GUT-scale inflation with sizeable tensor modes

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Motivation I: What if...

- Primordial fluctuations can be decomposed in scalar and tensor modes
- Scalars: well measured in CMB (COBE, WMAP, PLANCK...)
- Tensors: PLANCK constrains tensor-to-scalar ratio

$$r \lesssim 0.1$$

- single-field slow-roll inflation:

$$r = 16\,\epsilon \qquad \text{where} \quad \epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$$

- Observation fixes scalar power spectrum amplitude:

$$A_s = 2.2 \cdot 10^{-9} \simeq \frac{V}{24\pi^2 \epsilon M_P^4}$$

If $r \sim \mathcal{O}(0.1)$ **then** $\epsilon \sim \mathcal{O}(0.01)$ $\quad \Rightarrow \quad V \approx (10^{16}\ \text{GeV})^4$

Observable r $\quad \Rightarrow \quad$ scale of inflation $\approx$ SUSY GUT scale

Motivation II: BICEP2

- In March 2014 the BICEP2 collaboration reported

$$r \simeq 0.2$$

(maybe 0.15 depending on dust foreground model)

- The Law of 21st Century ENP (Exciting New Physics):
If there is a hint for ENP, it will
  - quickly become suspicious as all proposed explanations look implausible, and/or
  - eventually go away with more statistics, or
  - become dubious/untenable after re-examination of systematics or analysis methods
- Indications growing that BICEP2 “signal” is actually polarized dust
- PLANCK to put something out in autumn = around now. Not clear if this will settle the matter.

Back to Motivation I: What if...

Assume Planck will not just put a bound on r but find a signal,

$$r \simeq 0.1$$

Assume that the L21CENP will not hold in that case:

- Find a particle physics model connecting inflation with GUT breaking (easy & well known → Dvali/Shafi/Schaefer '94)
- Obtain a sizeable r in this model while keeping effects from Planck-scale physics under control
  (hard & well known to be hard → Lyth '96)

Slow-roll inflation toolbox

Consider a single-field inflation model with an inflaton rolling down its potential

Formulary:

FLRW metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{1}{M_P^2}\rho \qquad H = \dot{a}/a, \qquad \text{from now } k = 0 \text{ (flat universe)}$$

Homogeneous classical scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Klein-Gordon equation in this background:

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

Slow-roll parameters

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_P^2 \frac{V''}{V}$$

Slow-roll inflation toolbox

Formulary continued:

When slow-roll conditions hold,

$$\epsilon \ll 1 \qquad |\eta| \ll 1,$$

then Friedmann Eq. + Klein-Gordon Eq. become

$$3\,H^2 \approx \frac{1}{M_P^2} V \qquad \dot{\phi} \approx -\frac{1}{3H} V'$$

Quasi de Sitter metric: $a(t) \approx e^{Ht}$. Number of e-folds from integrating:

$$N = \int_{t_0}^{t_0+\delta t} H\,dt = -\frac{1}{M_P^2} \int_{\phi_0}^{\phi_0+\delta\phi} \frac{V}{V'} d\phi = -\frac{1}{M_P} \int_{\phi_0}^{\phi_0+\delta\phi} \frac{1}{\sqrt{2\epsilon(\phi)}} d\phi$$

The flatter the potential, the more e-folds generated for fixed field excursion $\delta\phi$

The Lyth(-Boubekeur) bound

Number of e-folds generated during field excursion $\delta\phi$:

$$N = -\frac{1}{M_P}\int_{\phi_0}^{\phi_0+\delta\phi}\frac{1}{\sqrt{2\epsilon(\phi)}}d\phi$$

Tensor-to-scalar ratio:

$$r = 16\epsilon_*$$

where $\epsilon_* = \epsilon(\phi_*)$ at scales relevant for CMB (50-60 e-folds before inflation ends)

If $\epsilon \approx$ const. or increasing, $\epsilon(\phi) \geq \epsilon_*$:

$$\delta\phi \geq \sqrt{\frac{r}{8}}\,N\,M_P$$

- Over $N = 60$ e-folds: $r \gtrsim 0.01$ implies $|\delta\phi| \gtrsim M_P$ $\rightarrow$ Lyth '96
- Over scales relevant for CMB (ca. $N \approx 5$): $r \gtrsim 0.3$ implies $|\delta\phi| \gtrsim M_P$ $\rightarrow$ Lyth/Boubekeur '01

Observable $r \Rightarrow$ need large-field model or nontrivial inflaton potential

Digression: The trouble with large-field models

Eta problem in SUGRA:

$$V = e^{K/M_P^2}\left(D_i W \bar{D}_j \bar{W} K^{i\bar{j}} - 3\frac{|W|^2}{M_P^2}\right)$$

$$K = |S|^2 \qquad \Rightarrow \qquad e^{K/M_P^2} = 1 + \frac{|S|^2}{M_P^2} + \ldots$$

$|S| > M_P \quad \Rightarrow$ large term in $\eta \sim V''$, hard to tune away: slow-roll violated

Proposed solution:

Shift symmetry $S \rightarrow S + ia$, $\qquad K = K(S + S^\dagger)$, $\qquad (S - S^\dagger)$ = inflaton

- Must be violated in W to build inflationary potential
- Should be justified by UV completion $\Rightarrow$ question for string cosmologists. . .

Review: F-term hybrid inflation

F-term hybrid inflation: $\rightarrow$ Dvali/Shafi/Schaefer '94

- vacuum energy provided by F-term of chiral inflaton superfield
- inflation ends not by violation of slow-roll but by a phase transition

$$W = \lambda S \left(\Lambda^2 - Q\tilde{Q} \right)$$

During inflation: $S \gg \Lambda$,

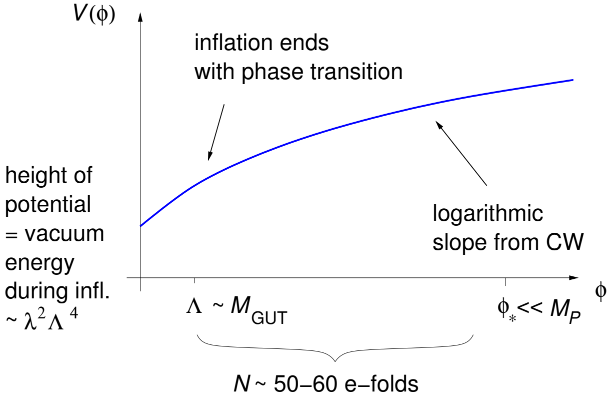
$$W_{\text{eff}} = \lambda \Lambda^2 S \quad \Rightarrow \quad V_{\text{tree}} = \lambda^2 \Lambda^4$$

Q and $\tilde{Q}$ are massive: 1-loop Coleman-Weinberg potential

$$V_{\text{CW}} = \frac{\lambda^4}{16\pi^2} \Lambda^4 \log \left(\frac{\lambda^2 |S|^2}{\mu^2} \right)$$

- After inflation ends, Q and $\tilde{Q}$ take VEVs $Q \sim \tilde{Q} \sim \Lambda$
- Identify with (or couple to) GUT Higgs $\Rightarrow$ GUT breaking at scale Λ
- Scale of inflation: $\sqrt{\lambda}\Lambda$

FHI in global SUSY: Small-field model, small r



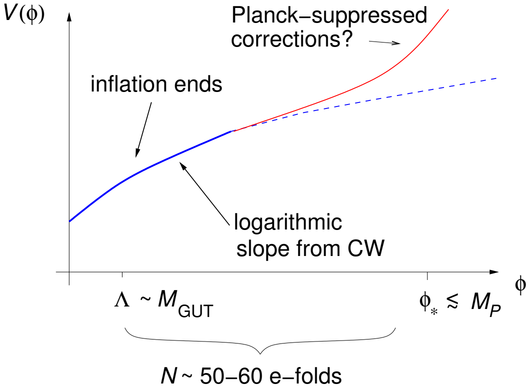
$$\epsilon_* = \frac{1}{2}\frac{M_P^2}{\phi_*^2}\left(\frac{\lambda^2}{16\pi^2}\right)^2 \qquad \eta_* = -\frac{M_P^2}{\phi_*^2}\frac{\lambda^2}{16\pi^2} \approx \frac{1}{2N}$$

- Wrong spectral index: $n_s - 1 = 2\eta_* - 6\epsilon_* \approx 2\eta_* \approx \frac{1}{N}$
- Unobservable tensor-to-scalar ratio: $r = 16\epsilon_* \approx \text{(loop factor)}/N$

FHI in SUGRA: Planck corrections

By choosing larger λ, can increase ϕ_* such that M_P-suppressed corrections become important

- SUGRA corrections: $V = |W_i|^2 \;\rightarrow\; V = e^{K/M_P^2}(|D_i W|^2 - 3\,|W|^2/M_P^2)$
- higher Kähler terms: $K = |\Phi|^2 \;\rightarrow\; K = |\Phi|^2 + \sum k_{nm}\Phi^n\Phi^{*m}/M_P^{n+m-2}$



FHI in SUGRA

Goal: Adjust higher-order terms such that

- spectral index $n_s = 0.96$ correctly reproduced? ✓
  $\rightarrow$ Bastero-Gil/King/Shafi '06
- potential is steep enough at ϕ_* for $r \sim \mathcal{O}(0.1)$? ✓
  (requires R-symmetry breaking)
- ϕ_*/M_P expansion is under control? ✓ ×
  (not really, in that $\phi_*/M_P \approx 0.5$ is hardly a good expansion parameter)
- other PLANCK constraints on the power spectrum are satisfied? ✓ ×
  (hard to say without further work)

Related recent work: $\rightarrow$ Ben-Dayan/Brustein '09, Hotchkiss/Nadathur/Mazumdar '11, Civiletti/Pallis/Shafi '14, Antusch/Nolde '14

FHI in SUGRA: our ansatz

Allow for non-minimal Kähler terms:

$$K = |S|^2 + \sum_{m+n \geq 3} \left(k_{mn} \frac{S^m (S^\dagger)^n}{M_P^{m+n-2}} + \text{h.c.} \right)$$

Allow for R-symmetry breaking constant term in superpotential:

$$W = W_0 + \lambda S \left(\Lambda^2 - Q\tilde{Q} \right)$$

- Higher-order terms involving Q, $\tilde{Q}$ unimportant
- No R-breaking relevant operators (underlying two-sector model?)
- In global SUSY limit $M_P \to \infty$, FHI recovered

Even more constraints from observation

Planck constrains not only the spectral index $(n_s - 1) \sim V''(\phi_*)$
but also the running of the spectral index $\alpha_s \sim V'''(\phi_*)$
as well as the running of the running $\kappa_s \sim V''''(\phi_*)$
etc. etc. ... to be small

Around ϕ_* the potential should be essentially linear

(also during the ~ 5 e-folds afterwards $\Rightarrow$ CMB scales)

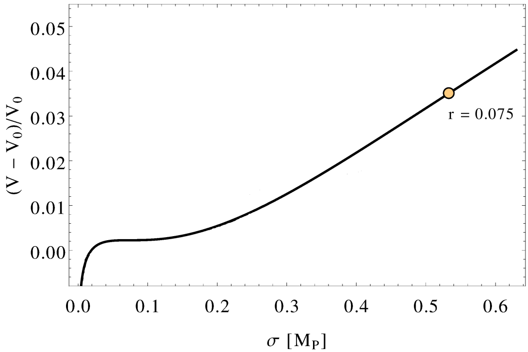
Also, need to ensure slow roll is preserved throughout

Our strategy:

- Taylor-expand scalar potential including M_P corrections
- Impose $n_s = 0.960 \pm 0.007$ (PLANCK)
- Impose $\alpha_s = 0.001^{+0.013}_{-0.014}$ and $\kappa_s = 0.022^{+0.016}_{-0.013}$ (PLANCK)
- Constrain higher moments of the potential $2|(V')^{n-2}(\partial^n V)/V^{n-1}| < \delta$
  Try $\delta = 0.3$ (optimistic), $\delta = 0.03$ (conservative)

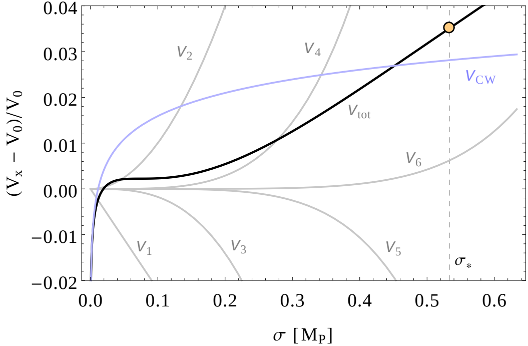
Find solution with 50 e-folds and as small as possible ϕ_* for given r

Shape of the potential



- steep(ish), $\sim$ linear around ϕ_*:
  ϵ sizeable, higher derivatives constrained by PLANCK
- flat at small ϕ: generate enough e-folds
- (Physical reason?)

Contributions from higher-order terms



- Previous picture of log + (corrections at large ϕ) does not hold
- Fine-tuning of W_0 and k_{mn} necessary
- Result still sensitive to even higher order terms (which we tuned to zero) $\phi_*/M_P \approx 0.5$, not a good expansion parameter

Dynamical model

Variation of → Dimopoulos/Dvali/Rattazzi '97:

$$W_{\text{tree}} = \lambda S Q_I \tilde{Q}^I \qquad \text{(R-symmetric)}$$

SU($N_c = n$) gauge theory with $N_f = n$ flavours Q_I, $\tilde{Q}^I$, scale Λ

$S \gg \Lambda \;\Rightarrow$ quark mass $\lambda S \;\Rightarrow$ SU(n) SYM at low energies, scale Λ'

Matching at scale λS:

$$2n \log \frac{\lambda S}{\Lambda} = 3n \log \frac{\lambda S}{\Lambda'} \quad \Rightarrow \quad W_{\text{gaugino cond.}} = (\Lambda')^3 = \lambda S \Lambda^2$$

Coupling to GUT Higgs:

$$W_{\text{tree}} = \lambda S Q_I \tilde{Q}^I - \frac{\lambda'}{2} S \operatorname{tr} \Sigma^2 + \frac{h}{3} \operatorname{tr} \Sigma^3 \qquad \text{(R-breaking)}$$

Isolated SUSY vacuum from deformed moduli space constraint → Seiberg '94

$$S \sim \Lambda, \qquad \Sigma \sim \operatorname{diag}(2, 2, 2, -3, -3)\Lambda$$

GUT breaking at same scale as inflation

More on dynamical model

To cancel vacuum energy after inflation:

$$W_0 = -\frac{h}{3}\langle \mathrm{tr}\,\Sigma^3\rangle \sim \Lambda^3$$

Energy density during inflation still $\approx \lambda^2\Lambda^4$ (important for self-consistency)

Other remarks:

- Monopoles no longer inflated away if GUT broken as inflation ends $\Rightarrow$ "safe" gauge group, e.g. flipped SU(5)?
- Model ends up in a SUSY vacuum. For realistic particle physics, want to break SUSY (but at a much lower scale)

Summary

- A nonzero tensor-to-scalar ratio may or may not have been observed
- It may or may not be observable
- In case it is: Difficult to construct a matching small-field inflation model
- F-term hybrid inflation + suitable M_P-suppressed operators can give sizeable r...
- ...at the price of considerable fine-tuning
- R-symmetry breaking necessary. Dynamical models exist with this feature.
- Physical reason for shape of potential?
- Detailed comparison with PLANCK results needs more dedicated work. Should be able to do better than imposing (higher derivatives) < (some δ)