Hard scale uncertainty in collinear factorization. Perspective from k_t-factorization

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(Simplified) equation (53) in *Factorization of hard processes in QCD (Collins, Soper and Sterman)*

$$F_2(x,Q) = f(x,\mu) + \int_x^1 \frac{d\xi}{\xi} f(\xi,\mu) \frac{\alpha_s}{2\pi} C\left(\frac{x}{\xi}, \frac{Q}{\mu}\right) + \mathcal{O}(\alpha_s^2)$$

with μ the factorization scale and $C()$ the coefficient factor. We ignore the renormalization scale and and take α_s constant

The definition of parton distributions is not unique. Any definitions allowing a correct factorization is acceptable. With the DIS definition one has (equation (58))

$$F_2(x,Q) = f^{DIS}(x,Q) + \mathcal{O}(\alpha_s^2)$$

But finite order observables depend on the factorization scale

$$F_2(x,Q^2;\mu^2) = f^{DIS}(x,Q^2;\mu^2) + \mathcal{O}(\alpha_s^2)$$

Comparing these 2 equations and making the coefficient factor slightly more explicit one has

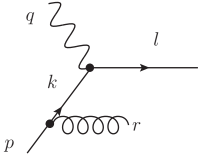
$$f(x,Q^2;\mu^2) = f(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f(\xi,\mu^2)\left(P\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu^2} + C(x)\right)$$

now following the notation in *"QCD and collider physics"*, *R. K. Ellis, W. J. Stirling and B. R. Webber*. The hard scale Q^2 has been included inside the parton density and the cross section gives

$$\frac{d^2\sigma^{NLO}}{dxdQ^2}(x,Q^2;\mu^2) = f(x,Q^2;\mu^2)\hat{\sigma}(x,Q^2)$$

I will keep this notation and the DIS definition in the following.

Remark :
Q^2 appears in the upper bound of the integration on $k_t \Rightarrow \ln\left(\frac{Q^2}{\mu^2}\right)$



Factorization scale and DGLAP equation

$$f(x,Q^2;\mu^2) = f(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f(\xi,\mu^2)\left(P\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu^2} + C(x)\right)$$

The factorization scale is an unphysical scale $\rightarrow$ DGLAP equation

$$\frac{df(x,Q^2;\mu^2)}{d\mu^2} = 0$$

In our case we obtain

$$\mu^2\frac{\partial f(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f(\xi,\mu^2) P\left(\frac{x}{\xi}\right) + \mathcal{O}(\alpha_s^2)$$

The uncertainty on the factorization scale is estimated by $Q^2/2 < \mu^2 < 2Q^2$ and is reduced by higher orders

Collinear and k_t-factorization formula in pp collision

Collinear factorization

$$\frac{d\sigma^{NLO}}{dx_1 dx_2 dp_t^2}(x_1,x_2,p_t^2,Q^2,\mu^2) = f(x_1,Q^2;\mu^2) f(x_2,Q^2;\mu^2) \hat{\sigma}\left(x_1,x_2,p_t^2\right)$$

A choice for the hard scale Q^2 has to be done, conventionally p_t^2 (or $p_t^2 + M^2$ for heavy quarks)

This choice is not necessary within the k_t-**factorization**

$$\frac{d\sigma}{dx_1 dx_1 d^2p_t}(s,x_1,x_2,p_t^2,\mu^2) = \int^{k^2_{max}} d^2k_{1t} d^2k_{2t} F(x_1,k_{1t}^2;\mu^2) F(x_2,k_{2t}^2;\mu^2) \times \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2)$$

with $k^2_{max} > \hat{s}/4$ the kinematical upper bound for k_t

Unintegrated parton densities are related to usual parton densities by

$$f(x,Q^2;\mu^2) = \int^{Q^2} F(x,k_t^2;\mu^2) d^2k_t$$

Goals and results

We want to discuss the choice $Q^2 = p_t^2$. We will show that :

1. It can be justified using k_t-factorization
2. It is accompanied by an uncertainty (not reduced by higher orders)

Remarks :

Collinear fact. $\Rightarrow k_{t,max}^2 \sim \hat{s}$.

But $Q^2 = p_t^2$ means $\int_{\mu^2}^{p_t^2} dk_t^2/k_t^2 = \ln(p_t^2/\mu^2)$

In the kinematical region $\lambda_{QCD}^2 \ll p_t^2 \ll \hat{s}/4$, the choice $Q^2 = p_t^2$ looks wrong

It is in this region that the k_t-factorization gives the larger correction

But in this region, the collinear factorization still gives "accurate" results (and we will explain why)

Relationship between collinear and k_t-factorization

An improvement of *E. M. Levin, M. G. Ryskin, Y. M. Shabelski, A. G. Shuvaev, Sov. J. Nucl.Phys. 54 (1991) 1420*

Split the k_t-factorization into two parts

$$\frac{d\sigma}{dx_1 dx_2 dp_t^2} = \int^{Q^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2) +$$
$$+ \int_{Q^2}^{k_{max}^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2) = I_1 + I_2$$

In the limit $k_t \ll Q^2$ the off-shell cross section reduces to the usual on-shell cross section. The first integral gives

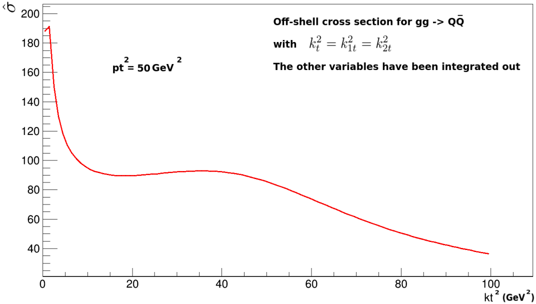
$$\begin{aligned} I_1 &\simeq \hat{\sigma}(x_1 x_2 s, p_t^2) \int^{Q^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \\ I_1 &\simeq f(x_1, Q^2; \mu^2) f(x_2, Q^2; \mu^2) \hat{\sigma}(x_1 x_2 s, p_t^2) \end{aligned}$$

and we define Q^2 such that I_2 is small. Then the k_t-factorization can be decomposed into collinear factorization + correction

Using the dynamical properties of the off-shell cross section and unintegrated parton density, we can show that $Q^2 = p_t^2$ is an acceptable choice (example with heavy quark production)

Case 1 : Large p_t

- $\hat{\sigma}$ decreases slowly with k_t^2 and has a local maximal close to p_t^2
- Unintegrated parton density strongly suppressed at large k_t^2



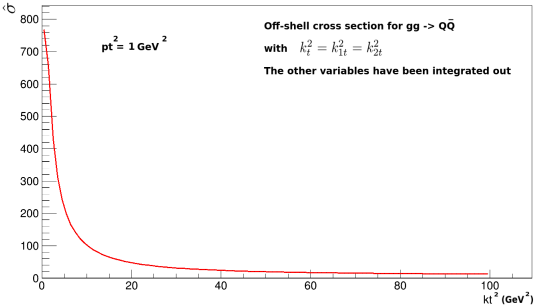
$$I_2 = \int_{p_t^2}^{k_{max}^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2)$$

small

Using the dynamical properties of the off-shell cross section and unintegrated parton density, we can show that $Q^2 = p_t^2$ is an acceptable choice (example with heavy quark production)

Case 2 : small p_t

- Suppression due to unintegrated parton density not enough
- $\hat{\sigma}$ decreases quickly with k_t^2



$$I_2 = \int_{p_t^2}^{k_{max}^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2)$$

small

First result

To summarize, the k_t-factorization formula can be decomposed into

$$I() = I_1(Q^2) + I_2(Q^2)$$

where the hard scale Q^2 verifies $I_2(Q^2) \ll I_1(Q^2)$. It corresponds to the effective upper bound for the k_t^2 integration

Using k_t-factorization we have shown that $Q^2 = p_t^2$ is a possible choice. It explains why the collinear factorization gives "accurate" results even in the region $\lambda_{QCD}^2 \ll p_t^2 \ll \hat{s}/4$

Remark : we do not consider exclusive processes like J/ψ production where the k_t-factorization can give large corrections (factor $K \gg 1$)

Second result

The choice of the hard scale Q^2 is not unique (but could be by fixing the ratio I_2/I_1)

$$I() = I_1(Q^2 = 2p_t^2) + I_2(Q^2 = 2p_t^2)$$

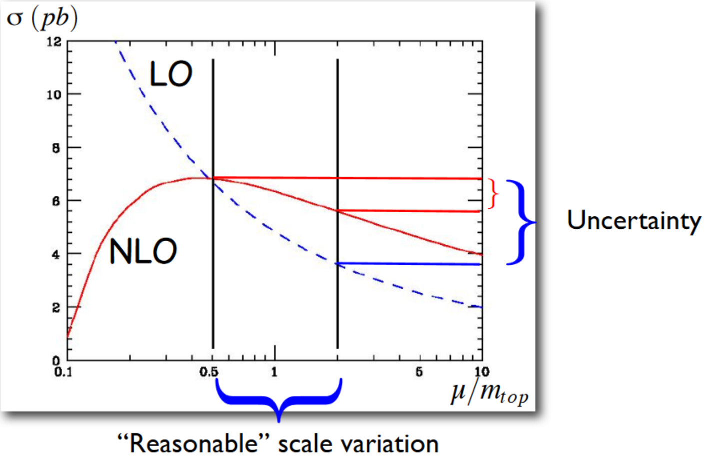
is also a possibility. Consequently, there is an uncertainty on this choice

$I()$ does not depend on $Q^2 \Rightarrow$ no hard scale uncertainty in the k_t-factorization

This uncertainty is not reduced by higher orders (contrary to the factorization scale uncertainty). Not surprising since Q^2 is a physical scale. Currently not taken into account

$$f(x, Q^2; \mu^2) = f(x, \mu^2) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} f(\xi, \mu^2) \left(P\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\mu^2} + C(x) \right)$$

Second result



Taken from *M. Cacciari, http://heavy-quarks.physi.uni-heidelberg.de/Agenda/Talks/M._Cacciari.pdf*

- Not true if one takes into account hard scale uncertainty (for collinear factorization)

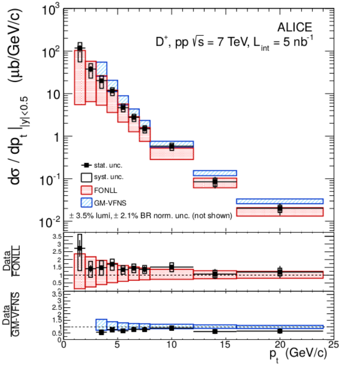
Final comments

At large p_t , poor sensitivity to Q^2 since in any case I_2 suppressed by the unintegrated pdf $\Rightarrow$ expected from the $\log Q^2$ dependence of parton densities.

At small p_t , larger uncertainty

Instead of $Q^2 = p_t^2$ it would be more rigorous to choose $Q^2 = h(p_t^2)p_t^2$ such that $I_2/I_1 = 0.05$ (for instance)

The fonction $h(p_t^2)$ could be used in order to improve collinear fact. results



$$f(x_1, h(p_t^2)p_t^2; \mu^2) f(x_2, h(p_t^2)p_t^2; \mu^2) \hat{\sigma}(x_1 x_2 s, p_t^2)$$

Summary

- $I() = I_1(Q^2) + I_2(Q^2)$, with $I_2(Q^2)$ being a correction
- There is an uncertainty on the choice of Q^2 which is not reduced by higher order corrections (presently not taken into account)
- One can use the k_t-factorization to justify the choice $Q^2 = p_t^2$
- The extraction of the function $h(p_t^2)$ could be used in order to improve the collinear factorization result

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