Extra Quarks in BSM Physics: Exotic or Not

Antonio Costantini

QCD@Work 2018

26 June 2018

Contents

Top Quark's Theoretical Prediction

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

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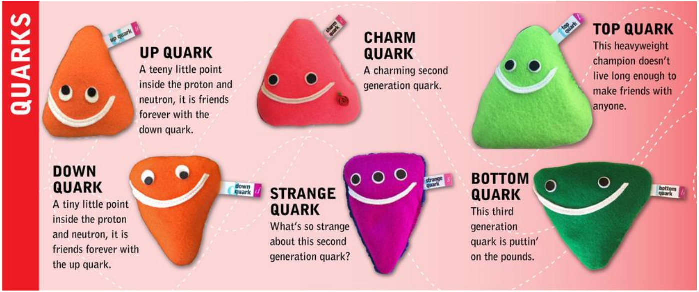

Top Quark's Discovery

VOLUME 74, NUMBER 14 PHYSICAL REVIEW LETTERS 3 APRIL 1995

Observation of Top Quark Production in $\bar{p}p$ Collisions with the Collider Detector at Fermilab

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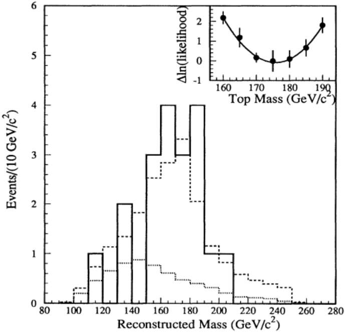
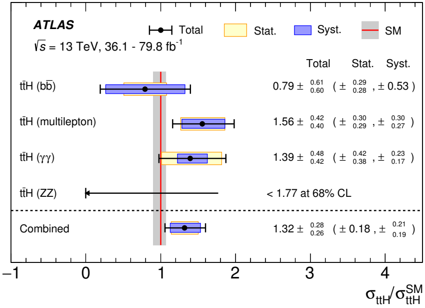


$$M_{\text{top}} = 176 \pm 8 \pm 10 \ \mathrm{GeV/c^2}$$

Top Quark's Coupling to the Higgs



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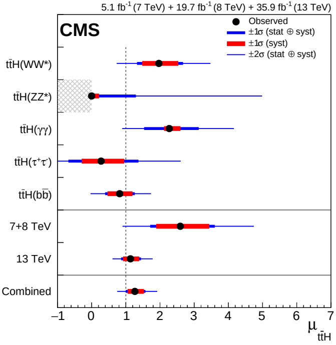
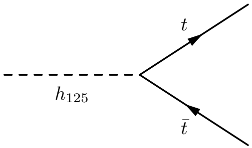


Phys. Rev. Lett. **120**, 231801



Contents

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

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$$Q_1 = \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix}, \quad Q_2 = \begin{pmatrix} c_L \\ s_L \\ S_L \end{pmatrix}, \quad Q_{1,2} \in (3, 3, -1/3)$$

$$Q_3 = \begin{pmatrix} b_L \\ t_L \\ T_L \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, 2/3)$$

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$$l = \begin{pmatrix} l_L \\ \nu_l \\ \bar{l}_R \end{pmatrix}, \quad l \in (1, \bar{3}, 0), \quad l = e,\ \mu,\ \tau$$

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$$l = \begin{pmatrix} l_L \\ \nu_l \\ \bar{l}_R \end{pmatrix}, \quad l \in (1, \bar{3}, 0), \quad l = e,\ \mu,\ \tau$$

$$\rho = \begin{pmatrix} \rho^{++} \\ \rho^{+} \\ \rho^{0} \end{pmatrix} \in (1, 3, 1), \quad \eta = \begin{pmatrix} \eta^{+} \\ \eta^{0} \\ \eta^{-} \end{pmatrix} \in (1, 3, 0), \quad \chi = \begin{pmatrix} \chi^{0} \\ \chi^{-} \\ \chi^{--} \end{pmatrix} \in (1, 3, -1)$$

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$Q_1 = \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix}, \quad Q_2 = \begin{pmatrix} c_L \\ s_L \\ S_L \end{pmatrix}, \quad Q_{1,2} \in (3, 3, -1/3)$$

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$$\mathcal{Q}_D^{\mathrm{em}} = \mathcal{Q}_S^{\mathrm{em}} = -4/3$$

$$\mathcal{Q}_T^{\mathrm{em}} = 5/3$$

Exotic Quarks!

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$SU(3)_L \times U(1)_X$

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$\begin{array}{c} SU(3)_L \times U(1)_X \\ \\ \| \\ \langle \rho \rangle \\ \Downarrow \\ \\ SU(2)_L \times U(1)_Y \end{array}$$

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$SU(3)_L \times U(1)_X$$

$$\begin{gathered} \| \\ \langle\rho\rangle \\ \Downarrow \end{gathered}$$

$$SU(2)_L \times U(1)_Y$$

$$\begin{gathered} \| \\ \langle\eta\rangle\,,\ \langle\chi\rangle \\ \Downarrow \end{gathered}$$

$$U(1)_{\mathrm{em}}$$

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$SU(3)_L \times U(1)_X \qquad\qquad W_1, \cdots, W_8\,,\ B_X$$

$$\Big\|$$
$$\langle \rho \rangle$$
$$\Downarrow$$

$$SU(2)_L \times U(1)_Y$$

$$\Big\|$$
$$\langle \eta \rangle\,,\ \langle \chi \rangle$$
$$\Downarrow$$

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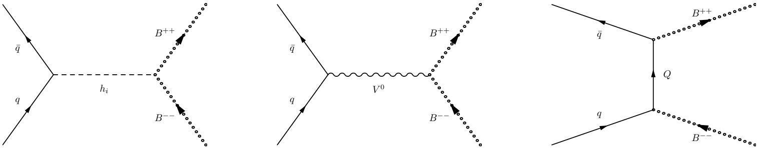
$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$
\begin{array}{ccc}
SU(3)_L \times U(1)_X & & W_1, \cdots, W_8\,,\ B_X \\
\Big\| & & \Big\| \\
\langle\rho\rangle & & \langle\rho\rangle \\
\Downarrow & & \Downarrow \\
SU(2)_L \times U(1)_Y & & W_1, W_2, W_3, B_Y, Y^{\pm}, Y^{\pm\pm}, Z' \\
\Big\| & & \\
\langle\eta\rangle\,,\ \langle\chi\rangle & & \\
\Downarrow & & \\
U(1)_{\rm em} & &
\end{array}
$$

$SU(3) \times SU(3) \times U(1)$: an exotic possibility

$$\begin{array}{ccc}
SU(3)_L \times U(1)_X & & W_1, \cdots, W_8\,,\; B_X \\
\Vert & & \Vert \\
\langle\rho\rangle & & \langle\rho\rangle \\
\Downarrow & & \Downarrow \\
SU(2)_L \times U(1)_Y & & W_1, W_2, W_3, B_Y, Y^{\pm}, Y^{\pm\pm}, Z' \\
\Vert & & \Vert \\
\langle\eta\rangle\,,\; \langle\chi\rangle & & \langle\eta\rangle\,,\; \langle\chi\rangle \\
\Downarrow & & \Downarrow \\
U(1)_{\mathrm{em}} & & \gamma, Z, Z', W^{\pm}, Y^{\pm}, Y^{\pm\pm}
\end{array}$$

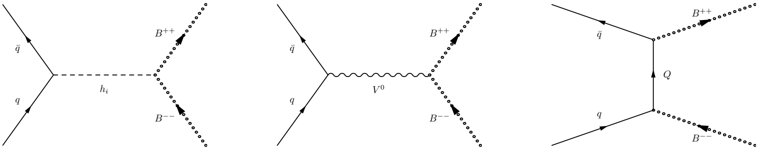
Production of Doubly-Charged States



Corcella, Corianò, C., Frampton arXiv:1806.04536

$$B^{\pm\pm} \equiv Y^{\pm\pm} \text{ or } H^{\pm\pm}$$

Production of Doubly-Charged States



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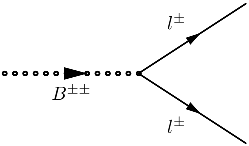
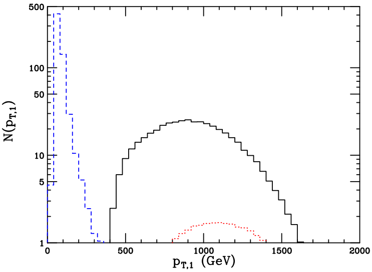
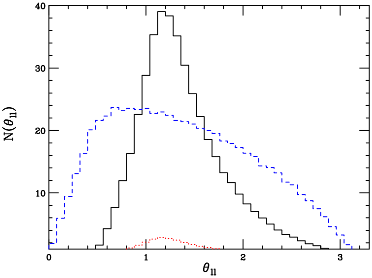


Distributions





Corcella, Corianò, C., Frampton arXiv:1806.04536

331 at various β

$$\mathcal{Q}^{\mathrm{em}} = \mathcal{T}_3 + \beta\,\mathcal{T}_8 + \mathcal{X}$$

TABLE I. *Electric charges of new particles for different choices of* β

particles	$Q(\beta)$	$\beta = -\frac{1}{\sqrt{3}}$	$\beta = \frac{1}{\sqrt{3}}$	$\beta = -\sqrt{3}$	$\beta = \sqrt{3}$
D, S	$\frac{1}{6} - \frac{\sqrt{3}\beta}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$-\frac{4}{3}$
T	$\frac{1}{6} + \frac{\sqrt{3}\beta}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{4}{3}$	$\frac{5}{3}$
E	$-\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$	-1	0	-2	1
V	$-\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$	-1	0	-2	1
Y	$\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$	0	1	-1	2
H_V	$-\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$	-1	0	-2	1
H_Y	$\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$	0	1	-1	2
H_W	1	1	1	1	1

Cao, Liu, Xie, Yan, Zhang Phys.Rev. D93(2016) no.7, 075030

Contents

Trinification

$$[SU(3)]^3 \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$$

....maximal subgroup of E_6

Trinification: Field Content

$$H = \begin{pmatrix} h^0_{11} & h^+_{12} & h^+_{13} \\ h^-_{21} & h^0_{22} & h^0_{23} \\ h^-_{31} & h^0_{32} & h^0_{33} \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$L = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix} \in (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$$

$$Q_R = \begin{pmatrix} \bar{u}_R & \bar{d}_R & \bar{D}_R \end{pmatrix} \in (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

Trinification: Field Content

$$H = \begin{pmatrix} h_{11}^0 & h_{12}^+ & h_{13}^+ \\ h_{21}^- & h_{22}^0 & h_{23}^0 \\ h_{31}^- & h_{32}^0 & h_{33}^0 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$L = \begin{pmatrix} L_1^1 & E^- & e^- \\ E^+ & L_2^2 & \nu \\ e^+ & \hat{\nu} & L_3^3 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix} \in (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$$

$$Q_R = \begin{pmatrix} \bar{u}_R & \bar{d}_R & \bar{D}_R \end{pmatrix} \in (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

$$\mathcal{Q}^{\mathrm{em}} = \mathcal{T}_L^3 + \mathcal{T}_R^3 + \frac{1}{\sqrt{3}}\mathcal{T}_L^8 + \frac{1}{\sqrt{3}}\mathcal{T}_R^8$$

Trinification: Field Content

$$H = \begin{pmatrix} h_{11}^0 & h_{12}^+ & h_{13}^+ \\ h_{21}^- & h_{22}^0 & h_{23}^0 \\ h_{31}^- & h_{32}^0 & h_{33}^0 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$L = \begin{pmatrix} L_1^1 & E^- & e^- \\ E^+ & L_2^2 & \nu \\ e^+ & \bar{\nu} & L_3^3 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

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$$\mathcal{Q}_{D_i}^{\mathrm{em}} = -1/3$$

non-Exotic Quarks!

$SU(3)^3 \to \ldots \to \mathcal{G}_{SM} \to SU(3)_c \times U(1)_{\mathrm{em}}$

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & M_1 \end{pmatrix}, \ \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M & M_2 \end{pmatrix}$$

$SU(3)^3 \to \ldots \to \mathcal{G}_{SM} \to SU(3)_c \times U(1)_{\mathrm{em}}$

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$SU(3)^3$

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$$SU(3)^3 \quad \xrightarrow{M_i} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$SU(3)^3 \to \ldots \to \mathcal{G}_{SM} \to SU(3)_c \times U(1)_{\mathrm{em}}$

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$$\begin{aligned} SU(3)^3 \quad & \xrightarrow{M_i} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \xrightarrow{M} \mathcal{G}_{SM} \end{aligned}$$

$SU(3)^3 \to \ldots \to \mathcal{G}_{SM} \to SU(3)_c \times U(1)_{\mathrm{em}}$

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & M_1 \end{pmatrix}, \; \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M & M_2 \end{pmatrix}$$

$$\begin{aligned} SU(3)^3 \quad & \xrightarrow{M_i} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \xrightarrow{M} \mathcal{G}_{SM} \\ & \xrightarrow{v_i,\, b_j} SU(3)_c \times U(1)_{\mathrm{em}} \end{aligned}$$

$SU(3)^3 \to \ldots \to \mathcal{G}_{SM} \to SU(3)_c \times U(1)_{\rm em}$

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$$\begin{aligned} SU(3)^3 \quad & \xrightarrow{M_i} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \xrightarrow{M} \mathcal{G}_{SM} \\ & \xrightarrow{v_i,\, b_j} SU(3)_c \times U(1)_{\rm em} \end{aligned}$$

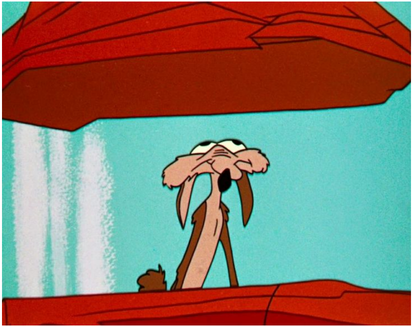
$$(\sqrt{2} G_F)^{1/2} \sim v_i,\, b_j < M < M_i \sim m_{GUT}$$

Hetzel, Stech, Phys.Rev.D91 (2015) 055026

Not-Exotic...but Heavy!

$$m_{D_i} = \frac{1}{\sqrt{2}} M_{1,2} Y_{Q_i} \lesssim m_{GUT}$$

Only indirect hints!



...work in progress...

Summary

$\Diamond$ Higgs mechanism give mass to heavier SM quark (experimentally established)

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- $\Diamond$ BSM quarks searched not only as resonances

Summary

$\Diamond$ Higgs mechanism give mass to heavier SM quark (experimentally established)

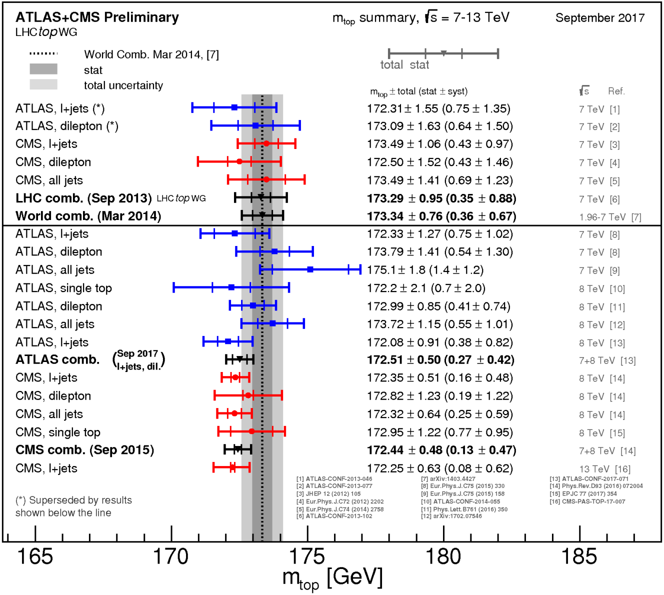
$\Diamond$ BSM quarks searched not only as resonances

$\Diamond$ exotic models $\not\Rightarrow$ exotic quarks

Thanks

Backup

ATLAS & CMS Top Mass



The triplet sector

$$\mathcal{L}^{Yuk.}_{q,triplet} = \big(y^1_d\ Q_1\eta^* d_R + y^2_d\ Q_2\eta^* s_R + y^3_d\ Q_3\chi\, b^*_R \\ + y^1_u\ Q_1\chi^* u^*_R + y^2_u\ Q_2\chi^* c^*_R + y^3_u\ Q_3\eta\, t^*_R \\ + y^1_E\ Q_1\,\rho^* D^*_R + y^2_E\ Q_2\,\rho^* S^*_R + y^3_E\ Q_3\,\rho\, T^*_R\big) + \mathrm{h.c.},$$

$v_\rho \gg v_{\eta,\chi} \Rightarrow$ the masses of the exotic quarks are $\mathcal{O}(\mathrm{TeV})$ whenever the relation $y^i_E \sim 1$ is satisfied.

The sextet sector

$$
\begin{aligned}
\mathcal{L}^{Yuk}_{l,\,triplet} &= G^{\eta}_{ab}(l^{i}_{a\alpha}\epsilon^{\alpha\beta}l^{j}_{b\beta})\eta^{*k}\epsilon^{ijk} + \mathrm{h.c.} \\
&= G^{\eta}_{ab}\, l^{i}_{a}\cdot l^{j}_{b}\,\eta^{*k}\epsilon^{ijk} + \mathrm{h.c.}
\end{aligned}
$$

$(l^i_a \cdot l^j_b)\eta^{*k}\epsilon^{ijk}$ is antisymmetric, implying that even G_{ab} has to be antisymmetric

$$
\sigma = \begin{pmatrix} \sigma_1^{++} & \sigma_1^{+}/\sqrt{2} & \sigma^0/\sqrt{2} \\ \sigma_1^{+}/\sqrt{2} & \sigma_1^0 & \sigma_2^{-}/\sqrt{2} \\ \sigma^0/\sqrt{2} & \sigma_2^{-}/\sqrt{2} & \sigma_2^{--} \end{pmatrix} \in (1,6,0)
$$

$$
\mathcal{L}^{Yuk.}_{l,sextet} = G^{\sigma}_{ab} l^i_a \cdot l^j_b \sigma^*_{i,j}
$$

G^{σ}_{ab} is symmetric in flavour space.

The potential

The (lepton-number conserving) potential of the model is given by

$$
\begin{aligned}
V = {} & m_1\, \rho^\dagger \rho + m_2\, \eta^\dagger \eta + m_3\, \chi^\dagger \chi + \lambda_1 (\rho^\dagger \rho)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\chi^\dagger \chi)^2 + \lambda_{12} \rho^\dagger \rho\, \eta^\dagger \eta \\
& + \lambda_{13} \rho^\dagger \rho\, \chi^\dagger \chi + \lambda_{23} \eta^\dagger \eta\, \chi^\dagger \chi + \zeta_{12} \rho^\dagger \eta\, \eta^\dagger \rho + \zeta_{13} \rho^\dagger \chi\, \chi^\dagger \rho + \zeta_{23} \eta^\dagger \chi\, \chi^\dagger \eta \\
& + m_4\, Tr(\sigma^\dagger \sigma) + \lambda_4 (Tr(\sigma^\dagger \sigma))^2 + \lambda_{14} \rho^\dagger \rho\, Tr(\sigma^\dagger \sigma) + \lambda_{24} \eta^\dagger \eta\, Tr(\sigma^\dagger \sigma) \\
& + \lambda_{34} \chi^\dagger \chi\, Tr(\sigma^\dagger \sigma) + \lambda_{44}\, Tr(\sigma^\dagger \sigma\, \sigma^\dagger \sigma) + \zeta_{14} \rho^\dagger \sigma\, \sigma^\dagger \rho + \zeta_{24} \eta^\dagger \sigma\, \sigma^\dagger \eta + \zeta_{34} \chi^\dagger \sigma\, \sigma^\dagger \chi \\
& + (\sqrt{2} f_{\rho\eta\chi} \epsilon^{ijk} \rho_i\, \eta_j\, \chi_k + \sqrt{2} f_{\rho\sigma\chi} \rho^T\, \sigma^\dagger\, \chi \\
& + \xi_{14} \epsilon^{ijk}\, \rho^{*l} \sigma_{li} \rho_j \eta_k + \xi_{24} \epsilon^{ijk} \epsilon^{lmn}\, \eta_i \eta_l \sigma_{jm} \sigma_{kn} + \xi_{34} \epsilon^{ijk}\, \chi^{*l} \sigma_{li} \chi_j \eta_k) + \mathrm{h.c.}
\end{aligned}
$$

Trinification's potential

The most general potential allowed by the gauge symmetry $SU(3)_c \times SU(3)_L \times SU(3)_R$ is

$$
\begin{aligned}
V = {} & \mu_1^2\,\mathtt{tr}(H_1^\dagger H_1) + \mu_2^2\,\mathtt{tr}(H_2^\dagger H_2) + \mu_{12}^2\left(\mathtt{tr}(H_1^\dagger H_2) + \mathtt{h.c.}\right) + \\
& \epsilon^{ijk}\epsilon_{lmn}\left(\mu_{112}(H_1)^i_l(H_1)^j_m(H_2)^k_n + \mu_{122}(H_1)^i_l(H_2)^j_m(H_2)^k_n + \mathtt{h.c.}\right) + \\
& \mu_{d1}\left(\mathtt{det}(H_1) + \mathtt{det}(H_1^\dagger)\right) + \mu_{d2}\left(\mathtt{det}(H_2) + \mathtt{det}(H_2^\dagger)\right) + \\
& \lambda_1\left(\mathtt{tr}(H_1^\dagger H_1)\right)^2 + \lambda_2\,\mathtt{tr}(H_1^\dagger H_1 H_1^\dagger H_1) + \lambda_3\left(\mathtt{tr}(H_2^\dagger H_2)\right)^2 + \lambda_4\,\mathtt{tr}(H_2^\dagger H_2 H_2^\dagger H_2) + \\
& \lambda_5\,\mathtt{tr}(H_1^\dagger H_1)\mathtt{tr}(H_2^\dagger H_2) + \lambda_6\,\mathtt{tr}(H_1^\dagger H_1 H_2^\dagger H_2) + \\
& \lambda_7\,\mathtt{tr}(H_1^\dagger H_2)\mathtt{tr}(H_2^\dagger H_1) + \lambda_8\,\mathtt{tr}(H_1^\dagger H_2 H_2^\dagger H_1) + \\
& \Big[\lambda_9\left(\mathtt{tr}(H_1^\dagger H_2)\right)^2 + \lambda_{10}\,\mathtt{tr}(H_1^\dagger H_2 H_1^\dagger H_2) + \lambda_{11}\,\mathtt{tr}(H_1^\dagger H_1)\mathtt{tr}(H_1^\dagger H_2) + \\
& \lambda_{12}\mathtt{tr}(H_1^\dagger H_1 H_1^\dagger H_2) + \lambda_{13}\,\mathtt{tr}(H_1^\dagger H_2)\mathtt{tr}(H_2^\dagger H_2) + \lambda_{14}\mathtt{tr}(H_1^\dagger H_2 H_2^\dagger H_2) + \mathtt{h.c.}\Big]
\end{aligned}
$$