Causality constraint on bound states and scattering with zero-range force

arXiv:1402.4973 [nucl-th].

do perturbative pions deserve another chance?

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@ 8th Chiral Dynamics Workshop, Pisa, Italy, June 29, 2015

Outline

Motivation

Chiral E**F**T of few-nucleon systems

Light-by-light scattering sum rules

general principles: unitarity, causality, etc.

Zero-range force:

Bound state, tachyon, K-matrix pole

using the sum rules as consistency (causality) criterion

phi^4 theory

(Relativistic) Wigner's inequality

positive effective range parameters

Conclusions and outlook

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Converges ?	Maybe	**Yes!**	**No!** Another conceptual problem: 3N force goes from NLO to LO

≈ Light by light scattering

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4} = \varepsilon^{*\mu_4}_{\lambda_4}(\vec{q}_4)\,\varepsilon^{*\mu_3}_{\lambda_3}(\vec{q}_3)\,\varepsilon^{\mu_2}_{\lambda_2}(\vec{q}_2)\,\varepsilon^{\mu_1}_{\lambda_1}(\vec{q}_1)\,\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}$$

HELICITY AMPL. FEYNMAN AMPL.

IN THE FORWARD DIRECTION ($t = 0, \quad s = 4\omega^2, \quad u = -s.$):

$$\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4} = A(s)\,g_{\mu_4\mu_2}g_{\mu_3\mu_1} + B(s)\,g_{\mu_4\mu_1}g_{\mu_3\mu_2} + C(s)\,g_{\mu_4\mu_3}g_{\mu_2\mu_1}\ ,$$

$$M_{++++}(s) = A(s) + C(s),$$
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1) CROSSING SYMMETRY (1 <-> 3, 2 <-> 4):

$$M_{+-+-}(s) = M_{++++}(-s), \quad M_{++--}(s) = M_{++--}(-s)$$

AMPLITUDES WITH DEFINITE PARITY UNDER CROSSING:

$$f^{(\pm)}(s) = M_{++++}(s) \pm M_{+-+-}(s)$$

$$g(s) = M_{++--}(s)$$

LbL sum rules

2) CAUSALITY => ANALYTICITY => DISPERSION RELATIONS:

$$\mathrm{Re}\left\{\begin{array}{l} f^{(\pm)}(s) \\ g(s) \end{array}\right\} = \frac{1}{\pi}\fint_{-\infty}^{\infty}\frac{ds'}{s'-s}\,\mathrm{Im}\left\{\begin{array}{l} f^{(\pm)}(s') \\ g(s') \end{array}\right\},$$

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$$\mathrm{Im}\, f^{(\pm)}(s) = -\frac{s}{8}\,[\,\sigma_0(s) \pm \sigma_2(s)\,],$$

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$$\mathrm{Re}\, f^{(+)}(s) = -\frac{1}{2\pi} \int\limits_0^\infty ds'\, s'^2 \, \frac{\sigma(s')}{s'^2 - s^2},$$

$$\sigma = (\sigma_0 + \sigma_2)/2 = (\sigma_{||} + \sigma_\perp)/2$$

$$\mathrm{Re}\, f^{(-)}(s) = -\frac{s}{4\pi} \int\limits_0^\infty ds' \, \frac{s'\, \Delta\sigma(s')}{s'^2 - s^2},$$

$$\Delta\sigma = \sigma_2 - \sigma_0$$

$$\mathrm{Re}\, g(s) = -\frac{1}{4\pi} \int\limits_0^\infty ds'\, s'^2 \, \frac{\sigma_{||}(s') - \sigma_\perp(s')}{s'^2 - s^2},$$

Light-by-light scattering sum rules

4) "LOW-ENERGY THEOREM": $$\mathcal{L}_{\mathrm{EH}} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2,$$

LOW-ENERGY EXPANSION

$$f^{(+)}(s) = -2(c_1 + c_2) s^2 + O(s^4)$$

$$f^{(-)}(s) = O(s^5)$$

$$g(s) = -2(c_1 - c_2) s^2 + O(s^4)$$

$O(s^1)$:

$$0 = \int_0^\infty \frac{\mathrm{d}s}{s} \left[\sigma_2(s) - \sigma_0(s) \right]$$

GERASIMOV & MOULIN, NPB (1976)
BRODSKY & SCHMIDT, PLB (1995)

$O(s^2)$:

$$c_1 = \frac{1}{8\pi} \int_0^\infty \frac{\mathrm{d}s}{s^2} \, \sigma_{||}(s) \, ,$$

$$c_2 = \frac{1}{8\pi} \int_0^\infty \frac{\mathrm{d}s}{s^2} \, \sigma_{\perp}(s)$$

V. P. & VANDERHAEGHEN, PRL (2010)

Zero-range force in light of the LbL sum rule

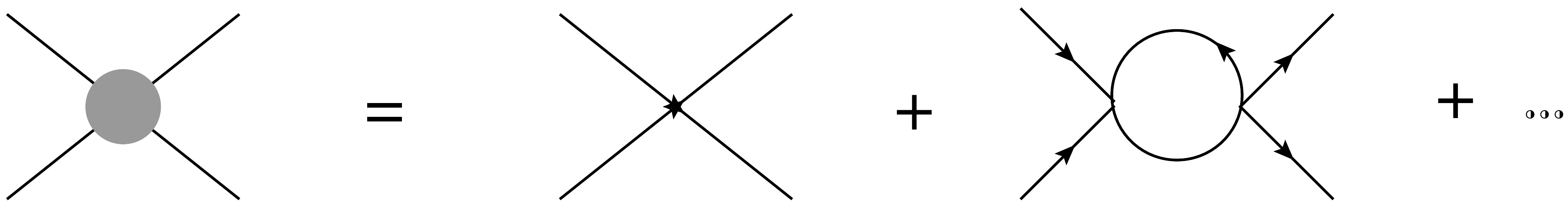
PAUK, V.P. & VANDERHAEGHEN, PLB 2014

$$T = V + V\,G\,T$$

$$V = \lambda$$

Bubble-chain sum:

$$T(s) = \frac{1}{\lambda^{-1} - G(s)}$$



$$G(s) = -i \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{[(p+\ell)^2 - m^2]\,(\ell^2 - m^2)} \qquad \text{with } p^2 = s.$$

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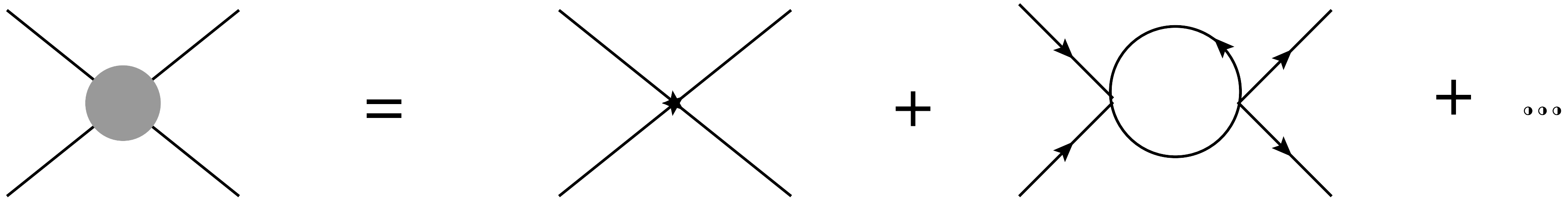

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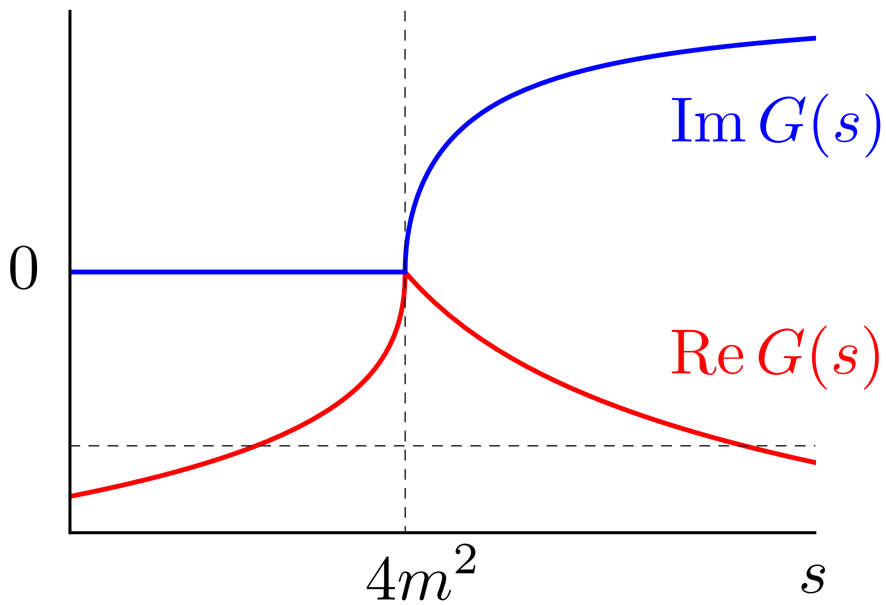


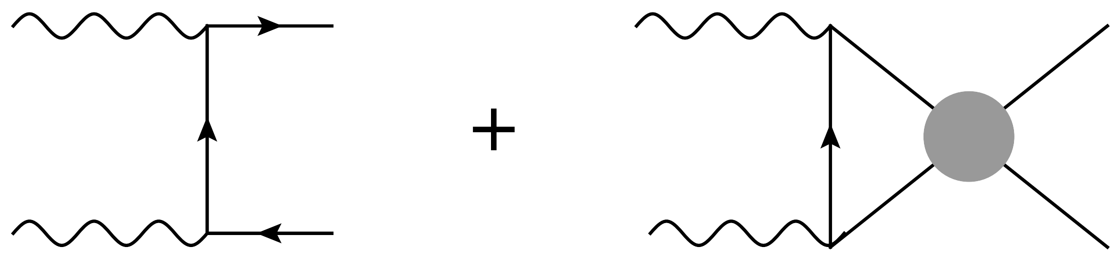
$\lambda > 0$: no poles

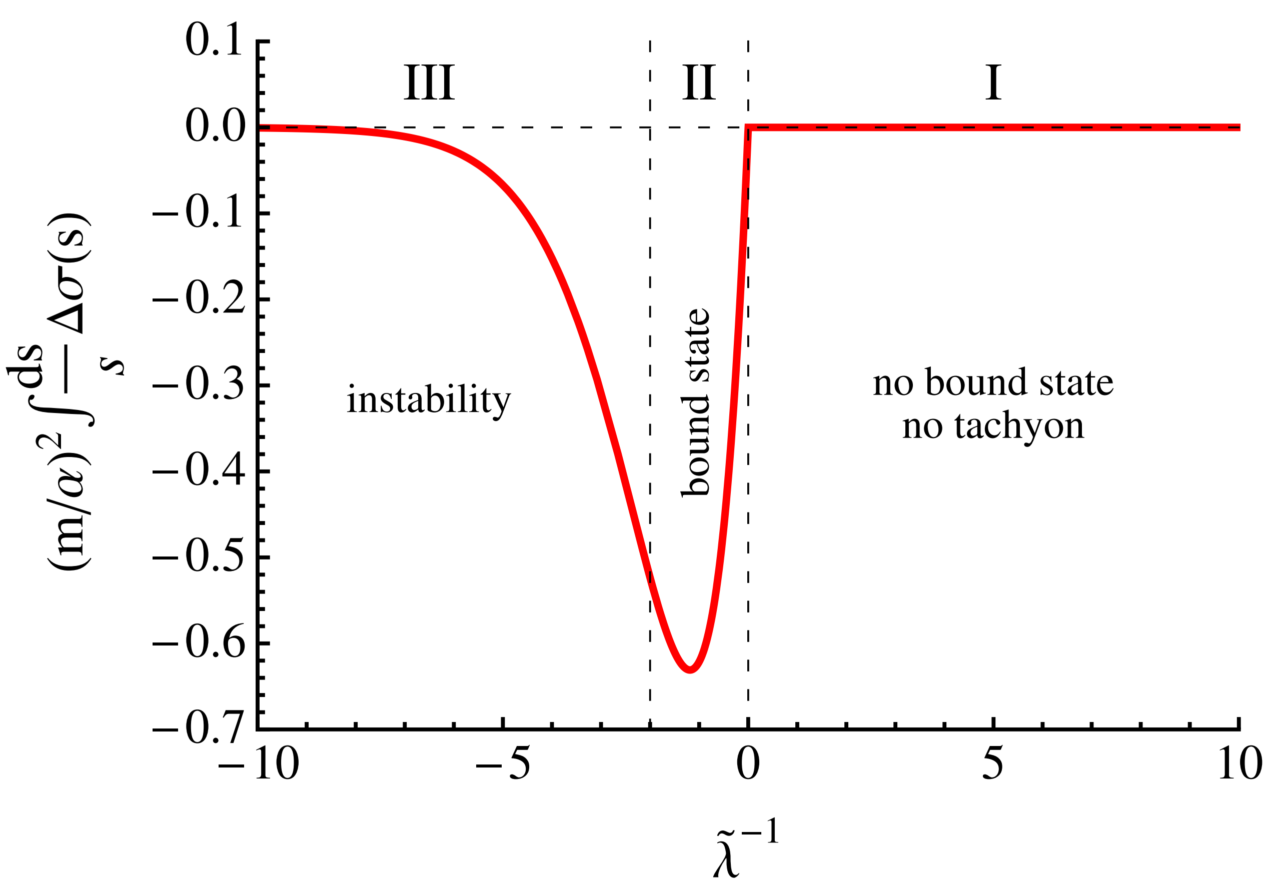
$\lambda < 0$: one pole and one K-matrix pole

$$T(s) = \frac{1}{K^{-1}(s) - i}$$

Light-by-light sum rule as causality criterion

$$\int_{s_0}^{\infty} \mathrm{d}s\, \frac{\Delta\sigma(s)}{s} = 0. \qquad \mathcal{L} = (D^\mu\phi)^* D_\mu\phi - m^2\phi^*\phi + \frac{\lambda}{4}(\phi^*\phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$





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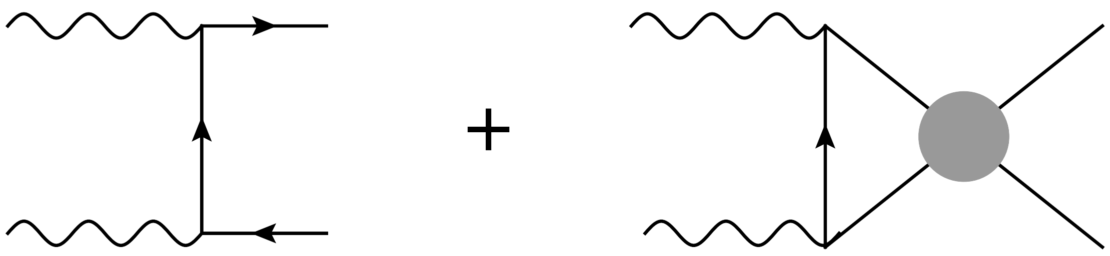
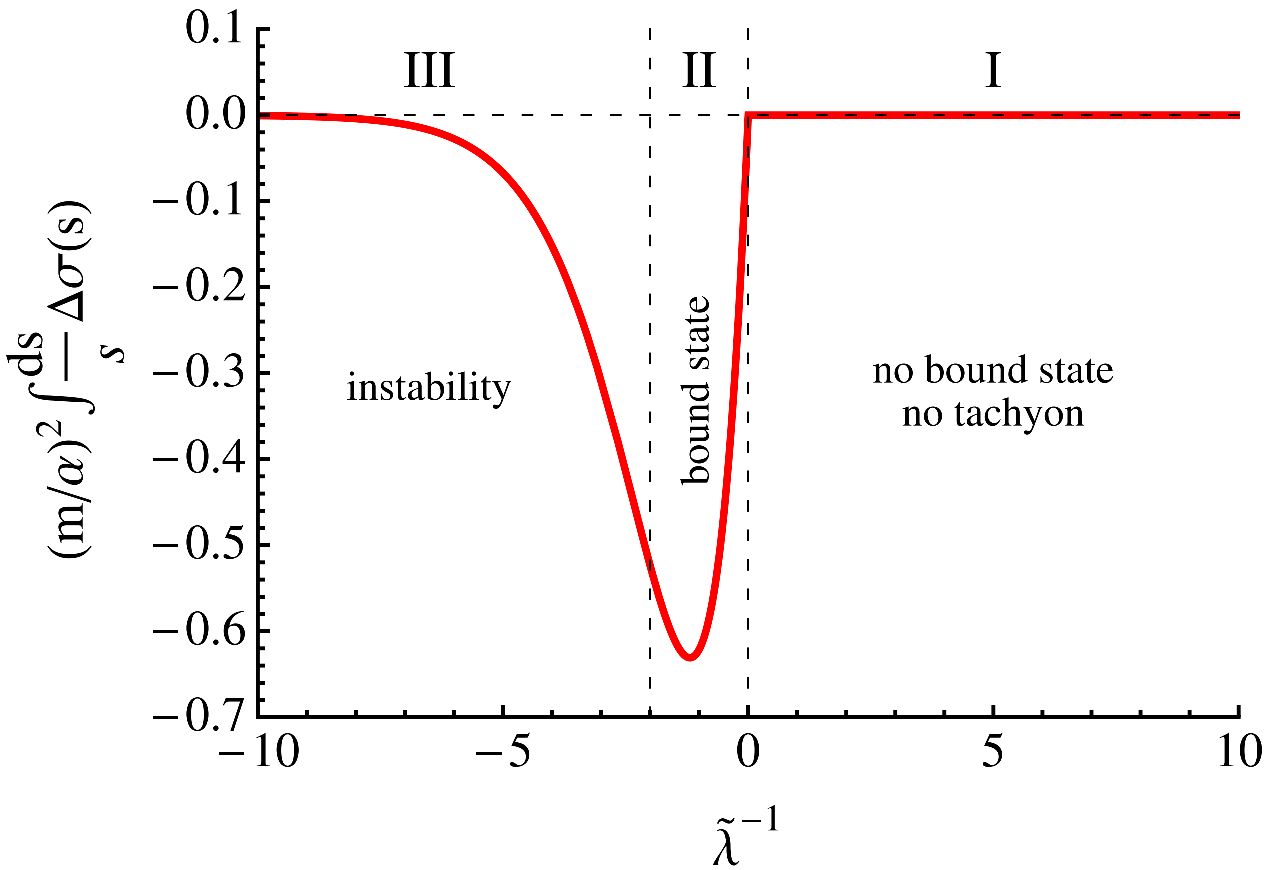
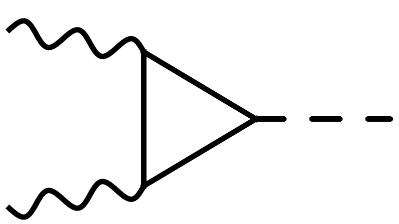




To cancel the integral one need to introduce the bound state as the asymptotic state i.e., new channel:



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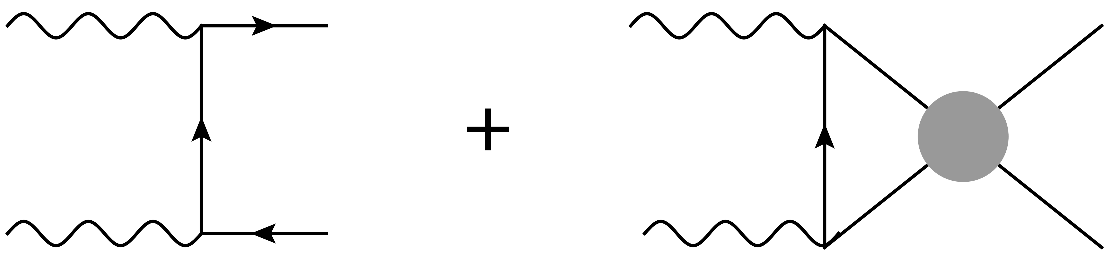

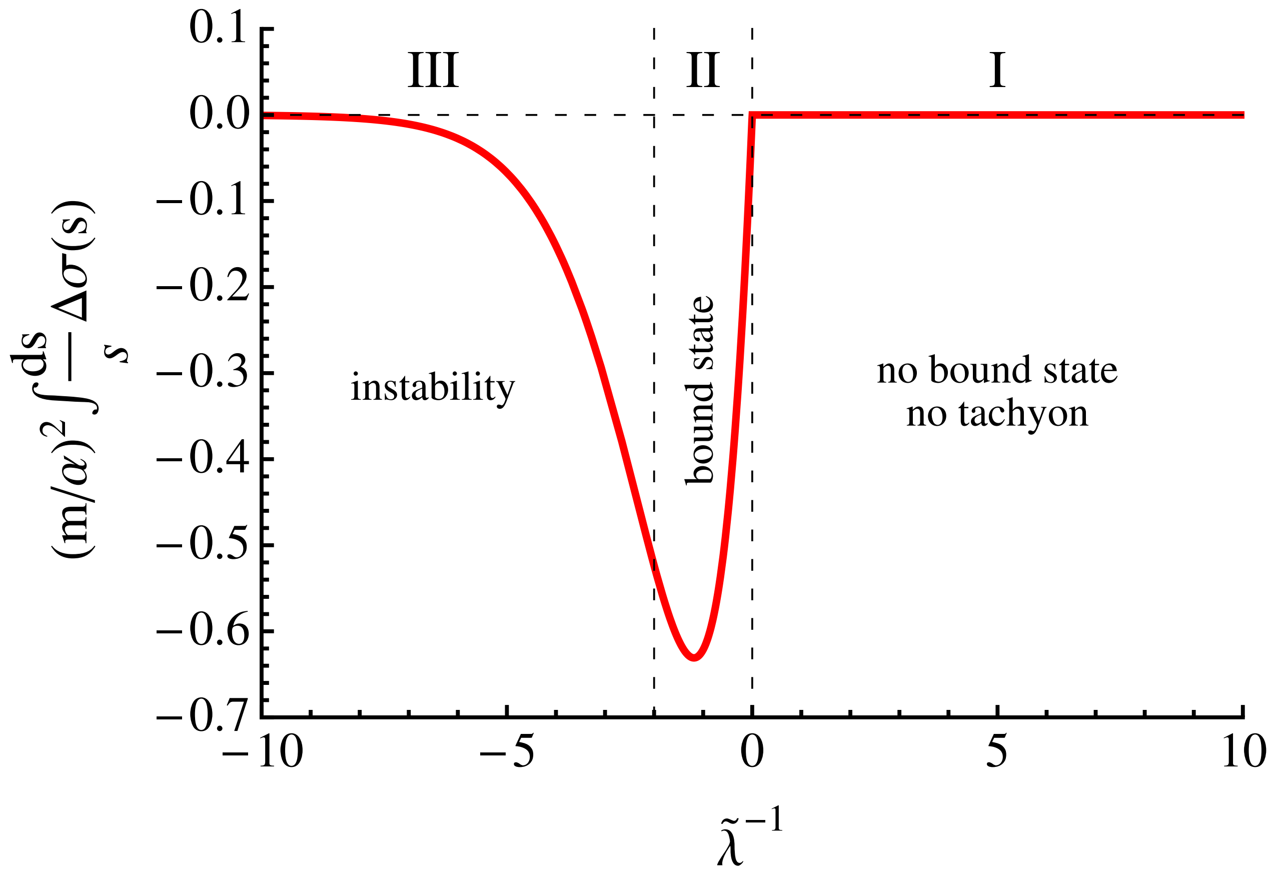

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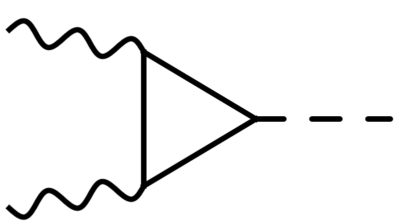

but not the K-matrix pole...

Phase shifts

Levinson's theorem:

$$\delta(0) = \pi N_{\text{bound states}}$$

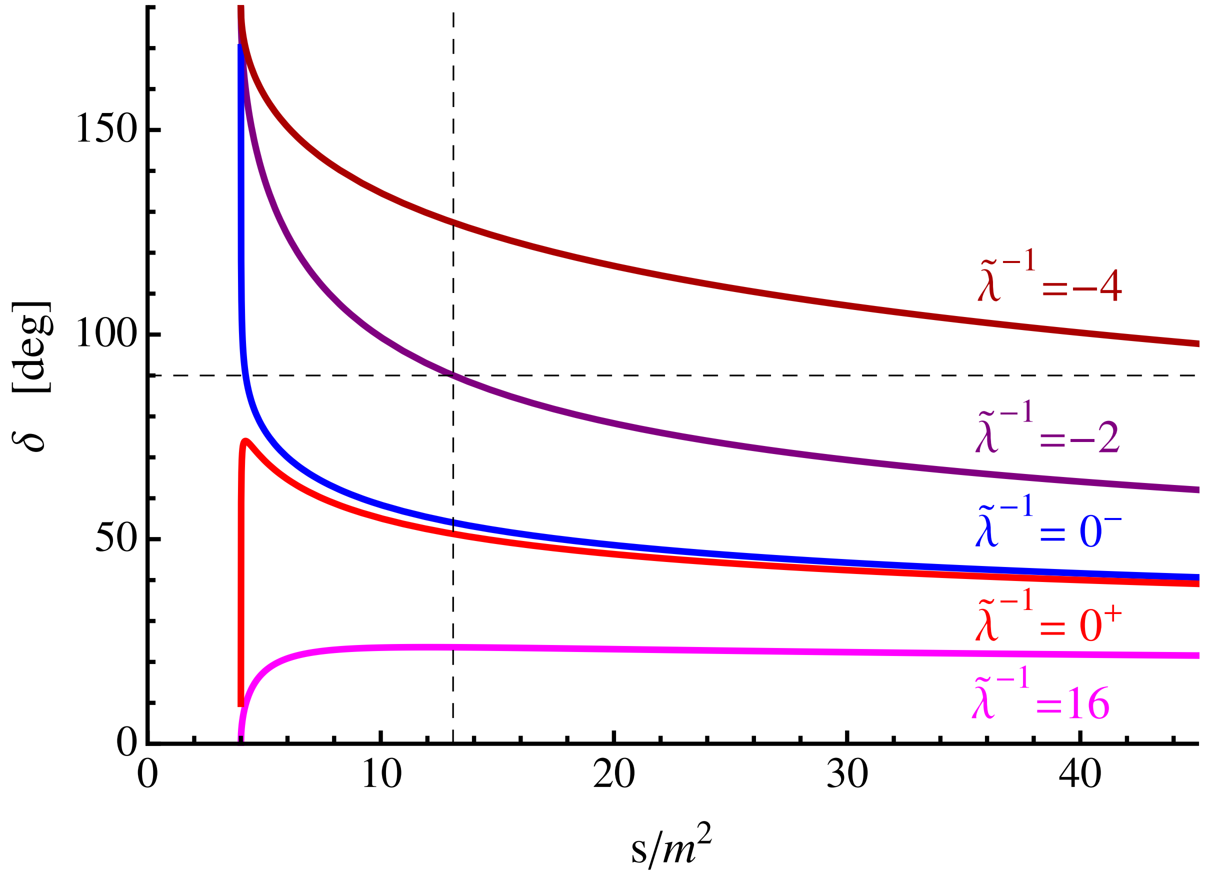

Figure 7: Phase shift for different values of $\tilde{\lambda}$.

The 90 degree crossing, i.e. the K-matrix pole does not correspond to any S-matrix pole in this case

Wigner's causality bound

$$r \leq 0$$

effective
range

WIGNER, PHYS REV (1955)

PHILLIPS & COHEN, PLB (1997);
HAMMER & D. LEE, ANN PHYS (2010); ...

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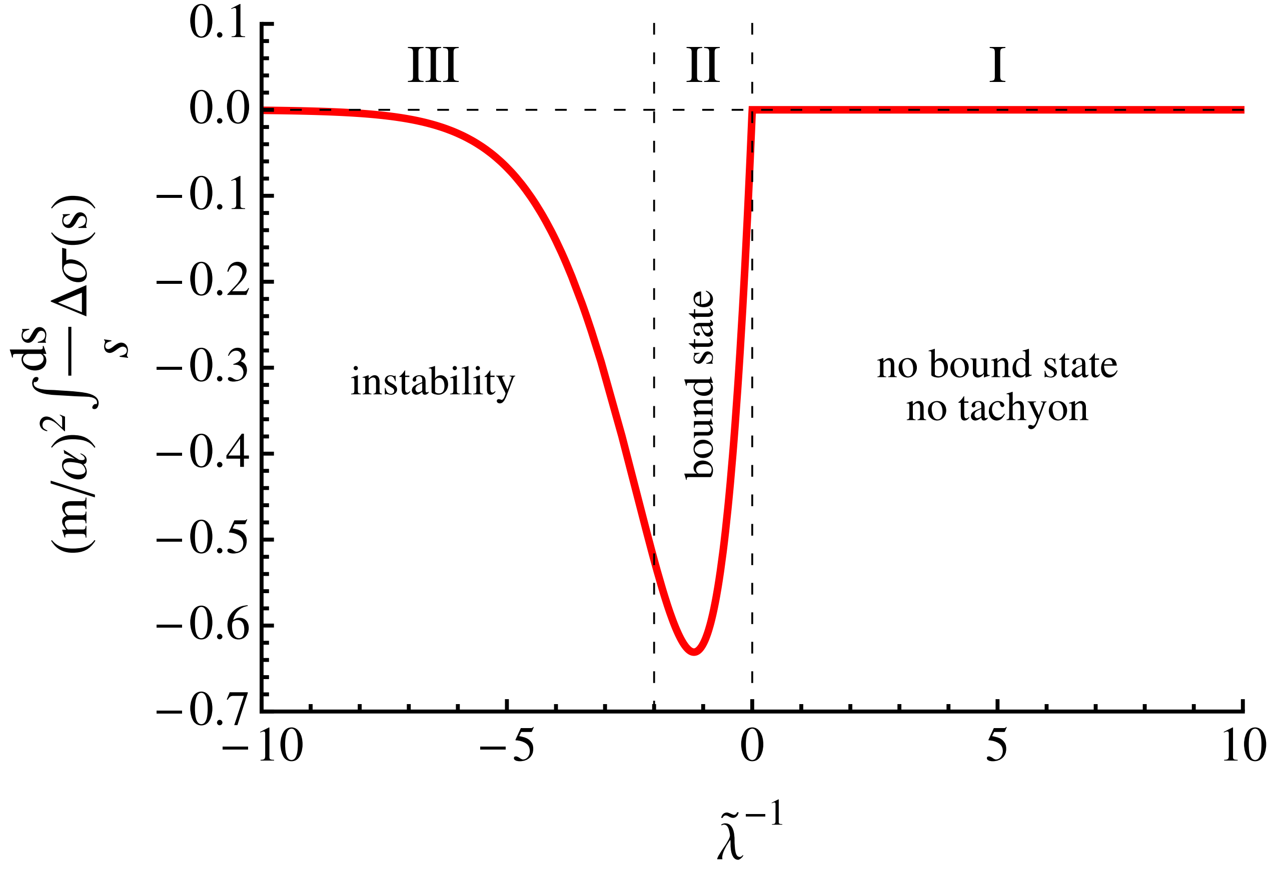
$$|\mathbf{k}| \cot \delta(s) = -\frac{1}{a} + \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n+1} r_n |\mathbf{k}|^{2n}$$

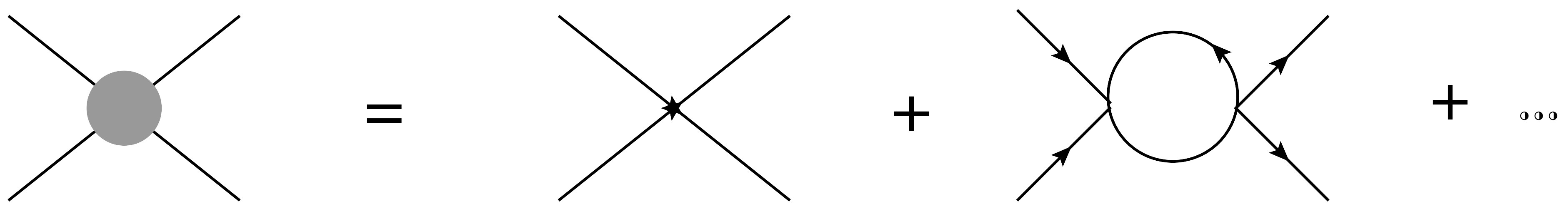


In the the tachyon (acausal) regime at least one of the effective range parameters is negative.

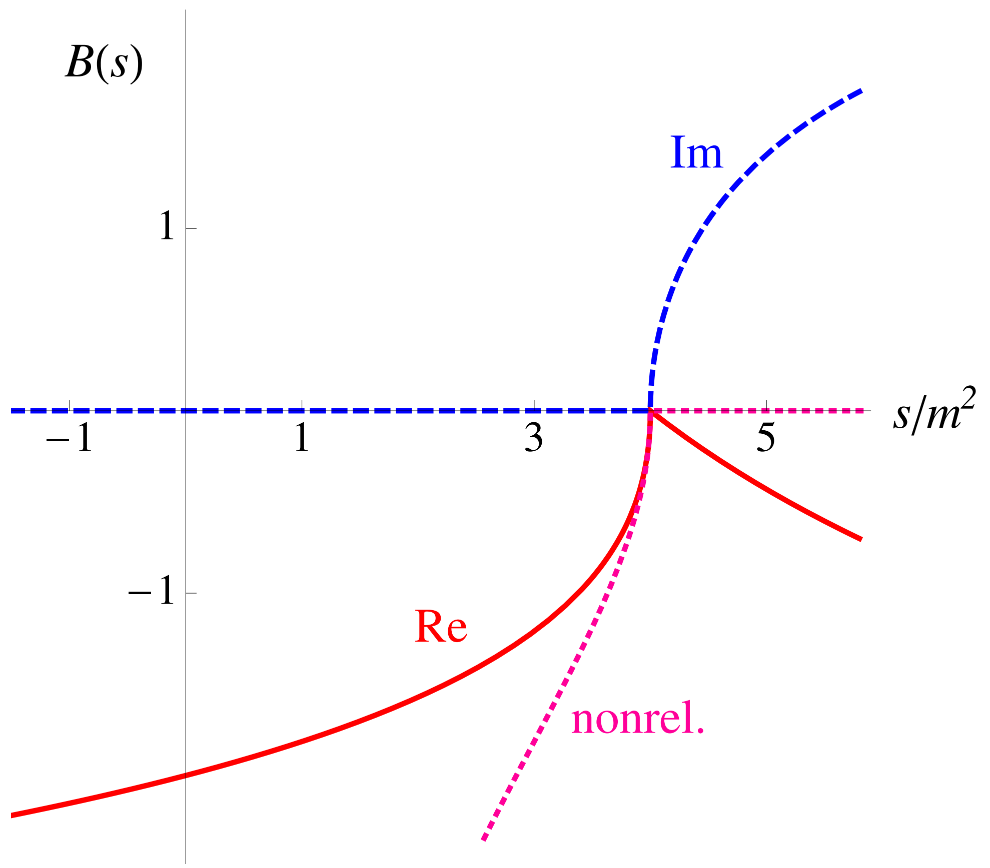
Therefore our causality criterion yields:

$$r_n \geq 0$$

Non-relativistic limit



$$T(s) = \frac{1}{\lambda^{-1} - (4\pi)^{-2} B(s)}$$



In non-rel. limi, K-matrix pole disappears and

$$r = 0$$

in agreement with Wigner's bound

Zero eff. range of 2-body force eventually leads to the problems with the 3-body force

[BEDAQUE, HAMMER, VAN KOLCK (1999)]

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