Strongly interacting matter in extreme conditions: insights from hydrodynamic modeling of heavy ion collisions

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Outline

LECTURE I

1. **Introduction**
   1.1 Standard model of heavy-ion collisions
   1.2 Basic hydrodynamic concepts
   1.3 Global and local equilibrium
   1.4 Navier-Stokes hydrodynamics
2. **Viscous fluid dynamics**
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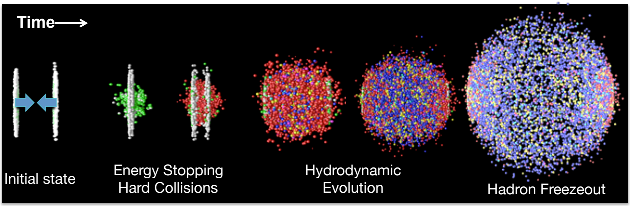
   2.2 Israel Stewart and MIS equations
   2.3 BRSSS approach
   2.4 DNMR approach
   2.5 Gradient expansion
3. **Hydrodynamic equations with spin**

1. Introduction

"Standard model" of heavy-ion collisions



T. K. Nayak, Lepton-Photon 2011 Conference

FIRST STAGE — HIGHLY OUT-OF EQUILIBRIUM ($0 < \tau_0 \lesssim 1\,\mathrm{fm}$)

- **initial conditions**, including fluctuations, reflect to large extent the distribution of matter in the colliding nuclei
- **emission of hard probes**: heavy quarks, photons, jets
- **hydrodynamization stage** – the system becomes well described by equations of viscous hydrodynamics

"Standard model" of heavy-ion collisions

SECOND STAGE — HYDRODYNAMIC EXPANSION ($1\,\mathrm{fm} \lesssim \tau \lesssim 10\,\mathrm{fm}$)

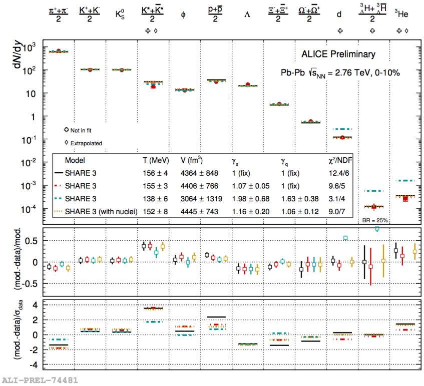
- expansion controlled by viscous hydrodynamics (effective description)
- **thermalization stage**
- **phase transition** from QGP to hadron gas takes place (encoded in the equation of state)
- **equilibrated hadron gas**

THIRD STAGE — FREEZE-OUT

- **freeze-out and free streaming of hadrons** ($10\,\mathrm{fm} \lesssim \tau$)

THIS TALK:
EFFECTS OF FINITE BARYON NUMBER DENSITY ARE NEGLECTED

Thermal fit to hadron multiplicity ratios

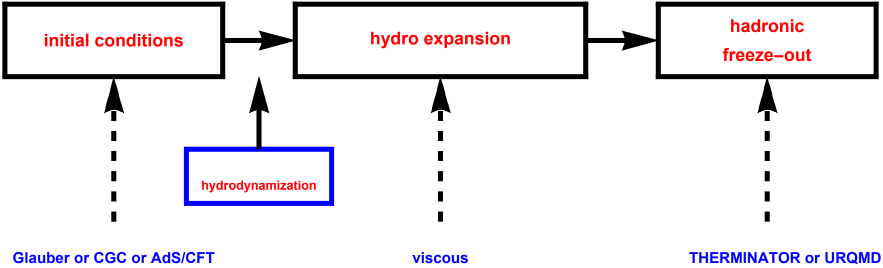


M. Floris, Nucl. Phys. A931 (2014) c103

elaborate studies by F. Becattini et al., P. Braun-Munzinger et al.,...

In the end of its space-time evolution, the system is close to local equilibrium

STANDARD MODEL (MODULES) of HEAVY–ION COLLISIONS



FLUCTUATIONS IN THE INITIAL STATE / EVENT–BY–EVENT HYDRO / FINAL–STATE FLUCTUATIONS

EQUATION OF STATE = lattice QCD

1 < VISCOSITY < 3 times the lower bound

W. Broniowski, M. Chojnacki, WF, A. Kisiel, Phys.Rev.Lett. 101 (2008) 022301

H. Song, S. Bass, U. Heinz, T. Hirano, and C. Shen, Phys. Rev. Lett. 106 (2011) 192301

Basic hydrodynamic concepts

WF, M. P. Heller, M. Spalinski, to be published

- **genuine hydrodynamic behaviour is a property of physical systems evolving toward equilibrium**
  1) one separates between transient (nonhydrodynamic) and slowly decaying (hydrodynamic) modes, 2) the latter are connected with real hydrodynamic behaviour, 3) typical modern hydrodynamic equations include both of them
- **hydrodynamics (set of hydrodynamic equations) may be formulated without explicit reference to microscopic degrees of freedom**
  1) this is important if we deal with strongly interacting matter —
  in this case neither hadronic nor partonic degrees of freedom seem to be adequate degrees of freedom, 2) such a general formulation of hydrodynamics may be limited – based on the gradient expansion, which does not converge
- **hydrodynamics (set of hydrodynamic equations) may be also constructed in a direct relation to some underlying, microscopic theory**
  1) the most common approaches refer to kinetic theory, 2) new developments based in the AdS/CFT correspondance

Basic hydrodynamic concepts

- **hydrodynamic equations describe the space-time evolution of the energy-momentum tensor components, $T^{\mu\nu}$, seems to be a limited knowledge but ...**
- **the information about the state of matter is, to large extent, encoded in the structure of its energy-momentum tensor**
  1) equation of state, kinetic (transport) coefficients including the shear and bulk viscosities, 2) this structure may be a priori determined by modelling of heavy-ion collisions, 3) we are lucky that this scenario has been indeed realised, this is largely so, because the created system evolves towards local equilibrium state

Global equilibrium

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1959

The equilibrium energy-momentum tensor in the **fluid rest-frame** is given by

$$T^{\mu\nu}_{\rm EQ} = \begin{vmatrix} \mathcal{E}_{\rm EQ} & 0 & 0 & 0 \\ 0 & \mathcal{P}(\mathcal{E}_{\rm EQ}) & 0 & 0 \\ 0 & 0 & \mathcal{P}(\mathcal{E}_{\rm EQ}) & 0 \\ 0 & 0 & 0 & \mathcal{P}(\mathcal{E}_{\rm EQ}) \end{vmatrix} \qquad (1)$$

assumption: the equation of state is known, so that the pressure $\mathcal{P}$ is a given function of the energy density $\mathcal{E}_{\rm EQ}$

in an arbitrary frame of reference

$$T^{\mu\nu}_{\rm EQ} = \mathcal{E}_{\rm EQ} u^\mu u^\nu - \mathcal{P}(\mathcal{E}_{\rm EQ}) \Delta^{\mu\nu}, \qquad (2)$$

where u^μ is a constant velocity, and $\Delta^{\mu\nu}$ is the operator that projects on the space orthogonal to u^μ, namely

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu. \qquad (3)$$

Local equilibrium – perfect fluid

The energy-momentum tensor of a perfect fluid is obtained by allowing the variables $\mathcal{E}$ and u^μ to depend on the spacetime point x

$$\boxed{T^{\mu\nu}_{\rm eq}(x) = \mathcal{E}(x)u^\mu(x)u^\nu(x) - \mathcal{P}(\mathcal{E}(x))\Delta^{\mu\nu}(x)} \tag{4}$$

the subscript “eq” refers to local thermal equilibrium.

local effective temperature $T(x)$ is determined by the condition that the equilibrium energy density at this temperature agrees with the non-equilibrium value of the energy density, namely

$$\boxed{\mathcal{E}_{\rm EQ}(T(x)) = \mathcal{E}_{\rm eq}(x) = \mathcal{E}(x)} \tag{5}$$

Perfect fluid

$T(x)$ and $u^\mu(x)$ are fundamental fluid variables

the relativistic perfect-fluid energy-momentum tensor is the most general symmetric tensor which can be expressed in terms of these variables without using derivatives.

dynamics of the perfect fluid theory is provided by the conservation equations of the energy-momentum tensor

$$\partial_\mu T^{\mu\nu}_{\rm eq} = 0 \tag{6}$$

four equations for the four independent hydrodynamic fields – a self-consistent (hydrodynamic) theory

DISSIPATION DOES NOT APPEAR!

$$\partial_\mu(\mathcal{S}u^\mu) = 0 \tag{7}$$

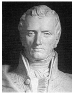
entropy conservation follows from the energy-momentum conservation and the form of the energy-momentum tensor

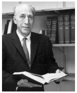
Navier-Stokes hydrodynamics

Claude-Louis Navier, 1785–1836, French engineer and physicist
Sir George Gabriel Stokes, 1819–1903, Irish physicist and mathematician

C. Eckart, Phys. Rev. 58 (1940) 919

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1959






complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu} \tag{8}$$

where $\Pi^{\mu\nu} u_\nu = 0$, which corresponds to the Landau definition of the hydrodynamic flow u^μ

$$T^{\mu}_{\ \nu} u^\nu = \mathcal{E}\, u^\mu . \tag{9}$$

It proves useful to further decompose $\Pi^{\mu\nu}$ into two components,

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Pi \Delta^{\mu\nu}, \tag{10}$$

which introduces the **bulk viscous pressure** Π (the trace part of $\Pi^{\mu\nu}$) and the **shear stress tensor** $\pi^{\mu\nu}$ which is symmetric, $\pi^{\mu\nu} = \pi^{\nu\mu}$, traceless, $\pi^{\mu}_{\ \mu} = 0$, and orthogonal to u^μ, $\pi^{\mu\nu} u_\nu = 0$.

Navier-Stokes hydrodynamics

in the Navier-Stokes theory, the **bulk pressure** and **shear stress tensor** are given by the gradients of the flow vector

$$\Pi = -\zeta\, \partial_\mu u^\mu, \quad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}. \tag{11}$$

Here ζ and η are the bulk and shear viscosity coefficients, respectively, and $\sigma^{\mu\nu}$ is the shear flow tensor defined as

$$\sigma^{\mu\nu} = 2\,\Delta^{\mu\nu}_{\alpha\beta}\partial^\alpha u^\beta, \tag{12}$$

where the projection operator $\Delta^{\mu\nu}_{\alpha\beta}$ has the form

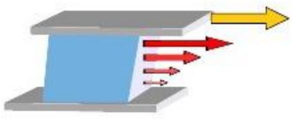
$$\Delta^{\mu\nu}_{\alpha\beta} = \frac{1}{2}\left(\Delta^\mu{}_\alpha \Delta^\nu{}_\beta + \Delta^\mu{}_\beta \Delta^\nu{}_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}. \tag{13}$$

Viscosity

shear viscosity η

$\Downarrow$

reaction to a change of shape

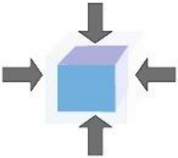


$$\pi^{\mu\nu}{}_{\mathrm{Navier-Stokes}} = 2\eta\, \sigma^{\mu\nu}$$

bulk viscosity ζ

$\Downarrow$

reaction to a change of volume



$$\Pi_{\mathrm{Navier-Stokes}} = -\zeta\theta$$

bulk viscosity and pressure vanish for conformal fluids

$$0 = T^{\mu}{}_{\mu} = \underbrace{\mathcal{E} - 3\mathcal{P}}_{=0} - 3\Pi + \underbrace{\pi^{\mu}{}_{\mu}}_{=0} = -3\Pi, \quad \Pi = 0$$

QGP shear viscosity: large or small?

John Mainstone (Wikipedia)

Pitch

Start 1927

1st drop 1938

8th drop 2000

$\eta \sim 2\,10^8$ Pa s

$\sim 10^{11} \eta_{H_2O}$

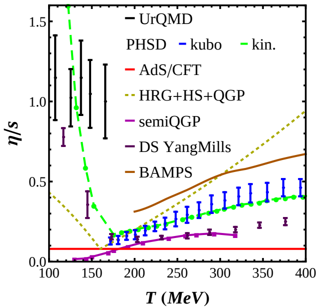
Wikipedia: The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

$$\eta_{\rm qgp} > \eta_{\rm pitch}$$

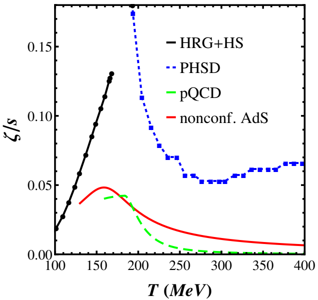
$$\eta_{\rm qgp} \sim 10^{11}\ \text{Pa s}, \qquad (\eta/s)_{\rm qgp} < 3/(4\pi)\hbar \qquad \text{(from experiment)}$$

Shear vs. bulk viscosity

$\eta/\mathcal{S}$ reaches minimum in the region of the phase transition



$\zeta/\mathcal{S}$ reaches maximum in the region of the phase transition



figures from: S. I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, JHEP 1502 (2015) 051

Navier-Stokes hydrodynamics

complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} = T^{\mu\nu}_{\rm eq} + 2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu} \tag{14}$$

again four equations for four unknowns

$$\partial_\mu T^{\mu\nu} = 0 \tag{15}$$

W. A. Hiscock and L. Lindblom, Phys.Rev. D31 (1985) 725

THIS SCHEME DOES NOT WORK IN PRACTICE!
ACAUSAL BEHAVIOR + INSTABILITIES!

NEVERTHELESS, THE GRADIENT FORM (14) IS A GOOD APPROXIMATION FOR SYSTEMS APPROACHING LOCAL EQUILIBRIUM

Gradient expansion

complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \underbrace{2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu}}_{\text{first order terms in gradients}} \tag{16}$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \underbrace{2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu}}_{\text{first order terms in gradients}} + \underbrace{\ldots\ldots\ldots\ldots\ldots\ldots\ldots}_{\text{second order terms in gradients}} + \ldots \tag{17}$$

HYDRODYNAMIC EXPANSION OF THE ENERGY-MOMENTUM TENSOR, ASYMPTOTIC SERIES

M.P. Heller, R. Janik, R. Witaszczyk, PRL 110 (2013) 211602

Pressure anisotropy

M. Strickland, Acta Phys.Polon. B45 (2014) 2355

space-time gradients in boost-invariant expansion
increase the transverse pressure and decrease the longitudinal pressure

$$\mathcal{P}_T = \mathcal{P} + \frac{\pi}{2}, \quad \mathcal{P}_L = \mathcal{P} - \pi, \quad \pi = \frac{4\eta}{3\tau} \tag{18}$$

$$\left(\frac{\mathcal{P}_L}{\mathcal{P}_T}\right)_{\rm NS} = \frac{3\tau T - 16\bar{\eta}}{3\tau T + 8\bar{\eta}}, \quad \bar{\eta} = \frac{\eta}{\mathcal{S}}$$

using the AdS/CFT lower bound for viscosity, $\bar{\eta} = \frac{1}{4\pi}$

RHIC-like initial conditions, $T_0 = 400$ MeV at $\tau_0 = 0.5$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.50$
LHC-like initial conditions, $T_0 = 600$ MeV at $\tau_0 = 0.2$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.35$

2. Viscous fluid dynamics

Relativistic Navier-Stokes equations

Navier-Stokes equations (NS)

$$\partial_\mu T^{\mu\nu}_{vis} \;=\; 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

# of unknowns: 5 + 6 ($\mathcal{E}, \mathcal{P}, u^\mu(3), \Pi, \pi^{\mu\nu}(5)$)
# of equations: 4 + 1 (equation of state $\mathcal{E}(\mathcal{P})$)
we need 6 extra equations - different methods possible

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta, \qquad \theta = \partial_\mu u^\mu \text{– expansion scalar}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu}, \qquad \sigma^{\mu\nu}\text{– shear flow tensor}$$

T, u^μ are the only hydrodynamic variables, $u^\mu{}_\mu = 1$
kinetic coefficients: $\tau_\Pi \beta_\Pi = \zeta \to$ bulk viscosity, $\tau_\pi \beta_\pi = \eta \to$ shear viscosity

Israel-Stewart equations

Israel-Stewart equations — $\Pi, \pi^{\mu\nu}$ promoted to dynamic variables —
non-hydrodynamic modes are introduced with the appropriate relaxation times τ_Π, τ_π

W. Israel and J.M. Stewart, *Transient relativistic thermodynamics and kinetic theory*, Annals of Physics 118 (1979) 341

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} - \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \end{aligned}$$

1) HYDRODYNAMIC EQUATIONS DESCRIBE BOTH HYDRODYNAMIC AND NON-HYDRODYNAMIC MODES

2) HYDRODYNAMIC MODES CORRESPOND TO GENUINE HYDRODYNAMIC BEHAVIOR

3) NON-HYDRODYNAMIC MODES (TERMS) SHOULD BE TREATED AS REGULATORS OF THE THEORY

4) NON-HYDRODYNAMIC MODES GENERATE ENTROPY

MIS equations

Müller-Israel-Stewart or Muronga-Israel-Stewart (MIS)

I. Müller, *Zum Paradoxon der Wärmeleitungstheorie*, Zeit. f. Physik 198 (1967) 329

A. Muronga, *Second-order dissipative fluid dynamics for ultra relativistic nuclear collisions*, PRL 88 (2002) 062302

$$
\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \frac{\zeta T}{2\tau_\Pi} \Pi \, \partial_\lambda \left(\frac{\tau_\Pi}{\zeta T} u^\lambda \right) \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} - \frac{\eta T}{2\tau_\pi} \pi^{\mu\nu} \, \partial_\lambda \left(\frac{\tau_\pi}{\eta T} u^\lambda \right)
\end{aligned}
$$

BRSSS equations

Baier, Romatschke, Son, Starinets, Stephanov (BRSSS)
symmetry arguments due to Lorentz and conformal symmetry, ...

R. Baier, P. Romatschke, D.T. Son, A. O. Starinets, M. A. Stephanov,

Relativistic viscous hydrodynamics, conformal invariance, and holography, JHEP 0804 (2008) 100

$$\partial_\mu T^{\mu\nu}_{vis} = 0 \qquad T^{\mu\nu}_{vis} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P} + \Pi) + \pi^{\mu\nu}$$

$$\begin{aligned} \Pi &= 0 \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + \frac{\lambda_1}{\tau_\pi \eta^2}\pi^{\langle\mu}_{\ \ \lambda}\pi^{\nu\rangle\lambda} \end{aligned}$$

(+ terms including vorticity and curvature)

DNMR equations

Denicol, Niemi, Molnar, Rischke (DNMR)
simultaneous expansion in the Knudsen number and inverse Reynolds number

approach based on the kinetic theory

$$\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta \underline{+ \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}} \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta \underline{- \tau_{\pi\pi} \pi_\gamma^{\langle\mu} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}}
\end{aligned}$$

the version valid for the RTA version of the Boltzmann kinetic equation, for standard form of the collision term additional terms (with new kinetic coefficients) appear

shear-bulk coupling $\eta - \zeta$

Review of different viscous-fluid frameworks

Bjorken viscous expansion
$\phi = -\pi^y_y$ component of the shear stress tensor (the only independent one)
energy-momentum conservation

$$\tau \dot{\epsilon} = -\frac{4}{3}\epsilon + \phi$$

BRSSS

$$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{\lambda_1 \phi^2}{2\eta^2} - \frac{4\tau_\pi \phi}{3\tau} - \phi \quad (19)$$

DNMR with RTA kinetic equation

$$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{38}{21}\frac{\tau_\pi \phi}{\tau} - \phi \quad (20)$$

MIS with RTA kinetic equation

$$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{4\tau_\pi \phi}{3\tau} - \phi \quad (21)$$

Exact solutions of RTA kinetic equation

- Boltzmann equation in the (RTA) relaxation time approximation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p^\mu u_\mu \, \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

Bhatnagar, Gross, Krook, Phys. Rev. 94 (1954) 511

- background distribution (Boltzmann statistics)

$$f^{\rm eq} = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{p^\mu u_\mu}{T}\right)$$

- implementation of boost invariance, exact solutions may be found

A. Bialas and W. Czyz, Phys. Rev. D30 (1984) 2371

$$\tau = \sqrt{t^2 - z^2}, \quad w = tp_\parallel - zE, \quad v = tE - zp_\parallel, \quad \frac{\partial f}{\partial \tau} = \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

G. Baym, Phys. Lett. B138 (1984) 18; Nucl. Phys. A418 (1984) 525c

WF, R. Ryblewski and M. Strickland, Nucl. Phys. A916 (2013) 249; Phys.Rev. C88 (2013) 024903

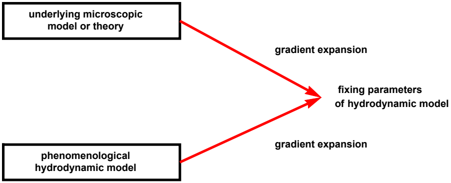
Gradient expansion

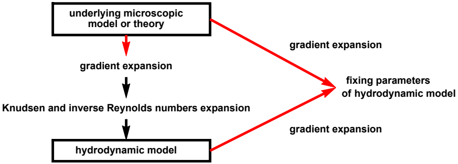
following the works by R. Janik, M. P. Heller, M. Spalinski, P. Witaszczyk

Formal expansion of $T^{\mu\nu}$ in gradients of hydrodynamic variables T and u^μ

$$T^{\mu\nu} \quad = \quad T^{\mu\nu}_{\rm eq} + \text{powers of gradients of } T \text{ and } u^\mu$$

Formal tool to make comparisons between different theories and check their close to equilibrium behaviour, no useful for finding approximate solutions of the theory, unless completed as a transseries





Gradient expansion

Simple structures for boost-invariant flow with the relaxation time $\tau_\pi = c/T$, for example, T is expanded around the Bjorken flow

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3} \left(1 + \sum_{n=1}^{\infty} \left(\frac{c}{T_0 \tau_0}\right)^n t_n \left(\frac{\tau_0}{\tau}\right)^{2n/3}\right)$$

similarly for ϕ, it is better to use $f(w)$

$$f = \frac{1}{T}\frac{dw}{d\tau}, \qquad w = \tau T, \quad \Delta = \frac{\Delta P}{P} = 3\frac{P_\parallel - P_\perp}{\varepsilon} = 12\left(f - \frac{2}{3}\right)$$

The gradient expansion for boost-invariant flow takes the form of an expansion

$$f(w) = \sum_{n=0}^{\infty} f_n w^{-n}, \quad f_0 = \frac{2}{3}$$

Gradient expansion

RTA - gradient expansion for the RTA kinetic-theory model

M. P. Heller, Kurkela, Spalinski, arXiv:1609.04803

WF, R. Ryblewski, M. Spalinski, Phys.Rev. D94 (2016) 114025

values of f_n

n	RTA	BRSSS	DNMR	MIS
0	$2/3$	$2/3$	$2/3$	$2/3$
1	$4/45$	$4/45$	$4/45$	$4/45$
2	$16/945$	$16/945$	$16/945$	$8/135$
3	$-208/4725$	$-1712/99225$	-304/33075	$112/2025$
3	-0.044	-0.017	-0.009	0.055

3. Relativistic fluid dynamics with spin

WF, Bengt Friman, Amaresh Jaiswal, Enrico Speranza, arXiv:1705.00587

Motivation

- **Non-central heavy-ion collisions create fireballs with large global angular momenta** which may generate a spin polarization of the hot and dense matter (Einstein-de Haas and Barnett effects)
- **Much effort has recently been invested in studies of polarization and spin dynamics of particles produced in high-energy nuclear collisions**, both from the experimental and theoretical point of view

L. Adamczyk et al. (**STAR**), (2017), arXiv:1701.06657, to appear in **Nature**
Global Λ hyperon polarization in nuclear collisions:
evidence for the most vortical fluid

www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

Local distribution functions

Our starting point: **phase-space distribution functions for spin-1/2 particles** and antiparticles in local equilibrium. In order to incorporate the spin degrees of freedom, they have been **generalized from scalar functions to two by two spin density matrices** for each value of the space-time position x and momentum p, **F. Becattini et al., Annals Phys. 338 (2013) 32**

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p) X^+ u_s(p), \quad f^-_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p) X^- v_r(p)$$

Following the notation used by F. Becattini et al., we introduce the matrices

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x) p^\mu\right] M^{\pm}$$

where

$$M^{\pm} = \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\hat{\Sigma}^{\mu\nu}\right]$$

Here we use the notation $\beta^\mu = u^\mu/T$ and $\xi = \mu/T$, with the temperature T, chemical potential μ and four velocity u^μ. The latter is normalized to $u^2 = 1$. Moreover, $\omega_{\mu\nu}$ is the spin tensor, while $\hat{\Sigma}^{\mu\nu}$ is the spin operator expressed in terms of the Dirac gamma matrices, $\hat{\Sigma}^{\mu\nu} = (i/4)[\gamma^\mu, \gamma^\nu]$.

Spin/polarization tensor

$$\omega_{\mu\nu} \equiv k_\mu u_\nu - k_\nu u_\mu + \epsilon_{\mu\nu\beta\gamma} u^\beta \omega^\gamma.$$

We can assume that both k_μ and ω_μ are orthogonal to u^μ, i.e., $k \cdot u = \omega \cdot u = 0$,

$$k_\mu = \omega_{\mu\nu} u^\nu, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta}\, \omega^{\nu\alpha} u^\beta.$$

It is convenient to introduce the dual spin tensor $\tilde{\omega}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta}$.
One finds $\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} = k \cdot k - \omega \cdot \omega$ and $\frac{1}{2}\tilde{\omega}_{\mu\nu}\omega^{\mu\nu} = 2k \cdot \omega$. Using the constraint

$$k \cdot \omega = 0$$

we find the compact form

$$M^\pm = \cosh(\zeta) \pm \frac{\sinh(\zeta)}{2\zeta}\, \omega_{\mu\nu} \hat{\Sigma}^{\mu\nu}, \tag{22}$$

where

$$\zeta \equiv \frac{1}{2}\sqrt{k \cdot k - \omega \cdot \omega}. \tag{23}$$

We now assume also that $k \cdot k - \omega \cdot \omega \geq 0$, which implies that ζ is real.

Charge current

The **charge current**

$$N^\mu = \int \frac{d^3p}{2(2\pi)^3 E_p} p^\mu \left[\mathrm{tr}(\mathrm{X}^+) - \mathrm{tr}(\mathrm{X}^-) \right] = n u^\mu$$

where 'tr' denotes the trace over spinor indices and n is the charge density

$$n = 4 \cosh(\zeta) \sinh(\xi)\, n_{(0)}(T) = 2 \cosh(\zeta) \left(e^{\xi} - e^{-\xi} \right) n_{(0)}(T)$$

Here $n_{(0)}(T) = \langle (u \cdot p) \rangle_0$ is the number density of spin 0, neutral Boltzmann particles, obtained using the thermal average

$$\langle \cdots \rangle_0 \equiv \int \frac{d^3p}{(2\pi)^3 E_p} (\cdots)\, e^{-\beta \cdot p},$$

where $E_p = \sqrt{m^2 + \mathbf{p}^2}$.

Energy-momentum tensor

The **energy-momentum tensor** for a perfect fluid then has the form

$$T^{\mu\nu} = \int \frac{d^3p}{2(2\pi)^3 E_p} p^\mu p^\nu \left[\mathrm{tr}(\mathrm{X}^+) + \mathrm{tr}(\mathrm{X}^-)\right] = (\varepsilon + \mathcal{P}) u^\mu u^\nu - \mathcal{P} g^{\mu\nu},$$

where the energy density and pressure are given by

$$\mathcal{E} = 4\,\cosh(\zeta)\cosh(\xi)\,\mathcal{E}_{(0)}(T)$$

and

$$\mathcal{P} = 4\,\cosh(\zeta)\cosh(\xi)\,\mathcal{P}_{(0)}(T),$$

respectively. In analogy to the density $n_{(0)}(T)$, we define the auxiliary quantities $\mathcal{E}_{(0)}(T) = \langle (u\cdot p)^2\rangle_0$ and $\mathcal{P}_{(0)}(T) = -(1/3)\langle \left[p\cdot p - (u\cdot p)^2\right]\rangle_0$.

Entropy current

The **entropy current** is given by an obvious generalization of the Boltzmann expression

$$S^\mu = -\int \frac{d^3p}{2(2\pi)^3 E_p} p^\mu \left(\mathrm{tr}\left[\mathrm{X}^+ (\ln \mathrm{X}^+ - 1) \right] + \mathrm{tr}\left[\mathrm{X}^- (\ln \mathrm{X}^- - 1) \right] \right)$$

This leads to the following entropy density

$$\mathcal{S} = u_\mu S^\mu = \frac{\mathcal{E} + \mathcal{P} - \mu n - \Omega w}{T},$$

where Ω is defined through the relation $\zeta = \Omega/T$ and

$$w = 4 \sinh(\zeta) \cosh(\xi)\, n_{(0)}.$$

This suggests that Ω should be used as a thermodynamic variable of the grand canonical potential, in addition to T and μ. Taking the pressure $\mathcal{P}$ to be a function of T, μ and Ω, we find

$$\mathcal{S} = \left.\frac{\partial \mathcal{P}}{\partial T}\right|_{\mu,\Omega}, \quad n = \left.\frac{\partial \mathcal{P}}{\partial \mu}\right|_{T,\Omega}, \quad w = \left.\frac{\partial \mathcal{P}}{\partial \Omega}\right|_{T,\mu}.$$

Basic conservation laws

The conservation of energy and momentum requires that

$$\partial_\mu T^{\mu\nu} = 0.$$

This equation can be split into two parts, one longitudinal and the other transverse with respect to u^μ:

$$\begin{aligned} \partial_\mu[(\mathcal{E}+\mathcal{P})u^\mu] &= u^\mu \partial_\mu \mathcal{P} \equiv \frac{d\mathcal{P}}{d\tau}, \\ (\mathcal{E}+\mathcal{P})\frac{du^\mu}{d\tau} &= (g^{\mu\alpha} - u^\mu u^\alpha)\partial_\alpha \mathcal{P}. \end{aligned} \tag{24}$$

Evaluating the derivative on the left-hand side of the first equation we find

$$T\,\partial_\mu(\mathcal{S}u^\mu) + \mu\,\partial_\mu(nu^\mu) + \Omega\,\partial_\mu(wu^\mu) = 0. \tag{25}$$

The middle term vanishes due to charge conservation,

$$\partial_\mu(nu^\mu) = 0. \tag{26}$$

Thus, in order to have entropy conserved in our system (for the perfect-fluid description we are aiming at), we demand that

$$\partial_\mu(wu^\mu) = 0. \tag{27}$$

Consequently, we self-consistently arrive at the equation for conservation of entropy, $\partial_\mu(\mathcal{S}u^\mu) = 0$.

Spin dynamics

Since we use a symmetric form of the energy-momentum tensor $T^{\mu\nu}$, the spin tensor $S^{\lambda,\mu\nu}$ satisfies the conservation law,

$$\partial_\lambda S^{\lambda,\mu\nu} = 0.$$

For $S^{\lambda,\mu\nu}$ we use

$$S^{\lambda,\mu\nu} = \int \frac{d^3p}{2(2\pi)^3 E_p}\, p^\lambda \, \mathrm{tr}\left[(X^+ - X^-)\hat{\Sigma}^{\mu\nu}\right] = \frac{w u^\lambda}{4\zeta}\omega^{\mu\nu}$$

Using the conservation law for the spin density and introducing the rescaled spin tensor $\bar{\omega}^{\mu\nu} = \omega^{\mu\nu}/(2\zeta)$, we obtain

$$u^\lambda \partial_\lambda \, \bar{\omega}^{\mu\nu} = \frac{d\bar{\omega}^{\mu\nu}}{d\tau} = 0,$$

with the normalization condition $\bar{\omega}_{\mu\nu}\,\bar{\omega}^{\mu\nu} = 2$.

With this definition of the spin tensor we obtain a consistent system of 10 differential equations for all 10 coefficients appearing in the local distribution function.