RADIATION FROM A WAKEFIELD DIELECTRIC STRUCTURE WITH OPEN END

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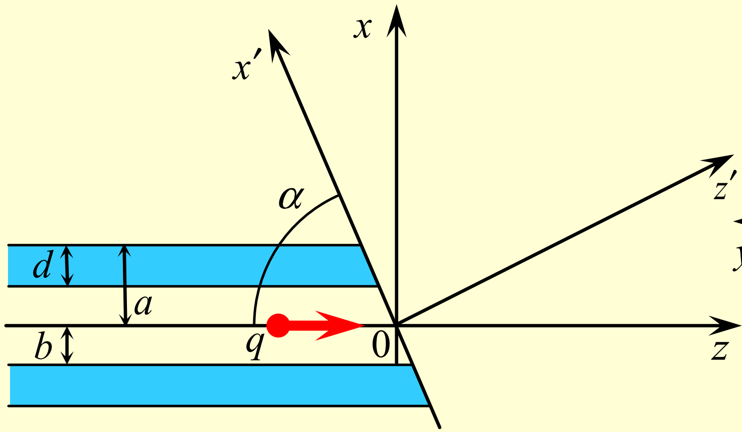
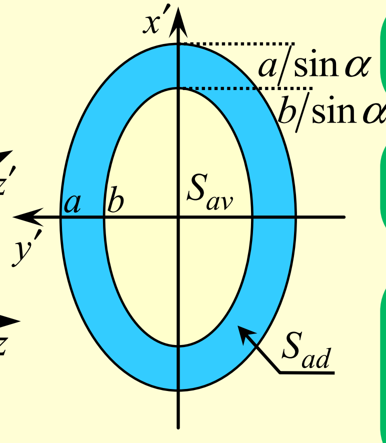
Recent results 1

S.N. Galyamin, A.V. Tyukhtin, S. Antipov, and S.S. Baturin // Opt. Express, **22**(8), 008902 (2014)





High-order TM$_{0m}$ mode, ultrarelativistic bunch

Vacuum channel: Kirchhoff approximation

Dielectric part of the aperture: approximate decomposition of a waveguide mode into two plane waves, refraction with Fresnel coefficients

$A k_0 g_0 \exp(-i\omega_m t)/2$

$g_0 = \exp(ik_0 R')/R'$

$R' >> 1/k_0$

$R' >> a/\sin\alpha$

$R' >> \dfrac{(a/\sin\alpha)^2}{\lambda_m}$

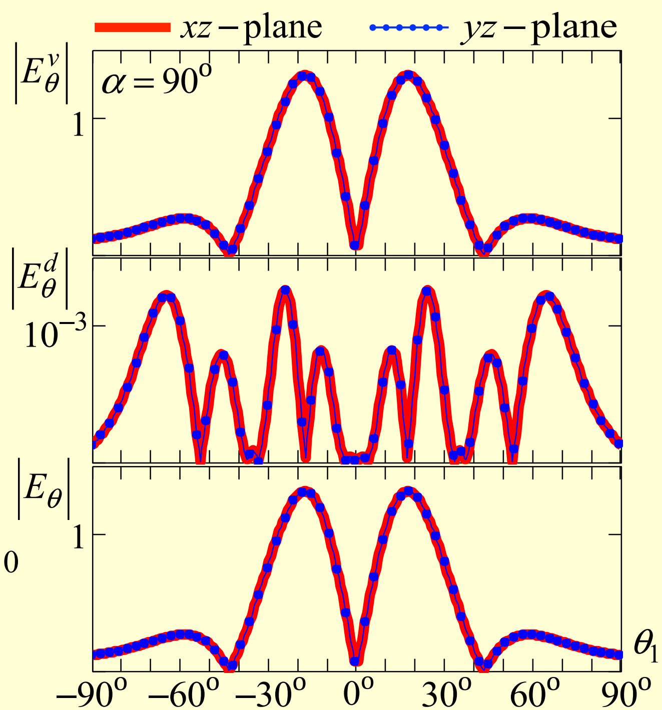
$\lambda_m = 2\pi/k_0 = 2d\sqrt{\varepsilon\mu - 1}/m$

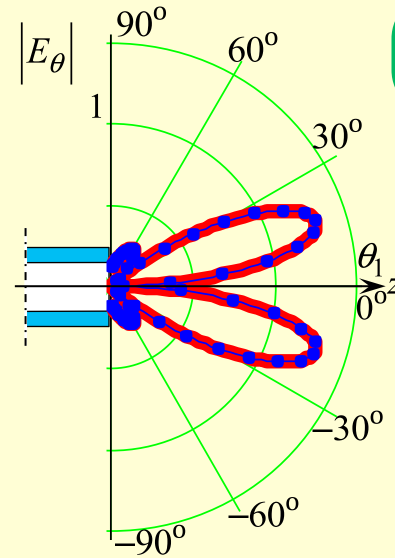
$a \approx 2\text{mm} \quad b \approx d \approx 1\text{mm} \quad \varepsilon \approx 10$

$m = 10$

$\nu_{10} = \omega_{10}/(2\pi) = 500\text{GHz}$

$R' >> 15\text{mm}$





Stratton-Chu formulas

Direction of maximum:

$$\sin\theta_1^{(0)} = \frac{d\sqrt{n^2-1}\,\psi_1}{\chi_m b}$$

$\psi_1 \approx 5.14 \quad J_2(\psi_1) = 0$

Recent results 2

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Days on Diffraction 2015

РАДИОТЕХНИКА И ЭЛЕКТРОНИКА

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КРАТКИЕ СООБЩЕНИЯ

УДК 621.372.81.09

ИЗЛУЧЕНИЕ ИЗ ОТКРЫТОГО КОНЦА ПЛОСКОГО ВОЛНОВОДА С ДИЭЛЕКТРИЧЕСКИМ ЗАПОЛНЕНИЕМ

Г. В. Воскресенский, С. М. Журав

Хотя волноводные излучающие системы, заполненные диэлектриком, используются на практике, теоретически они исследованы недостаточно. Основной интерес представляет выяснение влияния диэлектрического заполнения на частотные зависимости коэффициента отражения основной волны и на диаграмму излучения. В настоящей работе рассматривается задача об излучении из открытого конца плоского волновода и плоского волновода с фланцем, заполненных однородным диэлектриком. Способ решения, основанный на сшивании полей в плоскости раскрыва волновода и сведении функциональных уравнений типа Винера – Хопфа к системе линейных уравнений, аналогичен использованному в[1] при рассмотрении задачи об излучении из пустого волновода с фланцем.

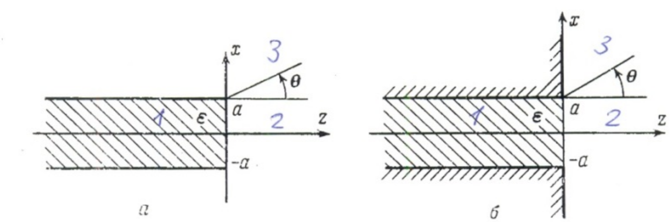
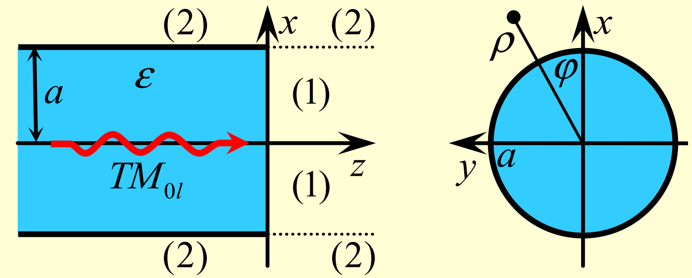


Рис. 1



$$E_{\omega z}^{(i)} = NJ_0(\rho j_{0l}/a)e^{ik_{zl}z}, \quad E_{\omega z}^{(r)} = \sum_{m=1}^{\infty} N_m J_0(\rho j_{0m}/a)e^{-ik_{zm}z},$$

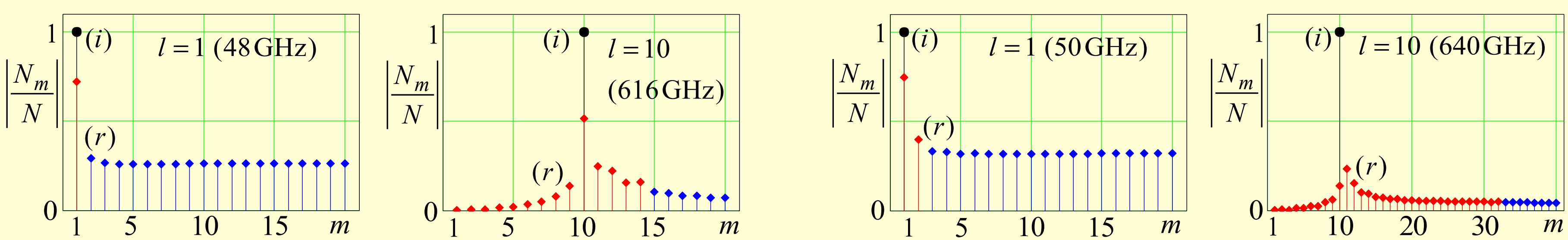
$$k_{zl} = \sqrt{k_0^2\varepsilon - j_{0l}^2 a^{-2}}, \qquad k_0 = \omega/c, \qquad J_0(j_{0l}) = 0$$

$$\sum_{m=1}^{\infty} N_m W_{mp} = N w_p,$$

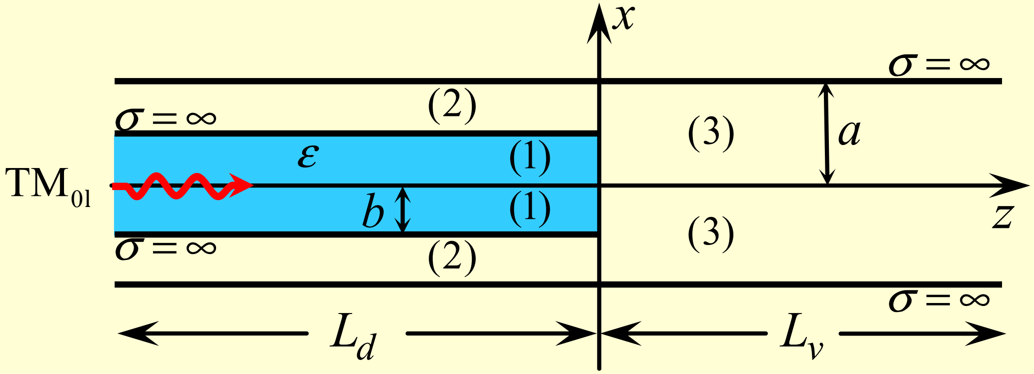
$$w_p = ia^2\delta_{lp} j_{0p}^{-1} J_1(j_{0p})(k_{zp} - \alpha_p\varepsilon) + \\ + \frac{G_+(\alpha_p)G_+(\alpha_m)J_1(j_{0l})(k_{zl} + \alpha_l\varepsilon)\kappa_+(\alpha_p)}{2\alpha_l\kappa_-(\alpha_l)(\alpha_p + \alpha_l)}, \quad \alpha_p = \kappa(j_{0p}/a),$$

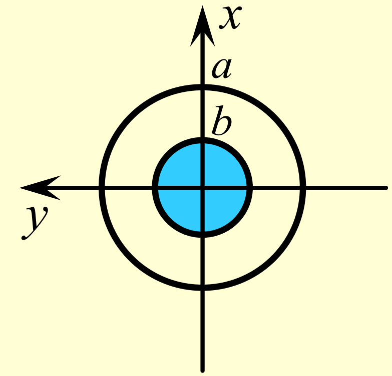
$$W_{mp} = ia^2\delta_{mp} j_{0p}^{-1} J_1(j_{0p})(k_{zp} + \alpha_p\varepsilon) + \frac{G_+(\alpha_p)G_+(\alpha_m)J_1(j_{0m})(k_{zm} - \alpha_m\varepsilon)\kappa_+(\alpha_p)}{2\alpha_m\kappa_-(\alpha_m)(\alpha_p + \alpha_m)}$$

$\varepsilon = 2$ $\varepsilon = 10$



Open-ended waveguide with uniform filling inside regular waveguide





$$H_{\omega\varphi}^{(i)} = J_1(\rho j_{0l}/b)e^{-\kappa_{zl}^{(1)}z},$$

$$H_{\omega\varphi}^{(1)} = \sum_{m=1}^{\infty} B_m J_1(\rho j_{0m}/b)e^{\kappa_{zm}^{(1)}z},$$

$$H_{\omega\varphi}^{(2)} = C_0\rho^{-1}e^{\gamma_{z0}^{(2)}z} + \sum_{m=1}^{\infty} C_m Z_m(\rho\chi_m)e^{\gamma_{zm}^{(2)}z},$$

$$H_{\omega\varphi}^{(3)} = \sum_{m=1}^{\infty} A_m J_1(\rho j_{0m}/a)e^{-\gamma_{zm}^{(3)}z},$$

R. Mittra, S.W. Lee, Analytical Techniques in the Theory of Guided Waves (Macmillan, 1971)

$$k_0 = \omega/c, \quad \kappa_{zl}^{(1)} = \sqrt{j_{0l}^2 b^{-2} - k_0^2\varepsilon}, \quad \gamma_{z0}^{(2)} = -ik_0, \quad \gamma_{zm}^{(2)} = \sqrt{\chi_m^2 - k_0^2}, \quad \gamma_{zm}^{(3)} = \sqrt{j_{0m}^2 a^{-2} - k_0^2}.$$

$$J_0(j_{0l}) = 0.$$

$$Z_m(\xi) = J_1(\xi) - N_1(\xi)J_0(a\chi_m)N_0^{-1}(a\chi_m), \qquad \chi_m: \; J_0(b\chi_m)N_0(a\chi_m) - J_0(a\chi_m)N_0(b\chi_m) = 0.$$

Open-ended waveguide with uniform filling inside regular waveguide

Boundary conditions for *z = 0*

$$\left\{H_{\omega\varphi}\right\}=0, \qquad \left\{E_{\omega\rho}\right\}=0.$$

$$\int_0^b \rho J_1\left(\rho\frac{j_{0m}}{b}\right)J_1\left(\rho\frac{j_{0p}}{b}\right)d\rho=\frac{b^2}{2}J_1^2\left(j_{0p}\right)\delta_{pm}$$

$$\int_0^b \rho J_1\left(\rho\frac{j_{0m}}{a}\right)J_1\left(\rho\frac{j_{0p}}{b}\right)d\rho=\frac{-bJ_1\left(j_{0p}\right)J_0\left(\frac{b}{a}j_{0m}\right)}{\left[\gamma_{zm}^{(3)}\right]^2-\left[\gamma_{zp}^{(1)}\right]^2}, \qquad \gamma_{zp}^{(1)}=\sqrt{j_{0p}^2b^{-2}-k_0^2}$$

$$\int_b^a J_1\left(\rho\frac{j_{0m}}{a}\right)d\rho=\frac{J_0\left(\frac{b}{a}j_{0m}\right)}{j_{0m}/a}$$

$$\int_b^a Z_m\left(\rho\chi_m\right)Z_p(\rho\chi_p)\rho\,d\rho=I_{pp}\delta_{pm}, \qquad I_{pp}=\frac{a^2}{2}Z_p^2\left(a\chi_p\right)-\frac{b^2}{2}Z_p^2\left(b\chi_p\right)$$

$$\int_b^a Z_p\left(\rho\chi_p\right)J_1\left(\rho\frac{j_{0m}}{a}\right)\rho\,d\rho=\frac{-\frac{b}{a}j_{0m}J_0\left(\frac{b}{a}j_{0m}\right)Z_p\left(b\chi_p\right)}{\left[\gamma_{zp}^{(2)}\right]^2-\left[\gamma_{zm}^{(3)}\right]^2}$$

$$Z_m(\xi)=J_1(\xi)-N_1(\xi)J_0(a\chi_m)N_0^{-1}(a\chi_m), \qquad \chi_m: \quad J_0(b\chi_m)N_0(a\chi_m)-J_0(a\chi_m)N_0(b\chi_m)=0.$$

Open-ended waveguide with uniform filling inside regular waveguide

$$\sum_{m=1}^{\infty}\left(\frac{\tilde{A}_m}{\gamma_{zm}^{(3)}-\gamma_{zp}^{(1)}}+\frac{\tilde{A}_m R_p}{\gamma_{zm}^{(3)}+\gamma_{zp}^{(1)}}\right)=\frac{-2\delta_{lp} b J_1(j_{0p})\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}}{\kappa_{zp}^{(1)}+\varepsilon\gamma_{zp}^{(1)}}, \quad p=1,2,\ldots$$

$$\sum_{m=1}^{\infty}\frac{\tilde{A}_m}{\gamma_{zm}^{(3)}-\gamma_{zn}^{(2)}}=0, \quad n=0,1,\ldots$$

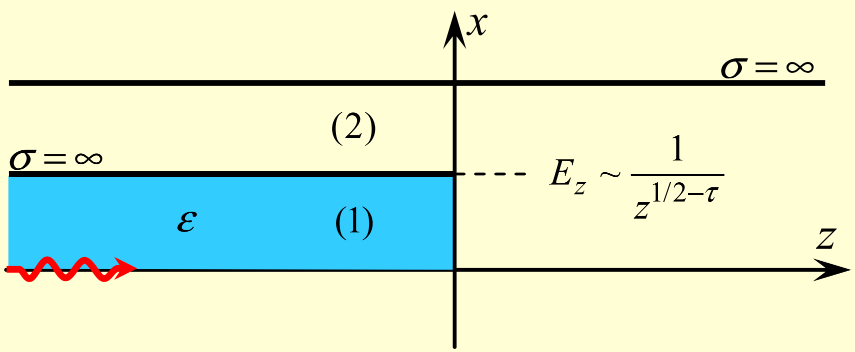
$$\sum_{m=1}^{\infty}\left(\frac{\tilde{A}_m R_p}{\gamma_{zm}^{(3)}-\gamma_{zp}^{(1)}}+\frac{\tilde{A}_m}{\gamma_{zm}^{(3)}+\gamma_{zp}^{(1)}}\right)=\frac{4\tilde{B}_p\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}}{\kappa_{zp}^{(1)}+\varepsilon\gamma_{zp}^{(1)}},$$

$$\sum_{m=1}^{\infty}\frac{\tilde{A}_m}{\gamma_{zm}^{(3)}+\gamma_{zn}^{(2)}}=-2\tilde{C}_n\gamma_{zn}^{(2)},$$

$$\gamma_{zm}^{(1)}=\sqrt{j_{0m}^2 b^{-2}-k_0^2}, \qquad R_p=\left(\varepsilon\gamma_{zp}^{(1)}-\kappa_{zp}^{(1)}\right)\left(\varepsilon\gamma_{zp}^{(1)}+\kappa_{zp}^{(1)}\right)^{-1},$$

$$\tilde{A}_m=A_m J_0\left(\frac{bj_{0m}}{a}\right)\frac{j_{0m}}{a}, \qquad \tilde{B}_p=B_p\frac{bJ_1(j_{0p})}{2}, \qquad \tilde{C}_0=C_0\ln\left(\frac{a}{b}\right), \qquad \tilde{C}_n=C_n\left[\frac{a^2}{2b}\frac{Z_n^2(a\chi_n)}{Z_n(b\chi_n)}-\frac{b}{2}Z_n(b\chi_n)\right].$$

Meixner edge condition:



$$\Rightarrow \qquad A_m \xrightarrow[m\to\infty]{} \frac{1}{m^{1+\tau}},$$

$$\sin(\pi\tau)=(\varepsilon-1)/(2\varepsilon+2).$$

Residue calculus technique

Known sets $\{x_i\}$ $\{y_i\}$ $i = 1,2,\ldots$ $F = const$

$$\sum_{i=1}^{\infty} \frac{A_i}{x_i - y_j} = 0, \qquad \sum_{i=1}^{\infty} \frac{A_i}{x_i + y_j} = F\delta_{jk}, \qquad j = 1,2,\ldots \qquad \{A_i\} - ?$$

$$f(w): \quad f(w) \to 0 \quad \text{for} \quad |w| \to \infty \quad \Rightarrow \quad \frac{1}{2\pi i}\oint_{C_\infty} \frac{f(w)}{w - y_j}\,dw = 0, \qquad \frac{1}{2\pi i}\oint_{C_\infty} \frac{f(w)}{w + y_j}\,dw = 0$$

On the other hand

$$f(w): \ \text{poles} \quad w = x_i, \quad \text{zeros} \quad w = y_i, \quad \text{zeros} \quad w = -y_i \quad \text{for} \quad i \neq k, \Rightarrow \quad f(w) = C_0 \frac{\prod_{i=1}^{\infty}(w - y_i)\prod_{i=1}^{\infty}{}^{(k)}(w + y_i)}{\prod_{i=1}^{\infty}(w - x_i)}$$

$$0 = \sum_{i=1}^{\infty} \frac{\operatorname{Res} f(x_i)}{x_i - y_j}, \qquad 0 = \sum_{i=1}^{\infty} \frac{\operatorname{Res} f(x_i)}{x_i + y_j} + f(y_k)$$

$$f(y_k) = F \quad \Rightarrow \quad C_0, \qquad A_i = \operatorname{Res} f(x_i)$$

R. Mittra, S.W. Lee, Analytical Techniques in the Theory of Guided Waves (Macmillan, 1971)

Open-ended waveguide with uniform filling inside regular waveguide

$$\frac{1}{2\pi i}\int_{C_\infty}\left(\frac{f(w)}{w-\gamma_{zp}^{(1)}}+\frac{R_p f(w)}{w+\gamma_{zp}^{(1)}}\right)dw=0, \qquad \frac{1}{2\pi i}\int_{C_\infty}\frac{f(w)}{w-\gamma_{zn}^{(2)}}dw=0,$$

$$\frac{1}{2\pi i}\int_{C_\infty}\left(\frac{R_p f(w)}{w-\gamma_{zp}^{(1)}}+\frac{f(w)}{w+\gamma_{zp}^{(1)}}\right)dw=0, \qquad \frac{1}{2\pi i}\int_{C_\infty}\frac{f(w)}{w+\gamma_{zn}^{(2)}}dw=0.$$

$f(w):\quad$ poles $\quad w=\gamma_{zm}^{(3)},\qquad$ zeros $\quad w=\gamma_{zm}^{(2)},$

$$f(\gamma_{zp}^{(1)})+R_p f(-\gamma_{zp}^{(1)})=0, \qquad p\neq l, \qquad f(\gamma_{zl}^{(1)})+R_l f(-\gamma_{zl}^{(1)})=\frac{2bJ_1(j_{0l})\gamma_{zl}^{(1)}\kappa_{zl}^{(1)}}{\kappa_{zp}^{(1)}+\varepsilon\gamma_{zl}^{(1)}}.$$

Meixner edge condition: $f(w)\xrightarrow[|w|\to\infty]{} w^{-(\tau+1/2)}, \qquad \sin(\pi\tau)=(\varepsilon-1)/(2\varepsilon+2).$

Shifted zeros: $\Gamma_s=\gamma_{zs}^{(1)}+\pi\Delta_s/b, \qquad \Gamma_s\xrightarrow[s\to\infty]{}\gamma_{zs}^{(1)}+\frac{\pi}{b}\tau\sim\frac{\pi}{b}\left(s-\frac{1}{4}+\tau\right)$

$$\tilde{A}_m=\operatorname{Res} f(\gamma_{zm}^{(3)}), \qquad \tilde{B}_p=-\frac{\left(R_p f(\gamma_{zp}^{(1)})+f(-\gamma_{zp}^{(1)})\right)}{4\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}}\left(\varepsilon\gamma_{zp}^{(1)}+\kappa_{zp}^{(1)}\right), \qquad \tilde{C}_n=f(-\gamma_{zn}^{(2)})(2\gamma_{zn}^{(2)})^{-1}$$

Open-ended waveguide with uniform filling inside regular waveguide

Infinite nonlinear system for shifted zeros:

$$\Delta_s = R_s\left(\frac{2b\gamma_{zs}^{(1)}}{\pi}+\Delta_s\right)\prod_{m=1}^{\infty}{}^{(l,s)}\frac{\gamma_{zm}^{(1)}+\gamma_{zs}^{(1)}+\frac{\pi}{b}\Delta_s}{\gamma_{zm}^{(1)}-\gamma_{zs}^{(1)}+\frac{\pi}{b}\Delta_s}\frac{\gamma_{zs}^{(1)}+\gamma_{z0}^{(2)}}{\gamma_{zs}^{(1)}+\gamma_{z0}^{(2)}}\prod_{n=1}^{\infty}\frac{1+\dfrac{\gamma_{zs}^{(1)}}{\gamma_{zn}^{(2)}}}{1-\dfrac{\gamma_{zs}^{(1)}}{\gamma_{zn}^{(2)}}}\prod_{q=1}^{\infty}\frac{1-\dfrac{\gamma_{zs}^{(1)}}{\gamma_{zq}^{(3)}}}{1+\dfrac{\gamma_{zs}^{(1)}}{\gamma_{zq}^{(3)}}}G^2(-\gamma_{zs}^{(1)}), \qquad \Delta_s \xrightarrow[s\to\infty]{} \tau$$

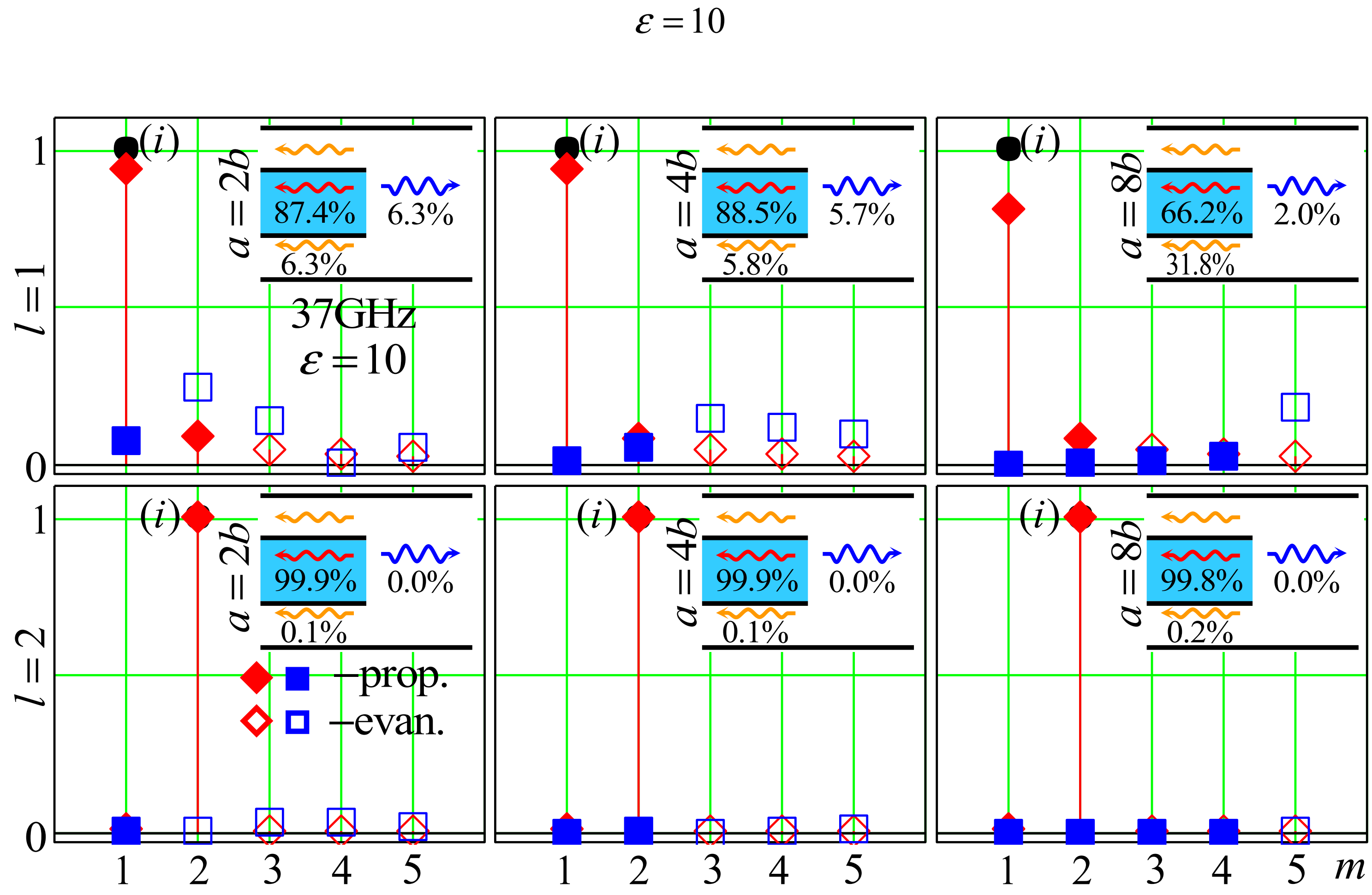
$$G(w)=\exp\left[-\frac{w}{\pi}\left(b\ln\left(\frac{b}{a-b}\right)+a\ln\left(\frac{a-b}{a}\right)\right)\right],$$

$$f(w)=P_0\frac{(w-\gamma_{z0}^{(2)})\prod\limits_{n=1}^{\infty}\left(1-\dfrac{w}{\gamma_{zn}^{(2)}}\right)}{\prod\limits_{m=1}^{\infty}\left(1-\dfrac{w}{\gamma_{zn}^{(3)}}\right)}\prod_{s=1}^{\infty}{}^{(l)}\left(1-\frac{w}{\Gamma_s}\right)G(w)$$

$$P_0: \qquad f(\gamma_{zl}^{(1)})+R_l f(-\gamma_{zl}^{(1)})=\frac{2bJ_1(j_{0l})\gamma_{zl}^{(1)}\kappa_{zl}^{(1)}}{\kappa_{zp}^{(1)}+\varepsilon\gamma_{zl}^{(1)}}.$$

Numerical results

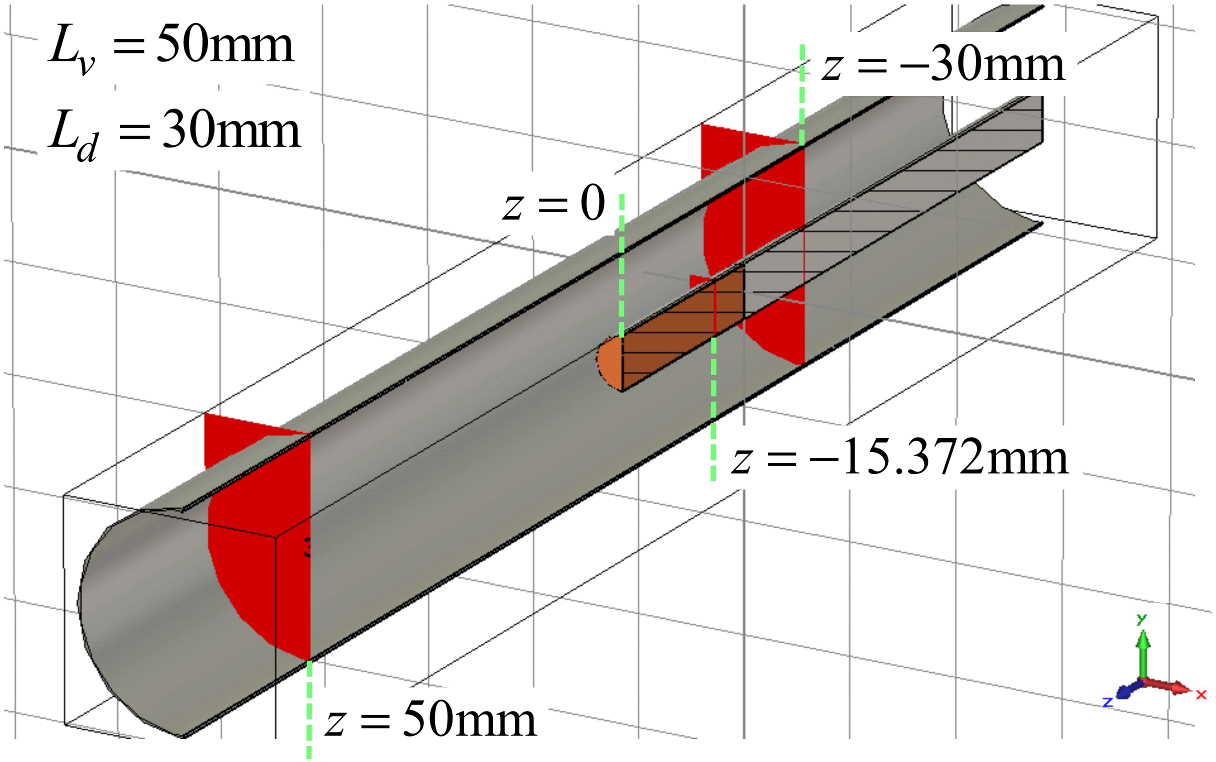
b=2.4mm @ 37GHz



Numerical results

b=2.4mm @ 37GHz

CST simulation



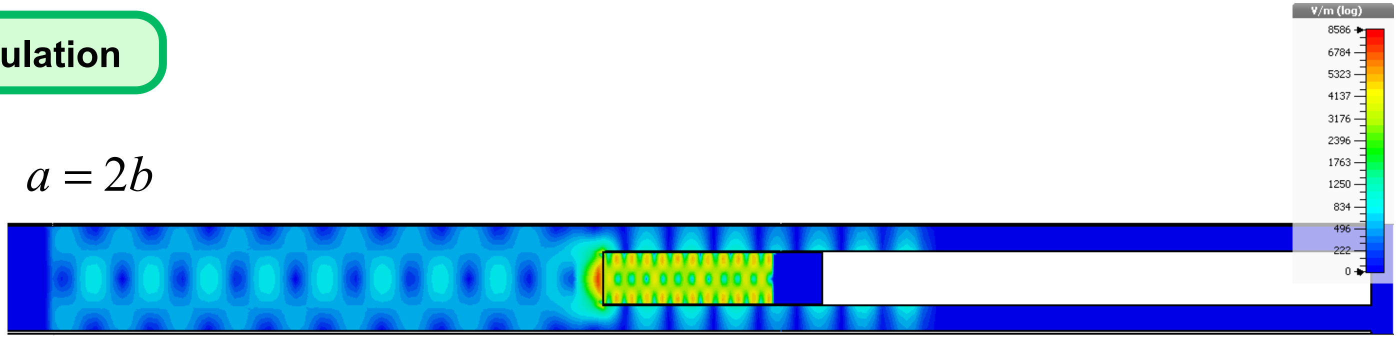
- 2 orthogonal *H*-planes: (*xz*), (*yz*).
- 3 axial symmetric modes (15 CST modes) at each port
- Time-domain solver (36-38GHz wave packet)

Numerical results

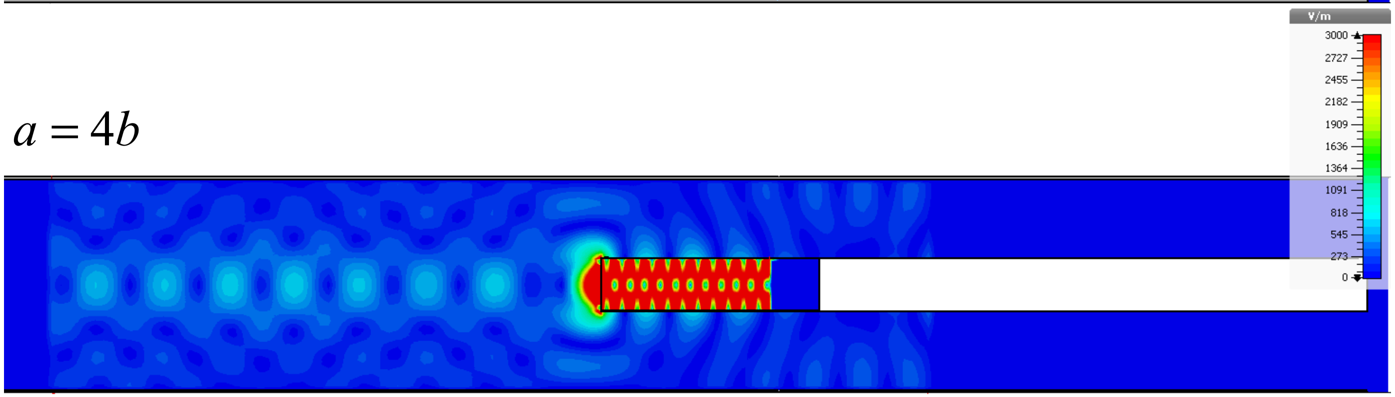
b=2.4mm @ 37GHz

CST simulation

$a = 2b$



$a = 4b$



$a = 2b$

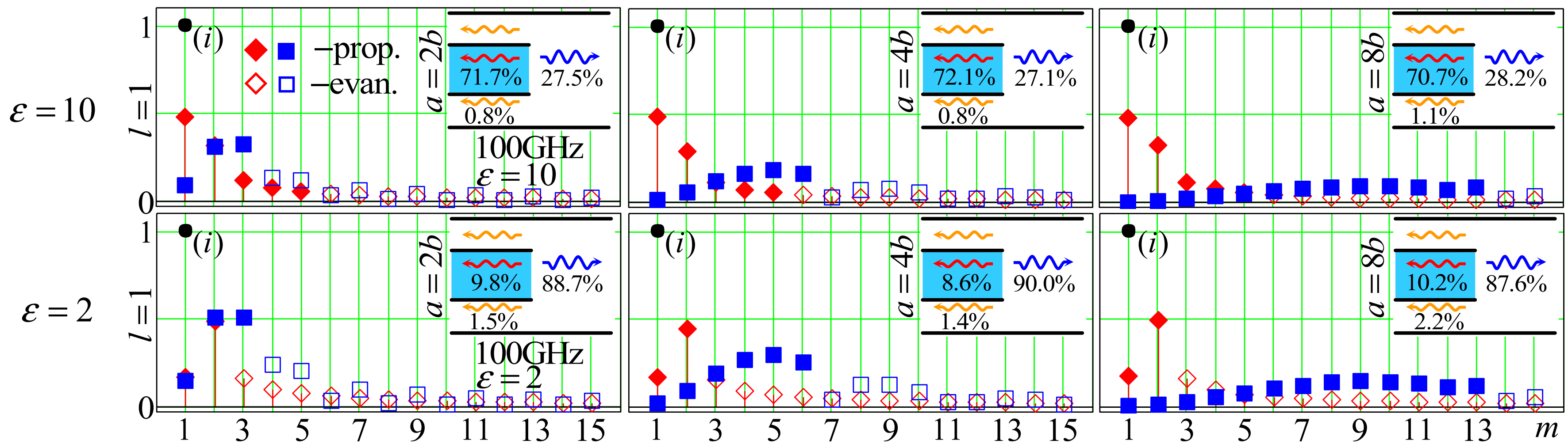
Mode (area)	CST	PTC Mathcad 15
1 (1)	87.9%	87.3%
2 (1)	0.1%	0.1%
0 (2)	6.0%	6.3%
1 (3)	6.0%	6.3%
Total:	100%	100.0%
Time:	23 min*	30 sec*

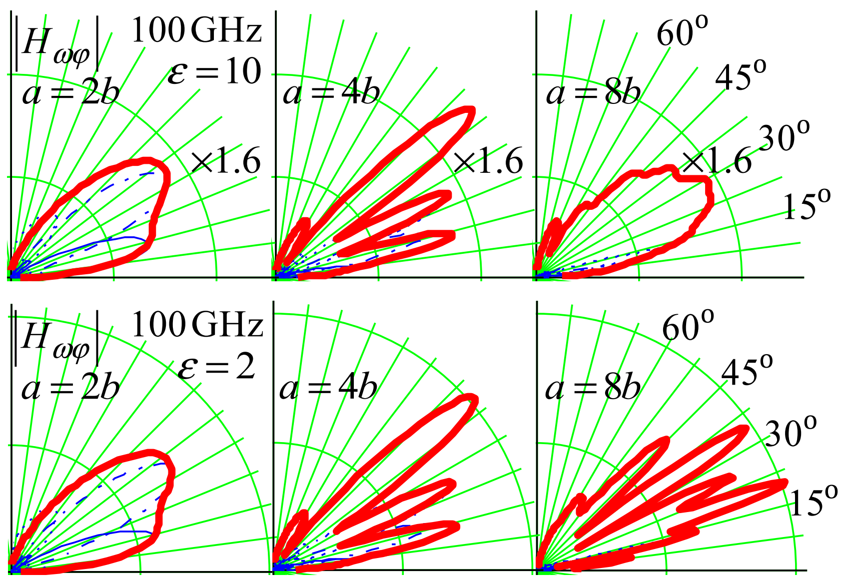
$a = 4b$

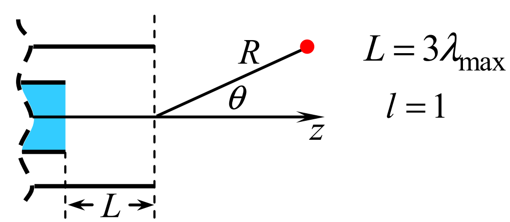
Mode (area)	CST	PTC Mathcad 15
1 (1)	88.4%	88.4%
2 (1)	0.05%	0.1%
0 (2)	2.8%	2.6%
1 (2)	2.7%	3.2%
1 (3)	2.9%	0.4%
2 (3)	5.2%	5.3%
Total:	102%	100.0%
Time:	-	34 sec*

Numerical results

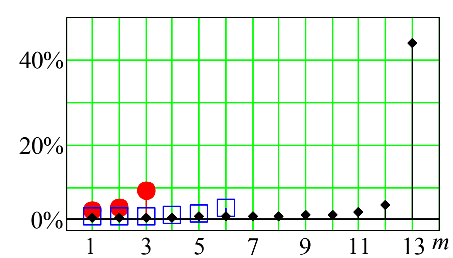
b=2.4mm @ 100GHz







Power reflected by the second open end



Thank you for your attention!