Recent results on QCD thermodynamics from lattice

Sayantan Sharma

BROOKHAVEN NATIONAL LABORATORY

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Outline

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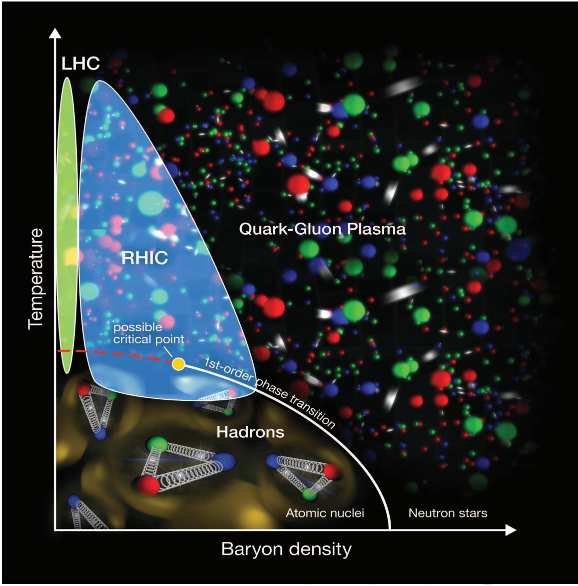
1. The QCD phase diagram: outstanding issues
2. Symmetries
3. Degrees of freedom in QCD
4. Lattice QCD and heavy ion experiments

The QCD phase diagram: outstanding issues

- Understanding QCD phase diagram is one of the most challenging problems in the recent years.
- The underlying physics of confinement and chiral symmetry breaking is not yet completely understood.
  [Schaefer and Shuryak, 96]
- Challenges bring in new opportunities!
- Beyond bulk thermodynamic quantities Lattice now can give a glimpse of the microscopic degrees of freedom.



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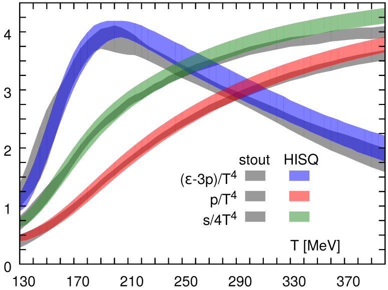
- How do we observe any hints of QCD thermodynamics from the rich set of experimental data from the heavy-ion colliders.
  - QCD thermodynamics at finite density.
  - Understanding freezeout conditions in HIC.
- Heavy quarks are also excellent probes of strongly coupled QCD plasma [Talks from Lattice perspective, by H. Ohno, S. Kim on Wednesday]

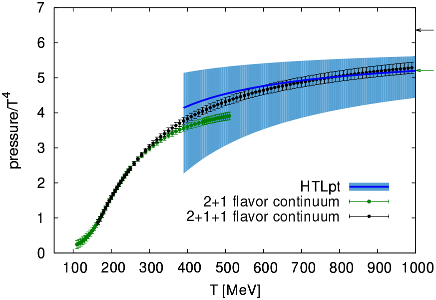
Outline

The phase diagram at $\mu_B = 0$

- For finite quark masses, no unique order parameter.
- It is now well established that $\mu_B = 0$ chiral symmetry restoration occurs via crossover transition with a $T_c = 154(9)$ MeV.

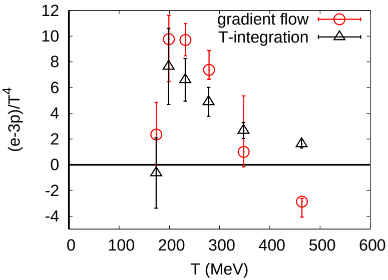
  [Budapest-Wuppertal collaboration, 1309.5258, HotQCD collaboration, Bazavov et. al, 1407.6387]
- The EoS for $2+1$ QCD is measured in the continuum and different lattice groups agree.
- The dynamical effects of charm quarks included till 1 GeV $\rightarrow$ EoS during cosmological evolution. [Borsanyi et. al, 1606.07494]

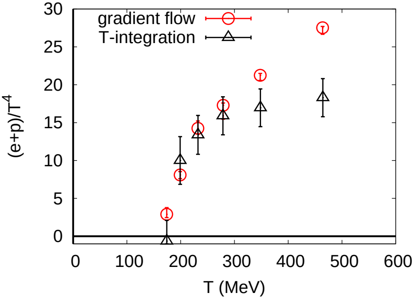




The phase diagram at $\mu_B = 0$

- Recently new advances made with EoS with Wilson fermions
  [WHOT QCD collaboration, Phys.Rev.D95, 054502 (2017), ETMC collaboration, 15]
- Energy Momentum tensor extracted using gradient flow. Good agreement with the integral method for $T < 2T_c$. A peak in chiral susceptibility observed even with Wilson fermions at $m_\pi \sim 400$ MeV. Now simulations extends to physical quark masses [See K. Kanaya's talk].
- EM Tensor correlators are now being calculated
  [See Y. Taniguchi's talk, poster by M. Kitazawa]





The phase diagram at $\mu_B = 0$

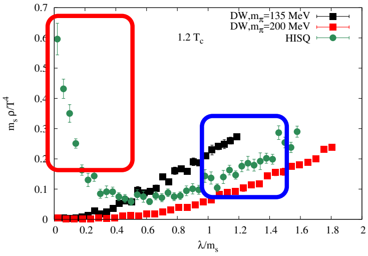
- However since $m_u, m_d << \Lambda_{QCD}$ there is an approximate $U_L(2) \times U_R(2)$ symmetry of QCD Lagrangian.
- $U_L(2) \times U_R(2) \rightarrow SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- At chiral crossover transition: $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$.
- Is $U_A(1)$ effectively restored at T_c? $\rightarrow$ can change the universality class of the second order phase transition at $\mu_B = 0$. Either O(4) or $U_L(2) \times U_R(2)/U_V(2)$

  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]

The phase diagram at $\mu_B = 0$

- Not an exact symmetry$\rightarrow$ what observables to look for? Degeneracy of the 2-point correlators [Shuryak, 94] $\rightarrow$ higher point correlation functions imp.

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$



[V. Dick, et. al, 1602.02197]

- $\rho(\lambda)$: non-analyticities+analytic part of the eigenvalue spectrum when chiral symm. restored.

  [HotQCD collaboration, 1205.3535, V. Dick et. al. 1502.06190]

- Analytic part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is necessary cond. for $U_A(1)$ breaking invisible in upto 6 pt correlators [Aoki, Fukaya & Taniguchi, 1209.2061]

- Near-zero modes need careful study: lattice cut-off + finite volume effects.

Understanding eigenvalue spectrum at finite T

$$\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^\gamma$$

- Zero modes has strong lattice cut-off dependence [G. Cossu et. al, 13, A. Tomiya et. al, 15,16]. These will not contribute in the thermodynamic limit!

$$a^2\left(\chi_\pi - \chi_\delta\right) = \sum_\lambda \frac{4m_f^2}{(\lambda^2 + m_f^2)^2} + \frac{|Q|\,T}{m^2 V}$$

- Non-analytic part still needs careful study. Analytic part of the spectrum strongly suggest that $U_A(1)$ is broken! [V. Dick, et. al, 1502.06190, 1602.02197, G. Cossu et. al., 1510.07395].
- Independent hints from study of screening masses (excitations) for π, δ. [Y. Maezawa et. al. 1411.3018, B. Brandt et. al. 1608.06882, See talk by A. Gomez Nicola]

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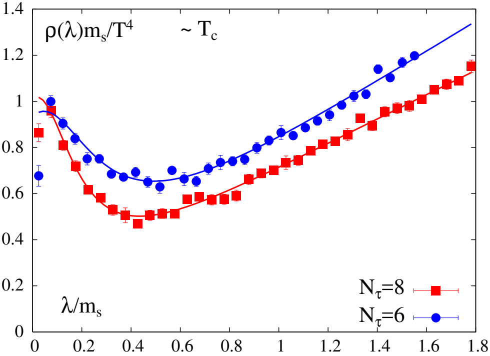

Bulk follows Random Matrix Theory level spacings, $\lambda = 1$

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, 1502.06190]

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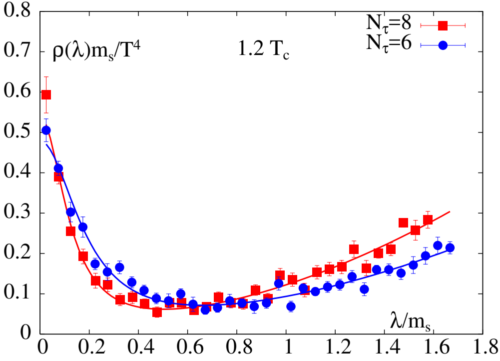

$$\lambda = 2$$

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, 1502.06190]

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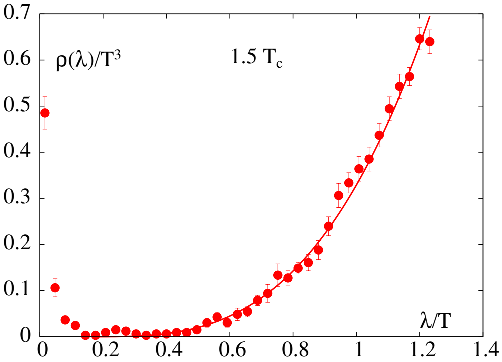
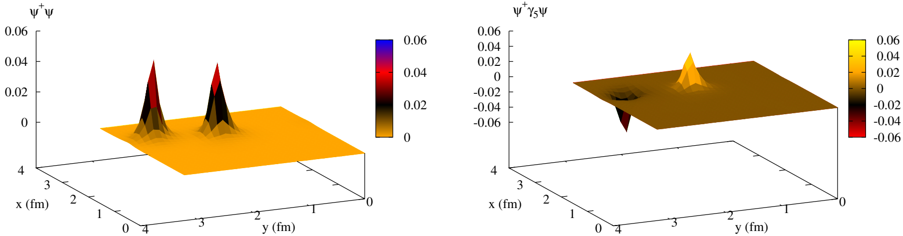


$\lambda = 3$. Gap opening up!

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, 1502.06190]

Microscopic origin of $U_A(1)$ breaking?



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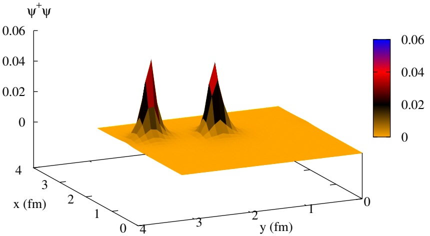
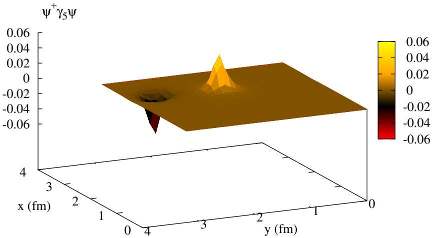




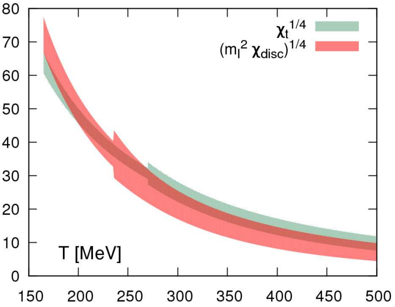
- Near-zero modes of QCD Dirac operator at $1.5\ T_c$ due to a weakly interacting instanton-antiinstanton pair!
- The density $\simeq 0.147(7) fm^{-4}$. This is much more dilute than an instanton liquid with density $1 fm^{-4}$.

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, 1502.06190].

Independent confirmation: Topological susceptibility

- Topological susceptibility measurement at high T on the lattice suffers from rare topological tunneling, lattice artifacts.
- Going towards continuum limit difficult due to freezing of topology.

- Fermionic observables
  [L. Giusti, G. C. Rossi, M. Testa, 0402027, HotQCD 1205.3535]
  shown to agree with standard definition of $\chi_t = \int d^4x \langle F\tilde{F}(x)F\tilde{F}(0)\rangle$ in the continuum even with staggered quarks.
  [P. Petreczky, H-P Schadler, SS, 1606.03145].
- Continuum extrapolated results now available for QCD!



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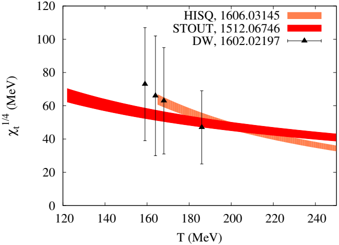


- Fit ansatz: $\chi_t^{1/4} = AT^{-b}$.
- $b = 0.9 - 1.2$ for $T < 250$ MeV from continuum extrapolated results with HISQ.

  [P. Petreczky, H-P Schadler, SS, 1606.03145].

  Agrees well with an independent study [Bonati et. al, 1512.06746] and with results with chiral fermions 1602.02197.
- Dilute gas prediction: $b = 4 - \frac{11 N_c}{3} - \frac{2 N_f}{3}$.

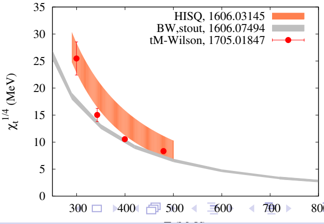
- $T > 300$ MeV: Continuum extrapolated $b = 1.85(15)$. Agreement with dilute instanton gas.

  Confirmed also in an independent study with reweighting techniques.

  [Borsanyi et. al, 1606.07494]

  Wilson type quarks with m rescaling agrees quite well

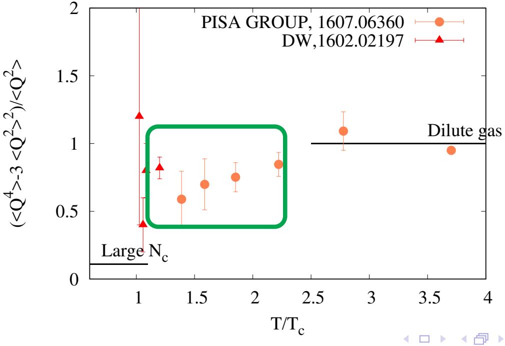
  [F. Burger et. al, 1705.01847, Y. Taniguchi et. al., 1611.02413]



More Diagnostics!

- Since θ is tiny, $F(\theta) = \frac{1}{2}\chi_t\theta^2\left(1 + b_2\theta^2 + \ldots\right)$.

  [L. D. Debbio, H. Panagopoulos, E. Vicari, 0407068]



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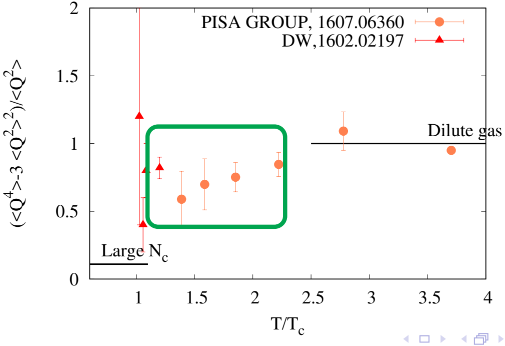
- Strong non-Gaussianity in higher order expansions. Hints about existence of dyons? Hints observed in lattice studies [M. Ilgenfritz, M-Mueller Pruessker, et. al. 14, 15].



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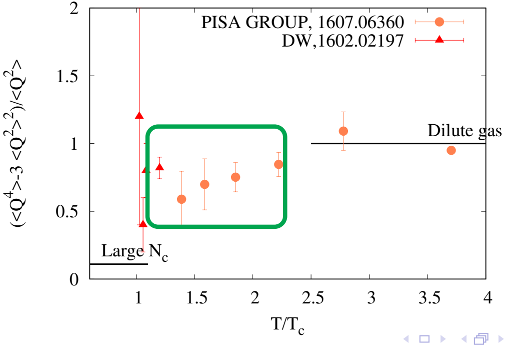
- Strong non-Gaussianity in higher order expansions. Hints about existence of dyons? Hints observed in lattice studies [M. Ilgenfritz, M-Mueller Pruessker, et. al. 14, 15].
- Evident also from the T-dependence of χ_t [P. Petreczky, H-P Schadler, SS, 1606.03145]. New lattice techniques are being discussed to explore them.
  [R. Larsen, E. Shuryak, 1703.02434].



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- Underlying microscopic origin is being studied in quite detail. $\rightarrow$ hints to interplay between topology in QCD and chiral phase transition [Shuryak, 82, Ilgenfritz & Shuryak, 88, Shuryak and Schaefer, 96].

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- Topological susceptibility has been measured in lattice QCD $\rightarrow$ suggests non-trivial top-fluctuations in hot QCD medium even at 1 GeV, consequences for axion cosmology.

- **Challenges** Is is possible to understand the intricate connection between chiral symmetry breaking and confinement through a detailed study of the topological constituents of QCD near T_c? What are the topological constituents near T_c?

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Basic observables: Fluctuations of conserved charges

- Correlations and fluctuations between different conserved quantum numbers like Baryon no, electric charge, strangeness give important information about nature of quasi-particles in different phases of QCD.

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- Conventional Monte-Carlo algorithms at finite μ in Lattice QCD suffer from sign problem.
- One of the methods to circumvent sign problem:
  Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T
  [Bielefeld-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0,T)}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0,T) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \ldots$$

  Different orders of fluctuations appear as Taylor coefficients

Challenges for Lattice computations

- The fluctuations of conserved charges can be expressed in terms of Quark no. susceptibilities (QNS).
- QNS χ_{ij}'s can be written as derivatives of the Dirac operator.

  Example:$\chi_2^u = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{''} - (D_u^{-1}D_u^{'})^2) + (Tr(D_u^{-1}D_u^{'}))^2\rangle$.
  $\chi_{11}^{us} = \frac{T}{V}\langle Tr(D_u^{-1}D_u^{'}D_s^{-1}D_s^{'})\rangle$.

- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !

- Why extending to higher orders so difficult?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.

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- Introduce μ such that it appears as a linear term multiplying the conserved number [Gavai & Sharma, 1406.0474] as in the continuum, a limit of conventional e^{μ} [Hasenfratz & Karsch, 83].
- Divergences exist for $\chi_n, n \leq 4$
- No divergences for χ_6 and beyond. Number of inversions significantly reduced for $6th$ and higher orders. Also see poster by B. Jaeger

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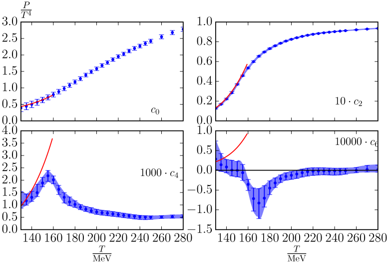
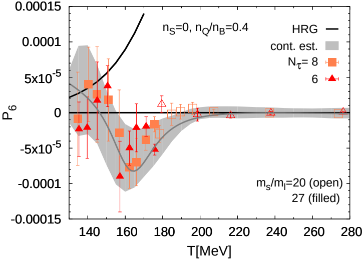
- Calculate n_B in imaginary μ and extract higher order fluctuations.
- Current state of the art: 6th order fluctuations known with very good precision. [Gunther et. al, 1607.02493] even 8th order known with reasonably good precision. [M. D'Elia et. al., 1611.08285, Poster by F. Sanfilippo]

Susceptibilities from lattice

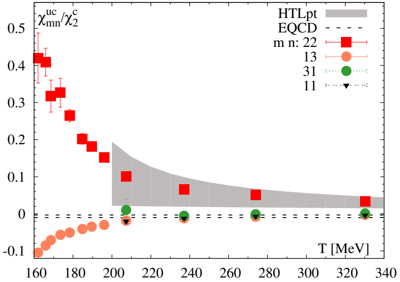




[Gunther et. al, 1607.02493, Bielefeld-BNL-CCNU, 1701.04325]

- Heavy-ion experiments at different collision energies sets non-trivial constraints: $n_s = 0$, n_B/n_Q=constant.
- χ_6^B has very distinct structure $\rightarrow$ deviates from Hadron Resonance gas picture for $T < T_c$. Weak coupling results cannot predict the dip at $T > T_c$ $\rightarrow$ signatures of a strongly coupled medium?

Can we understand degrees of freedom in hot QCD medium?



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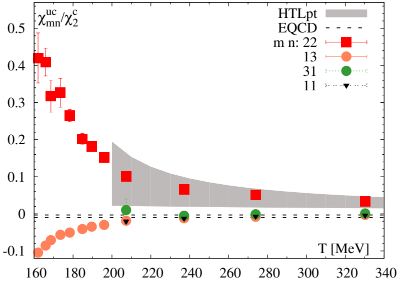

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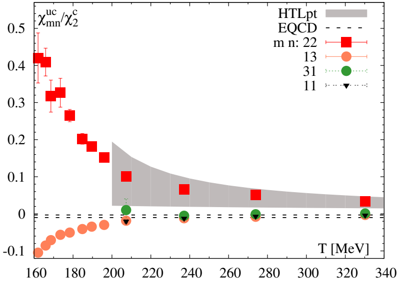

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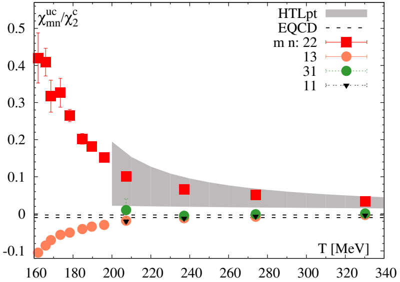

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- Look at a simple system: correlation between charm and light quarks
- Deviation from Hard Thermal Loop results between $160 - 200$ MeV.
- Charm quarks not a good quasi-particle below 200 MeV? What happens after charm hadron melts at T_c. [Mukherjee, Petreczky, SS, 1509.08887.].

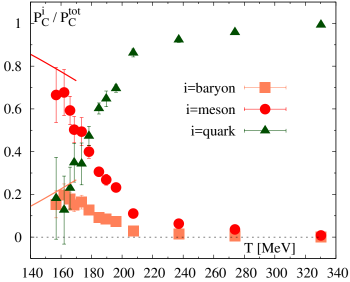
Charm d.o.f at deconfinement

- What are the microscopic constituents beyond T_c?
  Model charm d.o.f in QCD medium as charm meson+baryon+quark-like excitations.

$$p_C(T,\mu_B,\mu_C) = p_M(T)\cosh\left(\frac{\mu_C}{T}\right) + p_{B,C=1}(T)\cosh\left(\frac{\mu_C+\mu_B}{T}\right) + p_q(T)\cosh\left(\frac{\mu_C+\mu_B/3}{T}\right).$$

- Considering fluctuations upto 4th order there are 6 measurements and thus 2 trivial constraints $\chi_4^C = \chi_2^C$, $\chi_{11}^{BC} = \chi_{13}^{BC}$.
- A more non-trivial constraint:
  $c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0$.
- Non-trivial check: LQCD data agree with the constraints in the model thus validating it. [Mukherjee, Petreczky, SS, 1509.08887].

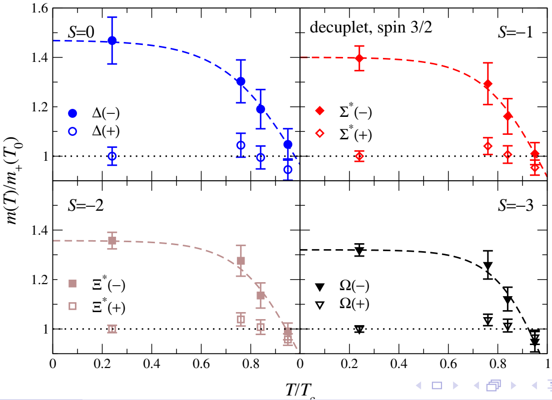
Charm d.o.f at deconfinement



- Meson and baryon like excitations survive upto $1.2\,T_c$.
- Quark-quasiparticles start dominating the pressure beyond $T \gtrsim 200$ MeV $\Rightarrow$ hints of strongly coupled QGP [Mukherjee, Petreczky, SS, 1509.08887].
- Introduce more sophistications: it is now possible to rule out di-quark excitations atleast for the charm sector for $T > T_c$..
- **Challenge** : Understand microscopics in strange sector.

Strange baryons across deconfinement

- χ_{11}^{BS}/χ_2^S show strong deviation from a hadron resonance gas model with experimentally known states. Contribution of the resonances in baryon sector. Flavor independent deconfinement temperature [Bielefeld-BNL collaboration, 1404.6511].
- Thermal masses of $-ve$ parity strange baryon show strong sensitivity to T [G. Aarts et. al., 1703.09246, more details in talk by G. Aarts].



Outline

Basic issues and requirements

- Do the expanding fireball formed in most central heavy-ion collisions attain local thermal equilibrium? $\rightarrow$ can be modelled by viscous hydrodynamics.

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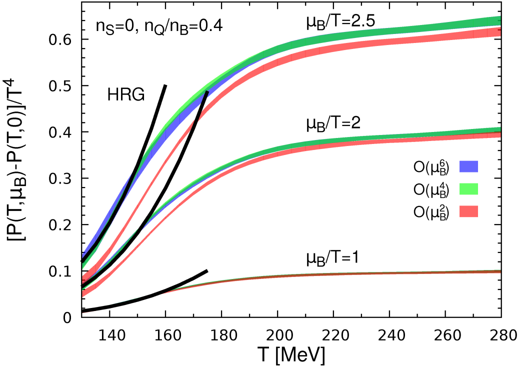
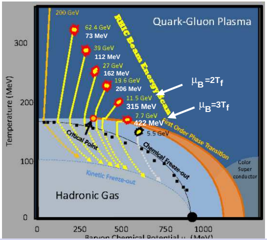
- For most RHIC energies: $n_S = 0$, $n_Q/n_B = 0.4$ need to calculate EOS for the constrained case.
- In order to disentangle the thermal fluctuations from non-thermal ones, need to measure suitable fluctuations of conserved charges on the lattice $\rightarrow$ then perform dynamical evolution in rapidity and relate to experimental measurements. [Asakawa & Kitazawa 1512.05038].
  Dynamical evolution of fluctuations near critical point in model studies show interesting patterns Mukherjee, Venugopalan, Yin, 1605.09341

EoS away from criticality

- The pressure already well determined by χ_B^6 for $\mu_B/T \le 2.5$ [Bielefeld-BNL-CCNU, 1701.04325].
- Extension to $\mu_B/T \sim 3$ is in progress to cover all the allowed range for energies of heavy-ion collisions to be probed in Beam Energy Scan II experiments $\rightarrow$ need to measure χ_8^B? Control errors on such measurements.



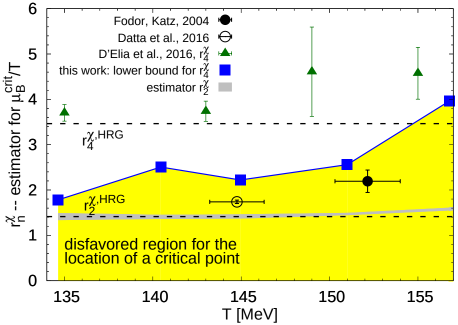


Critical-end point search from Lattice

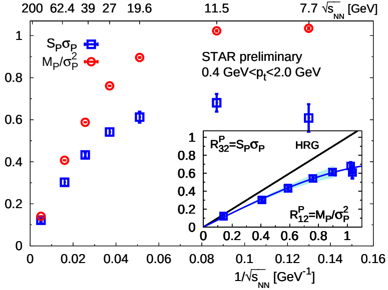
- The Taylor series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume.
- The radius of convergence will give the location of the critical point. [Gavai& Gupta, 03]
- Definition: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
  - Strictly defined for $n \to \infty$. How large n could be on a finite lattice?
  - Signal to noise ratio deteriorates for higher order χ_n^B.

Critical-end point search from Lattice

- Current bound for CEP: $\mu_B/T > 2$ for $135 \leq T \leq 160$ MeV [Bielefeld-BNL-CCNU, 1701.04325].
- The r_n extracted by analytic continuation of imaginary μ_B data [D'Elia et. al., 1611.08285] consistent with this bound.
- Results with a lower bound? [Datta et. al., 1612.06673, Fodor and Katz, 04] $\rightarrow$ need to understand the systematics in these studies. Ultimately all estimates will agree in the continuum limit!



Fluctuations measured at freezeout: Are these thermalized?



- Ratios of cumulants are independent of the volume of the fireball
- First to second moment: $\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}\left(\frac{\mu_B}{T}\right)^3$
- $S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}\left(\frac{\mu_B}{T}\right)^3$

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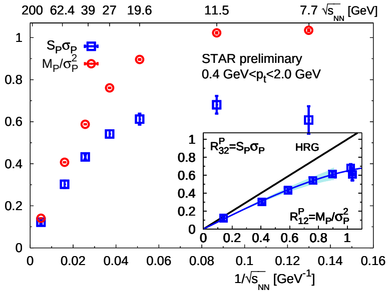

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  μ_B is unknown parameter and model dependent.
- Instead $S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \ldots$ removes model uncertainties!

  [Karsch et. al., arxiv:1512.06987

  , Many more applications from talk by F. Karsch]

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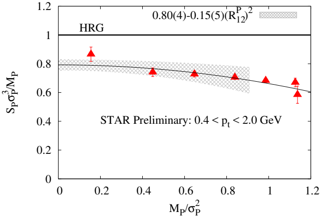
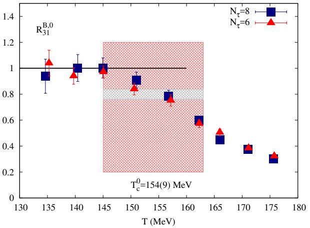
Fluctuations at freezeout and lattice

$$R_{31}^B = \frac{S_B \sigma_B^3}{M_B} = \frac{\chi_4^B}{\chi_2^B} + \frac{1}{6}\left[\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right]\left(\frac{M_B}{\sigma_B^2}\right)^2$$

[Karsch et. al., arxiv:1512.06987]

- Experimental data tantalizingly close to QCD prediction → Accidental coincidence or hints of thermalization?





- **Challenges** Need to perform dynamical evolution of the ratios of cumulants.
- Caveat: In experiments only charged baryons (protons) measured $n_P \neq n_B$!, take into account p_t cuts in the data. Look for suitable observables!

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- Efforts are on-going in understanding the rich set of data from heavy-ion experiments with input from lattice QCD.