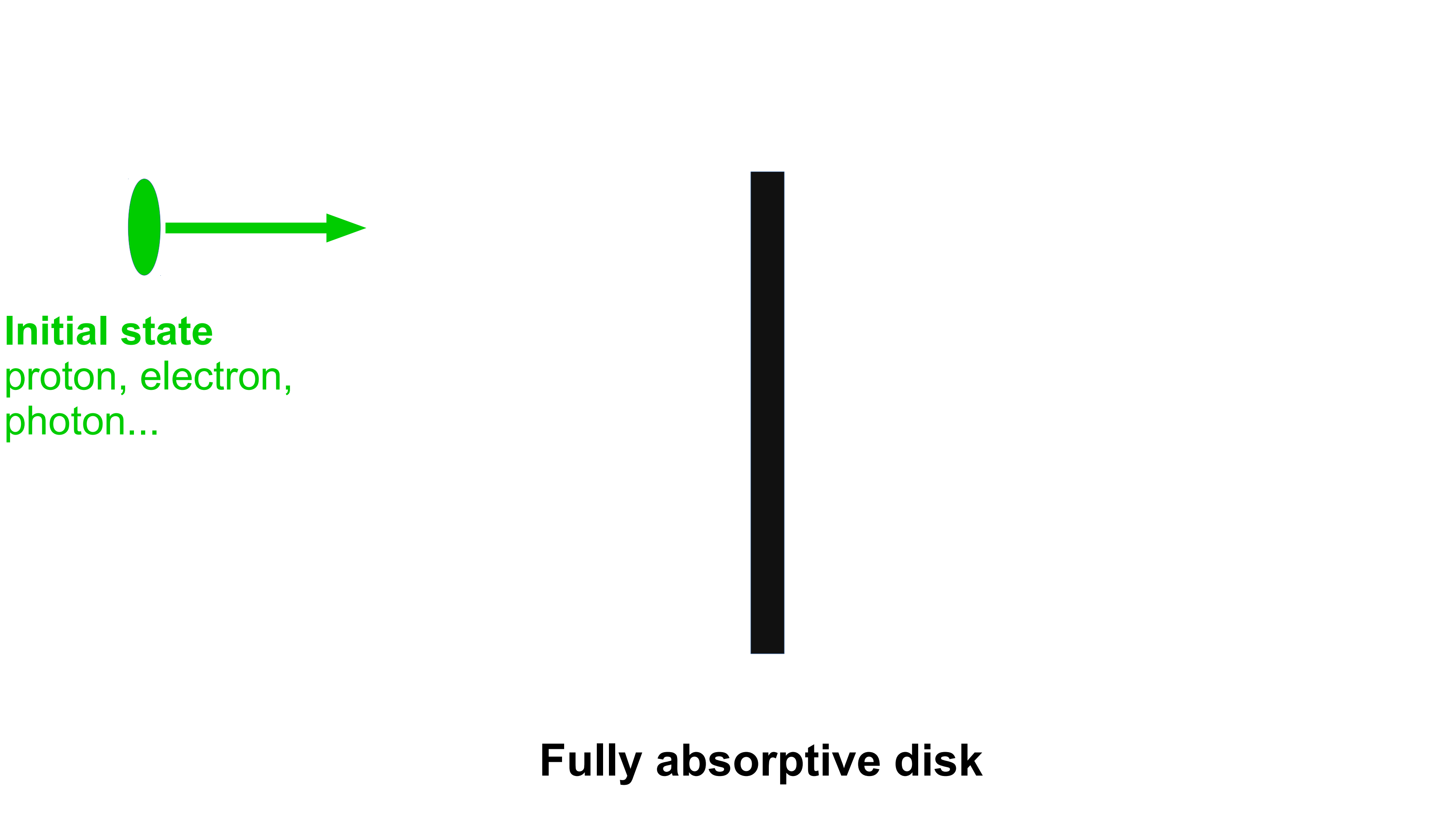
Diffraction in optics



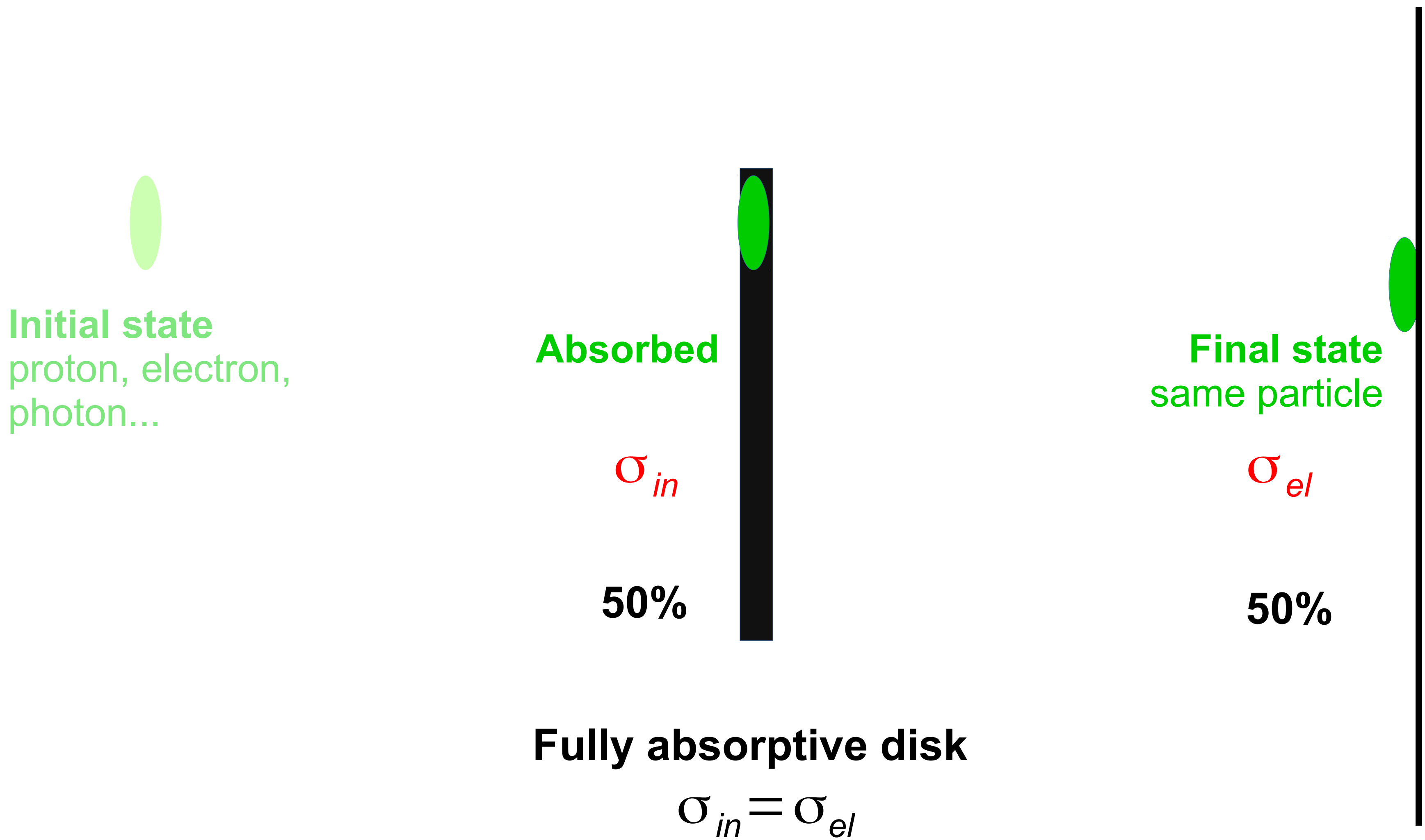
Diffraction in optics



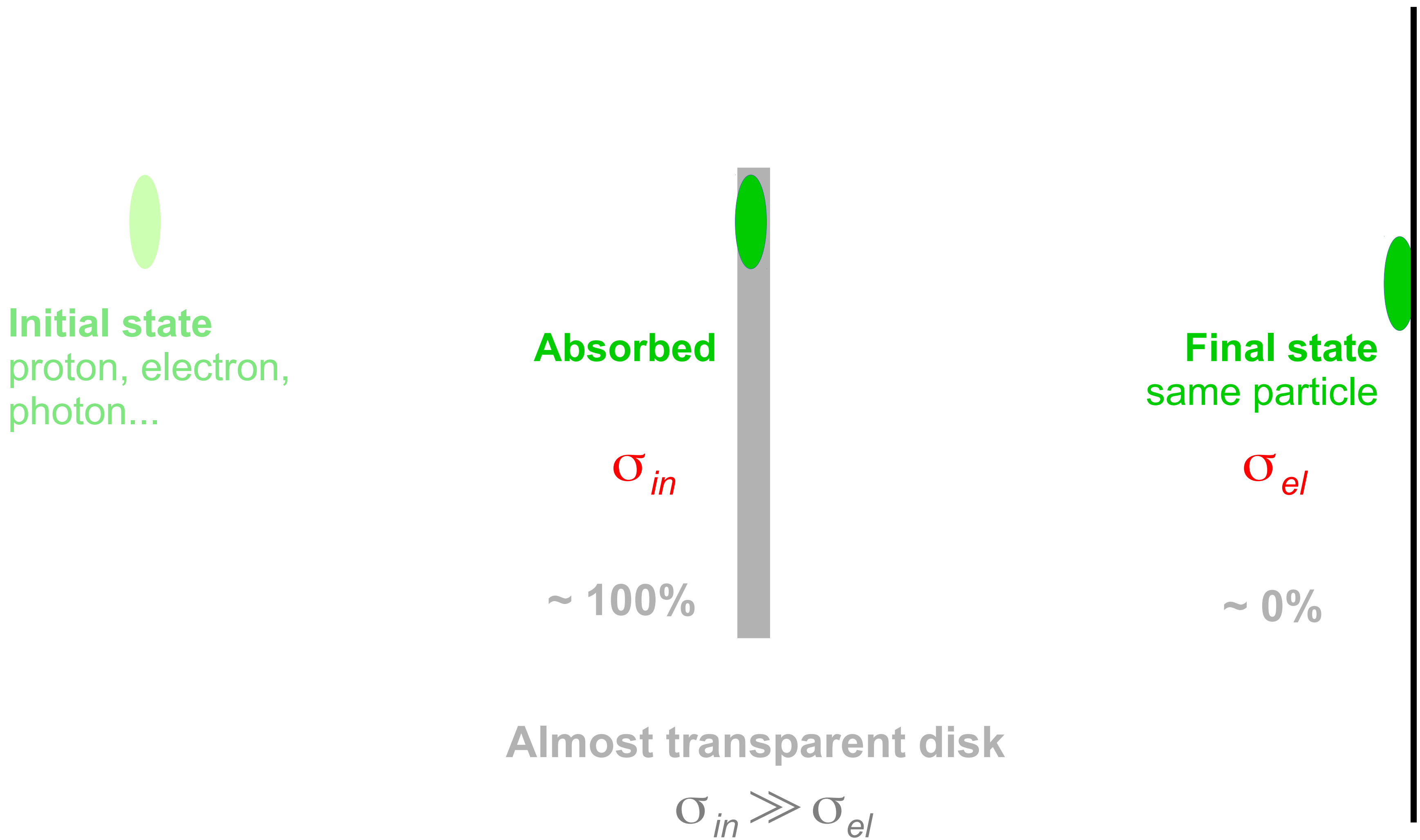
Diffraction in quantum mechanics



Diffraction in quantum mechanics

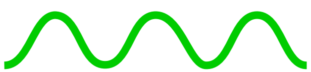


Diffraction in quantum mechanics

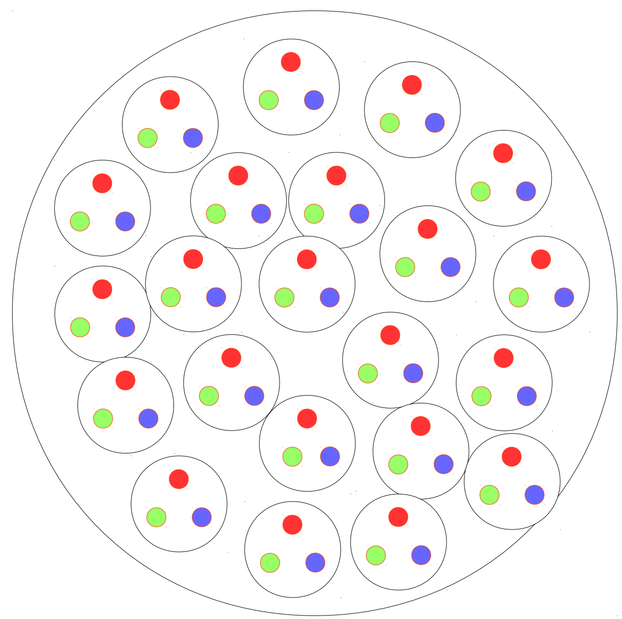


To have diffraction, one needs strong absorption!

Diffraction in particle physics

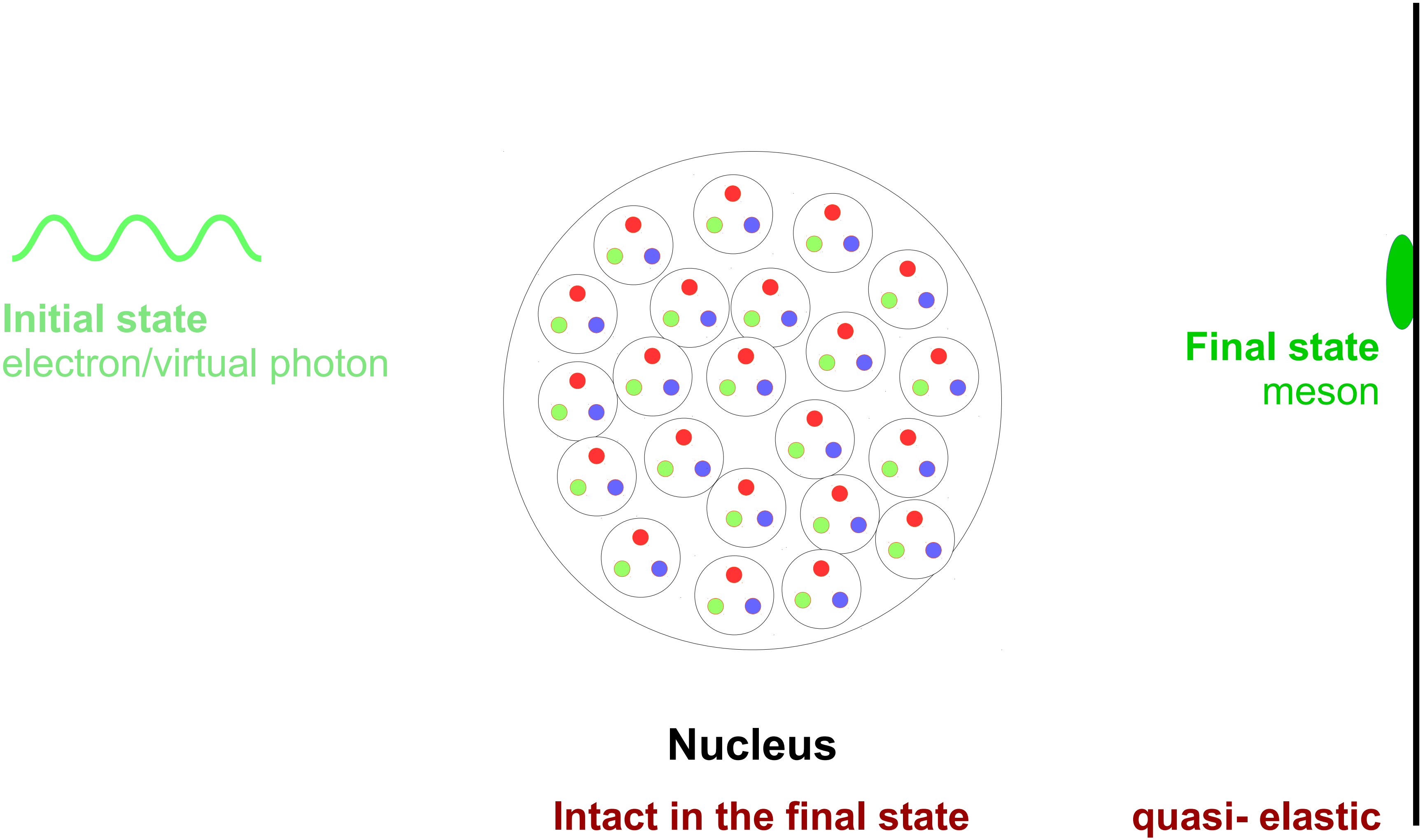


Initial state
electron/virtual photon

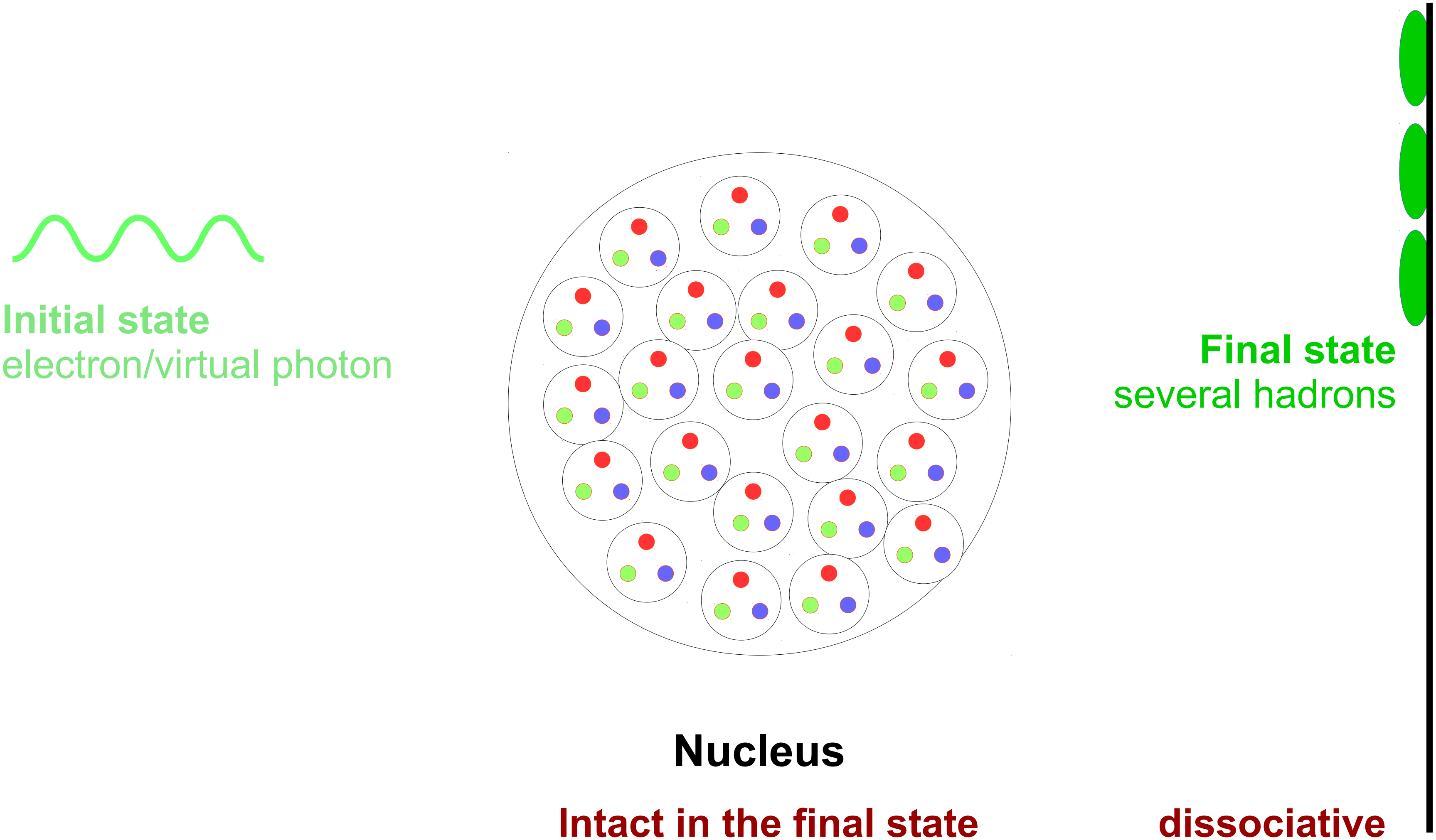


Nucleus

Diffraction in particle physics

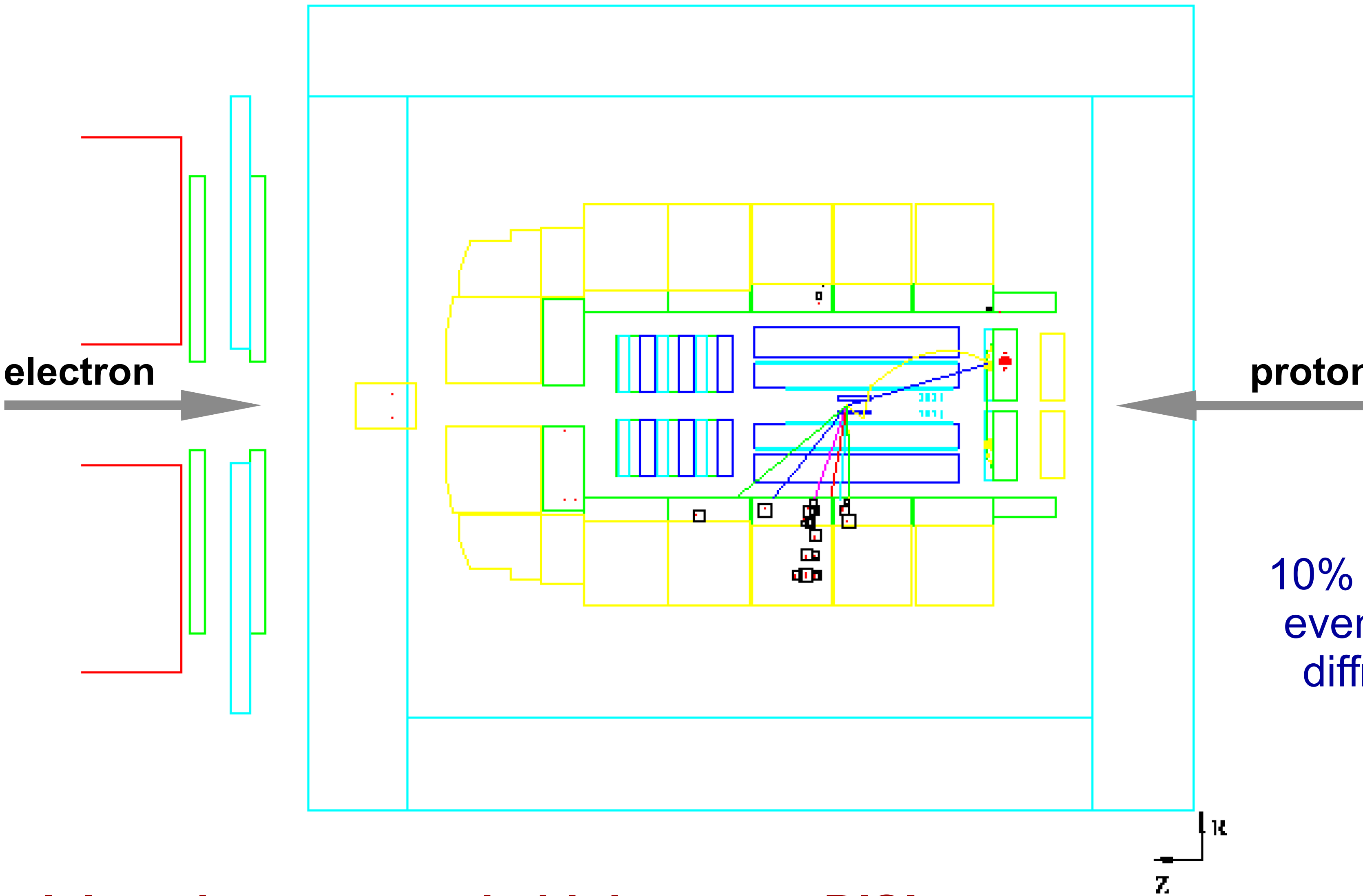


Diffraction in particle physics



Diffraction in deep-inelastic scattering

A major highlight of HERA!



10% of HERA events were diffractive!

Surprising phenomenon in high-energy DIS!

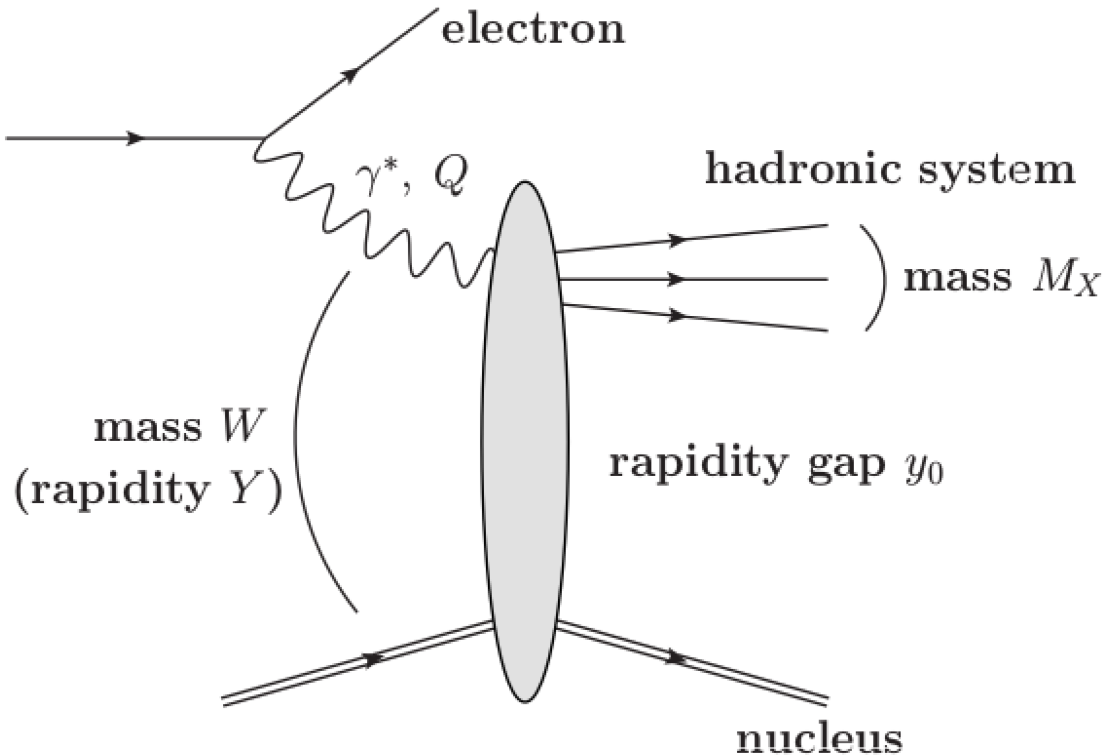
Its existence seems almost contradictory with the parton model...

Its observation boosted saturation physics!

[Golec-Biernat, Wüsthoff 1998]

Diffraction in deep-inelastic scattering

This talk: Diffractive dissociative electron-nucleus scattering



What is the distribution of the rapidity gap y_0?

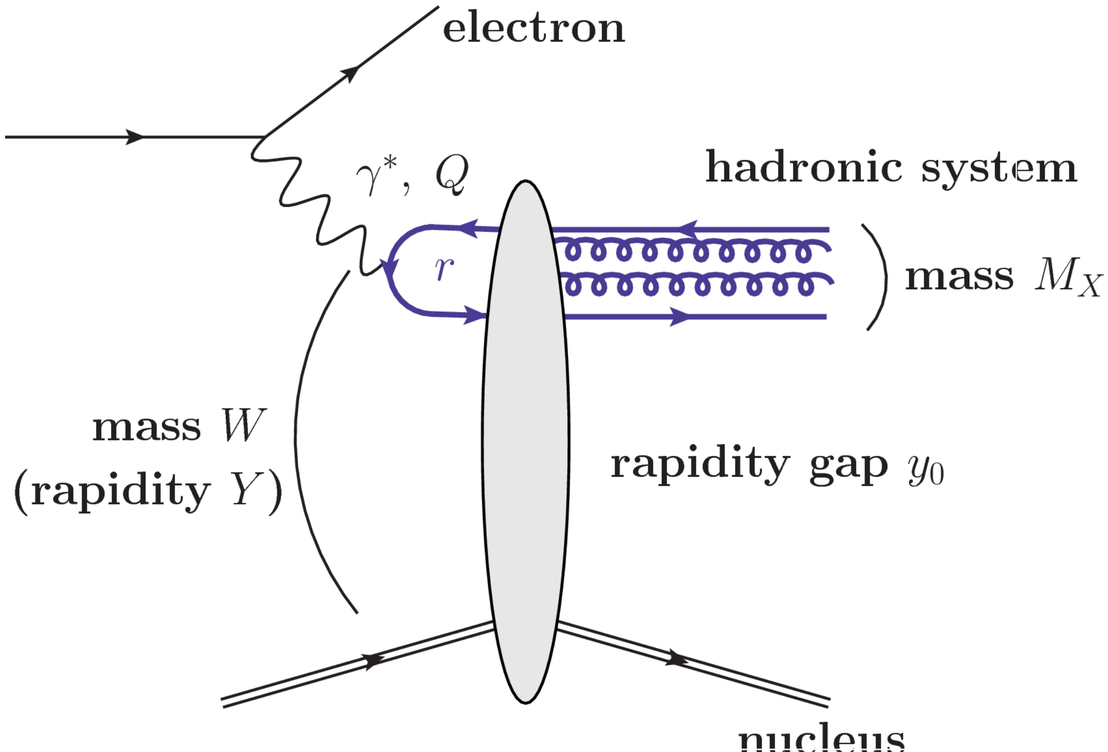
$$\frac{d\sigma_{diff}}{dy_0}$$

Diffraction in deep-inelastic scattering

This talk: Diffractive dissociative electron-nucleus scattering

~ $q\,\bar{q}$ dipole-nucleus

[Nikolaev,Zakharov '90
Golec-Biernat, Wüsthoff 1998]



What is the distribution of the rapidity gap y_0?

$$\frac{d\,\sigma_{diff}}{dy_0}$$

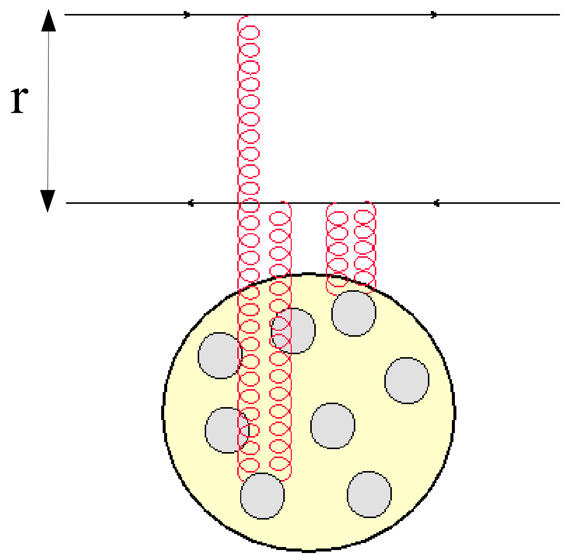
Outline

- Equation for the gap distribution
- Picture of onium-nucleus scattering
- Ancestry in branching random walks
- Numerical checks

Equation for the gap distribution

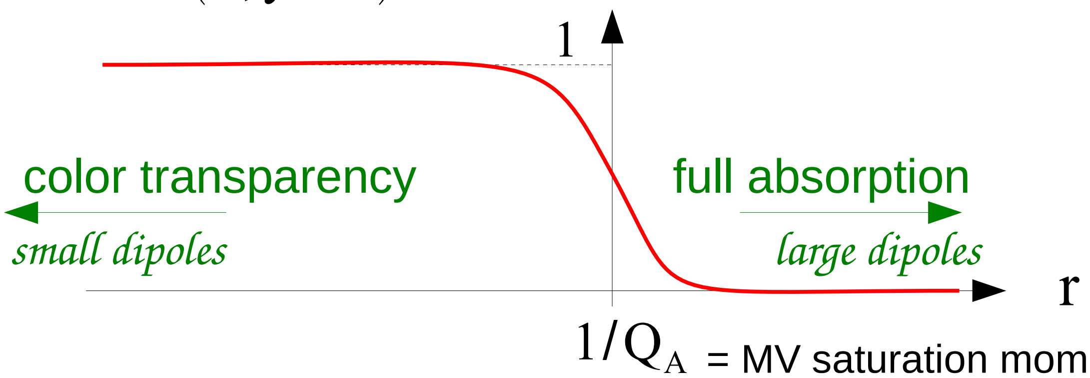
1)Total amplitude: the Balitsky-Kovchegov equation

Forward elastic S-matrix element for dipole-nucleus scattering: $\sigma_{tot}=2\left(1-S\right)$



$$S\left(r,y=0\right)=e^{-\frac{r^2Q_A^2}{4}}$$

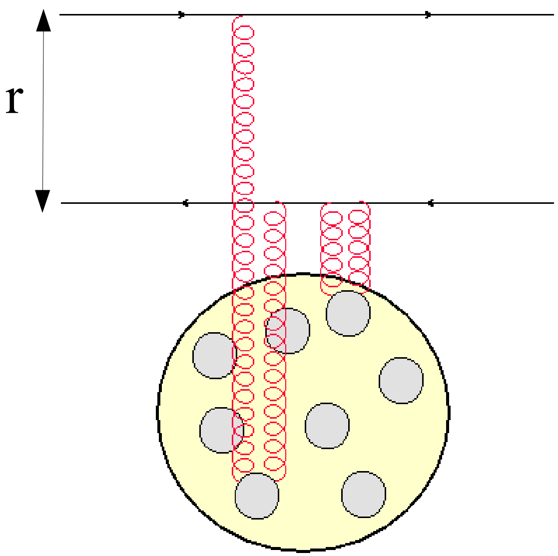
[McLerran, Venugopalan]



Equation for the gap distribution

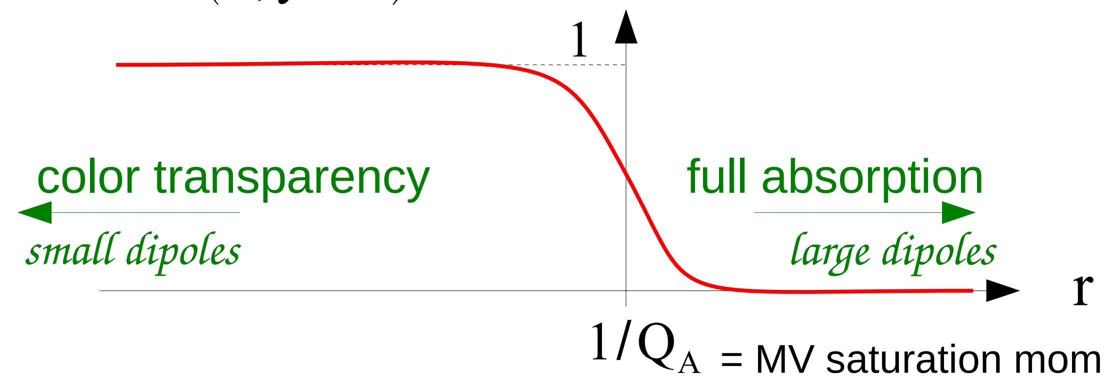
1)Total amplitude: the Balitsky-Kovchegov equation

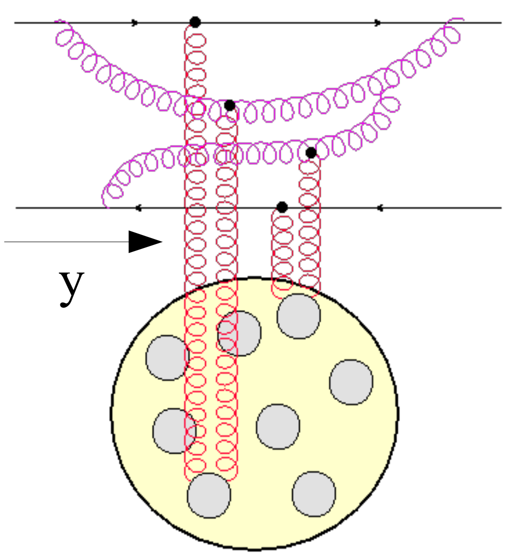
Forward elastic S-matrix element for dipole-nucleus scattering: $\sigma_{tot} = 2(1-S)$



$$S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

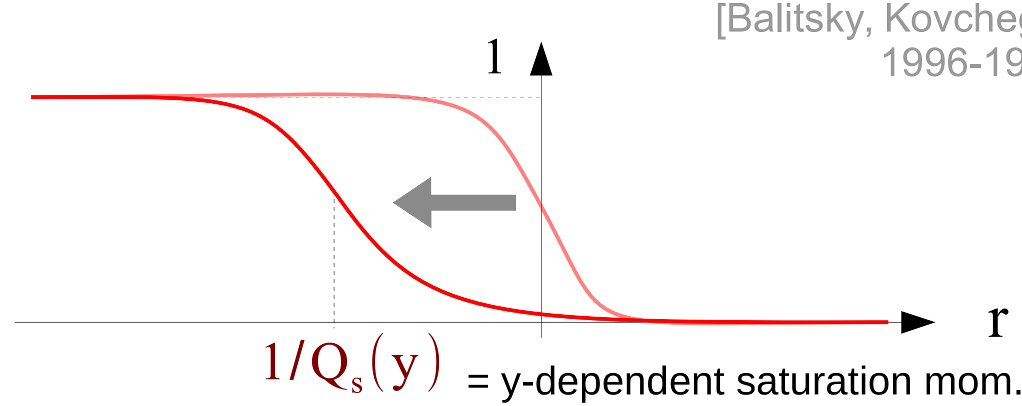
[McLerran, Venugopalan]





$$\partial_y S(r,y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \left[S(r',y) S(r-r',y) - S(r,y) \right]$$

[Balitsky, Kovchegov 1996-1999]



Equation for the gap distribution

2) Diffractive cross section: the Kovchegov-Levin equation

[Kovchegov, Levin 2000
Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos 2006]

$$S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

$$\partial_y S(r,y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \left[S(r',y) S(r-r',y) - S(r,y) \right]$$

Equation for the gap distribution

2) Diffractive cross section: the Kovchegov-Levin equation

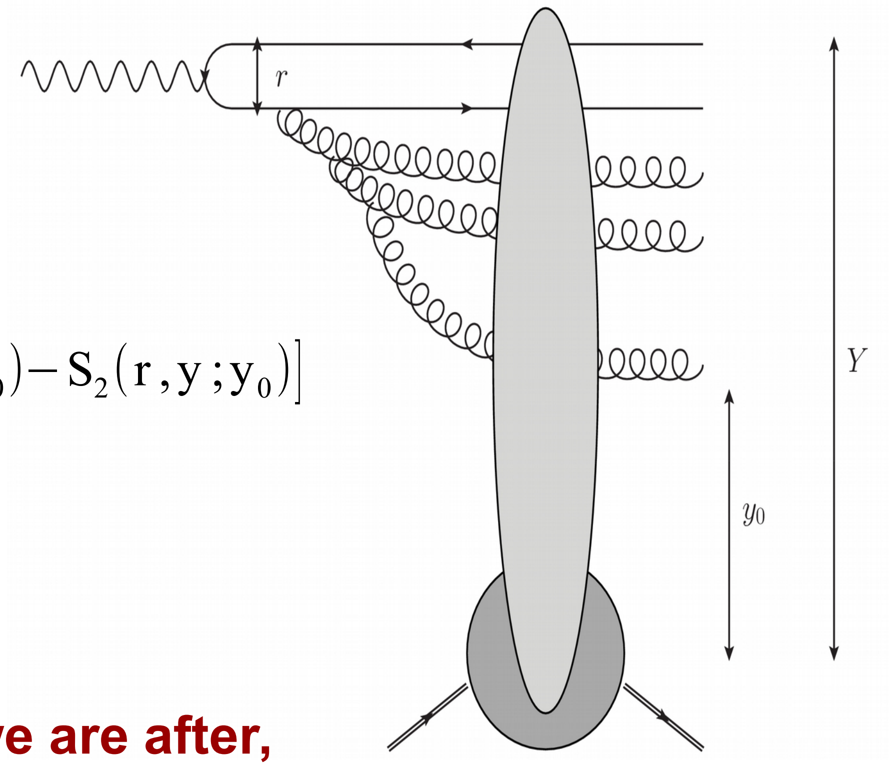
[Kovchegov, Levin 2000
Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos 2006]

$$S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

$$\partial_y S(r,y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \left[S(r',y) S(r-r',y) - S(r,y) \right]$$

Define S_2 as $$S_2(r, y_0; y_0) = \left[S(r, y_0) \right]^2$$

for $y>y_0$: $$\partial_y S_2(r, y; y_0) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \times \left[S_2(r', y; y_0) S_2(r-r', y; y_0) - S_2(r, y; y_0) \right]$$



$$\frac{d\sigma_{diff}}{dy_0} = -\frac{\partial}{\partial y_0} S_2(r, Y; y_0)$$

The solution to these equations is what we are after, but they are difficult to solve!

Outline

- ✯ Equation for the gap distribution
- ✯ Picture of onium-nucleus scattering
- ✯ Ancestry in branching random walks
- ✯ Numerical checks

Picture of onium-nucleus scattering

Total cross section

$$\sigma_{tot}(r,Y)=2[1-S(r,Y)] \qquad \partial_y S(r,y)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}[S(r',y)S(r-r',y)-S(r,y)]$$

Solution:

$$Q_s^2(Y)\simeq Q_A^2\frac{e^{\text{const}\times\bar{\alpha}Y}}{(\bar{\alpha}Y)^{3/2\gamma_0}} \qquad 1-S\underset{rQ_s(Y)\ll 1}{\widetilde{\;}}\ln\frac{1}{r^2Q_s^2(Y)}\underset{(\gamma_0\simeq 0.63)}{[r^2Q_s^2(Y)]^{\gamma_0}} \ll 1$$

Picture of onium-nucleus scattering

Total cross section

$$\sigma_{tot}(r,Y)=2[1-S(r,Y)] \qquad \partial_y S(r,y)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}[S(r',y)S(r-r',y)-S(r,y)]$$

Solution: $Q_s^2(Y)\simeq Q_A^2\frac{e^{\text{const}\times\bar{\alpha}Y}}{(\bar{\alpha}Y)^{3/2\gamma_0}}$ $\qquad 1-S \underset{rQ_s(\widetilde{Y})\ll 1}{\ln}\frac{1}{r^2Q_s^2(Y)}[r^2Q_s^2(Y)]^{\gamma_0} \ll 1$

$(\gamma_0\simeq 0.63)$

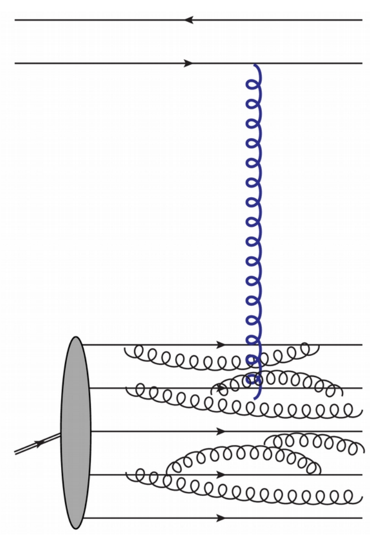
Picture of onium-nucleus scattering

Total cross section

$$\sigma_{tot}(r,Y)=2[1-S(r,Y)] \qquad \partial_y S(r,y)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}[S(r',y)S(r-r',y)-S(r,y)]$$

Solution: $Q_s^2(Y)\simeq Q_A^2\frac{e^{\text{const}\times\bar{\alpha}Y}}{(\bar{\alpha}Y)^{3/2\gamma_0}}$ $\quad 1-S \underset{rQ_s(\widetilde{Y})\ll 1}{\sim} \ln\frac{1}{r^2Q_s^2(Y)}[r^2Q_s^2(Y)]^{\gamma_0} \ll 1$ $\quad (\gamma_0\simeq 0.63)$

Onium restframe



1-S is like the transparency
of the boosted nucleus

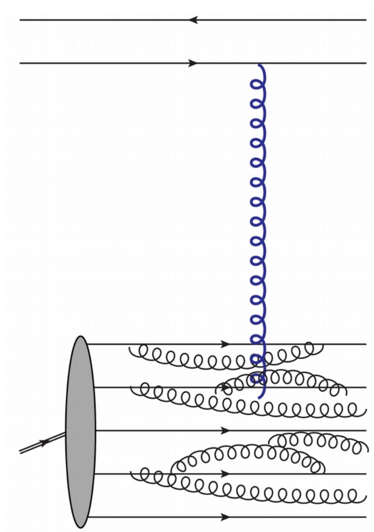
Picture of onium-nucleus scattering

Total cross section

$$\sigma_{tot}(r,Y)=2[1-S(r,Y)] \qquad \partial_y S(r,y)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}[S(r',y)S(r-r',y)-S(r,y)]$$

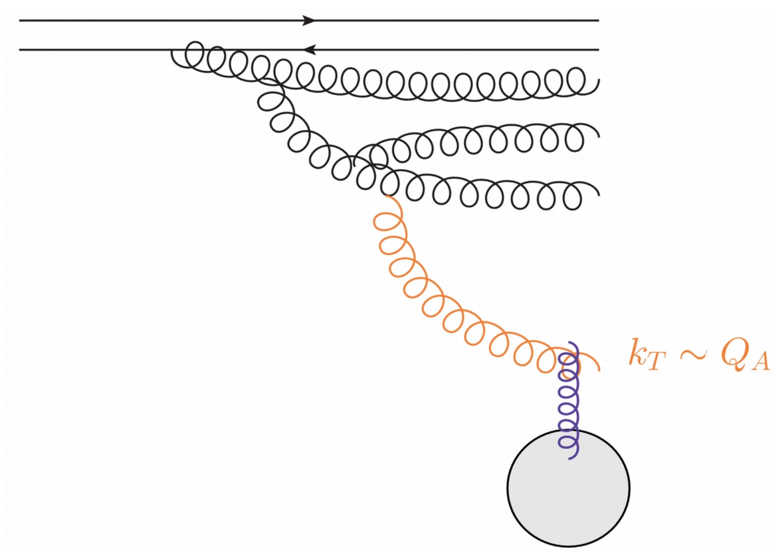
Solution: $Q_s^2(Y)\simeq Q_A^2\frac{e^{\text{const}\times\bar{\alpha}Y}}{(\bar{\alpha}Y)^{3/2\gamma_0}}$ $\qquad 1-S \underset{rQ_s(\tilde{Y})\ll 1}{\sim} \ln\frac{1}{r^2Q_s^2(Y)}\underbrace{[r^2Q_s^2(Y)]^{\gamma_0}}_{(\gamma_0\simeq 0.63)} \ll 1$

Onium restframe



1-S is like the transparency of the boosted nucleus

Nucleus restframe



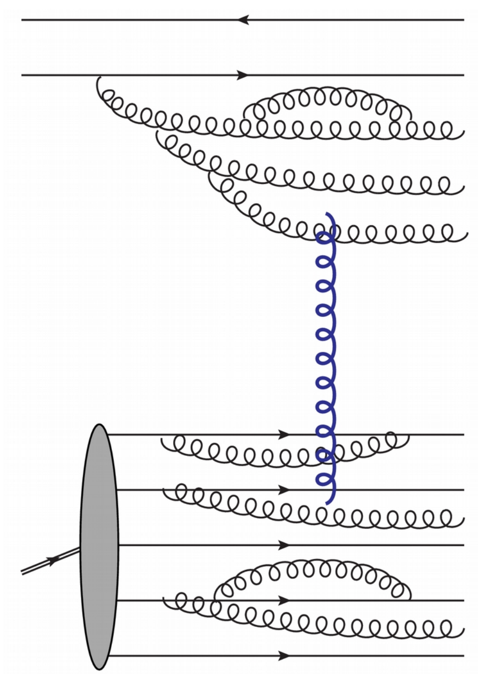
1-S is like the probability to have a gluon of transverse momentum larger than the nuclear saturation momentum in the boosted onium Fock state

Picture of onium-nucleus scattering

Total and diffractive cross section in the y_0 frame

in which the nucleus has rapidity y_0

Total

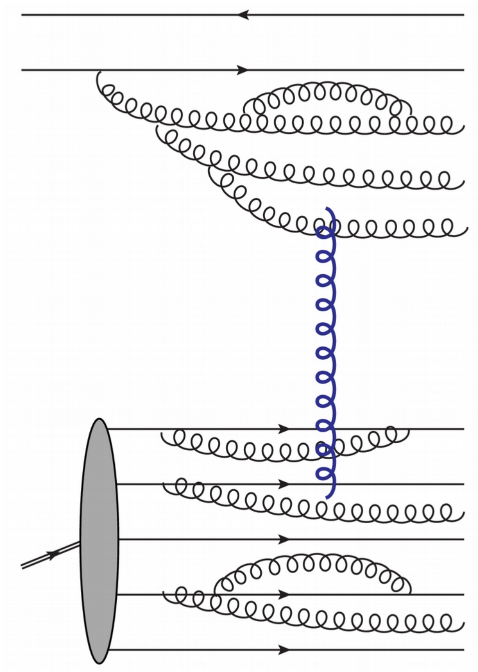


Picture of onium-nucleus scattering

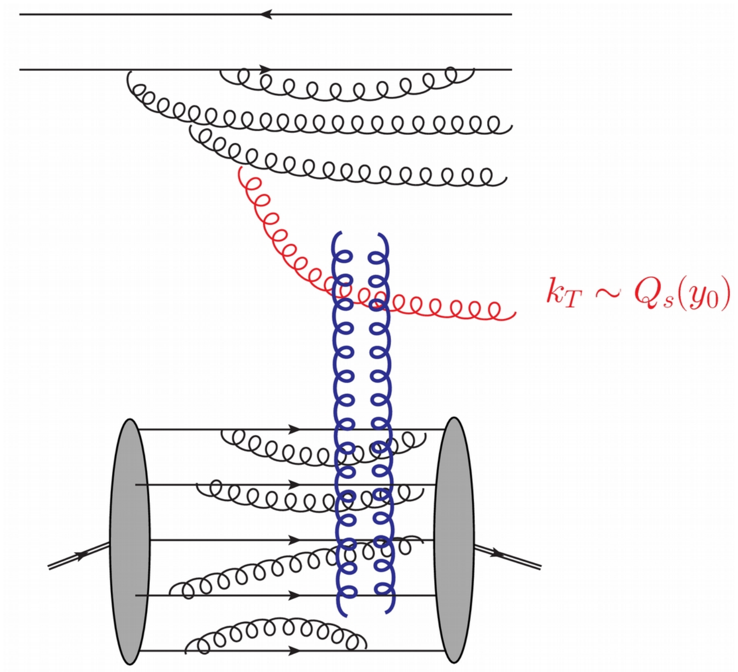
Total and diffractive cross section in the y_0 frame

in which the nucleus has rapidity y_0

Total



Diffractive



The scattering between the state of the onium and the evolved nucleus needs to be elastic

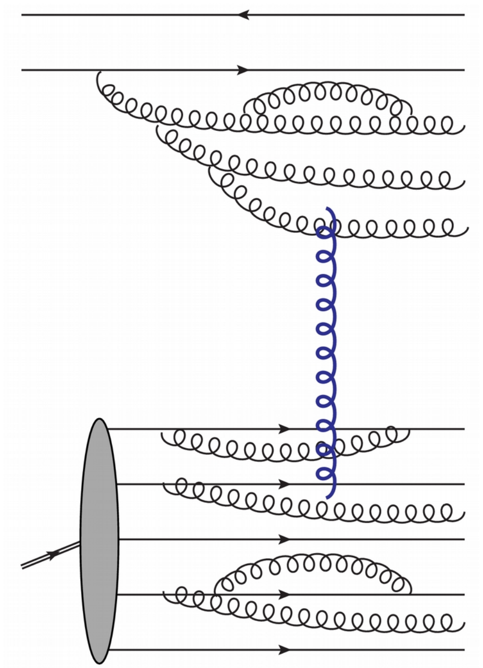
This requires an unusually low-k_T gluon at rapidity y_0

Picture of onium-nucleus scattering

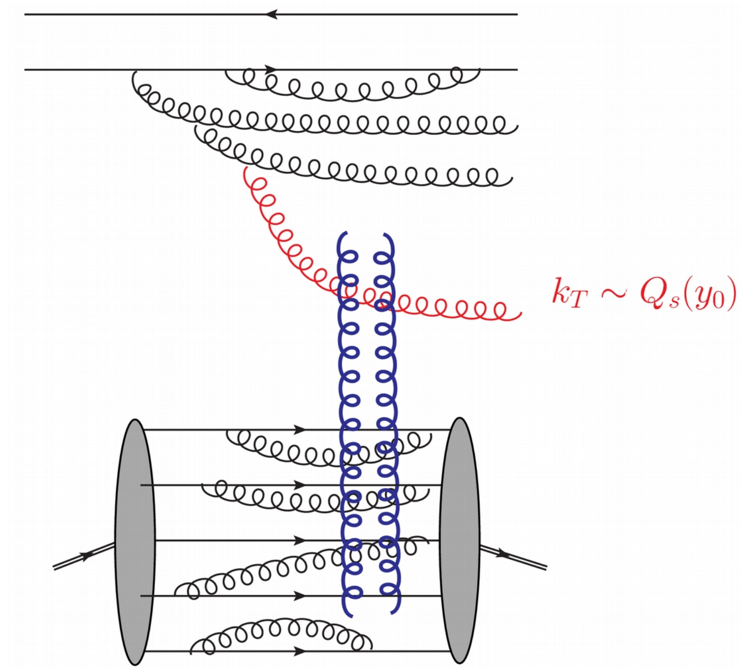
Total and diffractive cross section in the y_0 frame

in which the nucleus has rapidity y_0

Total



Diffractive



The scattering between the state of the onium and the evolved nucleus needs to be elastic

This requires an unusually low-k_T gluon at rapidity y_0

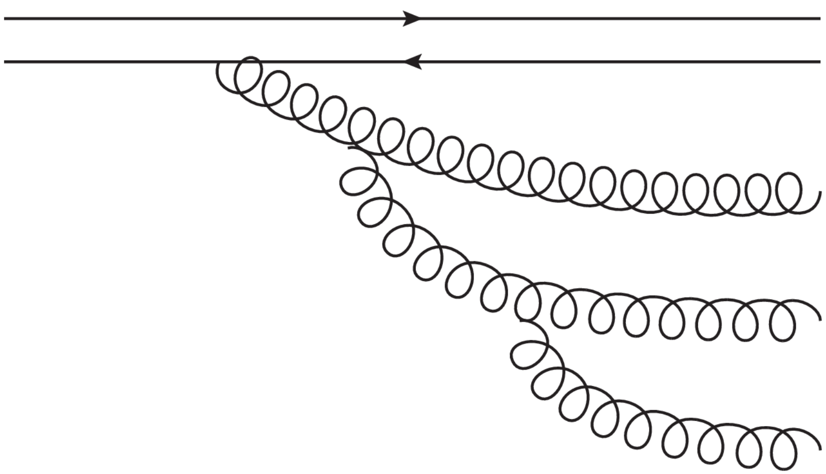
Probability given by the BK equation!

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{diff}}{dy_0} = \text{const} \times \left[\frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha}(Y - y_0))} \right]^{3/2}$$

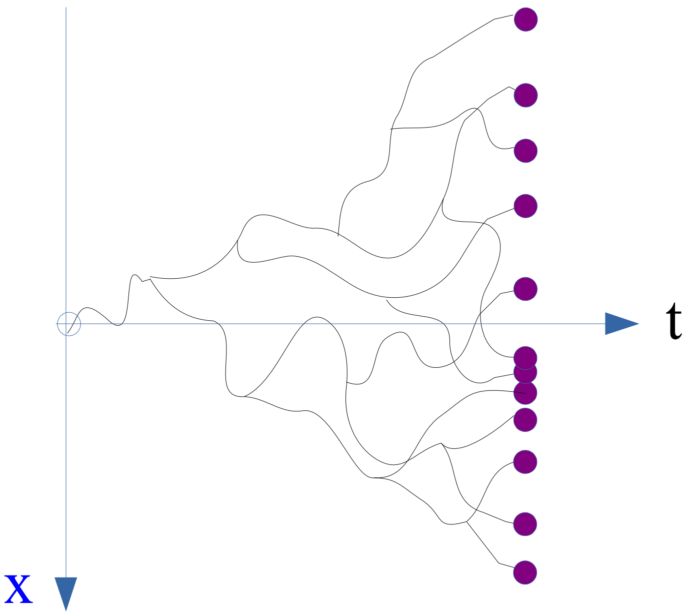
Outline

- ✯ Equation for the gap distribution
- ✯ Picture of onium-nucleus scattering
- ✯ Ancestry in branching random walks
- ✯ Numerical checks

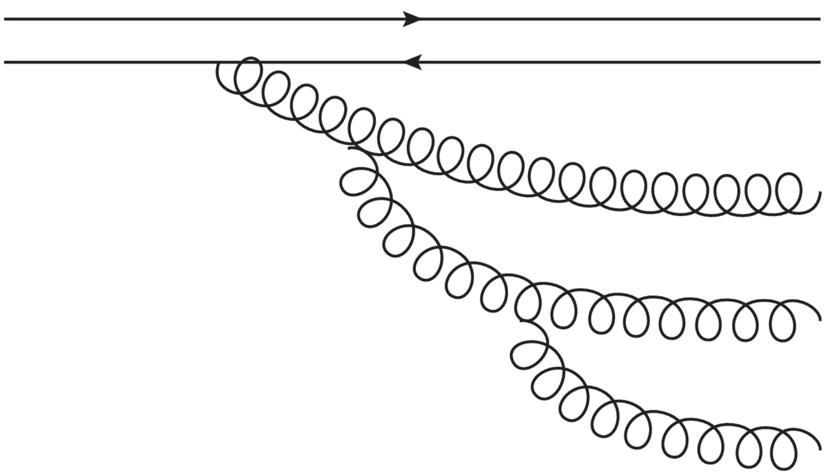
Branching random walks



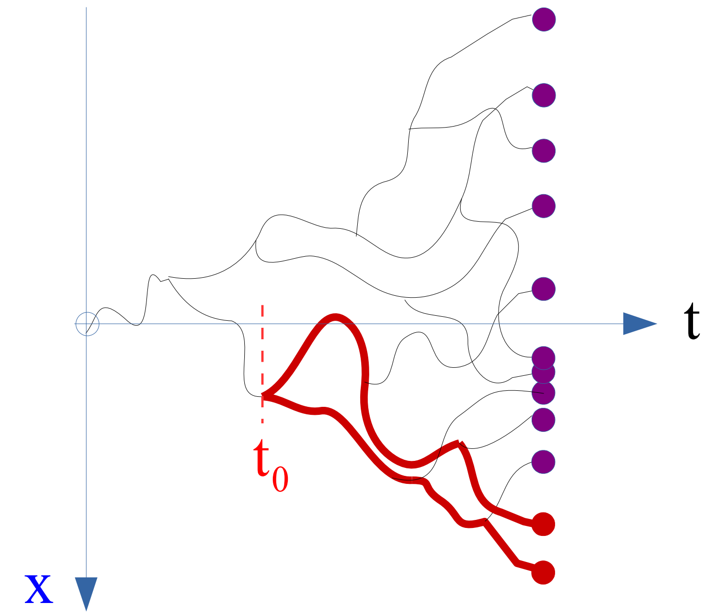
$\simeq$



Branching random walks

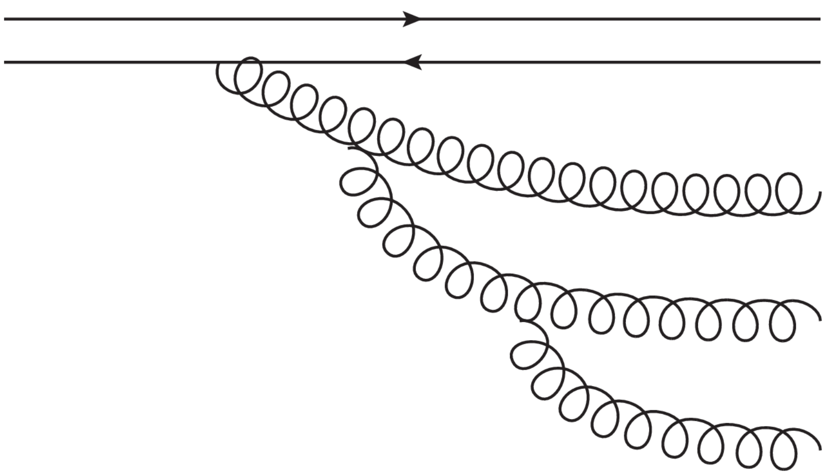


$\simeq$

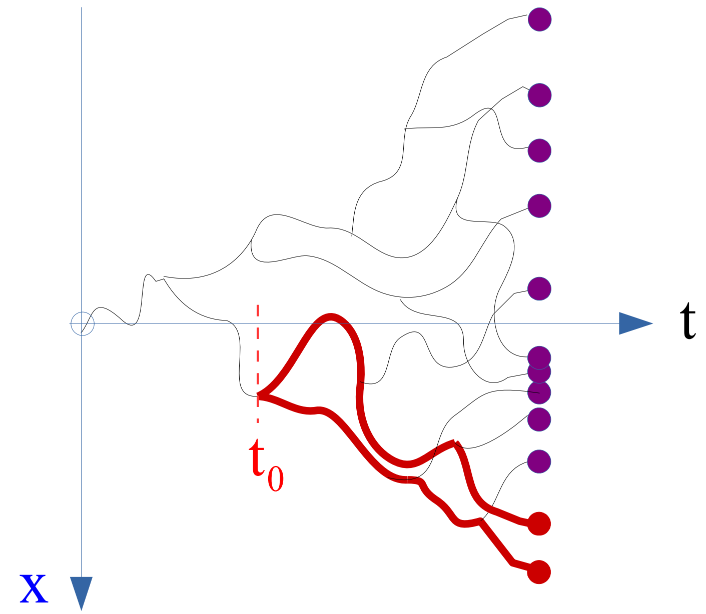


Distribution of decay time of most recent common ancestor?

Branching random walks



≃



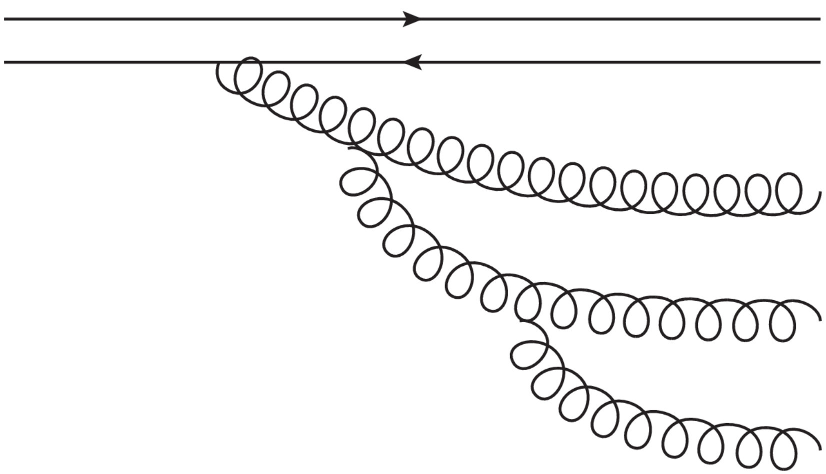
Distribution of decay time of most recent common ancestor?

$$\frac{dp}{dt_0} = \frac{1}{\sqrt{4\pi}} \times \left[\frac{t}{t_0(t-t_0)}\right]^{3/2}$$

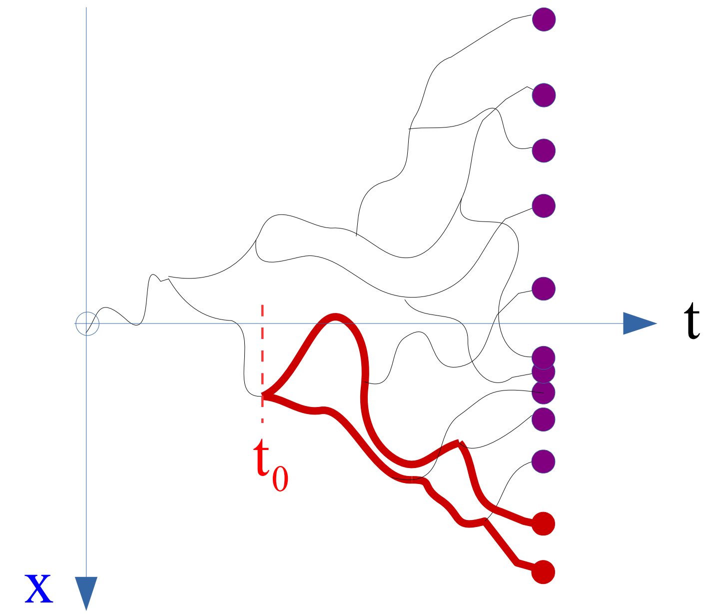
[Derrida, Mottishaw 2016]

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)

Branching random walks



$$\simeq$$



Distribution of rapidity gaps:

$$\frac{1}{\sigma_{tot}}\frac{d\sigma_{\text{diff}}}{dy_0}=\text{const}\times\left[\frac{\bar{\alpha}\,Y}{\bar{\alpha}\,y_0\left(\bar{\alpha}\left(Y-y_0\right)\right)}\right]^{3/2}$$

$$\simeq$$

$\bar{\alpha}\,Y \Leftrightarrow t$

$\bar{\alpha}\,y_0 \Leftrightarrow t_0$

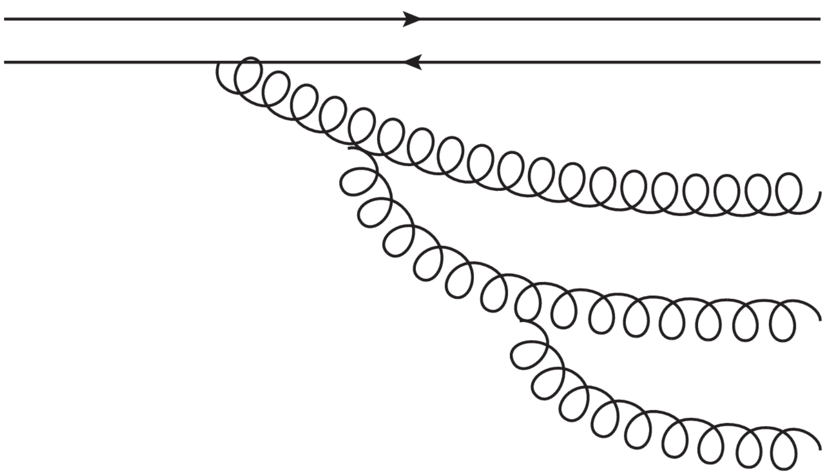
Distribution of decay time of most recent common ancestor?

$$\frac{dp}{dt_0}=\frac{1}{\sqrt{4\pi}}\times\left[\frac{t}{t_0\left(t-t_0\right)}\right]^{3/2}$$

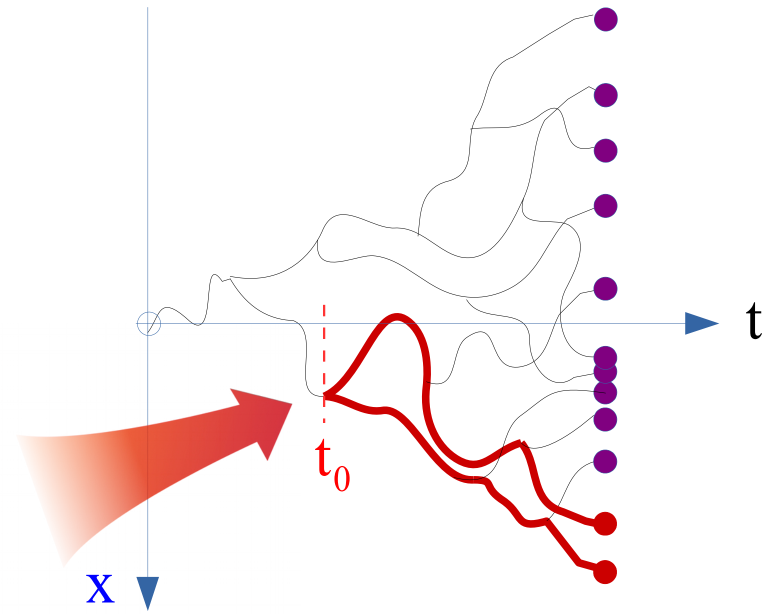
[Derrida, Mottishaw 2016]

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)

Branching random walks



$\simeq$



Distribution of rapidity gaps:

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma_{\text{diff}}}{dy_0}=\text{const}\times\left[\frac{\bar{\alpha}Y}{\bar{\alpha}y_0(\bar{\alpha}(Y-y_0))}\right]^{3/2}$$

$\simeq$

$\bar{\alpha}Y \Leftrightarrow t$

$\bar{\alpha}y_0 \Leftrightarrow t_0$

Distribution of decay time of most recent common ancestor?

$$\frac{dp}{dt_0}=\frac{1}{\sqrt{4\pi}}\times\left[\frac{t}{t_0(t-t_0)}\right]^{3/2}$$

[Derrida, Mottishaw 2016]

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)

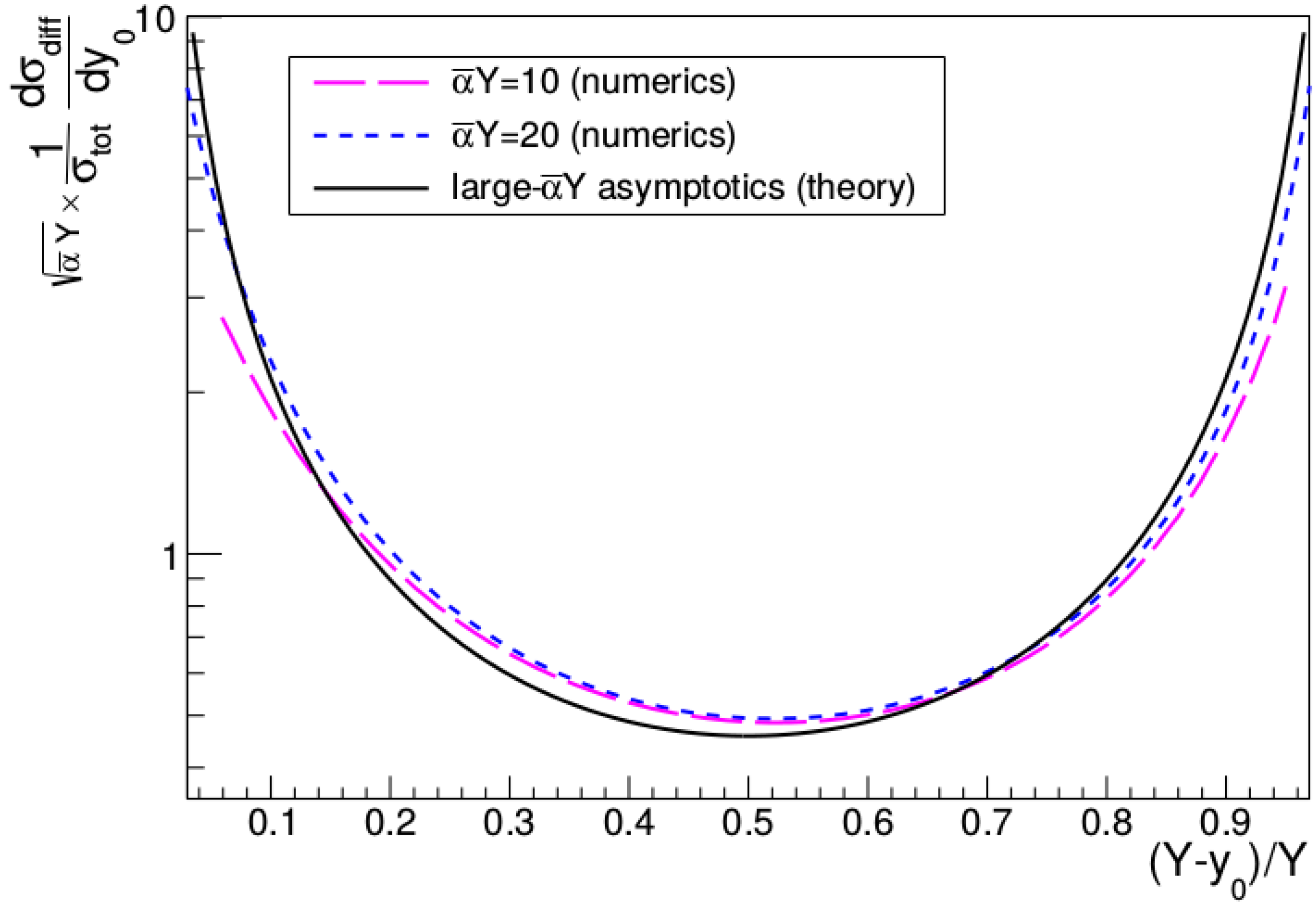
Physical reason for the correspondence:
The common ancestor is an unusually large fluctuation! (here: large value of x)

Outline

- ✯ Equation for the gap distribution
- ✯ Picture of onium-nucleus scattering
- ✯ Ancestry in branching random walks
- ✯ Numerical checks

Comparison to numerics

Very large-Y asymptotics

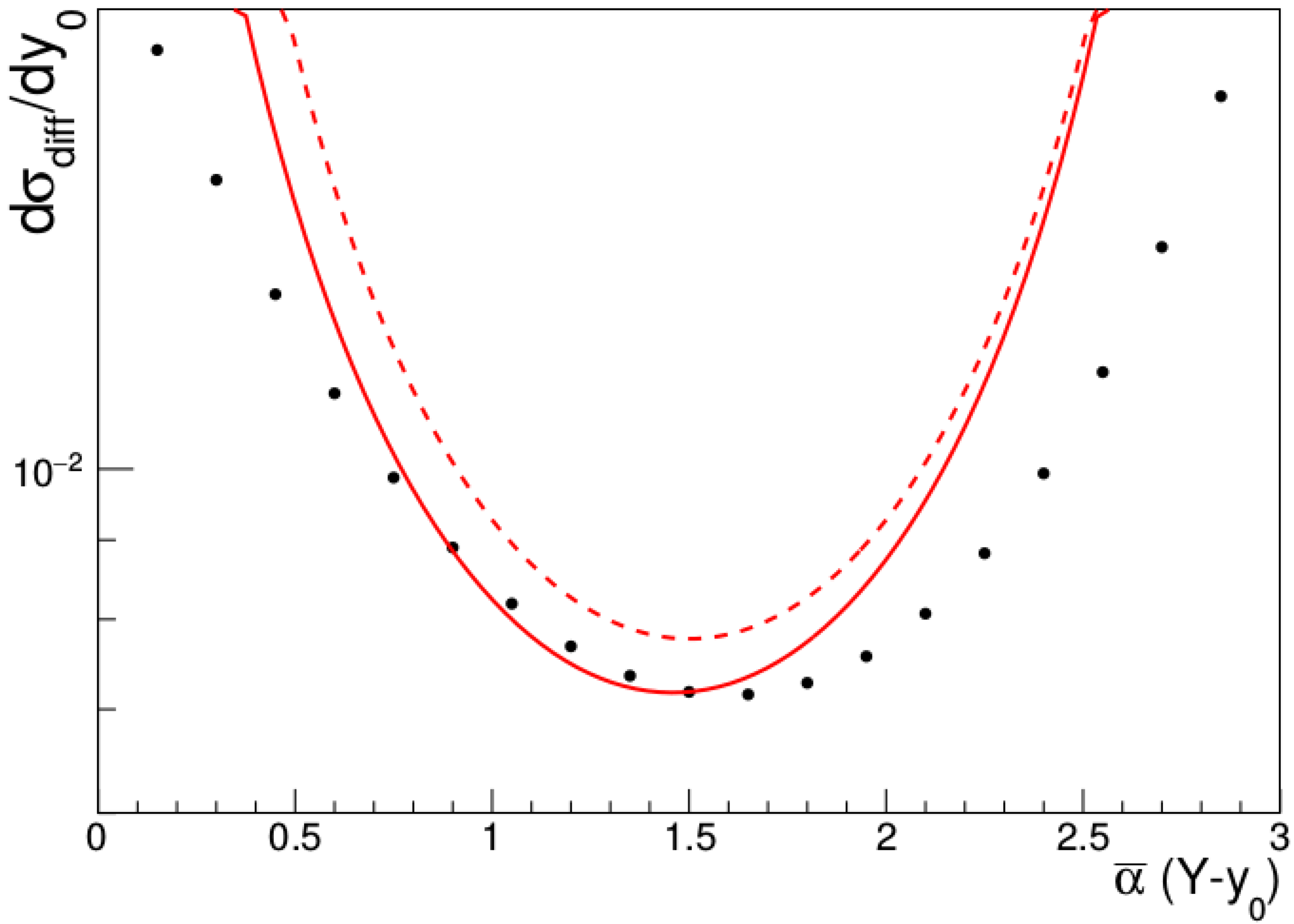


<u>Theory:</u> $\frac{1}{\sigma_{tot}} \frac{d\,\sigma_{diff}}{dy_0} = \text{const} \times \left[\frac{\bar{\alpha}\, Y}{\bar{\alpha}\, y_0 \left(\bar{\alpha} \left(Y - y_0 \right) \right)} \right]^{3/2}$

<u>Numerics</u>: Solution to the Kovchegov-Levin eq.

Comparison to numerics

Realistic rapidities (EIC) $\bar{\alpha}\,Y = 3$



Theory: $\frac{d\,\sigma_{diff}}{dy_0} \sim \left[\frac{\bar{\alpha}\,Y}{\bar{\alpha}\,y_0\left(\bar{\alpha}\left(Y-y_0\right)\right)}\right]^{3/2}$

Numerics: Solution to the Kovchegov-Levin eq.

Summary

- We have predicted, up to an overall constant, the rapidity gap distribution in diffractive dissociation of a virtual photon off a nucleus:

$$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma_{\mathrm{diff}}}{dy_0}=\mathrm{const}\times\left[\frac{\bar{\alpha}\,Y}{\bar{\alpha}\,y_0\left(\bar{\alpha}\left(Y-y_0\right)\right)}\right]^{3/2}$$

Partonic interpretation: the rapidity gap is due to a **large fluctuation** (unusually low-transverse momentum gluon) in the course of the QCD evolution of the onium Fock state

- There is a deep analogy between this distribution and the distribution of the time at which common ancestors of extreme particles in branching random walks decay

Outlook

- We may have an even closer analogy between diffraction and genealogy: The overall constant in the distributions may be the same!
Work in progress...