Modified Gravity. Problems and Observational Manifestations

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Results and Perspectives in Particle Physics

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Outline

- Cosmological acceleration, antigravity
- Data in favor of acceleration
- Problem of vacuum energy
- Dark energy or modifed gravity.
- Cosmological Evolution and Particle Production in $F(R) + R^2$ gravity
- Conclusions

Cosmological Acceleration

A large set of independent, different types astronomical data show that the universe today expands with acceleration (antigravity).

With cosmological inflation, at the very beginning, the picture would be:

- first acceleration (initial push)
- then normal deceleration
- and lastly (today) surprising acceleration again

Antigravity at the beginning was a source of expansion - inflation.

Cosmological Equations

Universe expansion is described by scale factor $\mathbf{a(t)}$, which satisfies the Friedmann equations. In particular, cosmological acceleration is given by:

$$\frac{\ddot{\mathbf{a}}}{\mathbf{a}} = -\frac{4\pi\,\mathbf{G_N}}{3}(\varrho + 3\mathbf{P})$$

NB: Pressure gravitates!

- Source of gravitational force $\varrho + 3\mathbf{P}$, not only ϱ.
- Negative pressure is the source of the cosmological expansion, cosmic antigravity.

Equation of State

- There are two independent cosmological equations for three functions $\mathbf{a(t)}$, $\varrho(\mathbf{t})$, $\mathbf{P(t)}$.
- These equations should be supplemented by the equation of state $\mathbf{P} = \mathbf{P}(\varrho)$, which is determined by physical properties of matter.

Usually matter is described by linear equation of state:

$$\mathbf{P} = \mathbf{w}\varrho,$$

$\mathbf{w} = \mathbf{const}$ is new cosmological parameter (for DE).

Simple Examples ($k = 0$, 3D flat Universe)

Non-relativistic matter: $w = 0$, $a \sim t^{2/3}$

$$\frac{\ddot{a}}{a} = -\frac{2}{9t^2} < 0$$

Matter dominant stage: **deceleration**

Relativistic matter: $w = 1/3$, $a \sim t^{1/2}$

$$\frac{\ddot{a}}{a} = -\frac{1}{4t^2} < 0$$

Radiation dominant stage: **deceleration**

Dark energy: $w = -1$, $a \sim \exp(Ht)$

$$\frac{\ddot{a}}{a} = H^2 = \mathrm{const} > 0$$

Cosmological acceleration

Universe Today

The present day value of the **H**:

$$\mathbf{H} = \mathbf{100\,h}\,\mathrm{km/sec/Mps}$$

$\mathbf{h} = \mathbf{0.73} \pm \mathbf{0.05}$ - dimensionless parameter.
$\mathbf{H}^{-1} = \mathbf{9.8}\,\mathrm{Gyr}/\mathbf{h} \approx \mathbf{13.4}\,\mathrm{Gyr}$ is approximately equal to the Universe age.

The present day value of the critical energy density:

$$\varrho_c = \frac{3H^2 m_{Pl}^2}{8\pi} = 1.88 \cdot 10^{-29} h^2 \frac{g}{cm^3}$$

It corresponds to 10 protons per $\mathbf{m}^3$, but the dominant matter is not the baryonic one and in reality there are about 0.5 protons per $\mathbf{m}^3$

Data in favor acceleration: Universe Age

Ages of stellar clusters and nuclear chronology:

$$t_U = 12 - 15\,\mathrm{Gyr}$$

In terms of the present day values of H and of dimensionless parameter $\Omega_j = \varrho_j/\varrho_c$:

$$t_U = \frac{1}{H}\int_0^1 \frac{dx}{\sqrt{1 - \Omega_t + \frac{\Omega_m}{x} + \frac{\Omega_r}{x^2} + x^2\Omega_v}}.$$

- Ω_t is the total Ω
- Ω_m, Ω_r, Ω_v are respectively contributions from relativistic and non-relativistic matter and from vacuum energy.

If $\Omega_v = 0$, then $t_U = 2/3H \approx 9\,\mathrm{Gyr}$ and the Universe is much younger than observed.

Matter Inventory

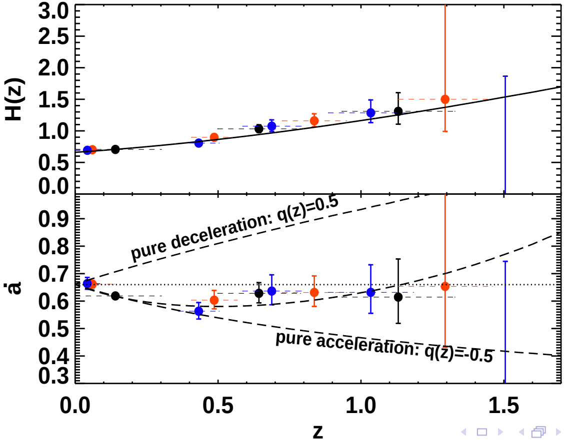
- Total energy density: $\Omega_{\text{tot}} = 1 \pm 0.02$
  from the position of the first peak of CMBR and LSS.
- Usual baryonic matter: $\Omega_{\text{B}} = 0.044 \pm 0.004$
  from heights of CMBR peaks, BBN, and onset of structure formation with small $\delta T/T$.
- Total dark matter: $\Omega_{\text{DM}} \approx 0.22 \pm 0.04$
  from galactic rotation curves, gravitational lensing, equilibrium of hot gas in rich galactic clusters, cluster evolution, LSS.
- The rest: $\Omega_{\text{DE}} \approx 0.76,\ w \approx -1$
  - induces accelerated expansion;
  measured from dimming of high-z supernovae, LSS, universe age.

Direct measurment of acceleration

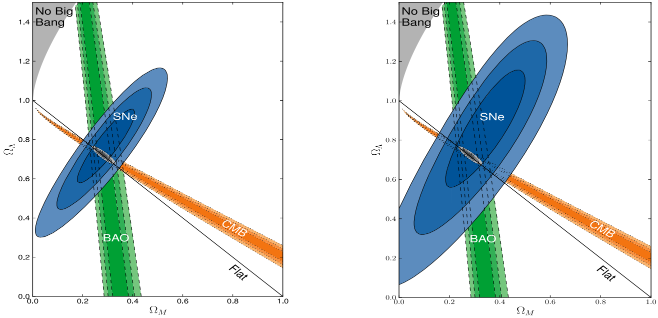
- Dimming of high redshift supernovae, if they are standard candles means that they are at a larger distance, i.e. the universe expands faster than expected.
- Nonmonotonic dependence on $\mathbf{z}$ excludes light absorption on the way.
- Nobel Prize of 2011: S. Perlmutter, B.P. Schmidt, and A.G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

Red shift dependence of dimming

The bottom panel shows $\dot{\mathbf{a}}$ versus redshift.
At some redshift deceleration turns into acceleration, because $\varrho_v = \mathbf{const}$ and $\varrho_m \sim (1 + z)^3$.

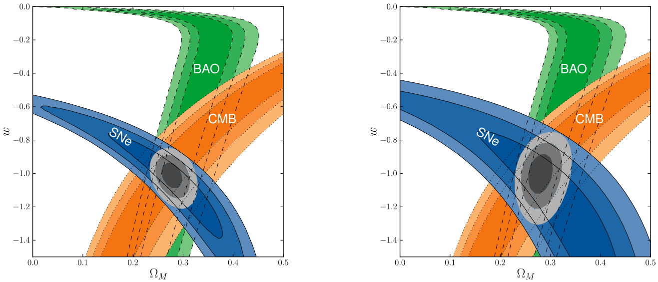


Summary Figures: $(\Omega_M, \Omega_\Lambda)$ plane



68.3%, **95.4%**, and **99.7%** confidence regions with constraints from BAO, CMB and high-**z** Supernovae without (left) and with (right) systematic errors. Cosmological constant dark energy ($\mathbf{w} = -\mathbf{1}$) has been assumed.

Summary Figures: (Ω_M, w) plane



68.3%, **95.4**%, and **99.7**% confidence regions of the $(\mathbf{\Omega_M}, \mathbf{w})$ plane from Supernovae combined with the constraints from BAO and CMB without (left) and with (right) systematic errors. Zero curvature and constant **w** have been assumed.

Fundamental Cosmological Mysteries

1. Tiny value of vacuum energy. Expected value is $10^{45} - 10^{123}$ larger than the observed one.
   - Observed cosmological energy density is $\mathbf{10^{-47}\,GeV^4}$.
   - Contributions to vacuum energy vary from $\mathbf{m_{Pl}^4 \sim 10^{76}\,GeV^4}$ (SUGRA) to $\mathbf{0.01\,GeV^4}$, theoretically and experimentally established with QCD.

2. Vacuum-like energy today is very close to the energy of matter despite different evolution in the course of expansion.

3. Source of accelerated expansion: remnants of vacuum energy, dark energy, or modified gravity?

Possible source of the cosmic acceleration?

- Dark energy (DE), $\mathbf{P} = \mathbf{w}\varrho$ with $\mathbf{w} < -\mathbf{1}/\mathbf{3}$.
- Vacuum energy, DE with $\mathbf{w} = -\mathbf{1}$.
- Non-compensated remnant of vacuum energy.
- Modification of gravity, which is considered below.

Gravity Modification

Action in **F(R)** theories:

$$
\begin{aligned}
S &= \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}f(R) + S_m \\
&= \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}[R + F(R)] + S_m\,,
\end{aligned}
$$

where $\mathbf{m}_{Pl} = 1.22\cdot 10^{19}\,GeV$ is the Planck mass and $\mathbf{S}_m$ is the matter action.

- Usual Einstein gravity: $\mathbf{f(R) = R}$
- Modified gravity: $\mathbf{f(R) = R + F(R)}$

Pioneering Papers

$\mathbf{F}(R)$ changes gravity at large distances and is responsible for cosmological acceleration

- *Quintessence without scalar fields.*
  S. Capozziello, S. Carloni, A. Troisi,
  Recent Res. Develop. Astron. Astrophys. 1 (2003) 625; astro-ph/0303041.
- *Is Cosmic Speed-Up Due to New Gravitational Physics?*
  S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner,
  Phys. Rev. D70 (2004) 043528, astro-ph/0306438.

Singular action:

$$\mathbf{F}(\mathbf{R}) = -\mu^4/\mathbf{R},$$

where $\mu^2 \sim \mathbf{R_c} \sim \mathbf{1/t_u^2}$ is a small parameter with dimension of mass squared.

$F(R) = -\mu^4/R$

Can modified gravity explain accelerated cosmic expansion?
A.D. Dolgov, M. Kawasaki, Phys.Lett. B573 (2003) 1.

- These modified gravity theories agree with the Newtonian limit of the standard gravity for sufficiently small μ.
- For the time dependent matter density such a choice of $\mathbf{F}(R)$ leads to exponential instability of small fluctuations or of regular time evolution.
- The characteristic time of instability:

$$\tau = \frac{\sqrt{6}\mu^2}{T^{3/2}} \sim 10^{-26}\mathrm{sec}\left(\frac{\varrho_m}{\mathrm{g/cm^3}}\right)^{-3/2},$$

where ϱ_m is the mass density of the body and $\mathbf{T}$ is the trace of the energy-momentum tensor of matter.

Modified modified gravity: free from exponential instability

W.Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

$$\mathbf{F_{HS}(R) = -\frac{R_{vac}}{2}\frac{c\left(\frac{R}{R_{vac}}\right)^{2n}}{1+c\left(\frac{R}{R_{vac}}\right)^{2n}}},$$

A.Appleby, R. Battye, Phys. Lett. B **654**, 7 (2007).

$$\mathbf{F_{AB}(R) = \frac{\epsilon}{2}\log\left[\frac{\cosh\left(\frac{R}{\epsilon}-b\right)}{\cosh b}\right] - \frac{R}{2}},$$

A.A. Starobinsky, JETP Lett. **86**, 157 (2007).

$$\mathbf{F_S(R) = \lambda R_0\left[\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right]}.$$

Necessary conditions for $f(R) = R + F(R)$

Condition of accelerated expansion in absence of matter: the existense of real positive root, $R_1 > 0$, of the equation

$$Rf'(R) - 2f(R) = 0$$

Conditions to avoid pathologies:

- Future stability of cosmological solutions:

$$f'(R_1)/f''(R_1) > R_1\,.$$

- Classical and quantum stability (gravitational attraction and absence of ghosts): $f'(R) > 0$.
- Absence of matter (DK) instability: $f''(R) > 0$.
- Existence of the stable Newtonian limit:

$$|f(R) - R| \ll R\,, |f'(R) - 1| \ll 1\,, Rf''(R) \ll 1\,.$$

Another troublesome feature

Past singularity:

- In cosmological background with decreasing energy density the system must evolve from a singular state with an infinite **R**.
- In other words, if we travel backward in time from a normal cosmological state, we come to singularity.

Future Singularity

The system with rising energy density will evolve to singularity, $\mathbf{R} \to \infty$, in finite (short) time.

- *Explosive phenomena in modified gravity.*
  E.V. Arbuzova, A.D. Dolgov, Phys.Lett.B700 (2011) 289.

HSS version in the limit $\mathbf{R} \gg \mathbf{R_0}$:

$$\mathbf{F(R)} \approx -\lambda \mathbf{R_0} \left[1 - \left(\frac{\mathbf{R_0}}{\mathbf{R}} \right)^{2n} \right] .$$

We analyze the evolution of $\mathbf{R}$ in massive object with time varying density, $\varrho_{\mathrm{matt}} \gg \varrho_{\mathrm{cosm}}$.

- $\varrho_{\mathrm{matt}} \sim 10^{-24}\mathrm{g/cm^3}$ is matter density of a dust cloud in a galaxy.
- $\varrho_{\mathrm{cosm}} \approx 10^{-29}\,\mathrm{g/cm^3}$ is the cosmological energy density at the present time

Unharmonic Oscillator

The equation of motion is very much simplified for $\mathbf{w \equiv F' = -2n\lambda\left(R_0/R\right)^{2n+1}}$:

$$(\partial_t^2 - \Delta)w + U'(w) = 0\,.$$

Potential $\mathbf{U(w)}$ is equal to:

$$U(w) = \frac{1}{3}\left(T - 2\lambda R_0\right)w +$$
$$\frac{R_0}{3}\left[\frac{q^\nu}{2n\nu}w^{2n\nu} + \left(q^\nu + \frac{2\lambda}{q^{2n\nu}}\right)\frac{w^{1+2n\nu}}{1+2n\nu}\right],$$

where $\nu = 1/(2n+1)$, $q = 2n\lambda$.
NB: Infinite $\mathbf{R}$ corresponds to $\mathbf{w = 0}$.

Further Simplification

Potential $\mathbf{U}$ would depend upon time, if the mass density of the object changes with time:

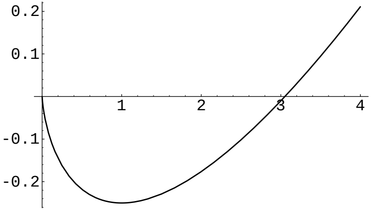
$$\mathbf{T} = \mathbf{T}(\mathbf{t}) = \mathbf{T_0}(1 + \kappa\tau)\,.$$

If only the dominant terms are retained and if the space derivatives are neglected, equation simplifies to:

$$\mathbf{z}'' - \mathbf{z}^{-\nu} + (1 + \kappa\tau) = 0$$

with dimensionless quantities $\mathbf{t} = \gamma\tau,\ \ \mathbf{w} = \beta\mathbf{z}\,.$
Here prime means differentiation with respect to τ

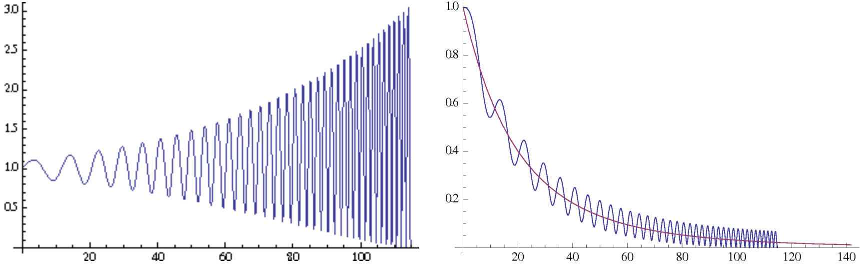
Potential
$U(z) = z(1+\kappa\tau) - z^{1-\nu}/(1-\nu),\ \nu = \frac{1}{5}, \tau = 0.$



- Minimum of the potential sits at $z_{min} = (1+\kappa\tau)^{-1/\nu}$.
- When the mass density rises, the minimum moves towards zero and becomes less deep.
- For small initial values $z(0)$ and $z'(0)$ $z(\tau)$ oscillates near the minimum of the potential.
- If at the process of "lifting" of the potential $z(\tau)$ happens to be at $U > 0$ it would overjump potential at $z = 0$ where it equals zero.

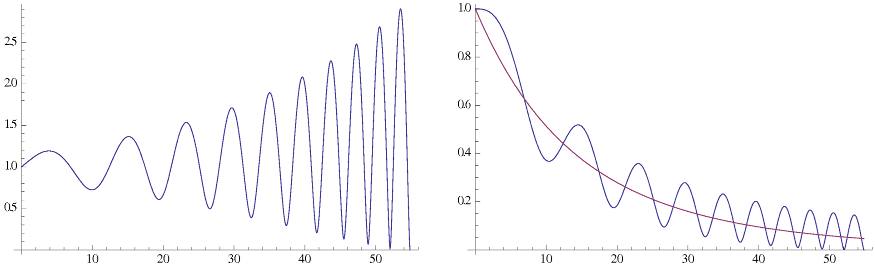
Numerical Solution: $\mathbf{n = 2}$, $\boldsymbol{\kappa} = \mathbf{0.01}$, $\boldsymbol{\varrho_m / \varrho_c} = \mathbf{10^5}$

So $\mathbf{z}(\tau)$ would reach zero, which corresponds to infinite **R**,and the singularity could appear in finite time.



Ratio $\mathbf{z}(\tau)/\mathbf{z}_{\mathbf{min}}(\tau)$ (left) and functions $\mathbf{z}(\tau)$ and $\mathbf{z}_{\mathbf{min}}(\tau)$ (right).
The initial conditions: $\mathbf{z(0) = 1}$ and $\mathbf{z'(0) = 0}$.

Numerical Solution: $\mathbf{n = 3}$, $\boldsymbol{\kappa = 0.01}$, $\boldsymbol{\varrho_m/\varrho_c = 10^5}$



Ratio $\mathbf{z(\tau)/z_{min}(\tau)}$ (left) and functions $\mathbf{z(\tau)}$ and $\mathbf{z_{min}(\tau)}$ (right).
The initial conditions: $\mathbf{z(0) = 1}$ and $\mathbf{z'(0) = 0}$.

R^2 term

The simplest way to avoid singularity is to introduce $\mathbf{R^2}$-term into the gravitational action:

$$\delta \mathbf{F}(\mathbf{R}) = -\mathbf{R}^2/6\mathbf{m}^2 \,,$$

where $\mathbf{m}$ is a constant parameter with dimension of mass.

- The natural value is $\mathbf{m} \sim \mathbf{m}_{\mathbf{Pl}}$.
- BBN demands $\mathbf{m} \geq \mathbf{10^5}$ GeV.

In the homogeneous case and in the limit of large ratio $\mathbf{R}/\mathbf{R}_0$ equation of motion for $\mathbf{R}$ is modified as:

$$\left[1 - \frac{\mathbf{R}^{2n+2}}{6\lambda n(2n+1)\mathbf{R}_0^{2n+1}\mathbf{m}^2}\right]\ddot{\mathbf{R}} - (2n+2)\frac{\dot{\mathbf{R}}^2}{\mathbf{R}} - \frac{\mathbf{R}^{2n+2}(\mathbf{R}+\mathbf{T})}{6\lambda n(2n+1)\mathbf{R}_0^{2n+1}} = 0 \,.$$

Dimensionless Equation

With dimensionless curvature and time

$$\mathbf{y} = -\frac{\mathbf{R}}{\mathbf{T_0}}, \quad \tau_1 = \mathbf{t}\left[-\frac{\mathbf{T_0^{2n+2}}}{\mathbf{6\lambda n(2n+1)R_0^{2n+1}}}\right]^{1/2}$$

equation for **R** is transformed into:

$$\left(\mathbf{1 + gy^{2n+2}}\right)\mathbf{y''} \\ -\mathbf{2(n+1)}\frac{\mathbf{(y')^2}}{\mathbf{y}} + \mathbf{y^{2n+2}}\left[\mathbf{y} - (\mathbf{1} + \kappa_1\tau_1)\right] = \mathbf{0}\,,$$

where prime means derivative with respect to τ_1.

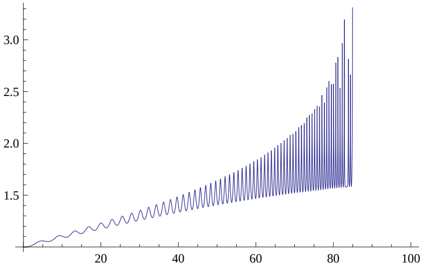
New Parameter

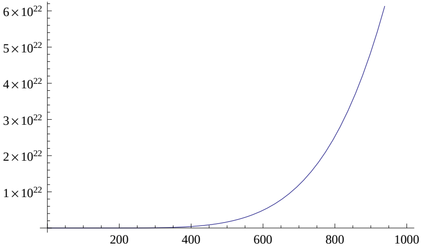
The new parameter which can stop approach to infinity:

$$\mathbf{g} = -\frac{\mathbf{T_0^{2n+2}}}{\mathbf{6\lambda n(2n+1)m^2R_0^{2n+1}}} > \mathbf{0}\,.$$

- For very large **m**, or small **g**, the numerical solution demonstrates that **R** would reach infinity in finite time in accordance with the results presented above.
- Nonzero **g** would terminate the unbounded rise of **R**.
- To avoid too large deviation **R** from the usual gravity coefficient **g** should be larger than or of the order of unity: $\mathbf{g} \geq \mathbf{1}$.

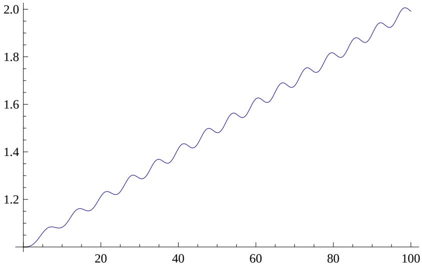
Numerical Solutions

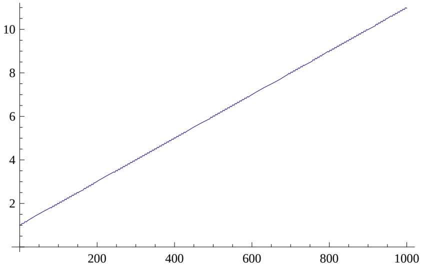




$$\mathbf{g = 0, n = 3, \kappa_1 = 0.01};$$
$$\mathbf{y(\tau_{in}) = 1 + \kappa_1 \tau_{in}, \quad y'(\tau_{in}) = 0}.$$

Numerical Solutions





$$\mathbf{g = 1, n = 3, \kappa_1 = 0.01};$$
$$\mathbf{y(\tau_{in}) = 1 + \kappa_1 \tau_{in}, \quad y'(\tau_{in}) = 0}.$$

Oscillation Frequency

The oscillation frequency in dimensionless time τ or τ_1 is about unity.

In physical time the frequency is:

$$\omega \sim \frac{g^{-1/2}}{t_U} \left(\frac{T_0}{R_0}\right)^{n+1} \leq m\,.$$

- For $n = 5$ and for a galactic gas cloud with $T_0/R_0 = 10^5$:

$$\omega \sim 10^{12}\ \mathrm{Hz} \approx 10^{-3}\ \mathrm{eV}.$$

- For higher density objects, e.g. with $\varrho = 1\ \mathrm{g/cm}^3$:

$$\omega \sim m.$$

Cosmological Evolution in $\mathbf{R}^2$ Gravity

- *Cosmological evolution and particle production in* $\mathbf{R}^2$ *cosmology.*
  E.V. Arbuzova, A.D. Dolgov, L.Reverberi
  e-Print: arXiv:1112.4995.

The epoch when $\mathbf{R} \gg \mathbf{R}_0$ is considered, so large scale modifications are neglected.
We study here the cosmological evolution of the early and not so early universe in the model with the action:

$$\mathbf{S} = -\frac{\mathbf{m}_{\mathbf{Pl}}^2}{16\pi} \int \mathbf{d}^4\mathbf{x}\sqrt{-\mathbf{g}}\left(\mathbf{R} - \frac{\mathbf{R}^2}{6\mathbf{m}^2}\right) + \mathbf{S}_{\mathbf{m}}\,,$$

Dimensionless Equations

Dimensionless quantities:

- $\tau = H_0\, t$, $h = H/H_0$, $r = R/H_0^2$,
- $y = 8\pi\varrho/(3m_{Pl}^2 H_0^2)$, $\omega = m/H_0$, where $H_0 = H(t_0)$

Two equivalent systems of equations for relativistic matter:

$$\begin{cases} h'' + 3hh' - \dfrac{h'^2}{2h} + \dfrac{\omega^2}{2}\dfrac{h^2 - y}{h} = 0\,, \\ y' + 4hy = 0\,, \end{cases} \tag{1}$$

and

$$\begin{cases} r'' + 3hr' + \omega^2 r = 0\,, \\ r + 6h' + 12h^2 = 0\,. \end{cases} \tag{2}$$

Approximate Analytical Solutions

Small deviations from GR $\mathbf{h} = 1/(2\tau) + \mathbf{h}_1$, $\mathbf{y} = 1/(4\tau^2) + \mathbf{y}_1$

- The complete asymptotic solution has the form:

$$\mathbf{h}(\tau) \simeq \frac{1}{2\tau} + \frac{c_1 \sin(\omega\tau + \varphi)}{\tau^{3/4}}.$$

- the Hubble parameter oscillates around GR value, $\mathbf{h}_0 \sim 1/(2\tau)$ with rising amplitude, $\mathbf{h}_1/\mathbf{h}_0 \sim \tau^{1/4}$,
- For sufficiently large τ the second term would start to dominate and the linear approximation breaks.

Full non-linear system can be solved in large frequency limit, $\omega\tau \gg 1$ using truncated Fourier expansion.

- It is found $\mathbf{h}_1/\mathbf{h} \to$ const and oscillation center is shifted above $1/2$.

Numerical Integration

Equations are numerically integrated from $\tau_0 = 1/2$ with initial conditions

$$\mathbf{h_0 = 1 + \delta h_0, h'_0 = -2 + \delta h'_0,\ y_0 = 1 + \delta y_0},$$

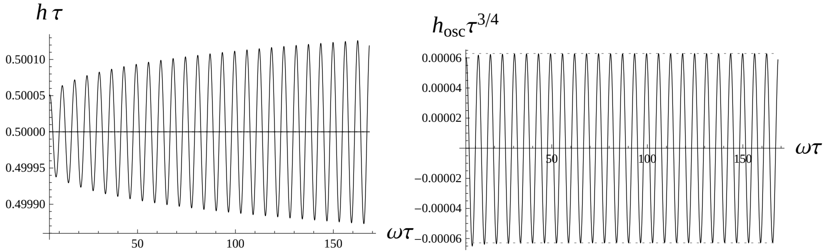
where initial deviations from GR do not vanish simultaneously.

For small deviations from GR the solutions are quite accurately fitted by

$$\mathbf{h}(\tau) \simeq \frac{\alpha + \beta\, \tau^{1/4} \sin(\omega\tau + \varphi)}{2\tau}$$

Numerical solutions well agree with analitical results.

Numerical Solutions: small deviations from GR

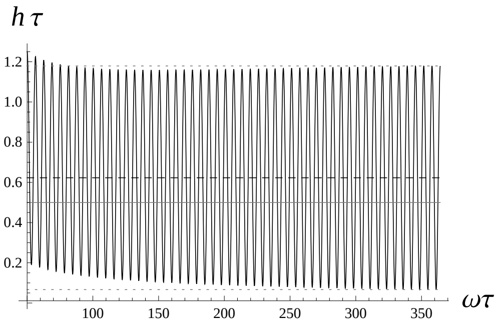


Numerical solution with initial conditions

$$\delta \mathbf{h_0} = \mathbf{10^{-4}},\ \delta \mathbf{h'_0} = \mathbf{0},\ \mathbf{y_0} = \mathbf{1},\ \omega = \mathbf{10}.$$

The best fit is given by is $\alpha \simeq \mathbf{1}$, $\beta \simeq \mathbf{6.29} \times \mathbf{10^{-5}}$.

Numerical Solutions: non-linear regime, high frequency



Numerical solution for the dimensionless Hubble parameter $\mathbf{h}$ with initial conditions

$$\delta \mathbf{h_0} = \mathbf{1.5},\ \delta \mathbf{h}'_0 = \mathbf{0},\ \mathbf{y_0} = \mathbf{0},\ \omega = \mathbf{100}.$$

Central value **0.6** is shifted from GR value **0.5**

Particle Production in R^2 Gravity

Massless scalar field:

$$\mathbf{S}_\phi = \frac{1}{2}\int \mathbf{d^4x}\,\sqrt{-\mathbf{g}}\,\mathbf{g}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\,.$$

For rescaled field, $\chi \equiv \mathbf{a(t)}\phi$, and conformal time $\mathbf{a\,d}\eta = \mathbf{dt}$ the equations of motion read:

$$\mathbf{R}'' + 2\frac{\mathbf{a}'}{\mathbf{a}}\mathbf{R}' + \mathbf{m^2a^2R} =$$

$$\frac{8\pi \mathbf{m}^2}{\mathbf{a}^2\mathbf{m}_{\mathbf{Pl}}^2}\left[(\chi')^2 - (\nabla\chi)^2 + \frac{\mathbf{a}'^2}{\mathbf{a}^2}\chi^2 - \frac{2\mathbf{a}'}{\mathbf{a}}\chi\chi'\right],$$

$$\chi'' - \Delta\chi + (1/6)\,\mathbf{a^2R}\,\chi = 0\,.$$

Particle Production in R^2 Gravity

Taking average values of χ^2 over **R**-background we find the dominant contribution of particle production:

$$\ddot{R} + 3H\dot{R} + m^2 R \simeq -\frac{1}{12\pi}\frac{m^2}{m_{Pl}^2}\int_{t_0}^{t} dt' \, \frac{\ddot{R}(t')}{t-t'} \, .$$

The decay rate is:

$$\Gamma_R = -\frac{\pi g m}{4} = \frac{m^3}{48 m_{Pl}^2} \, .$$

The characteristic decay time of the oscillating curvature is

$$\tau_R = \frac{1}{2\Gamma_R} = \frac{24 m_{Pl}^2}{m^3} \simeq 2\left(\frac{10^5 \text{ GeV}}{m}\right)^3 \text{ seconds} \, .$$

CONCLUSIONS

- $\mathbf{1/R}$-theories are unstable, excluded.
- Stable and ghost free modifications are both past and future singular.
- $\mathbf{R^2/m^2}$ eliminates singularity.
- In $\mathbf{R^2}$-cosmology $\mathbf{R}$ and $\mathbf{H}$ oscillate with frequency $\mathbf{m}$ and with initially rising amplitude.
- Cosmological evolution differs from GR even if $\mathbf{m} \to \infty$, e.g. $\langle \mathbf{H} \rangle \neq \mathbf{1/2t}$.
- Particle production by oscillating $\mathbf{R}$ damps oscillations, returning to GR.

CONCLUSIONS

- Oscillating **R** might be source of non-thermal DM, e.g. very heavy LSP.
- In contemporary astronomical objects oscillation frequency could vary from **m** down to very low frequency. The oscillations may produce radiation from high energy cosmic rays down to radio waves.

Gravity modification could be useful.

THE END

THANK YOU FOR THE ATTENTION!