## $b \rightarrow c \tau \nu_{\tau}$ Decays : A Catalogue to Compare, Constrain, and Correlate New Physics

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## Direct and Indirect Search

- New physics search can follow one of two tracks :
- Direct detection of new particles at the collider
- Indirect probes for new physics from precision measurements
- No direct evidence for physics beyond SM by LHC.
- Indirect hints for new physics (NP) in the flavour sector.
- NP can show up as a deviation of the experimental data from SM prediction.


## $\mathcal{R}(D), \mathcal{R}\left(D^{*}\right)$ : Experimental Status

- Observables with
less theoretical uncertainty :

$$
\begin{aligned}
\mathcal{R}(D) & =\frac{\mathcal{B}\left(\bar{B} \rightarrow D \tau \nu_{\tau}\right)}{\mathcal{B}\left(\bar{B} \rightarrow D \ell \nu_{\ell}\right)} \\
\mathcal{R}\left(D^{*}\right) & =\frac{\mathcal{B}\left(\bar{B} \rightarrow D^{*} \tau \nu_{\tau}\right)}{\mathcal{B}\left(\bar{B} \rightarrow D^{*} \ell \nu_{\ell}\right)}
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Stefania Vecchi's talk today morning


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$$
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[^0]S. Fajfer, J. F. Kamenik, and I. Nisandzic, Phys. Rev. D85 (2012) 094025

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$$

Stefania Vecchi's talk today morning
 S. Jaiswal, S. Nandi, and S. K. Patra, JHEP 12, 060(2017). D. Bigi and P. Gambino, Phys. Rev. D94 (2016), no. 9094008.

$$
\text { S. Fajfer, J. F. Kamenik, and I. Nisandzic, Phys. Rev. D85 (2012) } 094025
$$

- For both $\mathcal{R}(D), \mathcal{R}\left(D^{*}\right)$ : Deviations $4.1 \sigma$ (Global) and $3.5 \sigma\left(\mathcal{R}\left(D^{*}\right)\right)$.


## More Observables...

- Present experimental status of these observables with their correlation:

|  | $\mathcal{R}_{D}$ | $\mathcal{R}_{D^{*}}$ | $\rightarrow$ Correlation | $P_{\tau}\left(D^{*}\right)$ | $\mathcal{R}_{J / \Psi}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BABAR | $0.440(58)(42)$ | $0.332(24)(18)$ | -0.27 | - | - |
| Belle (2015) | $0.375(64)(26)$ | $0.293(38)(15)$ | -0.49 | - | - |
| Belle (2016) | - | - | - | - |  |
| Belle (2016) | - | $0.302(30)(11)$ | 0.33 | $-0.38(51)(26)$ | - |
| LHCb (2015) | - | - | - | - |  |
| LHCb (2017) | - | $0.336(35)(37)(30)$ | - | - | $0.71(17)(18)$ |
| LHCb (2017) | - | - | - | $-286(19)(25)$ |  |

$$
P_{\tau}\left(D^{(*)}\right)=\frac{\Gamma^{(*) \lambda_{\tau}=1 / 2}-\Gamma^{(*) \lambda_{\tau}=-1 / 2}}{\Gamma^{(*) \lambda_{\tau}=1 / 2}+\Gamma^{(*) \lambda_{\tau}=-1 / 2}}
$$

$$
\mathcal{R}(J / \psi)=\frac{\mathcal{B}\left(B_{c} \rightarrow J / \psi \tau \nu_{\tau}\right)}{\mathcal{B}\left(B \rightarrow J / \psi \ell \nu_{\ell}\right)}
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$P_{\tau}\left(D^{*}\right)$ : Large uncertainty, Consistent with SM

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$$

## $\mathcal{R}_{J / \Psi}$ : Large uncertainty, $2 \sigma$ above SM prediction.

## More Channels... More Observables...



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Correlations among observables Important!!

$F_{L}^{D^{*}}$
Precise measurements: constraint on model specific NP parameters
$\mathcal{A}_{F B}^{(*)}$

## More Channels... More Observables...



Correlations among observables Important!!

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Precise measurements: constraint on model specific NP parameters

Prediction in NP model : Consistency check with future measurements $\mathcal{A}_{\text {FB }}$

## SM prediction (Exclusive)

- For SM calculation in $B \rightarrow D^{(*)} \tau \nu_{\tau}$ : CLN parametrization is used. (Nucl. Phys. B530 (1998) 153-181)
- For SM calculation in $\Lambda_{B} \rightarrow \Lambda_{c} \tau \nu_{\tau}$ : Lattice QCD in relativistice heavy quark limit. (Phys. Rev. D92 (2015), no. 3 034503)
- Unavailability of precise calculation of $B_{c} \rightarrow J / \psi$ form factors :
- Option to choose different parametrization.
- Two different parametrizations are considered
- Light-front Covariant Quark Model (LFCQ) (Phys. Rev. D79 (2009) 054012)
- Perturbative QCD (pQCD) (Chin. Phys. C37 (2013) 093102)
- SM central value varying within range $0.25-0.29$


## Inclusive SM prediction

- For Inclusive decay :

$$
\mathcal{R}_{X_{c}}=\frac{\mathcal{B}\left(B \rightarrow X_{c} \tau \bar{\nu}_{\tau}\right)}{\mathcal{B}\left(B \rightarrow X_{c} \ell \bar{\nu}_{\ell}\right)},
$$

- Upto NNLO corrections in $\alpha_{s}$ are considered (Phys. Lett. B346 (1995) 335-341, JHEP 02 (2010) 089).
- The contributions, both at the order $1 / m_{b}{ }^{2}$ and $1 / m_{b}{ }^{3}$ are considered separately. (Phys. Lett. B326 (1994) 145-153, Nucl. Phys. B921 (2017) 211-224)

SM prediction for $\mathcal{R}_{X_{C}}$

| $m_{c}$ in scheme: |  |  |  |
| :---: | :---: | :---: | :---: |
| $\overline{M S}$ upto order | Kinetic up to order |  |  |
| $\mathcal{O}\left(1 / m_{b}^{2}\right)$ | $\mathcal{O}\left(1 / m_{b}^{3}\right)$ | $\mathcal{O}\left(1 / m_{b}^{2}\right)$ | $\mathcal{O}\left(1 / m_{b}^{3}\right)$ |
| $0.242(8)$ | $0.218(8)$ | $0.232(3)$ | $0.209(4)$ |

Phys. Rev. Lett. 114, 061802 (2015).

## Inclusive SM prediction

${ }^{b}$-quark mass: Kinetic scheme, $c$-quark mass: both Kinetic and $\bar{M} S$ scheme

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SM prediction for $\mathcal{R}_{X_{c}}$

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- The contributions, both at the order $1 / m_{b}{ }^{2}$ and $1 / m_{b}{ }^{3}$ are considered separately. (Phys. Lett. B326 (1994) 145-153, Nucl. Phys. B921 (2017) 211-224) scheme dependence deviates the central value $\approx 4 \%$ (consistent within error bar) SM prediction for $\mathcal{R}_{X_{c}}$
$m_{c}$ in scheme:
$\overline{M S}$ upto order $\quad$ Kinetic up to order

| $\mathcal{O}\left(1 / m_{b}^{2}\right)$ | $\mathcal{O}\left(1 / m_{b}^{3}\right)$ | $\mathcal{O}\left(1 / m_{b}^{2}\right)$ | $\mathcal{O}\left(1 / m_{b}^{3}\right)$ |
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## New Physics Analysis

- Varieties of NP models can contribute to $B \rightarrow D^{(*)} \tau \nu_{\tau}$
- An observable not equally sensitive to all types of NP.
- Useful to know :
- Which type of new physics can best explain the present experimental data??
- Data- based Model Selection $\rightarrow$ a multi-scenario analysis on the experimentally available binned data, to obtain a data-based selection of a best NP scenario and ranking and weighting of the remaining models.


## Model Independent Analysis

- Most general effective Hamiltonian describing the $b \rightarrow c \tau \nu_{\tau}$ [Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe,PRD 91, no. 11, 114028 (2015)]

$$
\begin{aligned}
\mathcal{H}_{e f f} & =\frac{4 G_{F}}{\sqrt{2}} V_{c b}\left[\left(1+C_{V_{1}}\right) \mathcal{O}_{V_{1}}+C_{V_{2}} \mathcal{O}_{V_{2}}\right. \\
& \left.+C_{S_{1}} \mathcal{O}_{S_{1}}+C_{S_{2}} \mathcal{O}_{S_{2}}+C_{T} \mathcal{O}_{T}\right]
\end{aligned}
$$

Operator basis :

$$
\begin{aligned}
\mathcal{O}_{V_{1}} & =\left(\bar{c}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{\tau L}\right), \mathcal{O}_{V_{2}}=\left(\bar{c}_{R} \gamma^{\mu} b_{R}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{\tau L}\right) \\
\mathcal{O}_{S_{1}} & =\left(\bar{c}_{L} b_{R}\right)\left(\bar{\tau}_{R} \nu_{\tau L}\right), \quad \mathcal{O}_{S_{2}}=\left(\bar{c}_{R} b_{L}\right)\left(\bar{\tau}_{R} \nu_{\tau L}\right) \\
\mathcal{O}_{T} & =\left(\bar{c}_{R} \sigma^{\mu \nu} b_{L}\right)\left(\bar{\tau}_{R} \sigma_{\mu \nu} \nu_{\tau L}\right)
\end{aligned}
$$

- Neutrinos are assumed to be left handed.


## Data- Based Model Selection

- Work Plan : Data-based selection of a 'best' case and ranking the remaining cases.


## Data- Based Model Selection

- Work Plan: Data-based selection of a 'best' case and ranking the remaining cases.
- Akaike Information criteria(Second Order) [N. Sugiura, Commun. Stat. Theor. Meth. A 7, 13 (1978).]

$$
\mathrm{AIC}_{c}=\chi_{\text {min }}^{2}+2 K+\frac{2 K(K+1)}{n-K-1}
$$

$K=$ number of parameters ; $n=$ sample size; $n / K<40$.

- $\Delta_{i}^{A I C}\left(\mathrm{AIC}_{c}^{i}-\mathrm{AIC}_{c}^{\min }\right) \Rightarrow$ Comparison and ranking of candidate models
- 'Best' model $\Rightarrow \Delta_{i}^{A I C} \equiv \Delta_{\text {min }}^{A I C}=0$.

| $\Delta_{i}^{\text {AIC }}$ | Level of Empirical Support for Model $i$ |
| :---: | :---: |
| $0-2$ | Substantial |
| $4-7$ | Considerably Less |
| $>10$ | Essentially None |

- Akaike Weight : weight of evidence in favor of model i

$$
w_{i}=\frac{e^{\left(-\Delta_{i}^{A I C} / 2\right)}}{\sum_{r=1}^{R} e^{\left(-\Delta_{r}^{A I C} / 2\right)}}
$$

## Model Selection

- Model Independent multi-scenario analysis with experimentally available results $\rightarrow$ data-based selection of a 'best' scenario.
- Four different combination of datasets :

- 3 variations of similar combinations of datasets.
- Without $\mathcal{R}_{J / \psi}$
- With $\mathcal{R}_{J / \psi}$ in LFCQ
- With $\mathcal{R}_{J / \psi}$ in pQCD
- Apparent tension among experimental and SM value $\Rightarrow \mathcal{R}_{J / \psi}$ treated separately.


## Results

| Index | Data Without $\mathcal{R}_{J / \Psi}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|l} \hline x_{\min }^{2} \\ / \operatorname{DoF} \end{array}$ | $p$-val (\%) | Param.s | $w^{\mathrm{AIC}_{c}}$ | $\begin{gathered} B_{c} \rightarrow \\ \tau \nu \\ \hline \end{gathered}$ |
| 1 | 4.05/8 | 85.3 | $\mathcal{R e}\left(C_{T}\right)$ | 35.85 | $\checkmark$ |
| 2 | $4.58 / 8$ | 80.13 | $\mathcal{R e}\left(C_{V_{1}}\right)$ | 20.99 | $\checkmark$ |
| 3 | 4.64/8 | 79.54 | $\mathcal{R e}\left(C_{S_{2}}\right)$ | 19.82 | $\times$ |
| 4 | $3.54 / 7$ | 83.07 | $\mathcal{I} m\left(C_{S_{2}}\right), \mathcal{R e}\left(C_{S_{2}}\right)$ | 1.92 | $\times$ |
| 5 | 3.54/7 | 83.07 | $\mathcal{R e}\left(C_{S_{1}}\right), \mathcal{R e}\left(C_{S_{2}}\right)$ | 1.92 | $\times$ |
| 6 | $3.56 / 7$ | 82.9 | $\mathcal{R e}\left(C_{S_{2}}\right), \mathcal{R} e\left(C_{V_{1}}\right)$ | 1.89 | $\checkmark$ ! |
| 7 | $3.56 / 7$ | 82.9 | $\mathcal{R e}\left(C_{S_{2}}\right), \mathcal{R e}\left(C_{T}\right)$ | 1.89 | $\checkmark$ ! |
| 8 | 3.56/7 | 82.88 | $\mathcal{R e}\left(C_{S_{2}}\right), \mathcal{R e}\left(C_{V_{2}}\right)$ | 1.89 | $\checkmark$ ! |
| 9 | $3.62 / 7$ | 82.23 | $\mathcal{R e}\left(C_{T}\right), \mathcal{R e}\left(C_{V_{2}}\right)$ | 1.78 | $\checkmark$ |
| 10 | $3.69 / 7$ | 81.45 | $\mathcal{R e}\left(C_{S_{1}}\right), \mathcal{R e}\left(C_{T}\right)$ | 1.66 | $\checkmark$ ! |
| 11 | $3.7 / 7$ | 81.31 | $\mathcal{R e}\left(C_{S_{1}}\right), \mathcal{R} e\left(C_{V_{2}}\right)$ | 1.64 | $\checkmark$ ! |
| 12 | $3.76 / 7$ | 80.71 | $\mathcal{R e}\left(C_{S_{1}}\right), \mathcal{R e}\left(C_{V_{1}}\right)$ | 1.55 | $\checkmark$ ! |
| 13 | $3.79 / 7$ | 80.37 | $\mathcal{R e}\left(C_{V_{1}}\right), \mathcal{R e}\left(C_{V_{2}}\right)$ | 1.5 | $\checkmark$ |
| 14 | 3.79/7 | 80.37 | $\operatorname{Im}\left(C_{V_{2}}\right), \mathcal{R} e\left(C_{V_{2}}\right)$ | 1.5 | $\checkmark$ |
| 15 | $3.82 / 7$ | 80.08 | $\mathcal{R e}\left(C_{T}\right), \mathcal{R} e\left(C_{V_{1}}\right)$ | 1.46 | $\checkmark$ |
| 16 | $3.87 / 7$ | 79.49 | $\mathcal{I} m\left(C_{T}\right), \mathcal{R} e\left(C_{T}\right)$ | 1.39 | $\checkmark$ |
| 17 | $4.58 / 7$ | 71.09 | $\mathcal{I} m\left(C_{V_{1}}\right), \mathcal{R} e\left(C_{V_{1}}\right)$ | 0.68 | $\checkmark$ |



## Results

## Best One operator scenarios: $\mathcal{O}_{T} / \mathcal{O}_{V_{1}}$ with $\operatorname{Re}\left(C_{W}\right)$

| Index | $/ \mathrm{DoF}$ | $(\%)$ |  |  | $\tau \nu$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $4.05 / 8$ | 85.3 | $\mathcal{R} e\left(C_{T}\right)$ | 25.85 | $\checkmark$ |
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## Results

Best One operator scenarios: $\mathcal{O}_{T} / \mathcal{O}_{V_{1}}$ with $\operatorname{Re}\left(C_{W}\right)$


## Results

Best One operator scenarios: $\mathcal{O}_{T} / \mathcal{O}_{V_{1}}$ with $\operatorname{Re}\left(C_{W}\right)$

$\mathcal{O}_{V_{2}}$ : Less favored, allowed with complex $C_{W}$

| 14 | $3.79 / 7$ | 80.37 | $\mathcal{I} m\left(C_{V_{2}}\right), \mathcal{R} e\left(C_{V_{2}}\right)$ | 1.5 | $\checkmark$ |
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## Results

- In absence of $P_{\tau}\left(D^{*}\right)$ the conclusions remain same.
- Without considering BABAR data : two more one-operator scenarios $\mathcal{R} e\left(C_{V_{2}}\right)$ and $\mathcal{R} e\left(C_{S_{1}}\right)$ are allowed.
- Considering only $\mathcal{R}_{D^{*}}$ data : All of $\mathcal{O}_{V_{1}}, \mathcal{O}_{V_{2}}, \mathcal{O}_{S_{1}}, \mathcal{O}_{S_{2}}, \mathcal{O}_{T}$ are allowed with $\mathcal{R} e\left(C_{W}\right) . \mathcal{B}\left(B_{c} \rightarrow \tau \nu_{\tau}\right)$ disfavors the scenarios with scaler operators.
- In all these analysis, conclusions remain unchanged in presence of $\mathcal{R}_{J / \psi}$ data.
- For all the scenarios allowed by $\Delta A I C_{c}$ as well as $\mathcal{B}\left(B_{c} \rightarrow \tau \nu_{\tau}\right)$ constraints the values of NP parameters with their uncertainties and correlations are estimated.


## Results

| Data Without $\mathcal{R}_{J / \Psi}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Index | Param.s | Best-fit | Correlation |
| 1 | $\mathcal{R e} e\left(C_{T}\right)$ | $0.387(11)$ | - |
| 2 | $\mathcal{R e} e\left(C_{V_{1}}\right)$ | $0.098(22)$ | - |
| 6 | $\mathcal{R e}\left(C_{S_{2}}\right)$ | $0.073(79)$ | -0.409 |
|  | $\mathcal{R e}\left(C_{V_{1}}\right)$ | $0.089(24)$ |  |
| 7 | $\operatorname{Re}\left(C_{S_{2}}\right)$ | $0.181(67)$ | 0.075 |
|  | $\mathcal{R e}\left(C_{T}\right)$ | -0.043(11) |  |
| 8 | $\mathcal{R} e\left(C_{S_{2}}\right)$ | $0.279(68)$ | -0.302 |
|  | $\mathcal{R e}\left(C_{V_{2}}\right)$ | -0.111 (29) |  |
| 9 | $\mathcal{R e}\left(C_{T}\right)$ | -0.112 (26) | -0.93 |
|  | $\mathcal{R e}\left(C_{V_{2}}\right)$ | $0.196(74)$ |  |
| 10 | $\operatorname{Re}\left(C_{S_{1}}\right)$ | $0.179(66)$ | 0.351 |
|  | $\mathcal{R} e\left(C_{T}\right)$ | -0.033(12) |  |
| 11 | $\mathcal{R} e\left(C_{S_{1}}\right)$ | $0.245(60)$ | -0.01 |
|  | $\mathcal{R} e\left(C_{V_{2}}\right)$ | -0.075(28) |  |
| 12 | $\mathcal{R e}\left(C_{S_{1}}\right)$ | $0.086(90)$ | -0.684 |
|  | $\mathcal{R e}\left(C_{V_{1}}\right)$ | $0.078(30)$ |  |
| 13 | $\mathcal{R e}\left(C_{V_{1}}\right)$ | $0.117(31)$ | 0.709 |
|  | $\mathcal{R e}\left(C_{V_{2}}\right)$ | $0.037(41)$ |  |
| 14 | $\operatorname{Im}\left(C_{V_{2}}\right)$ | $0.497(68)$ | 0.716 |
|  | $\mathcal{R} e\left(C_{V_{2}}\right)$ | $0.042(46)$ |  |
| 15 | $\mathcal{R e}\left(C_{T}\right)$ | $0.030(34)$ | 0.917 |
|  | $\mathcal{R} e\left(C_{V_{1}}\right)$ | $0.142(54)$ |  |
| 16 | $\operatorname{Im}\left(C_{T}\right)$ | $0.16(15)$ | -0.995 |
|  | $\mathcal{R e}\left(C_{T}\right)$ | $0.32(15)$ |  |
| 17 |  | See Plot |  |



## Results

- Using these NP results, the values of all the observables are predicted.
- Trying to explain the deviation in $\mathcal{R}_{D^{(*)}}$ for a specific NP $\Rightarrow$ Information about the expected deviations in other associated observables.
- Any result, inconsistent with SM, but consistent with a future prediction of some observable $\Rightarrow$ indirect evidence in support for that specific scenario.
- The correlations between the observables will play an important role.


## Correlation Plots



| SM |  |
| :---: | :---: |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{T}}\right)$ |  |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{1}}\right)$ |  |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{2}}\right)$ |  |
| . $\cdot$ R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{S}_{1}}\right)$ |
| $\begin{aligned} & \text { Belle }+\mathrm{I} \\ & \text { (w/o } \mathcal{R} \end{aligned}$ | $\begin{aligned} & \mathrm{le}+\mathrm{LHCb} \\ & \left.o \mathcal{R}_{\mathrm{J} / \Psi}\right) \end{aligned}$ |

## Correlation Plots

Only with $\mathcal{O}_{V_{2}} \mathcal{R}_{D}$ and $\mathcal{R}_{D^{*}}$ are negatively correlated


## Correlation Plots

Only with $\mathcal{O}_{V_{2}} \mathcal{R}_{D}$ and $\mathcal{R}_{D^{*}}$ are negatively correlated


For $\operatorname{Re}\left(C_{V_{2}}\right), \operatorname{Re}\left(C_{S_{1}}\right)$ and $\operatorname{Re}\left(C_{V_{1}}\right)$ : simultaneously consistency with SM


| SM |  |
| :---: | :---: |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{T}}\right)$ |  |
| $\cdots \operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{1}}\right)$ |  |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{2}}\right)$ |  |
| R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{S}_{1}}\right)$ |
| Belle+ (w/o | $\begin{aligned} & \text { e+LHCb } \\ & \left.\mathcal{R}_{\mathrm{J} / \Psi}\right) \end{aligned}$ |

## Correlation Plots

Only with $\mathcal{O}_{V_{2}} \mathcal{R}_{D}$ and $\mathcal{R}_{D^{*}}$ are negatively correlated
$\square$
For $\operatorname{Re}\left(C_{V_{2}}\right), \operatorname{Re}\left(C_{S_{1}}\right)$ and $\operatorname{Re}\left(C_{V_{1}}\right)$ : simultaneously consistency with SM 0 ¢
For $\mathcal{R e}\left(C_{T}\right)$ : consistency of $\mathcal{R}_{D^{*}}$, large deviation of $\mathcal{R}_{D}$ with SM


| SM |  |
| :---: | :---: |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{T}}\right)$ |  |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{1}}\right)$ |  |
| $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{2}}\right)$ |  |
| $\cdots \cdots \operatorname{Re}\left(\mathrm{C}_{\mathrm{S}_{1}}\right)$ |  |
| Belle+ (w/o | $\begin{aligned} & \mathrm{e}+\mathrm{LHCb} \\ & \left.\mathrm{o} \mathcal{R}_{\mathrm{J} / \Psi}\right) \end{aligned}$ |

## Correlation Plots

- asymmetric and angular observables : insensitive to $\mathcal{O}_{V_{1}} \Rightarrow$ canceled in the ratios.




## Correlation Plots

- asymmetric and angular observables : insensitive to $\mathcal{O}_{V_{1}} \Rightarrow$ canceled in the ratios.



## Correlation Plots

- asymmetric and angular observables : insensitive to $\mathcal{O}_{V_{1}} \Rightarrow$ canceled in the ratios.

measured value, above SM: explained by the scalar operator.


|  | SM |
| :---: | :---: |
| - R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{T}}\right)$ |
| ... R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{1}}\right)$ |
| R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{2}}\right)$ |
| R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{S}_{1}}\right)$ |
| $\begin{aligned} & \text { Belle+LHCb } \\ & \left(\mathrm{w} / \mathrm{o} \mathcal{R}_{\mathrm{J} / \Psi}\right) \end{aligned}$ |  |

## Correlation Plots

- asymmetric and angular observables : insensitive to $\mathcal{O}_{V_{1}} \Rightarrow$ canceled in the ratios.

measured value, above SM: explained by the scalar operator.


In future measured value consistent with $\mathcal{R}_{D^{*}}$ large deviation in $P_{\tau}(D)$ : tensor NP



## Correlation Plots



## Correlation Plots

presence of $\mathcal{O}_{T}: \mathcal{R}_{D^{*}}$ is with negative correlation with $P_{\tau}(D)$


## Correlation Plots

presence of $\mathcal{O}_{T}: \mathcal{R}_{D^{*}}$ is with negative correlation with $P_{\tau}(D)$


presence of $\mathcal{O}_{T}: \mathcal{R}_{D^{*}}$ is with positive correlation with $P_{\tau}\left(D^{*}\right)$


| $\cdots$ | SM |
| :--- | :--- |
| $\cdots$ | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{T}}\right)$ |
| $\cdots$ | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{\mathrm{L}}}\right)$ |
|  | $\operatorname{Im}\left(\mathrm{C}_{\mathrm{V}_{2}}\right), \operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{2}}\right)$ |
|  | All Data |
|  | $\left(\mathrm{W} / \mathrm{o} R_{\mathrm{J} / \Psi}\right)$ |

( $\mathrm{w} / \mathrm{o} \mathcal{R}_{\mathrm{J} / \mathrm{\Psi}}$ )
presence of $\mathcal{O}_{T}: P_{\tau}(D)$ and $P_{\tau}\left(D^{*}\right)$ below and above SM predictions

## Correlation Plots




|  | SM |
| :---: | :---: |
| - R | $\operatorname{Re}\left(C_{T}\right)$ |
| - R | $\operatorname{Re}\left(\mathrm{C}_{\mathrm{V}_{1}}\right)$ |
| . R | $\mathrm{Re}\left(\mathrm{CV}_{2}\right)$ |
| . . . . . R | $\operatorname{Re}\left(C_{S_{1}}\right)$ |
| Belle + LHCb |  |
| (w/o R | - $\mathcal{R}_{\mathrm{J} / \Psi}$ ) |



## Correlation Plots



## Correlation Plots



Large uncertainty in SM for $\mathcal{R}_{J / \psi} \Rightarrow$ NP predictions consistent with its SM


## Correlation Plots



Large uncertainty in SM for $\mathcal{R}_{J / \psi} \Rightarrow$ NP predictions consistent with its SM

$\mathcal{R e}\left(C_{T}\right)$ : allow a large deviation in $\mathcal{R}_{\Lambda}^{\mu}$, a sizeable effect in $\mathcal{R}_{D^{*}}$

## Conclusion

- In the first part of analysis :
- Following the result of up-to-date analysis on $B \rightarrow D^{(*)} \ell \nu_{\ell} \Rightarrow \mathrm{SM}$ prediction of angular observables associated with $B \rightarrow D^{(*)} \tau \nu_{\tau}$
- The SM prediction of inclusive semitaunic observable $\mathcal{R}_{X_{c}}$ is updated. These predictions are based on two different schemes of the charm quark mass ( $\bar{M} S$ and Kinetic). These include the NNLO perturbative corrections, and power-corrections up to order $1 / m_{b}^{3}$.
- In the next part :
- we have analysed the semitaunic $b \rightarrow c \tau \nu_{\tau}$ decays in a model independent framework.
- Among all the data sets the one operator scenario with real Wilson coefficient can best explain the available data.
- Scalar operators are not allowed by the constraint $\mathcal{B}\left(B_{c} \rightarrow \tau \nu_{\tau}\right) \leq 30 \%$
- The most favoured scenarios are the ones with tensor $\left(\mathcal{O}_{T}\right)$ or $(V-A)$ ( $\mathcal{O}_{V_{1}}$ ) type of operators.
- These one operator scenarios are easily distinguishable from each other by studying the correlations of $\mathcal{R}_{D^{*}}$ with $\mathcal{R}_{D}$ and all the other asymmetric and angular observables.


## Thank You

## SM prediction (Exclusive)

| Observable | SM Prediction | Correlation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathcal{R}_{D^{*}}$ | 0.260(6) | 1. | 0.118 | 0.617 | 0.118 | 0.604 | 0.628 | -0.118 |
| $\mathcal{R}_{D}$ | 0.305(3) |  | 1. | -0.023 | 1. | 0.021 | 0.007 | -1. |
| $P_{\tau}\left(D^{*}\right)$ | -0.491(25) |  |  | 1. | -0.023 | 0.803 | 0.895 | 0.023 |
| $P_{\tau}(D)$ | 0.3355(4) |  |  |  | 1. | 0.021 | 0.007 | -1. |
| $F_{L}^{D^{*}}$ | 0.457(10) |  |  |  |  | 1. | 0.921 | -0.021 |
| $\mathcal{A}_{F B}^{*}$ | -0.058(14) |  |  |  |  |  | 1. | -0.007 |
| $\mathcal{A}_{\text {F } B}$ | 0.3586(3) |  |  |  |  |  |  | 1. |
| $\mathcal{R}_{J / \Psi}(\mathrm{LFCQ})$ | 0.249(42) |  |  |  |  |  |  |  |
| $\mathcal{R}_{J / \Psi}(\mathrm{PQCD})$ | 0.289(28) |  |  |  |  |  |  |  |
| $\mathcal{R}_{\Lambda}^{\mu}$ | 0.329(13) |  |  |  |  |  |  |  |
| $\mathcal{R}_{\Lambda}^{e}$ | 0.328(13) |  |  |  |  |  |  |  |
| $\mathcal{B}\left(B_{c} \rightarrow \tau \nu\right)$ | 0.0208(18) |  |  |  |  |  |  |  |

## Formalism

- $q^{2}$-distributions of the differential decay rates in $B \rightarrow D^{(*)} \tau \nu_{\tau}$ decays are given by

$$
\begin{aligned}
\frac{d \Gamma\left(\bar{B} \rightarrow D \tau \bar{\nu}_{\tau}\right)}{d q^{2}}= & \frac{G_{F}^{2}\left|V_{c b}\right|^{2}}{192 \pi^{3} m_{B}^{3}} q^{2} \sqrt{\lambda_{D}\left(q^{2}\right)}\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times\{ \\
& \left|1+C_{V_{1}}+C_{V_{2}}\right|^{2}\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right) H_{V, 0}^{s}+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{s}\right] \\
& +\frac{3}{2}\left|C_{S_{1}}+C_{S_{2}}\right|^{2} H_{S}^{s 2}+8\left|C_{T}\right|^{2}\left(1+\frac{2 m_{\tau}^{2}}{q^{2}}\right) H_{T}^{s 2} \\
& +3 \operatorname{Re}\left[\left(1+C_{V_{1}}+C_{V_{2}}\right)\left(C_{S_{1}}{ }^{*}+C_{S_{2}}{ }^{*}\right)\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{S}^{s} H_{V, t}^{s} \\
& \left.-12 \operatorname{Re}\left[\left(1+C_{V_{1}}+C_{V_{2}}\right) C_{T}{ }^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{T}^{s} H_{V, 0}^{s}\right\}
\end{aligned}
$$

## Formalism

$$
\begin{aligned}
& \frac{d \Gamma\left(\bar{B} \rightarrow D^{*} \tau \bar{\nu}_{\tau}\right)}{d q^{2}}=\frac{G_{F}^{2} \mid V_{c b}{ }^{2}}{192 \pi^{3} m_{B}^{3}} q^{2} \sqrt{\lambda_{D^{*}}\left(q^{2}\right)}\left(1-\frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times\{ \\
& \left(\left|1+C_{V_{1}}\right|^{2}+\left|C_{V_{2}}\right|^{2}\right)\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right)\left(H_{V,+}^{2}+H_{V,-}^{2}+H_{V, 0}^{2}\right)+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{2}\right] \\
& -2 \operatorname{Re}\left[\left(1+C_{V_{1}}\right) C_{V_{2}}{ }^{*}\right]\left[\left(1+\frac{m_{\tau}^{2}}{2 q^{2}}\right)\left(H_{V, 0}^{2}+2 H_{V,+} H_{V,-}\right)+\frac{3}{2} \frac{m_{\tau}^{2}}{q^{2}} H_{V, t}^{2}\right] \\
& +\frac{3}{2}\left|C_{S_{1}}-C_{S_{2}}\right|^{2} H_{S}^{2}+8\left|C_{T}\right|^{2}\left(1+\frac{2 m_{\tau}^{2}}{q^{2}}\right)\left(H_{T,+}^{2}+H_{T,-}^{2}+H_{T, 0}^{2}\right) \\
& +3 \operatorname{Re}\left[\left(1+C_{V_{1}}-C_{V_{2}}\right)\left(C_{S_{1}}{ }^{*}-C_{S_{2}}{ }^{*}\right)\right] \frac{m_{\tau}}{\sqrt{q^{2}}} H_{S} H_{V, t} \\
& -12 \operatorname{Re}\left[\left(1+C_{V_{1}}\right) C_{T}{ }^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}}\left(H_{T, 0} H_{V, 0}+H_{T,+} H_{V,+}-H_{T,-} H_{V,-}\right) \\
& \left.+12 \operatorname{Re}\left[C_{V_{2}} C_{T}^{*}\right] \frac{m_{\tau}}{\sqrt{q^{2}}}\left(H_{T, 0} H_{V, 0}+H_{T,+} H_{V,-}-H_{T,-} H_{V,+}\right)\right\}
\end{aligned}
$$

## Backup Slides

- A true model with true parameter values :

$$
\chi^{2}=\text { d.o.f i.e. } \chi_{\text {red }}^{2}=1 \text { (no fit involved) }
$$

- Not sufficient to assess convergence or compare different models ! (noise present in the data)
- For the true model, with a-priori known measurement errors:

Distribution of normalized residuals (in our case, $\frac{R_{b i n}^{t h}-R_{b i n}^{e x p}}{\delta R_{b i n}}$ ) is a Gaussian with mean $\mu=0$ and variance $\sigma^{2}=1$.

- Test of significance of the fit $\rightarrow$ Fitting the distribution of residuals to the Gaussian.
- Validity of a hypothesis : $p$-value of the goodness of fit test $\geq 5 \%$.
- $p$-value : probability that a random variable having a $\chi^{2}$-distribution with d.o.f $\geq 1$ assumes a value which is larger than a given value of $\chi^{2}(\geq 0)$


## Backup Sides

- To compare the latest BABAR and Belle binned data with a specific model, we devise a $\chi^{2}$ defined as:

$$
\chi_{N P}^{2}=\sum_{i, j=1}^{n_{b}}\left(R_{i}^{\text {exp }}-R_{i}^{t h}\right)\left(V^{\text {exp }}\right)_{i j}^{-1}\left(R_{j}^{e x p}-R_{j}^{t h}\right)+\chi_{N u i s a n c e}^{2}
$$

- $V_{i j}^{\text {exp }}=\delta_{i j} \delta R_{i}^{\text {exp }} \delta R_{j}^{e x p}$, where $\delta_{i j}$ is the Kronecker delta. (Assumptions : correlations negligible)
- Total 10 unknown NP parameters and 26 observables for BABAR (14 bins for $B \rightarrow D \tau \nu$ and 12 bins for $B \rightarrow D^{*} \tau \nu$ ) and 17 observables for Belle.
- Minimize the $\chi_{N P}^{2}$ for different cases and different set of observables.
- Define reduced statistic $\chi_{\text {red }}^{2}=\chi_{\text {min }}^{2} /$ d.o.f where d.o.f $=N_{\text {Obs }}-N_{\text {Params }}$
- In information theory, the Kullback-Leibler (K-L) Information or measure $I(f, g) \Rightarrow$ information lost when $g$ is used to approximate $f$. Here $f$ is a notation for full reality or truth and $g$ denotes an approximating model in terms of probability distribution.
- Akaike proposed the use of the K-L information as a fundamental basis for model selection.
- This is a rigorous way to estimate K-L information, based on the empirical log-likelihood function at its maximum point.
'Akaike's information criterion'(AIC) with respect to our analysis can be defined as,

$$
\begin{equation*}
\mathrm{AIC}=\chi_{\min }^{2}+2 K \tag{1}
\end{equation*}
$$

where $K$ is the number of estimable parameters.
AIC may perform poorly if there are too many parameters in relation to the size of the sample. second-order variant of AIC,

$$
\begin{equation*}
\mathrm{AIC}_{c}=\chi_{\min }^{2}+2 K+\frac{2 K(K+1)}{n-K-1} \tag{2}
\end{equation*}
$$

where $n$ is the sample size. As a rule of thumb, Use of $\mathrm{AIC}_{c}$ is preferred in literature when $n / K<40$.
$5 C_{W}$ 's $\rightarrow C_{V_{1}}, C_{V_{2}}, C_{S_{1}}, C_{S_{2}}, C_{T}$.
Each one complex $\rightarrow$ total 10 parameters.
We took a severl such combinations.
Which one fits the data best?
Standard method in Heavy Flavor physics: $\Delta \chi^{2}$ test (Likelihood-Ratio test ):

- Can only be applied to nested models.
- $\Delta \chi^{2}=\chi_{\text {min }, S}^{2}-\chi_{m i n, L}^{2}$.
- When model $S$ (fewer parameters: null) is true (under certain conditions), Wilks' Theorem $\rightarrow \Delta \chi^{2}$ has a $\chi^{2}$ distribution with the d.of $=p_{L}-p_{S}$.
- compute a $p$-value, compare it to a critical value $\rightarrow$ decide to reject the null in favor of the alternative.


[^0]:    S. Jaiswal, S. Nandi, and S. K. Patra, JHEP 12, 060(2017).

