Electromagnetic properties of viscous charged fluids

Davide Forcella
(ULB, FNRS)
Holographic Methods for Strongly Coupled Systems: “Gauge/Gravity Duality 2015”,
GGI, 13th April 2015

based on:
Phys.Rev. B90 (2014) 035143, arXiv:1406.1356 [cond-mat.str-el]
D.F., J. Zaanen, D. Valentinis, D.van der Marel

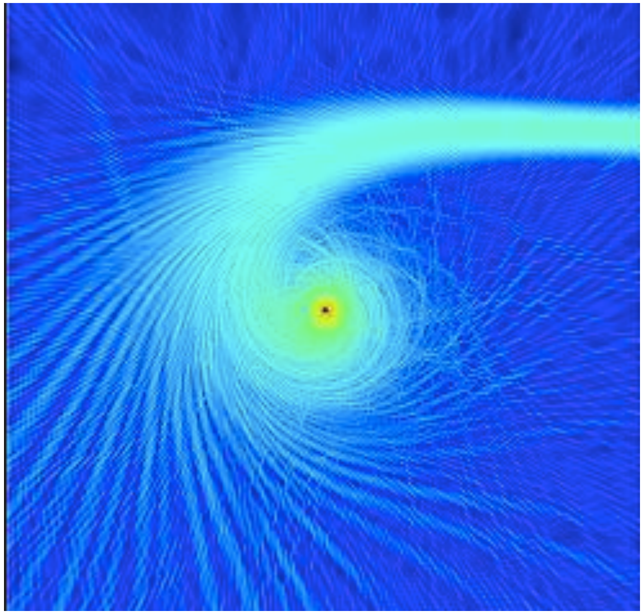
a long path in collaboration with:

A. Amariti, A. Mariotti, A. Mezzalira, D. Musso, G. Policastro,
M. Siani, D. Valentinis, D.V.D. Marel, J. Zaanen.

Motivations I (em devices)

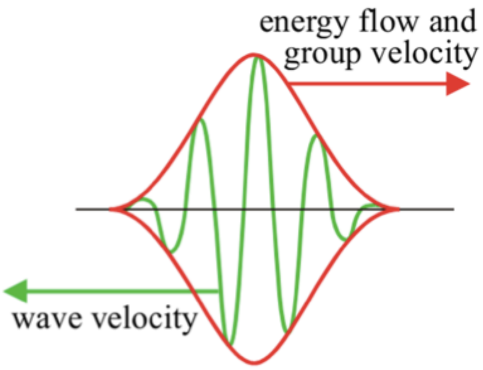
- impressive advances in the present ability to engineer devices that can bend em waves almost at will.

ex: photonic black holes, cloaking devices, etc.



- exotic phenomena predicted long ago are now realised in the laboratory

ex: negative refraction



Motivations II *(why was it difficult ?)*

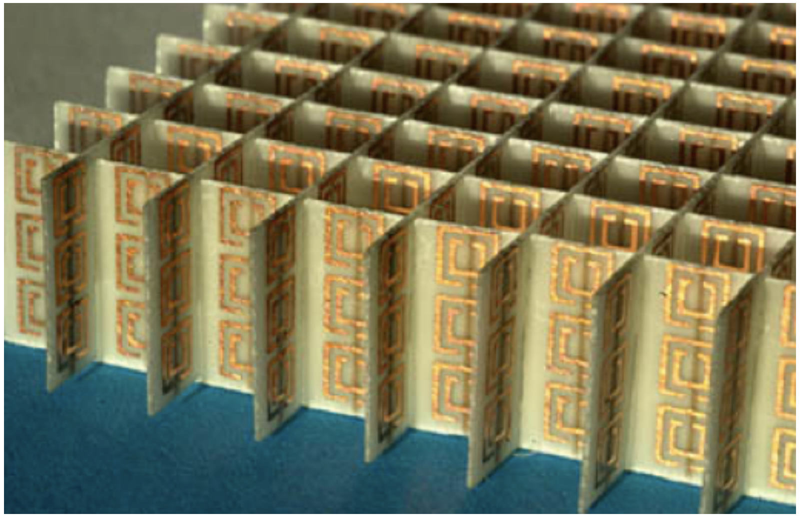
There exists an analogy between em waves in media in flat space time and em waves in empty curved space time:

at geometric optic level: $\delta s(n_{ij}) = 0 \leftrightarrow \delta s(g_{\mu\nu}) = 0$

at wave equation level: $\epsilon^{ij} = \mu^{ij} = -\frac{\sqrt{-g}g^{ij}}{g_{00}}$

the easiest requirement for negative refraction: $\epsilon < 0$ and $\mu < 0$

such phenomena were indeed realised in artificial materials "metamaterials"



Motivations III (string theory and holography)

- recently string theory (surprisingly) has been able to provide a quite accurate description of some aspects of high energy strongly coupled plasmas (e.g. quark-gluon plasma) and condensed matter strongly coupled systems (e.g. high Tc superconductors, strange metals, …)

- It is an inspiring ground to look for new materials: e.g. electrically charged fluids in hydrodynamical regime (the main actor of this talk)

- It could be an inspiring ground for new universal phenomena and for ideas to be tested in the laboratories

Bonus: 1) a universal prediction for electrically charged viscous fluids
2) new "natural" materials with exotic em properties
3) experimental signatures for electrons viscosity in metals, very rarely discussed and never measured before

Main Results

Theory

Theorem: homogeneous and isotropic charged fluids described by hydrodynamics equations have negative refraction for frequencies below the plasma frequency

Phenomenology

1- fluid viscosity, or spatial non-locality, is the main responsible for negative refraction

2- exotic em response in spectroscopy experiments provides sharp signatures to measure spatial non-localities and viscosity in electron fluids in condensed matter

The Setup I (em)

Maxwell eq. in spatially dispersive media

$$\nabla \cdot D = 0 \quad , \quad \nabla \cdot B = 0 \quad , \quad \nabla \wedge E = -\partial_t B \quad , \quad \nabla \wedge B = \partial_t D$$

with (if homogeneous): $D_i = \epsilon_{ij}(\omega, k) E_j$

for isotropic media: $\epsilon_{ij} = \epsilon_T(\omega, k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \epsilon_L(\omega, k)\frac{k_i k_j}{k^2}$

two dispersion relations: $\epsilon_T(\omega, k) = \frac{k^2}{\omega^2}, \qquad \epsilon_L(\omega, k) = 0$

for small spatial dispersion we find the usual ϵ, μ formalism:

$$\epsilon_T(\omega, k) = \epsilon(\omega) + \frac{k^2}{\omega^2}\left(1 - \frac{1}{\mu(\omega)}\right) + \mathcal{O}(k^4) \qquad \frac{k^2}{\omega^2} = n^2(\omega) = \epsilon(\omega)\mu(\omega)$$

The Setup II (linear response)

$$J_i = G_{ij} A_j$$

$$G_{ij}(x - x', t - t') = -i\theta(t - t') \langle [J_i(x,t), J_j(x',t')] \rangle$$

It Implies that:

$$\epsilon_T(\omega, k) = 1 - \frac{4\pi}{\omega^2} q^2 G_T(\omega, k) \quad , \quad \epsilon_L(\omega, k) = 1 - \frac{4\pi}{\omega^2} q^2 G_L(\omega, k)$$

For small spatial dispersion $G_T(w,k) = G_T^{(0)}(w) + k^2 G_T^{(2)}(w)$

$$\epsilon(\omega) = 1 - \frac{4\pi}{\omega^2} q^2 G_T^{(0)}(\omega)$$

$$\mu(\omega) = \frac{1}{1 + 4\pi\, q^2 G_T^{(2)}(\omega)}$$

How to check for negative refraction?

$$e^{-i\omega t+ikx} = e^{-i\omega t+iRe(k)x-Im(k)x}$$

for a passive media the energy propagates in the direction of attenuation
Negative refraction if:

$$\mathrm{Re}(k)\mathrm{Im}(k) < 0$$

for small spatial dispersion it is analogous to check if the Poynting vector S and the phase velocity are in opposite directions:

$$\vec{S} = \mathrm{Re}\,(n/\mu)\,\hat{k}|E_T|^2 \qquad \vec{v}_{ph} = 1/\mathrm{Re}(n)\hat{k}$$

$$\mathrm{Re}\,(n) < 0, \qquad \mathrm{Re}\left(\frac{n}{\mu}\right) > 0$$

The strategy

1. Chose a medium

2. Compute the two points retarded correlator for the transverse currents

3. Impose the electromagnetic dispersion relation: k(w)

4. Plot Im(k(w)) and Re(k(w))

String theory and holography: an inspiring medium

Holography naturally describes finite temperature charged media that for long time and long distances have a description in term of hydrodynamics and that can be weakly coupled to em field.

Charged back holes in AdS5

$$S = \frac{1}{2e^2 l^2}\int d^5x\sqrt{-g}\left(R+\frac{6}{l^2}\right) - \frac{1}{4e^2}\int d^5x\sqrt{-g}F_{mn}F^{mn}$$

$$ds^2 = \frac{(2-a)^2 l}{16\,b^2}\frac{1}{u}\left(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2-f(u)\mathrm{d}t^2\right)+\frac{l^2}{4}\frac{\mathrm{d}u^2}{u^2 f(u)}$$

$$A_t = -\frac{u}{2b}\sqrt{\frac{3}{2}a}+\Sigma$$

$$f(u) = (1-u)(1+u-au^2) \qquad T = \frac{2-a}{4\pi b} \qquad \Sigma = \frac{1}{2\,b}\sqrt{\frac{3}{2}a}$$

Hydrodynamics and negative refraction

Linear hydro and linear em respor $G_{JT}(w,k) = \dfrac{iw\mathcal{B}}{-iw + \mathcal{D}k^2} - \mathcal{C}$

A. non-superfluid $\mathcal{C}$ = 0

- non-relativistic $\mathcal{B}$ = $\frac{e^2 n}{m}$ $\mathcal{D}$ = $\frac{\eta}{nm}$,

- relativistic $\mathcal{B} = \rho^2/(\epsilon + P)$ and $\mathcal{D} = \eta/(\epsilon + P)$

B. superfluid

- non-relativistic $\mathcal{B} = e^2 n_n/m$ $\mathcal{D} = \eta/mn_n$ and $\mathcal{C} = e^2 n_s/m$

- relativistic $\mathcal{B} = \rho_n^2/(\epsilon + P - \rho_s\mu)$

$\mathcal{D} = \eta/(\epsilon + P - \rho_s\mu)$ and $\mathcal{C} = \rho_s\rho_n/(\epsilon + P - \rho_s\mu)$

Multiple em waves

Due to spatial dispersion the dispersion relation for the transverse em waves

$$\frac{k^2}{\omega^2} = 1 - \frac{4\pi}{\omega^2}\Big(\frac{i\omega\mathcal{B}}{-i\omega + \mathcal{D}k^2} - \mathcal{C}\Big)$$

has two solutions and hence there exist **two waves with different propagation and dissipation pattern inside the medium** !

Result I

It is easy to check that for $w < 2\sqrt{\pi(\mathcal{B}+\mathcal{C})}$.
one of the two waves has negative refraction!

Theorem:
charged fluids describe by hydrodynamics have negative refraction for low frequencies !

It is a feature that comes from two main ingredients:
finite charge & viscosity

The result is "universal", but do we have actual systems that satisfy these conditions ?

Electrons in Metals

- the dynamics is dominated by the presence of the atomic lattice that breaks the translation invariance

  => **momentum dissipation**

- viscosity has never been measured and very rarely studied

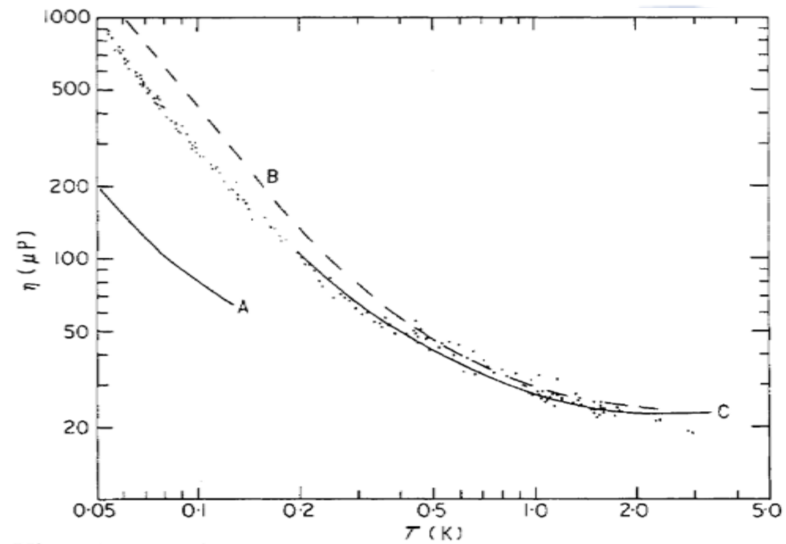
- two main characteristic times τ_{coll} , τ_K that control establishment of hydro and dissipation of momentum

- our previous model should be generalised to describe actual material !

Strategy: we use peculiar em properties to study/measure viscosity or spatial non localities for electrons in CM !

Example of uncharged viscous fluid:

He3 in normal phase



Black, Hall & Thompson, J Phys C 4, 129 (1971)

Fermi liquid theory:

hydrodynamic regime (high T) $\nu \sim \dfrac{A}{T^2}$

collisionless regime (high w)

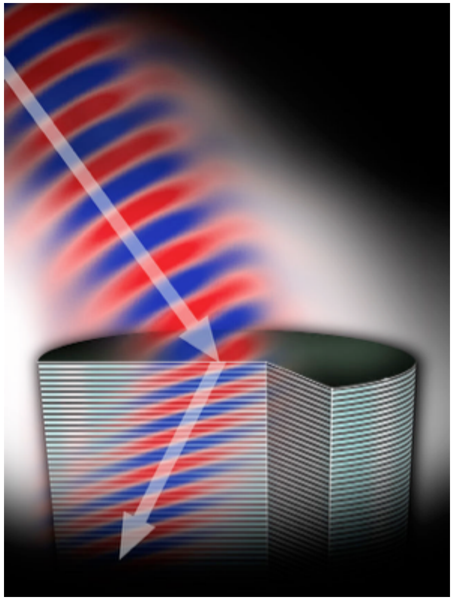
$$\nu \sim i\frac{B}{\omega} + CT^2$$

A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksperim. Teor. Fiz. 33, 110 (1957)

Example of charged viscous fluid:

Quark Gluon Plasma

Consequences: negative refraction, multiple waves



A. Amariti, D. Forcella, A. Mariotti and G. Policastro, JHEP 1104, 036 (2011).
A. Amariti, D. Forcella and A. Mariotti, JHEP 1301, 105 (2013).
A. Amariti, D. Forcella and A. Mariotti, arXiv:1010.1297.

What do we know about viscosity in electrons fluids?

One of the challenges yet to be met is to determine the value of the viscosity in a non-Gallilean invariant setting, and its dependence on temperature and frequency. Clear condition for the applicability of hydrodynamics in electrons fluid are not clearly understood.

Here we will simply assume that local non-spatial correction exists, and work out a number of physical consequences.

For this we will adopt values for the viscosity which seem plausible at this moment, **but which need to be determined ultimately on the basis of first principles and/or experimental data.**

The theoretical setup

Maxwell Equations => $\left[c^2\Delta+\omega^2\right]E(\mathbf{r})=-4\pi i\omega J(\mathbf{r})$

Generalised Navier-Stokes => $\left[\tau_K^{-1}-i\omega-\nu(\omega)\Delta\right]v=eE(\mathbf{r})/m$ $\quad J(t,\mathbf{r})=nev(t,\mathbf{r})$

$$\frac{q^2c^2}{\omega^2}=1-\frac{\omega_p^2}{\omega\left(\omega+i\tau_K^{-1}+i\nu(\omega)q^2\right)}$$

...phenomenological consequences

Two waves for each frequency $\quad 2n_j^2=1-\frac{1-i\omega\tau_K}{\omega^2\nu_c\tau_K}\pm\sqrt{\left[1+\frac{1-i\omega\tau_K}{\omega^2\nu_c\tau_K}\right]^2+\frac{i4\omega_p^2}{\omega^3\nu_c}}$

$\nu_c=\nu(\omega)/c^2$

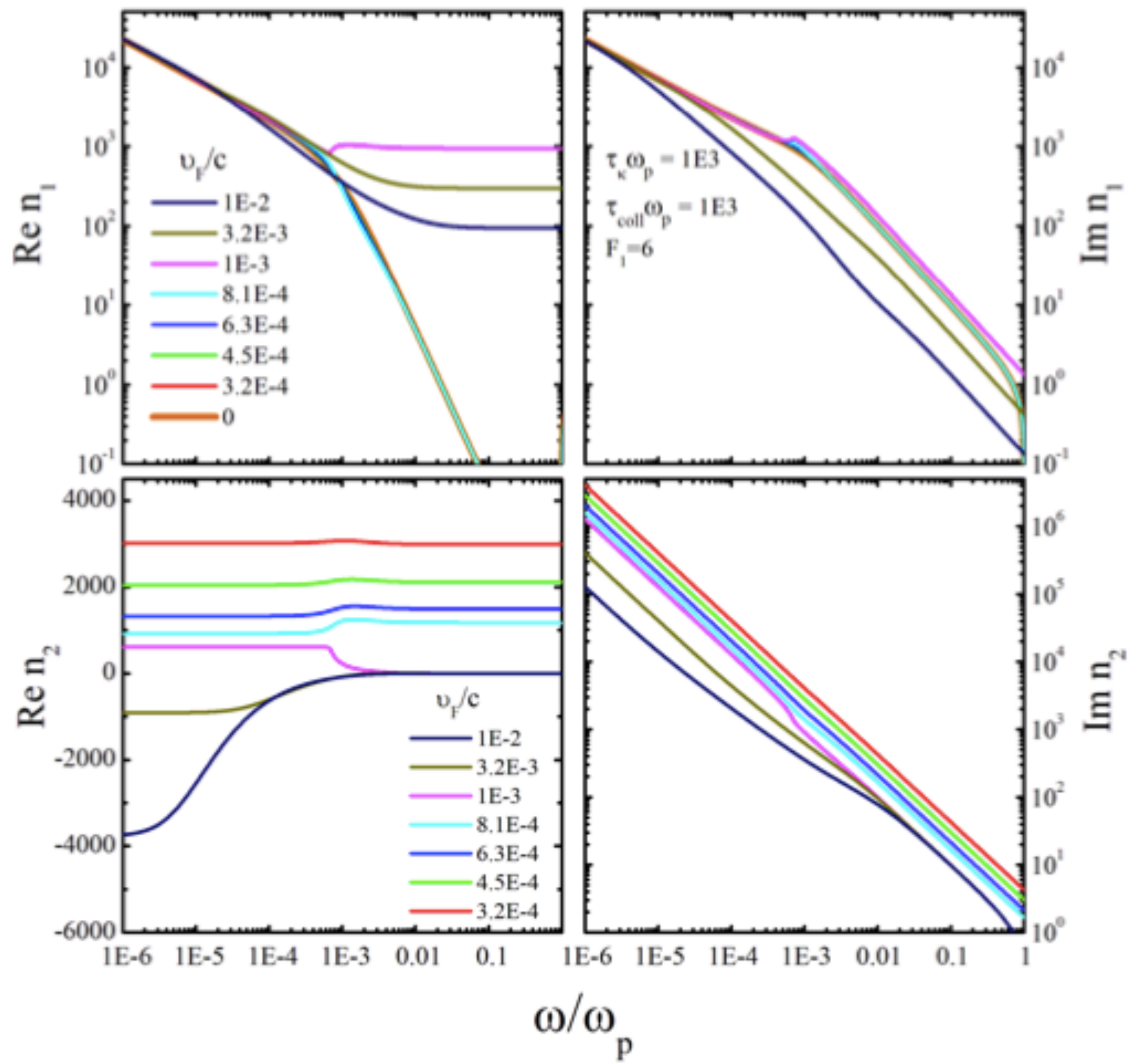
Negative refraction for one of the two mode if $\nu_c\tau_K\omega_p^2>1$

Consequences

$$\frac{\nu_c(\omega)}{\tilde{\tau}} \sim \frac{(\tilde{\upsilon}_F/c)^2}{1-i\omega\tilde{\tau}}$$

multiple waves

negative refraction



A boundary problem

To perform an experiment we should introduce a boundary in the material and compute observables.

On top of the usual em boundary condition it exists an additional condition for NS that fix the 2 waves

$$E(0-\delta) = E(0+\delta)$$
$$\left.\frac{\partial E}{\partial z}\right|_{0-\delta} = \left.\frac{\partial E}{\partial z}\right|_{0+\delta}$$
$$\frac{1}{\lambda} = \left.\frac{\partial \ln v}{\partial z}\right|_{0+\delta}$$

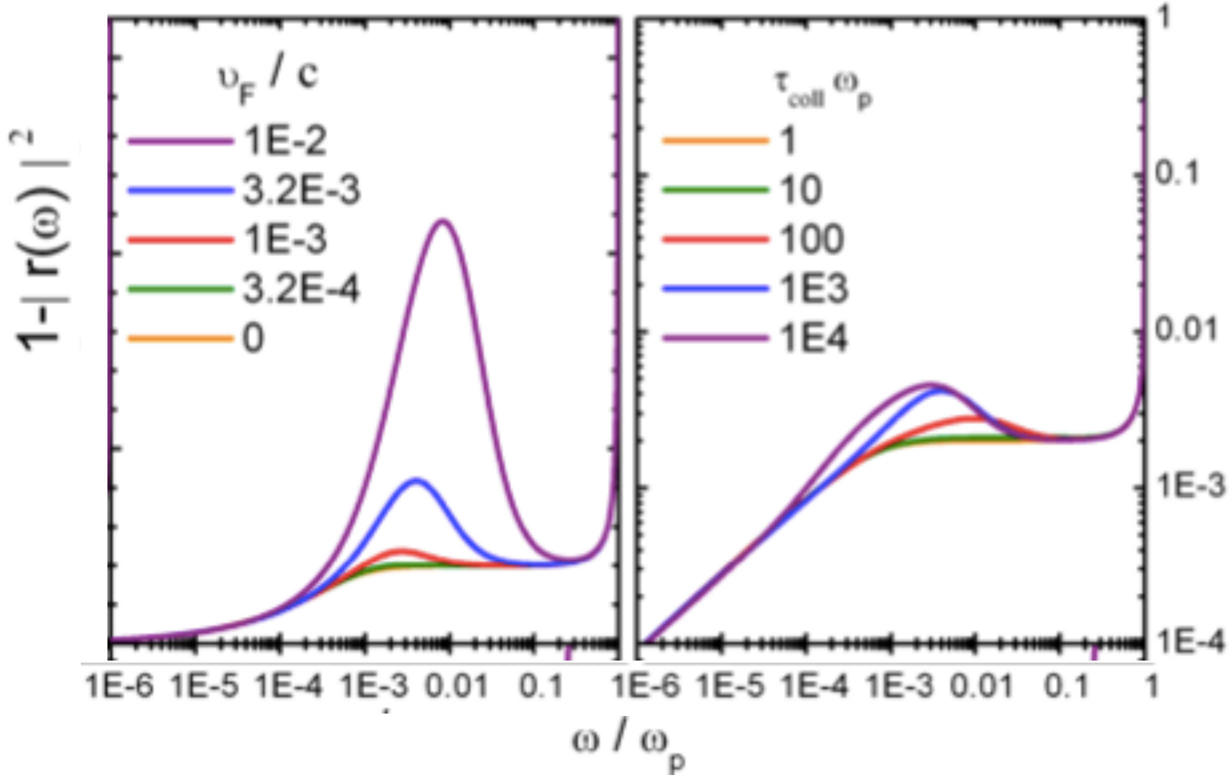
$$E(d-\delta) = E(d+\delta)$$
$$\left.\frac{\partial E}{\partial z}\right|_{d-\delta} = \left.\frac{\partial E}{\partial z}\right|_{d+\delta}$$
$$\left.\frac{\partial \ln v}{\partial z}\right|_{d-\delta} = \frac{-1}{\lambda}$$

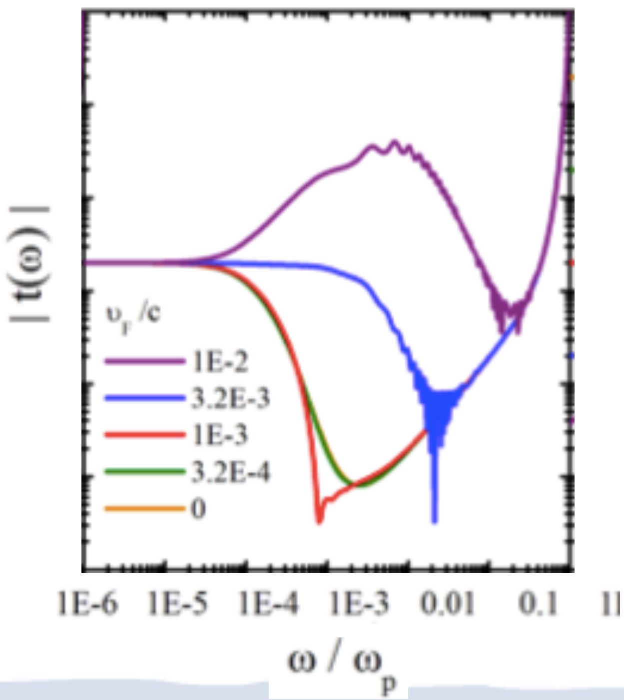
$$
\begin{aligned}
E(z)/E(0) \quad &= e^{ikz} + re^{-ikz} && (z<0)\\
&= t_1 e^{in_1kz} + \theta_1 e^{-in_1kz} + t_2 e^{in_2kz} + \theta_2 e^{-in_2kz} && (0<z<d)\\
&= te^{ikz} && (z>d)
\end{aligned}
$$

Some observables

Absorption coefficient at boundary from vacuum to matter interface

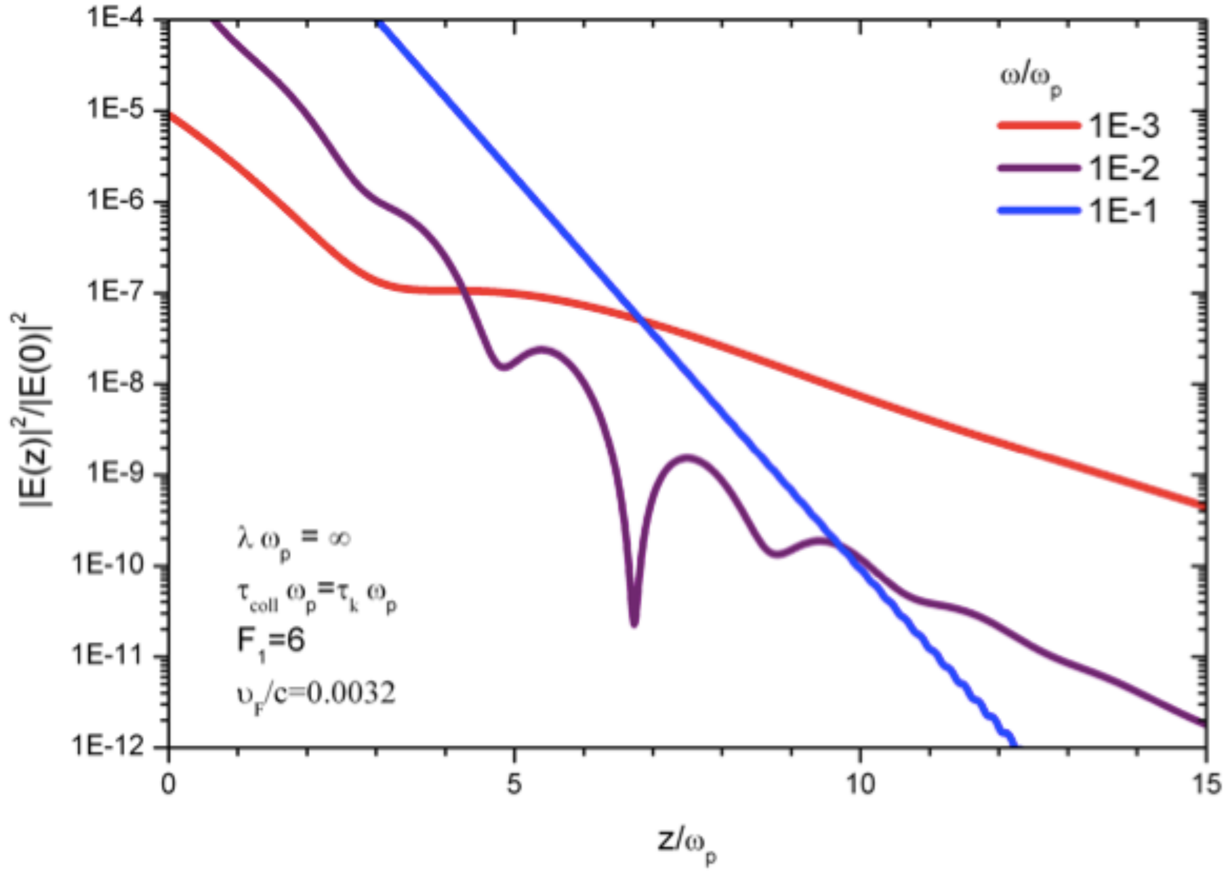
Peak at $\omega\tau_K \approx 1$





<- Thin film transmission

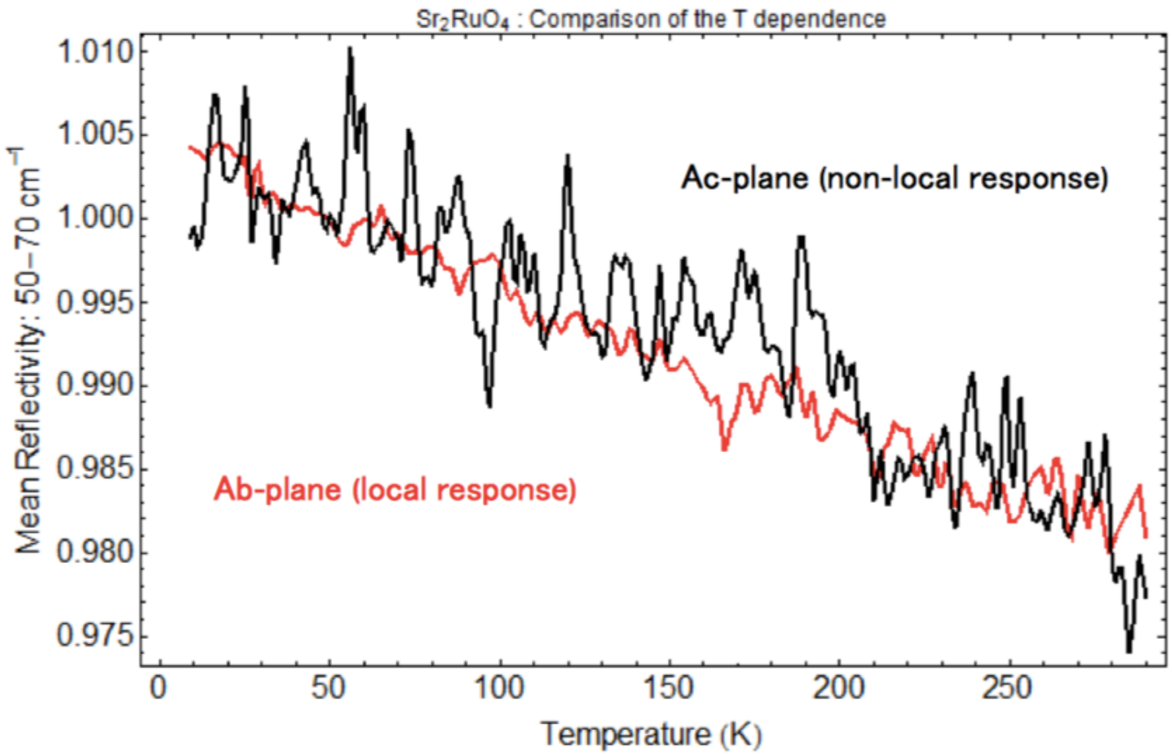
Some observables



Interference of two modes in the electrified intensity penetrating inside a viscous material

A first experimental test

Experimental test on Sr_2RuO_4



The overlap of both signals with and without viscosity implies an upper limit for η / τ

Results II

- There are sharp signatures of the finite viscosity in charged fluids that can be compared with present and future experiments:

  Decrease reflectivity, increase transmission of em field at vacuum sample interface, peak in the surface absorption at $\omega\tau = 1$, EM-field oscillations due to interference of the two modes,…

- Spectroscopy is used as a tool to hunt viscosity/spatial non-localities in electron systems

- Challenge: quantitative predictions and exact functional form of the viscosity in term of frequency and temperature is not presently known

Conclusions

- We have demonstrated the existence of a exotic em phenomena in viscous charged fluids
- Holography has been the inspiring ground to look for such systems
- Such exotic em response is a nice proxy for studying dissipation and viscosity/spatial dispersion of strongly correlated systems
- Application: electrons in metals: Fermi and non-Fermi liquids
- We provided sharp signatures for the existence of the viscosity in charged fluids that could be actually checked in experiments

- …a path from string theory to real electrons….
- **Experiments in progress** to observe the viscosity

Future directions:

- Look for systems in which the establishment of the hydrodynamics is more favourable
- Detailed comparison of Fermi and non-Fermi liquids spectroscopy
- Compute currents correlators in strongly interacting dissipative (translation invariant broken) models with spatial non localities => holography..
- Experimental measurements of the viscosity and its frequency and temperature dependence.

Thanks a lot !