J.G. Körner

Institute of Physics
PRISMA Cluster of Excellence,
University of Mainz

QCD@Work
International Workshop on QCD Theory and Experiment
25 - 28 June 2018 Matera, Italy

NLO and NNLO corections to polarized top quark decays

Acknowledgements

Thanks to my collaborators A. Czarnezki (Alberta), M. Fischer (Mainz), S. Groote (Tartu) and J. Piclum (Siegen).

In the time alloted to this talk I cannot review the subject of polarized top decays in any depth. Instead, I will take the opportunity to report on the achievements of our group in this field. I will share with you my insights into the problem, why we did the calculations and how we did them.

Motivation

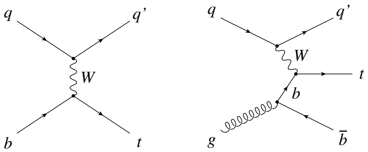

Figure: Weak production of single top quarks.

- **Singly produced top quarks are highly polarized ($P_t \sim 90\%$)**
- **Production rates of singly produced top quarks**
  - **Up to date the LHC detectors have seen $\sim 10^7$ singly produced top quarks**
  - **The projected luminosity of the HL-LHC is 3 ab^{-1} which corresponds to $\sim 10^9$ singly produced polarized top quarks**
- **Top quarks retain their polarization at birth when it decays since the top quark decays so rapidly**

Polarized top decay in the three-body decay (TBD) approach

The decay $t \to b + \mu^+ + \nu_\mu$ is described by

$$M = \bar{u}(b)\gamma^\mu(1-\gamma_5)u(t)\,\bar{u}(\mu)\gamma_\mu(1-\gamma_5)v(\nu)$$

After a Fierz transformation of the second kind one has

$$M = 2\bar{u}(b)(1+\gamma_5)v(\nu)\bar{u}(\mu)(1-\gamma_5)u(t)$$

The polarized angular decay distribution reads (set $|\vec{P}_t| = 1$)

$$\begin{aligned} W^P(\cos\theta_P) &= tr\{(\not{p}_t + m_t)(1+\gamma_5\not{s}_t)\not{p}_\mu\} \\ &= 4m_t|\vec{p}_\mu|(1+\cos\theta_P) \end{aligned}$$

where θ_P is the angle between the muon momentum direction and the polarization vector of the top quark. Positivity of the rate is (barely!) guaranteed at the Born term level!

Polarized top decay in the three-body decay (TBD) approach cont'd

At $O(\alpha_s)$ this becomes

$$W^P(\cos\theta_P) \sim \Big((1-8.54\%) + (1-8.71\%)\cos\theta_P\Big)$$

Again, positivity is (barely!) guaranteed at $O(\alpha_s)$!

When one adds azimuthal dependencies one can translate this sensitivity into tight bounds on the size of New Physics (NP) contributions.

Azimuthal dependence

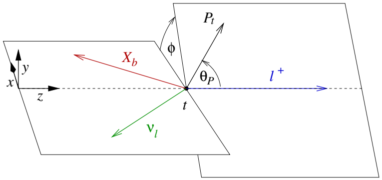

Figure: Definition of the polar angle θ_P and the azimuthal angle ϕ.

The full angular decay distribution for polarized top quark decay $t(\uparrow) \to b + \ell^+ + \nu_\ell$ in the top quark rest frame is given by

$$\frac{d\Gamma}{d\cos\theta d\phi} = A + B\,P_t\cos\theta_P + C\,P_t\sin\theta_P\cos\phi + D\,P_t\sin\theta_P\sin\phi$$

$$= A\Big(1 + \frac{B}{A}\,P_t\cos\theta_P + \frac{C}{A}P_t\sin\theta_P\cos\phi + \frac{D}{A}P_t\sin\theta_P\sin\phi\Big)$$

Remember $B/A = 1$ at NLO which implies $C/A = 0$ and $D/A = 0$ as confirmed by explicite calculation.

Number of structure functions

There are two complex amplitudes that describe polarized top quark decays:

$$M_{\lambda_t=1/2} \qquad M_{\lambda_t=-1/2}$$

$$\begin{aligned} A &\sim |M_{1/2}|^2 + |M_{-1/2}|^2 & B &\sim |M_{1/2}|^2 - |M_{-1/2}|^2 \\ C &\sim \mathrm{Re}\, M_{1/2} M^*_{-1/2} & D &\sim \mathrm{Im}\, M_{1/2} M^*_{-1/2} \end{aligned}$$

$D = \mathrm{Im}\, M_{1/2} M^*_{-1/2}$ **is a T-odd observable. D multiplies a T-odd angular dependence as can be seen by writing (see Figure)**

$$\sin\theta_P \sin\phi \propto \vec{p}_\nu \cdot (\vec{p}_\ell \times \vec{s}_t)$$

In the following we will concentrate on the T-odd observable D.

Absorptive parts of electroweak one-loop amplitudes

There is a SM source for the T-odd observable D from the absorptive parts of electroweak one-loop amplitudes. The diagrams are

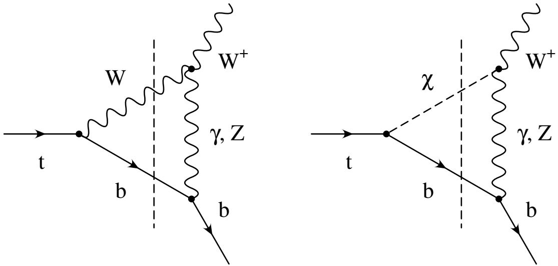

Figure: Absorptive parts of the four Feynman diagrams that contribute to T-odd correlations in polarized top quark decays

The T-odd contributions from these diagrams are rather small [Mainz, Tartu].

Bounds on the T-odd observable D

Effective Lagrangian for the $b \to c$ transition:

$$\mathcal{J}^{\mu}_{\rm eff} = -\frac{g_w}{\sqrt{2}}\bar{b}\Big\{\gamma_\mu((V^*_{tb} + f_L)P_L + f_R P_R) + \frac{i\sigma^{\mu\nu}q_\nu}{m_W}(g_L P_L + g_R P_R)\Big\}t$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The SM structure of the tbW^+ vertex is obtained by dropping all terms except for the contribution proportional to $V^*_{tb} \sim 1$.

We set $P_t = 1$ and $\sin\phi = \pm 1$, and expand around $\theta_P = \pi$. For $m_b = 0$ one then obtains the bounds

- **LO:** $$\mathrm{Im}\, g_R = 0$$
- **$O(\alpha_s)$:**

$$-0.0420 \leq \mathrm{Im}\, g_R \leq 0.0420$$

compared to the ATLAS bound

$$-0.18 \leq \mathrm{Im}\, g_R \leq 0.06$$

We mention that the electroweak T-odd contribution is

$$\mathrm{Im}\, g_R(\gamma + Z) = -2.175 \times 10^{-3}$$

Two-stage sequential two-body decays $t(\uparrow) \to X_b + W^+$ followed by $W^+ \to \ell^+ + \nu_\ell$

There are two ways to describe polarized top quark decays.

- **The three-body decay $t(\uparrow) \to X_b + \ell^+ + \nu_\ell$ (4 observables)**
- **The two-body decay $t(\uparrow) \to X_b + W^+$ followed by $W^+ \to \ell^+ + \nu_\ell$ (10 observables)**

The count is best done by considering the independent spin density matrix elements $H^{\lambda_t\,\lambda'_t}_{\lambda_W\,\lambda'_W}$ of the W^+ which form a hermitian (3×3) matrix

$$\left(H^{\lambda_t\,\lambda'_t}_{\lambda_W\,\lambda'_W} \right)^\dagger = \left(H^{\lambda'_t\,\lambda_t}_{\lambda'_W\,\lambda_W} \right)$$

There are ten independent double spin density matrix elements

$$H^{++}_{++},\ H^{--}_{++},\ H^{++}_{--},\ H^{--}_{--},\ H^{++}_{00},\ H^{--}_{00},\ \mathrm{Re}\,H^{+-}_{+0},\ \mathrm{Im}\,H^{+-}_{+0},\ \mathrm{Re}\,H^{-+}_{-0},\ \mathrm{Im}\,H^{-+}_{-0}.$$

One has 8 T-even and 2 T-odd observables.

Two-stage sequential two-body decays $t(\uparrow) \to X_b + W^+(\to \ell^+ + \nu_\ell)$

The two-stage sequential two-body decay process is described by two polar angles θ and θ_P, and the azimuthal angle ϕ.

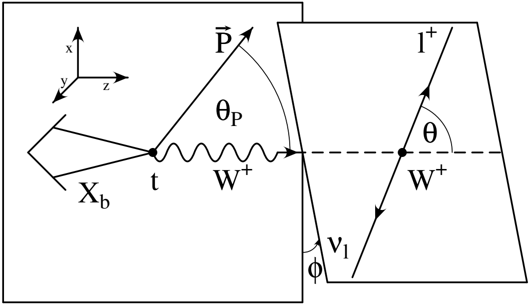

Figure: Definition of the polar angles θ and θ_P, and the azimuthal angle ϕ in the two-stage sequential two-body decay $t(\uparrow) \to X_b + W^+(\to \ell^+ + \nu_\ell)$.

The two-fold polar angular decay distribution

Integrating over the azimuthal angle ϕ one obtains

$$\frac{1}{\hat{\Gamma}}\frac{d\hat{\Gamma}}{d\cos\theta_P d\cos\theta} = \frac{1}{2}\left\{\frac{3}{8}\Big(1+\cos\theta\Big)^2\Big(\hat{\Gamma}_+ + \hat{\Gamma}_+^P P_t\cos\theta_P\Big) + \frac{3}{8}\Big(1-\cos\theta\Big)^2\Big(\hat{\Gamma}_- + \hat{\Gamma}_-^P P_t\cos\theta_P\Big) + \frac{3}{4}\sin^2\theta\Big(\hat{\Gamma}_L + \hat{\Gamma}_L^P P_t\cos\theta_P\Big)\right\}$$

Known results for $m_b = 0$

- **LO:** $\Gamma_- = -\Gamma_-^P$, $\Gamma_L = +\Gamma_L^P$, $\Gamma_+ = 0$ **positivity of rate (barely) satisfied. We have checked at $O(\alpha_s)$ that positivity is fulfilled.**
- **NLO:** $\Gamma_\pm$, Γ_L, $\Gamma_\pm^P$, Γ_L^P **[Mainz]**
- **NNLO:** $\Gamma_+, \Gamma_-, \Gamma_L$ **[Alberta,Mainz]**
  $\Gamma_+^P + \Gamma_-^P + \Gamma_L^P$ **[Alberta,Mainz,Siegen,Tartu]**

Convert two-scale problem (m_t, m_W) to a one scale problem (m_t)

The idea behind our approach is to convert a two-scale problem (m_t, m_W) to a one scale problem (m_t) bei expanding in the ratio $x = m_W/m_t$.

$$\Gamma(m_t,\, m_W) \to \Gamma(m_t,\, \sum a_i\, x^i)$$

In practise we terminate the expansion at $i = 10$. Very good convergence. For example, at NLO one has

$$\begin{aligned}
\hat{\Gamma}^{(1)}_{U+L} &= C_F \Bigg[\frac{5}{4} + \frac{3}{2}x^2 - 6x^4 + \frac{46}{9}x^6 - \frac{7}{4}x^8 - \frac{49}{300}x^{10} + \\
&\quad - 2(1-x^2)^2(1+2x^2)\zeta(2) + \left(3 - \frac{4}{3}x^2 + \frac{3}{2}x^4 + \frac{2}{5}x^6\right) x^4 \ln x \Bigg], \\
\hat{\Gamma}^{(1)}_{(U+L)^P} &= C_F \Bigg[-\frac{15}{4} - \frac{17}{8}x^4 - \frac{1324}{225}x^5 - \frac{31}{36}x^6 + \\
&\quad + \frac{48868}{11025}x^7 - \frac{23}{288}x^8 + \frac{884}{6615}x^9 - \frac{3}{100}x^{10} + (1+4x^2)\zeta(2) \Bigg].
\end{aligned}$$

Elements of the NNLO calculation

One needs the whole arsenal of multi-loop tools

- **The results are obtained from the absorptive parts of three-loop top quark self-energy diagrams.**

$$\Gamma + \Gamma^P = \frac{1}{m_t}\,\mathrm{Im}\,tr\left\{(\not p_t + m_t)(1+\gamma_5 \not s^{\ell}_t)\,\Sigma\right\}$$

  Σ is the sum of 36 top quark self-energy diagrams. For a representative set see next slide.

- **by cutting through the diagrams one can always replace $q^2 = m_W^2$**
- **expansion by regions of the loop momenta**
- **dimensional regularization / γ_5 problem**
- **Check of gauge invariance through use of the covariant R_ξ gauge. Dependence on arbitrary gauge parameter ξ drops out in the final result**
- **We use the unitary gauge for the gauge boson. No need for Goldstone bosons.**
- **We used the method also for a NLO calculation. Full agreement with the results calculated before by us in Mainz using the classical approach (gluon mass IR regulator etc..).**

Sample three-loop QCD diagrams

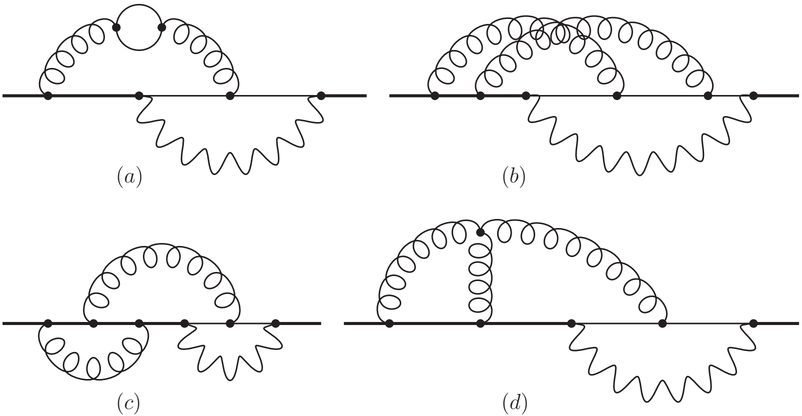

Figure: Sample three-loop diagrams. Thick and thin lines denote top and bottom quarks, respectively. Wavy lines denote W bosons and curly lines denote gluons. In the closed fermion loop all quark flavors have to be considered.

Elements of the NNLO calculation cont'd

- s_t^ℓ is the longitudinal polarization four-vector of the top quark. Need a covariant representation of s_t^ℓ given by

$$s_t^{l,\mu} = \frac{1}{|\vec{q}\,|}\left(q^\mu - \frac{p_t \cdot q}{m_t^2} p_t^\mu\right),$$

The unwieldy denominator factor $|\vec{q}\,|$ comes in through the normalization condition $s_t \cdot s_t = -1$.
Express $|\vec{q}\,|$ through (fictituous) inverse propagator of the top quark $N = (p_t + q)^2 - m_t^2$. Then expand

$$\frac{1}{|\vec{q}\,|} = \frac{2m_t}{N} \sum_{i=0}^{\infty} \binom{2i}{i} \left(\frac{2q^2 N - q^4 + 4m_t^2 q^2}{4\,N^2}\right)^i$$

- Empirical rule: Parity-even structure functions have only even powers of x in the m_W/m_t expansion while parity-odd structure functions feature even and odd powers of x.

Overall result: The numerical evaluation shows that perturbation theory is well behaved.

Thanks

Thanks for your attention!