Dirac and Majorana Leptonic CP Violation

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There have been remarkable discoveries in neutrino physics in the last ~ 16 years.

Compellings Evidence for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** μ**–like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

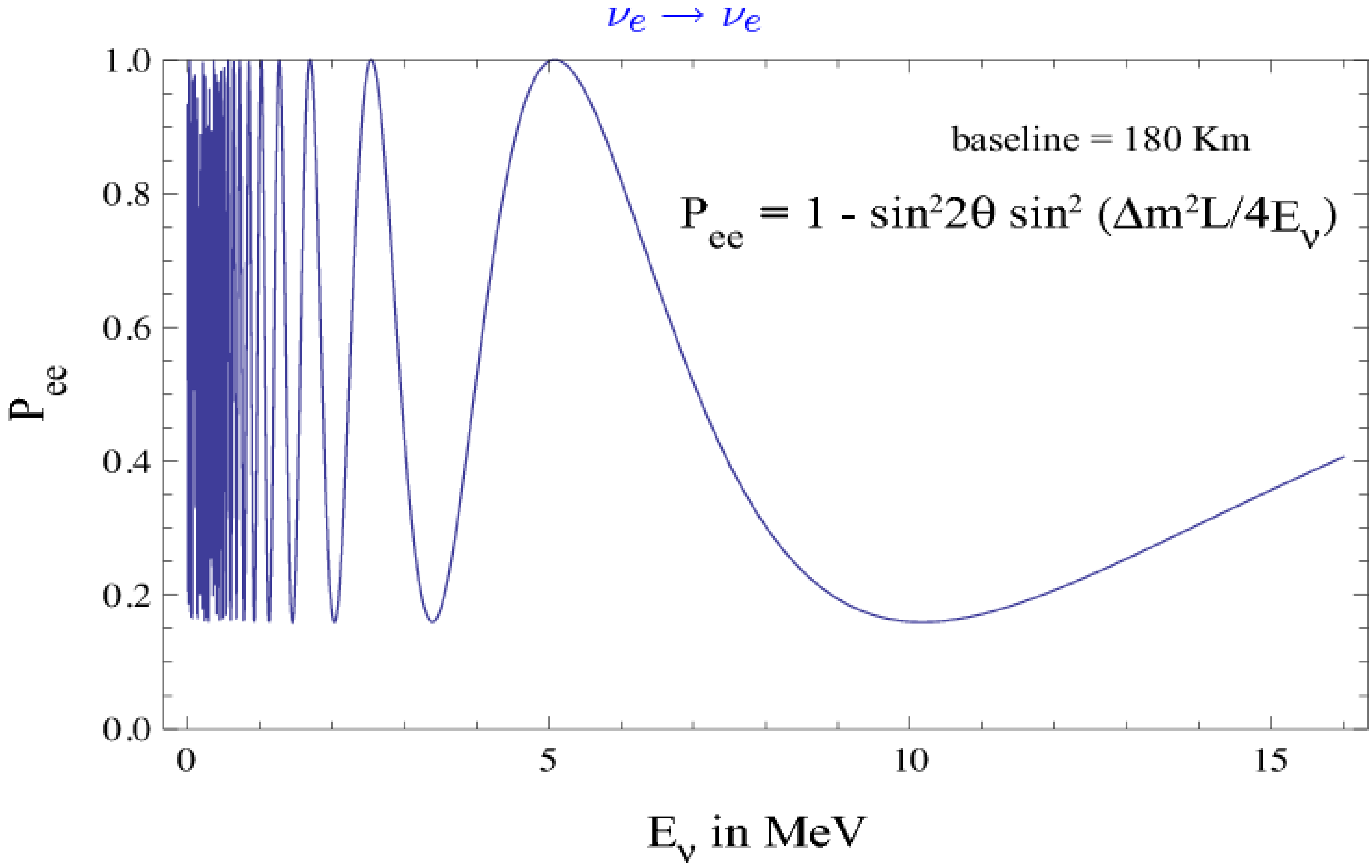
Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO**

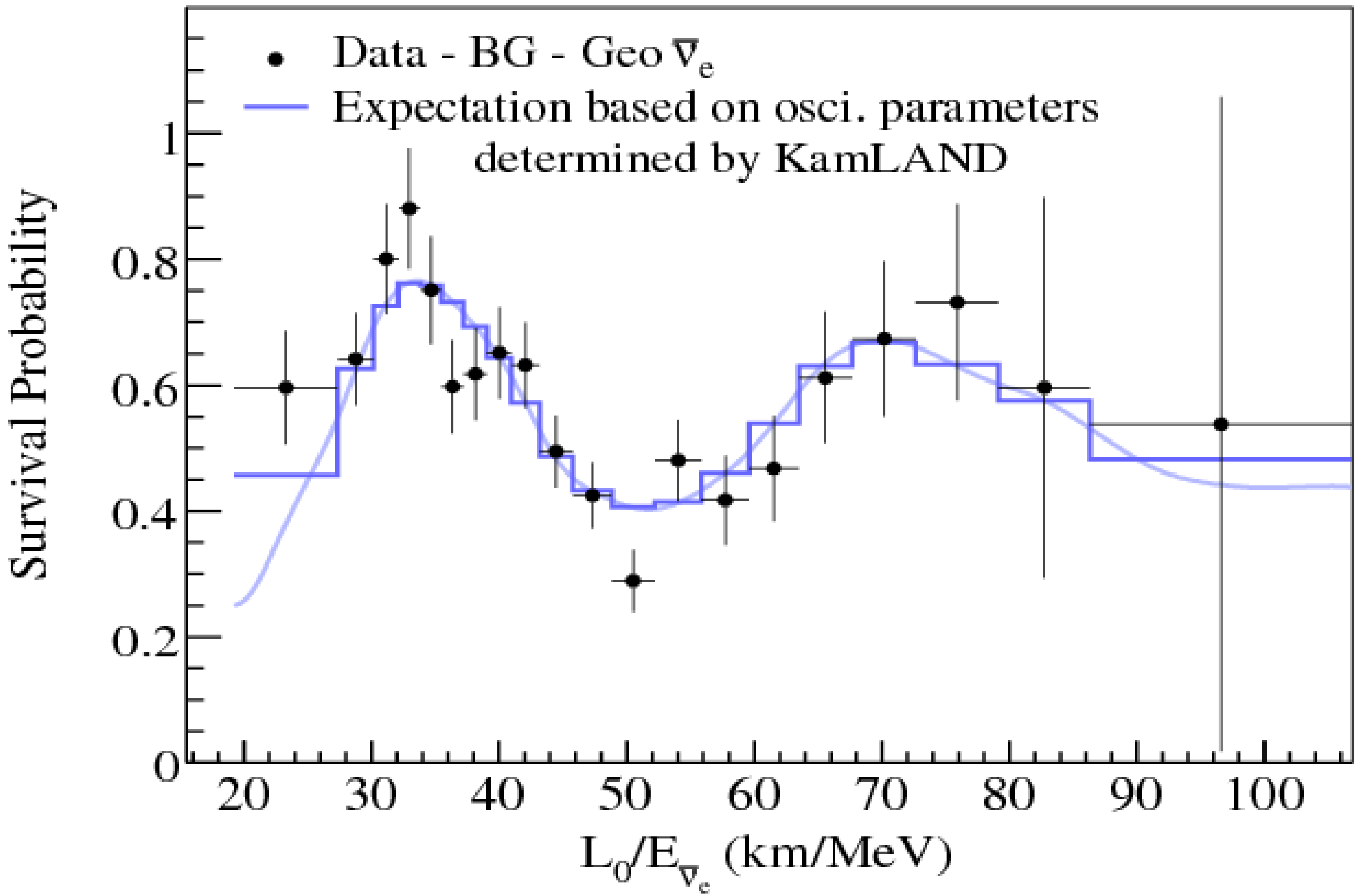
$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \longrightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_μ **from accelerators):** $\nu_\mu \longrightarrow \nu_e$



$\nu_e \longrightarrow \nu_e$
baseline = 180 Km
$P_{ee} = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 L/4E_\nu)$
P_{ee}
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
E_ν in MeV



KamLAND: L/E−Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, E = (1.8 - 10) MeV)

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj}\, |\nu_j>\,, \quad \nu_j:\; m_j \neq 0\,; \quad l = e, \mu, \tau\,; \quad n \geq 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\; m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \ldots$) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\ldots} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"));

ii) $M_{4,5,\ldots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
$M_{4,5,\ldots} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

We can also have, in principle:

$m_4 \sim 1$ eV ($\nu_{e(\mu)} \rightarrow \nu_S$), $m_5 \sim 5$ keV (DM), $M_6 \sim (10 - 10^3)$ GeV (seesaw).

• Data (relativistic ν's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard Theory: ν_l, $\tilde{\nu}_l$ - $\nu_{lL}(x)$;

$\nu_{lL}(x)$ form doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix} \quad l = e, \mu, \tau \,.$$

• No (compelling) evidence for existence of (relativistic) ν's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$.)

If ν_R, $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l, $\tilde{\nu}_l$: ν_R, $\tilde{\nu}_L$ - "sterile", "inert".

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν's can play crucial role
i) in the generation of $m(\nu) \neq 0$,
ii) in understanding why $m(\nu) \ll m_l, m_q$,
iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = const.$, $l = e, \mu, \tau$;
$L \equiv L_e + L_\mu + L_\tau = const.$

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Data: the 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}}\ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \ \chi_k(x) \to e^{i\alpha}\chi_k(x) - \ \text{impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ \ Q_{\text{el}} = 0,\ \ L_l = 0,\ \ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47$ $(2.42) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0240), NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

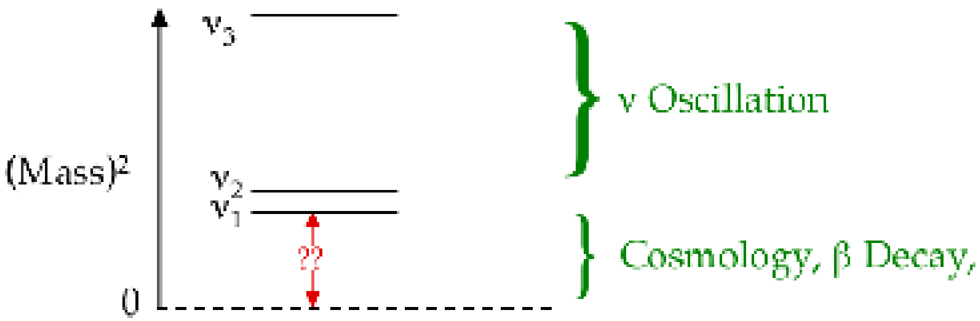
- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241$ (0.0244), NO (IO).

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47$ (2.42) $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0240), NO (IO).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 2.6\%$, $1\sigma(\sin^2\theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2\theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m^2_{21}):\ (6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12}):\ (0.259 - 0.359)$;
- $3\sigma(|\Delta m^2_{31(23)}|):\ 2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23}):\ 0.374(0.380) - 0.628(0.641)$;
- $3\sigma(\sin^2\theta_{13}):\ 0.0176(0.0178) - 0.0295(0.0298)$.

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero
is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

Due to B. Kayser

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = -\,\Delta m^2_{32}(IO)$$

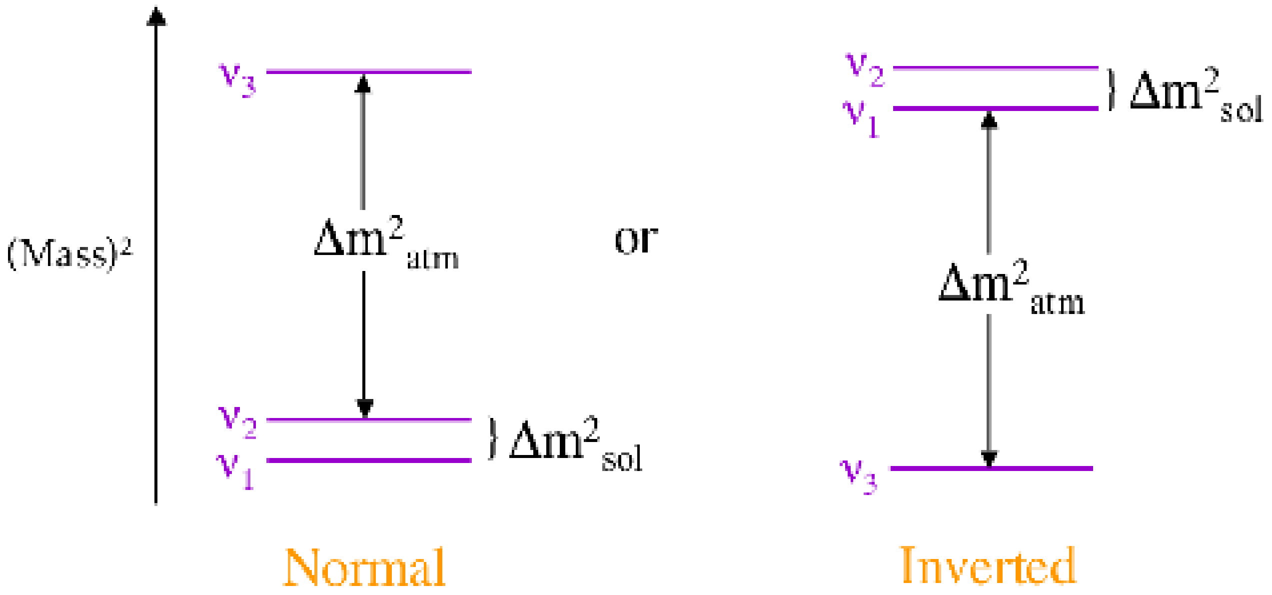
$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;

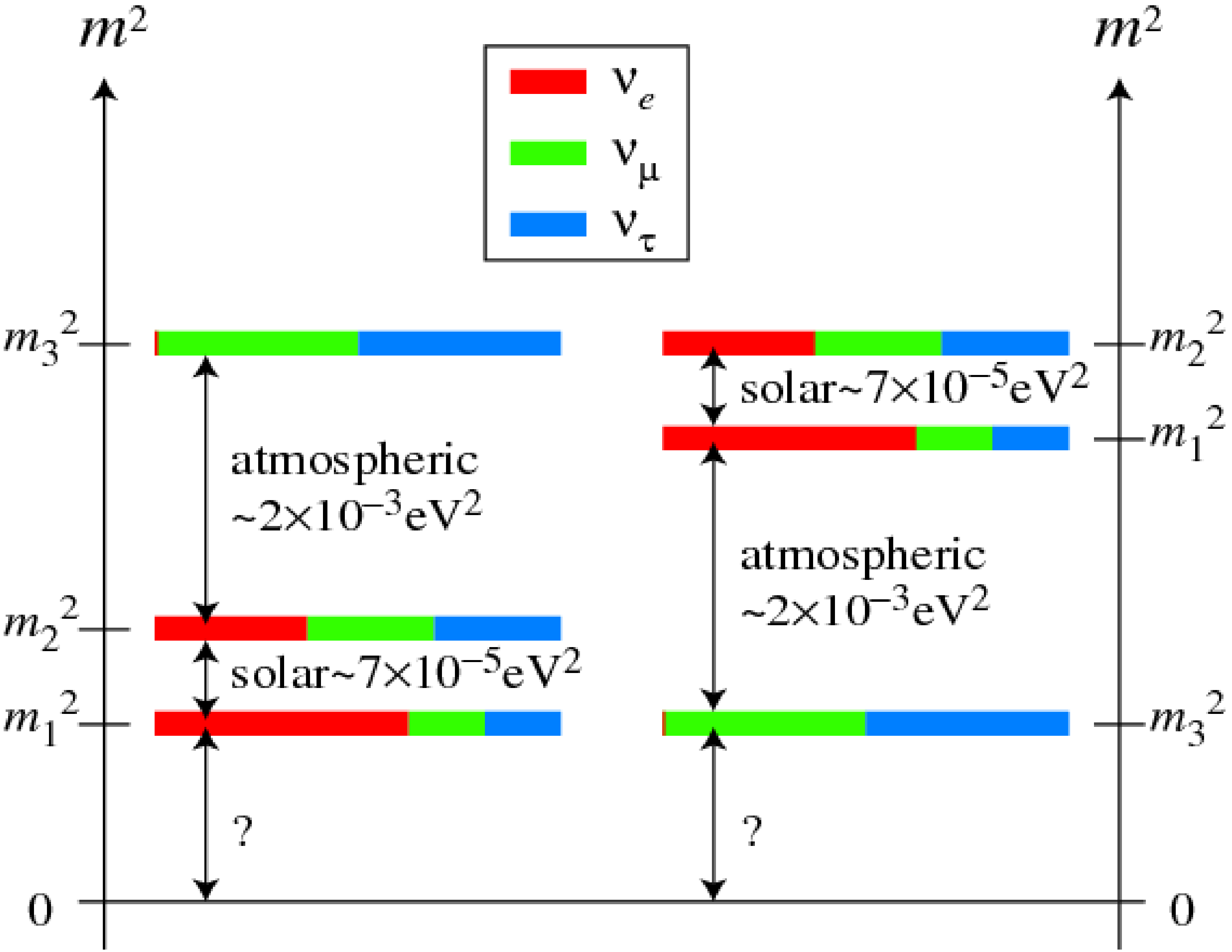
The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ \mathrm{eV}^2$, $\quad \Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ \mathrm{eV}^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

Due to B. Kayser



S.F. King and C. Luhn, 2013

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = {\rm Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.040$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.030$.

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left| < m > \right|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, θ_{12}, $\Delta m^2_{\rm atm}$, θ_{23}, θ_{13}

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

  – the origin of the observed patterns of ν-mixing and ν-masses ;

  – the physical origin of CPV phases in U_{PMNS} ;

  – Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

  – Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

  – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

  – Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

- Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

  – Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

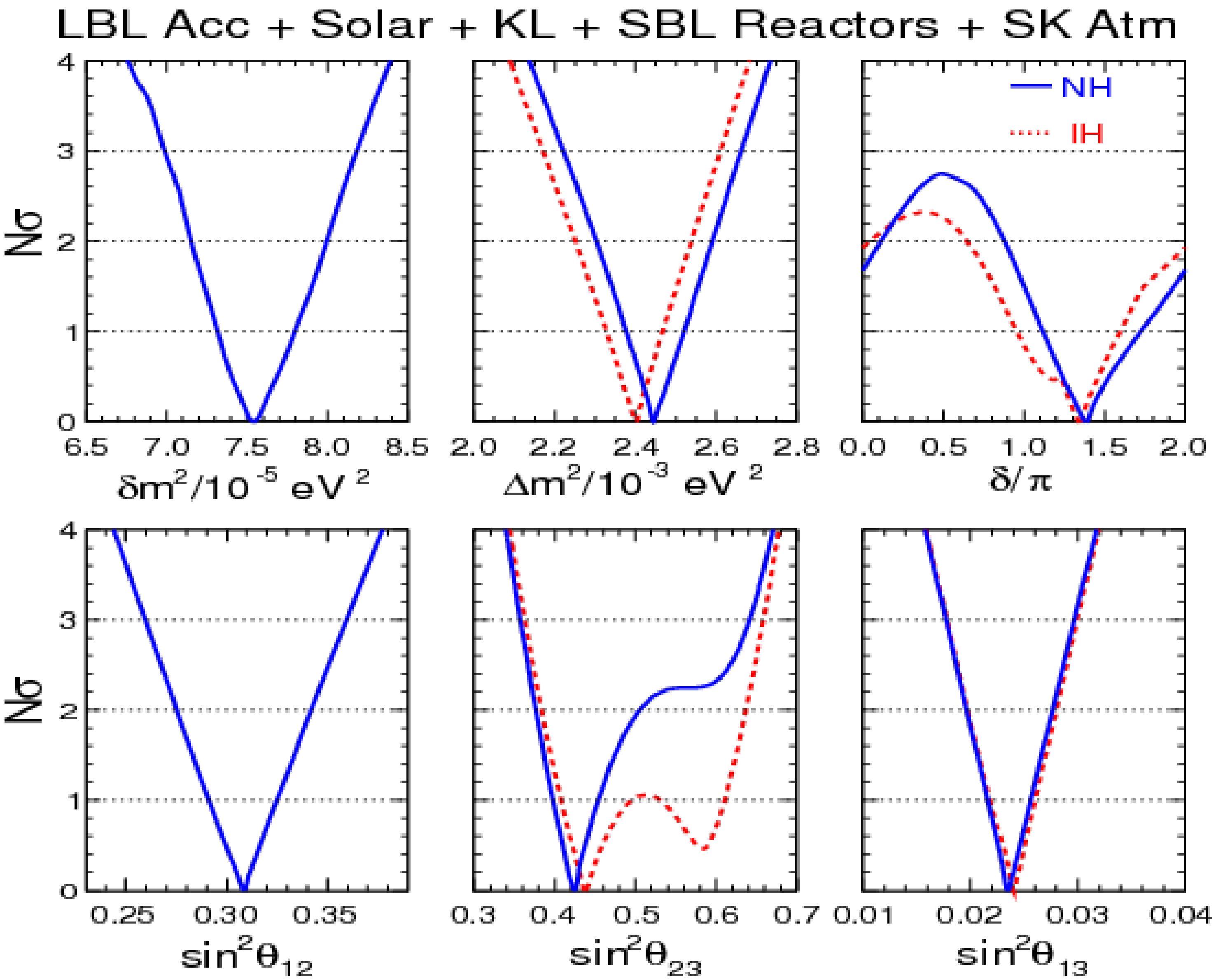
- determination of the nature - Dirac or Majorana, of massive neutrinos.

- determination of the neutrino mass hierarchy;

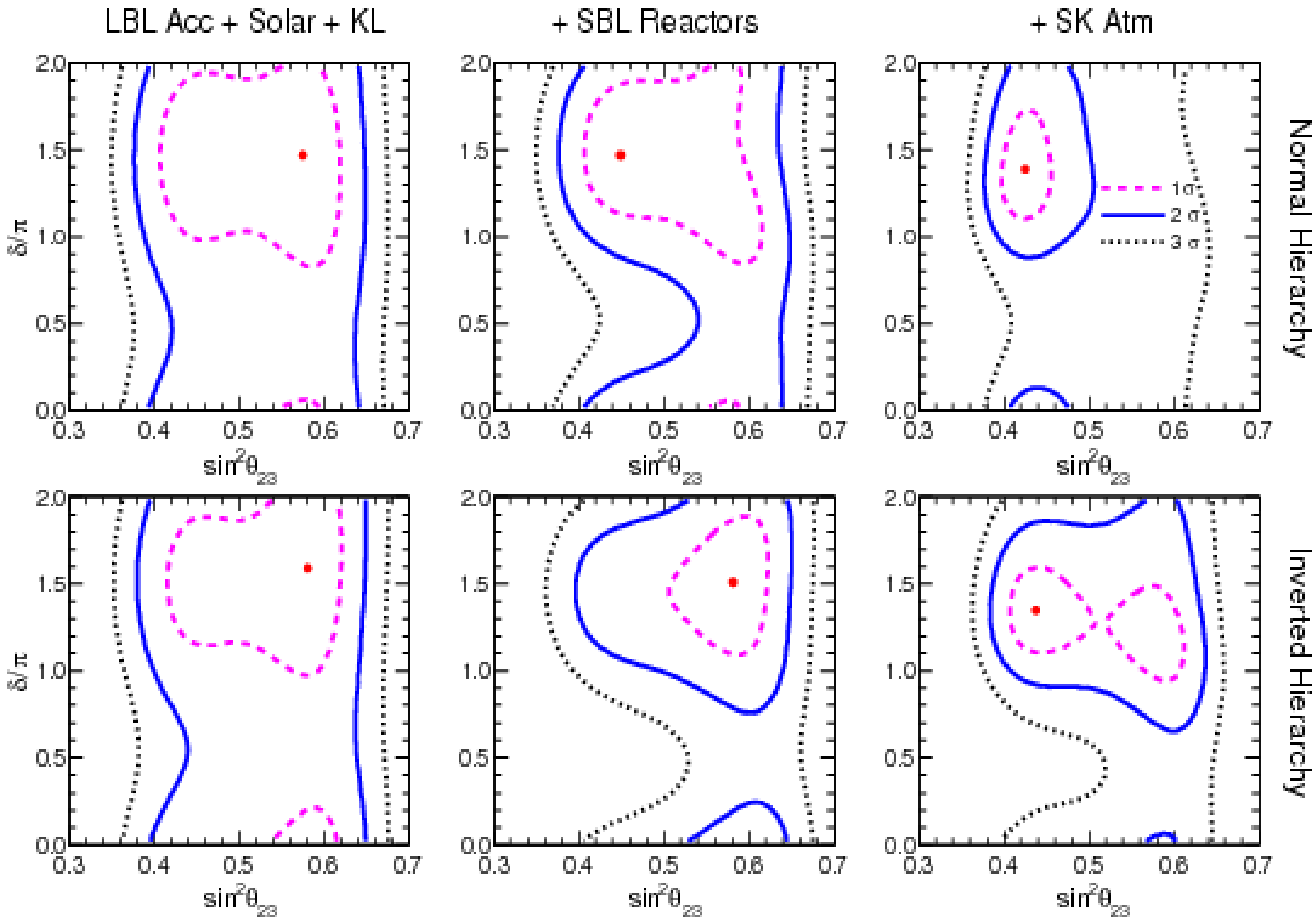
- determination of the absolute neutrino mass scale (or min(m_j));

- determination of the status of the CP symmetry in the lepton sector.

Hints for Dirac CP Violation: $\delta \cong 3\pi/2$



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

Large $\sin\theta_{13} \cong 0.15$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.030$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13}\,\sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ ,\quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ ,\qquad l,l' = e,\mu,\tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger, S. Pakvasa et al.,1980.

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

CPT-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

$$l = l':\quad P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

$3\nu -$mixing:

$$A_{\rm CP}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

$$A_{\rm T}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l),\ l \neq l'$$

$$A_{\rm T(CP)}^{(e,\mu)} = A_{\rm T(CP)}^{(\mu,\tau)} = -A_{\rm T(CP)}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa *et al.*, 1980

In vacuum: $A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin\left(\frac{\Delta m^2_{21}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E} L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\mathrm{CP}: \quad P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})$$

$$\mathrm{CPT}: \quad P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

In matter with constant density: $\mathrm{A}^{(e,\mu)}_{\mathrm{T}} = \mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} \mathrm{F}^{\mathrm{mat}}_{\mathrm{osc}}$

$$\mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} = \mathrm{J}^{\mathrm{vac}}_{\mathrm{CP}}\ \mathrm{R}_{\mathrm{CP}}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\}\ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\{U_{e1}U^*_{e3}\}\,,\quad S_2 = \text{Im}\{U_{e2}U^*_{e3}\}\quad \text{(not unique);}\quad \text{or}$$
$$S'_1 = \text{Im}\{U_{\tau 1}U^*_{\tau 2}\}\,,\quad S'_2 = \text{Im}\{U_{\tau 2}U^*_{\tau 3}\}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im}\{U_{e1}U^*_{e3}\} \neq 0$ and $\text{Re}\{U_{e1}U^*_{e3}\} \neq 0$.

S_1, S_2 appear in $|\!<m>\!|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.

- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.

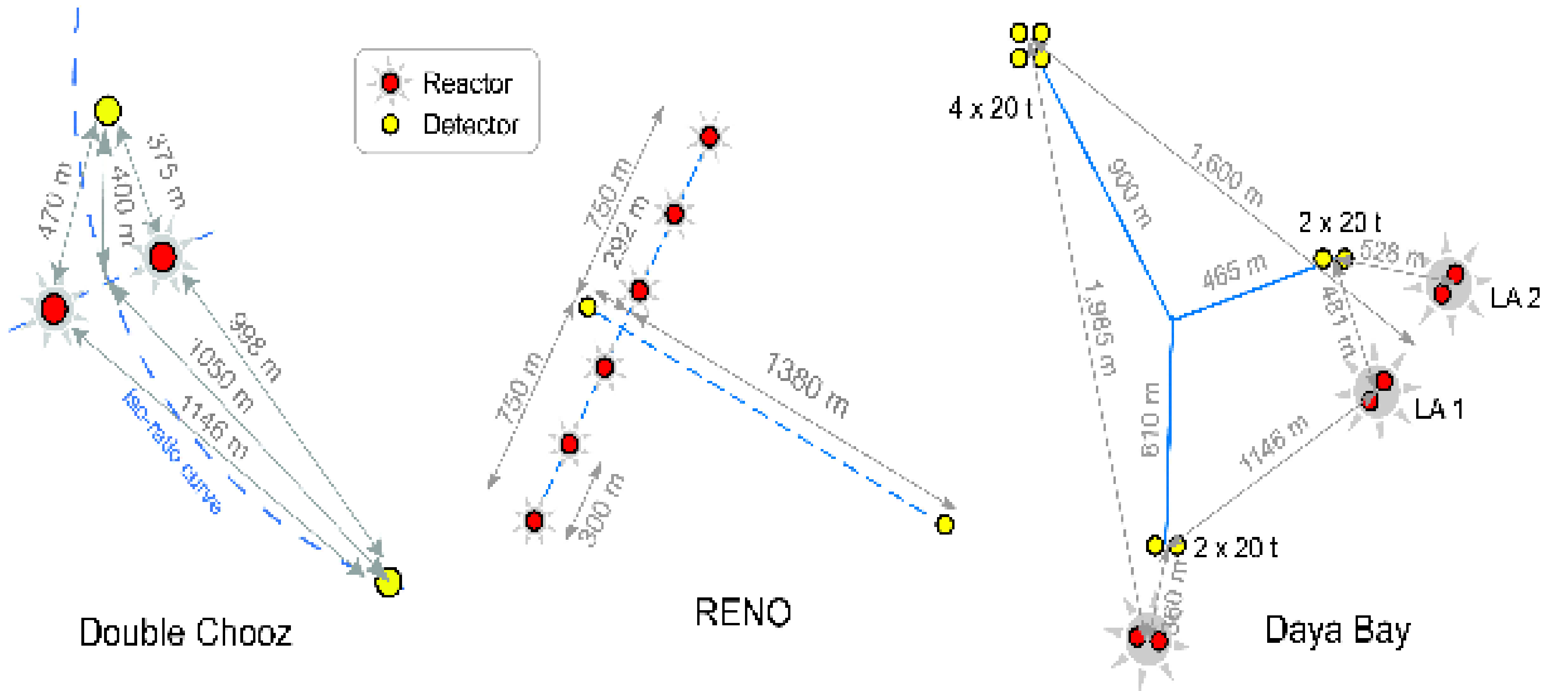
- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ and 2.9σ evidence for $\theta_{13} \neq 0$.

- Daya Bay, 23/08/2013: $\sin^2 2\theta_{13} = 0.090 \pm 0.009$.

- RENO, 12/09/2013 (TAUP 2013): $\sin^2 2\theta_{13} = 0.100 \pm 0.010\ (stat.) \pm 0.012$.

$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m^2_{31(32)}; \theta_{12}, \Delta m^2_{21}) \cong$

$1 - \sin^2 2\theta_{13} \sin^2(\frac{\Delta m^2_{31(32)}}{4E} L)$, no dependence on θ_{23}, δ.



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ;
July, 2013 (28 events).

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (1.7), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the value obtained in the Daya Bay and RENO experiments.

$$P^{3\nu}_m(\nu_\mu \rightarrow \nu_e) = P^{3\nu}_m(\theta_{13}, \Delta m^2_{31(32)}, \theta_{12}, \Delta m^2_{21}, \theta_{23}, \delta).$$

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P^{3\nu\ man}_m(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E},\;\; A = \sqrt{2}G_{\mathrm{F}}N^{man}_e\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: $\delta,\; A \to (-\delta),\; (-A)$

Predictions for the CPV Phase δ

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20, \quad \theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 - 0.10.$
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \psi)\ Q(\phi, \varphi) U_{\text{TBM,BM,LC}}\ P(\alpha_{21}, \alpha_{31}),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \psi)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM,BM,LC}}\ P(\alpha_{21}, \alpha_{31})$ - from diagonalization of the ν mass matrix;
- $Q(\phi, \varphi)$, - from diagonalization of both the l^- and ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\text{TBM(BM)}}$: Groups A_4, S_4, T', ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- $U_{\text{LC(BM)}}$: alternatively $U(1)$, $\quad L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

- U_{TBM}: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected

- U_{BM}: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

- U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^{ν}_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

None of the symmetries leading to U_{TBM}, U_{BM} or other approximate forms of U_{PMNS} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In the two cases of U_ν given by U_{TBM}, or U_{BM}, the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\mathrm{TBM,BM}}$ and on the form of U_{lep}, one obtaines diifferent experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and Δm^2_{ij}.

Predictions for δ

Assume:

- $U_{PMNS} = U_{\rm lep}^{\dagger}(\theta^{\ell}_{ij}, \psi)\ Q(\phi, \varphi) U_{\rm TBM,BM}\ P(\alpha_{21}, \alpha_{31})$,
- $U_{\rm lep}^{\dagger}$ - minimal, such that

i) $\sin\theta_{13} \cong 0.16$; BM: $\sin^2\theta_{12} \cong 0.31$;
ii) $\sin^2\theta_{23}$ can deviate significantly (by more than $\sin^2\theta_{13}$) from 0.5 (b.f.v. = 0.42-0.43) .

From i), ii) + $m_e << m_\mu << m_\tau$:

$$U_{\rm lep}^{\dagger}(\theta^{\ell}_{ij}, \psi) = R_{12}(\theta^{\ell}_{12})\, R_{23}(\theta^{\ell}_{23})\ ,\ \ Q(\phi, \varphi) = {\rm diag}(1, e^{i\phi}, 1)$$

Leads to $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$ - new sum rules for δ!

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

For U_{TBM}:

$$\cos\delta = \frac{\tan\theta_{23}}{3\sin 2\theta_{12}\sin\theta_{13}}\left[1 + \left(3\sin^2\theta_{12} - 2\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]$$

For U_{TBM} + b.f.v. of $\theta_{12}, \theta_{23}, \theta_{13}$:

$$\delta \cong 3\pi/2 \text{ or } \pi/2 \qquad (\delta = 266^\circ \text{ or } \delta = 94^\circ)$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

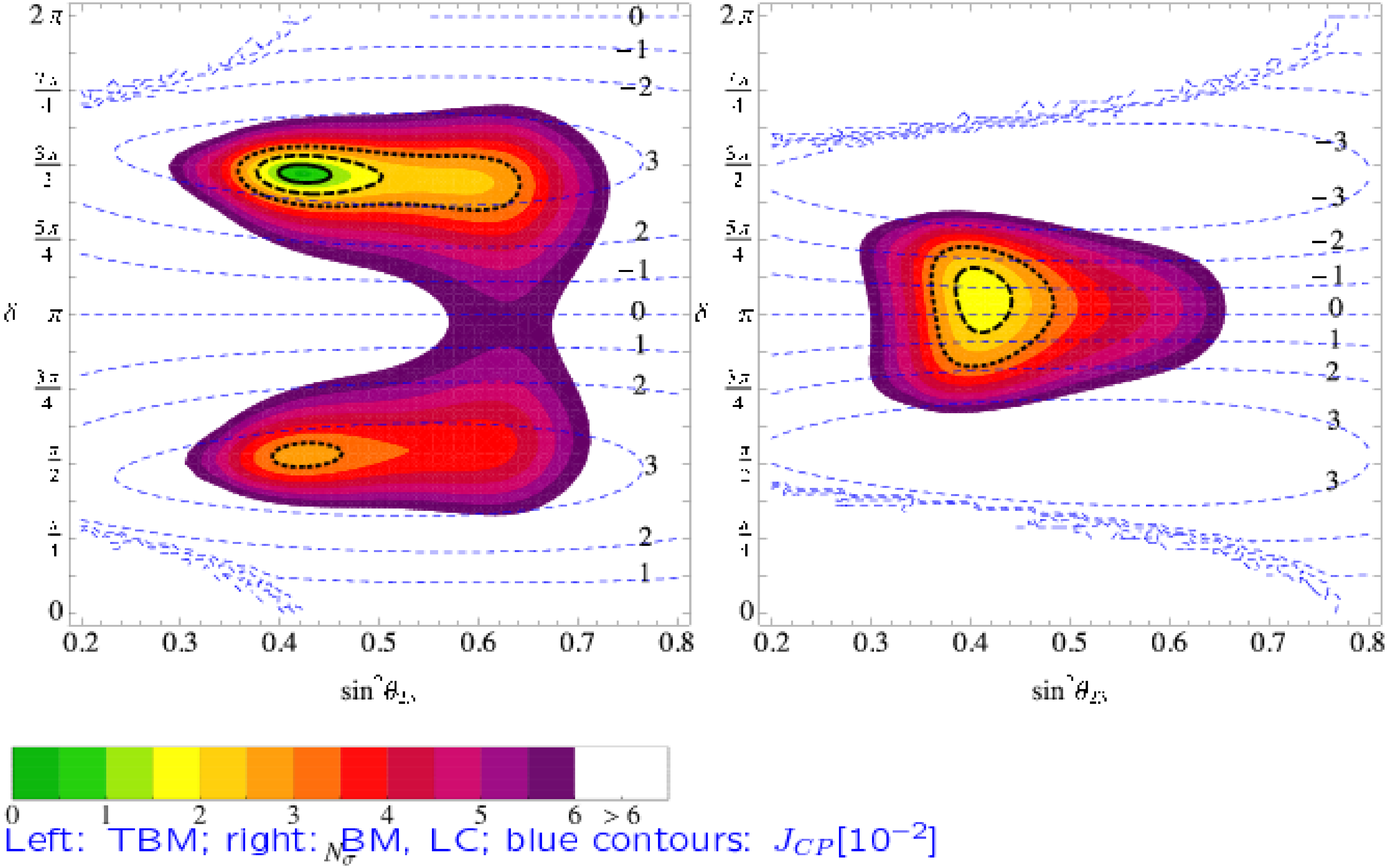
For $U_{\rm BM}$:

$$\cos\delta = -\frac{1}{2\sin\theta_{13}}\cot 2\theta_{12}\tan\theta_{23}\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right) .$$

For $U_{\rm BM}$ + b.f.v. of $\theta_{12}, \theta_{23}, \theta_{13}$:

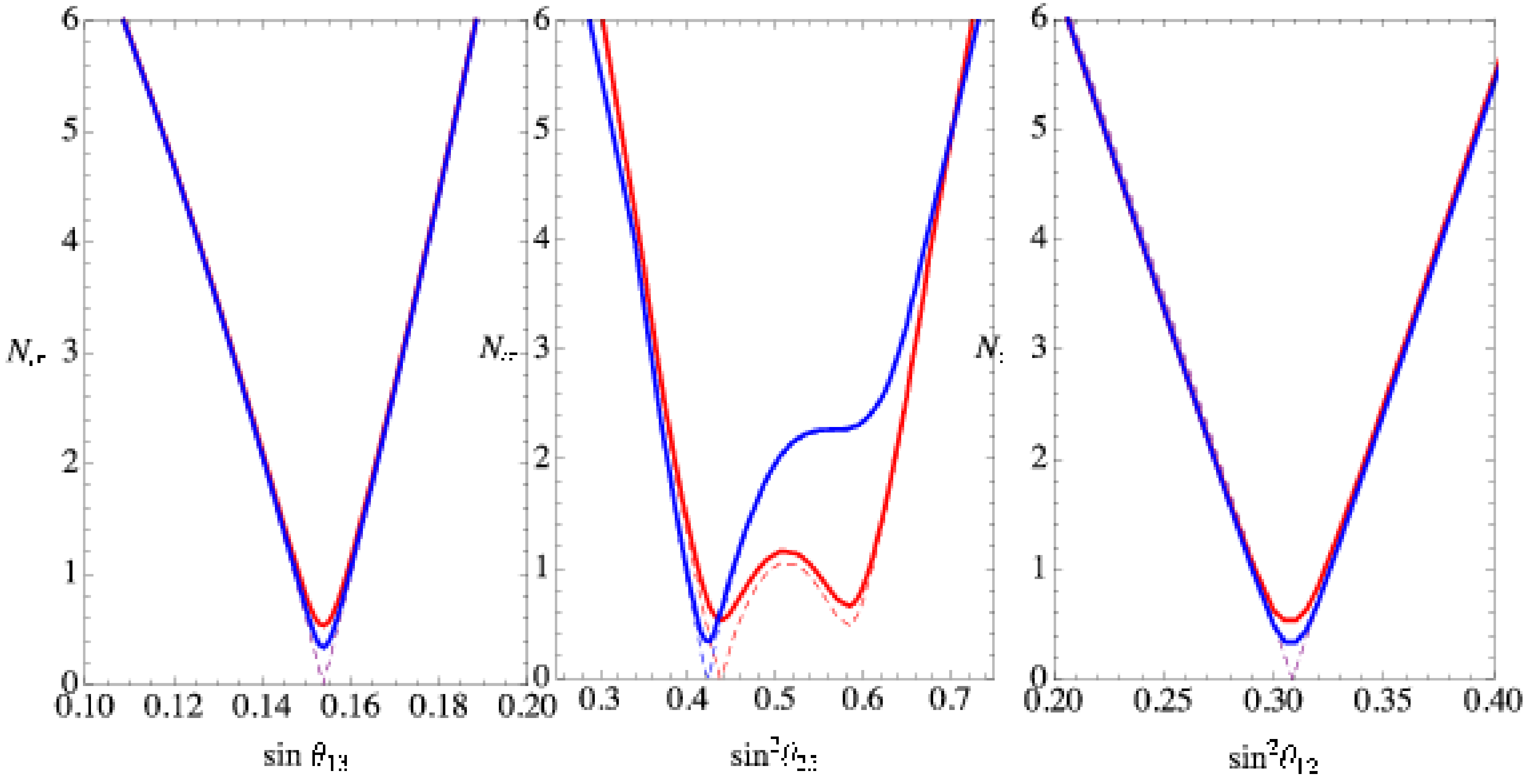
$$\delta \cong \pi$$

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.



Left: TBM; right: BM, LC; blue contours: $J_{CP}[10^{-2}]$

Standard Ordering - TBM

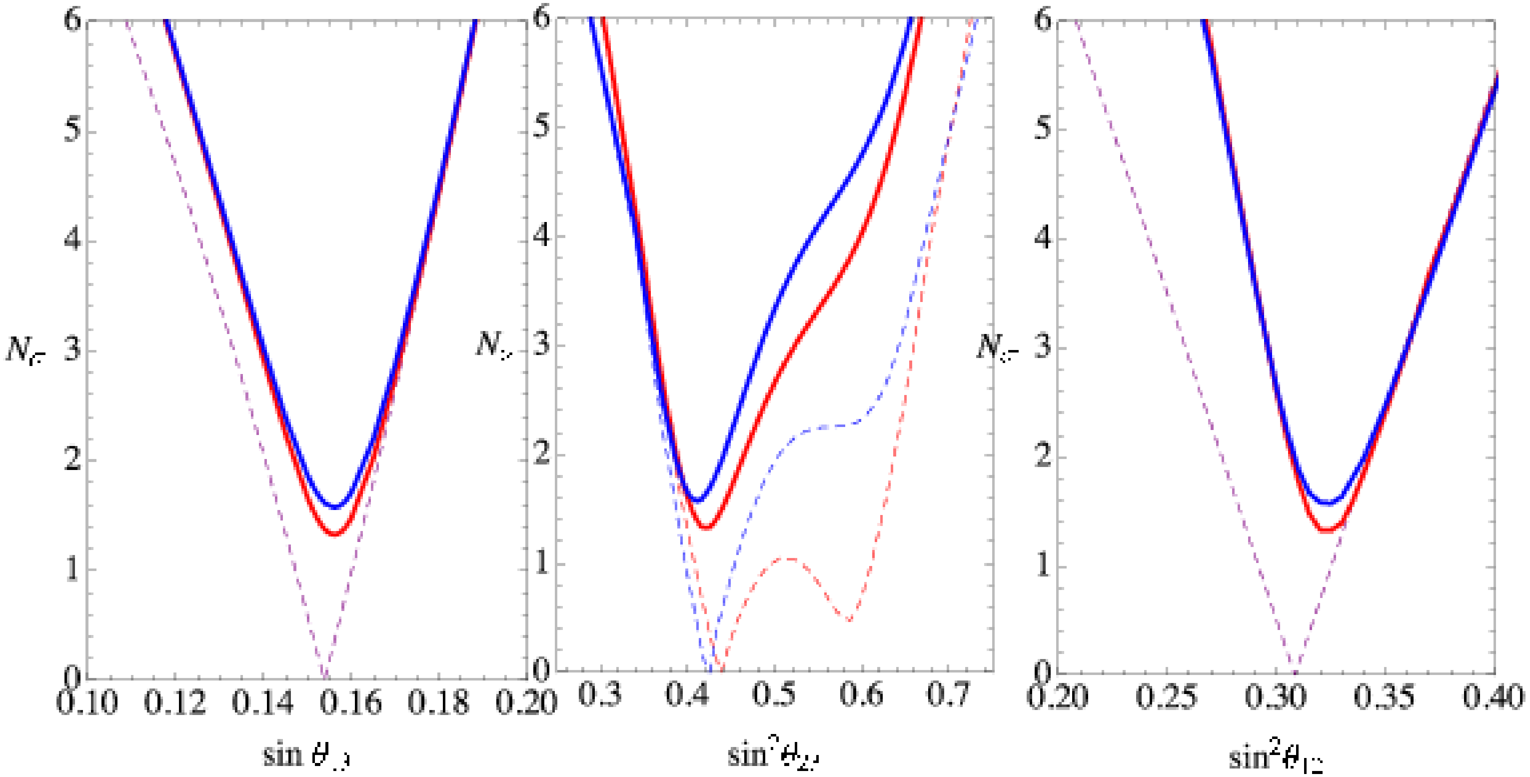


N_σ **as a function of** sin θ_{13}, sin$^2\theta_{12}$, sin$^2\theta_{23}$.

dashed lines - Fogli et al., solid lines - our analysis.

Blue lines - NO, red lines - IO; NO: $\sin^2\theta_{23} \leq 0.5$ **at** $\sim 2\sigma$.

Standard Ordering - BM, LC

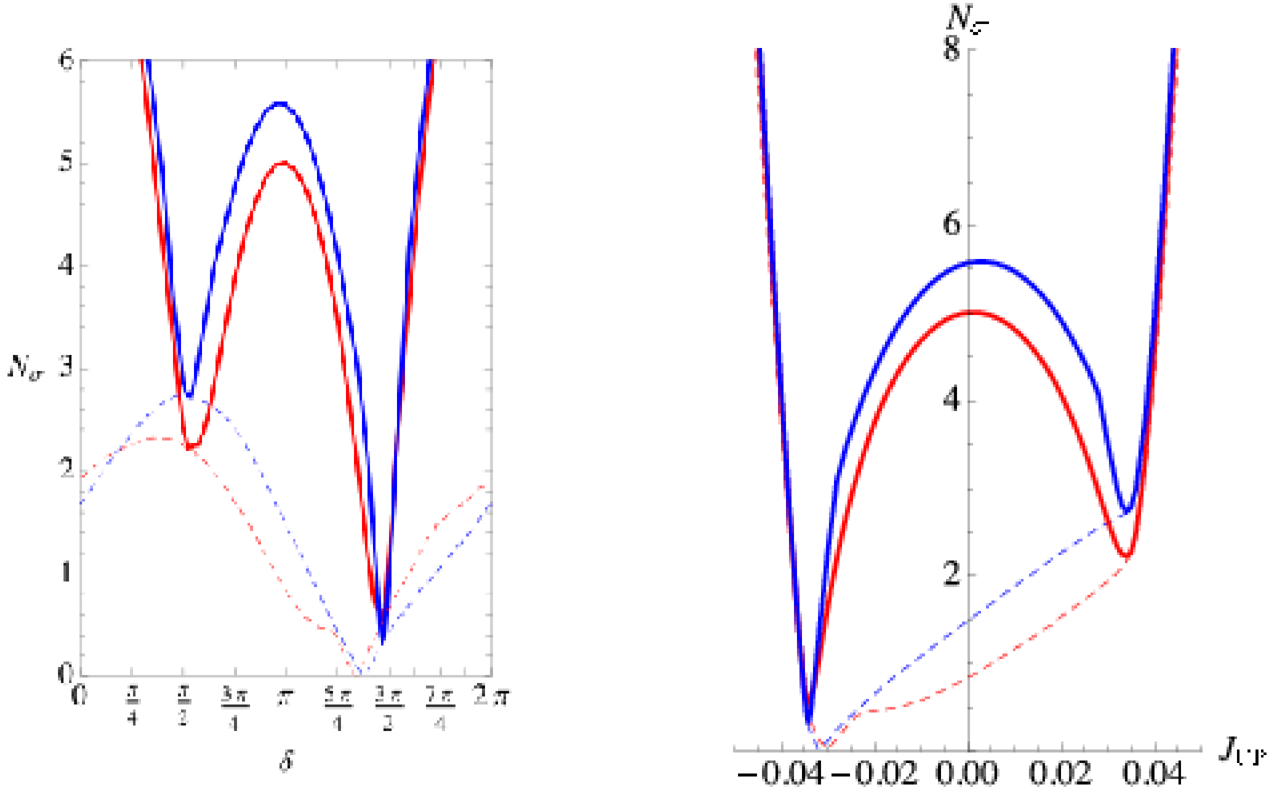


N_σ **as a function of** $\sin\theta_{13}$**,** $\sin^2\theta_{12}$**,** $\sin^2\theta_{23}$**.**

dashed lines - Fogli et al., solid lines - our analysis.

Blue lines - NO, red lines - IO; NO, IO: $\sin^2\theta_{23} \leq 0.5$ **at** $\sim 3.0\sigma$**.**

Standard Ordering - TBM



N_σ as a function of δ, J_{CP}. Blue lines - NO, red lines - IO.

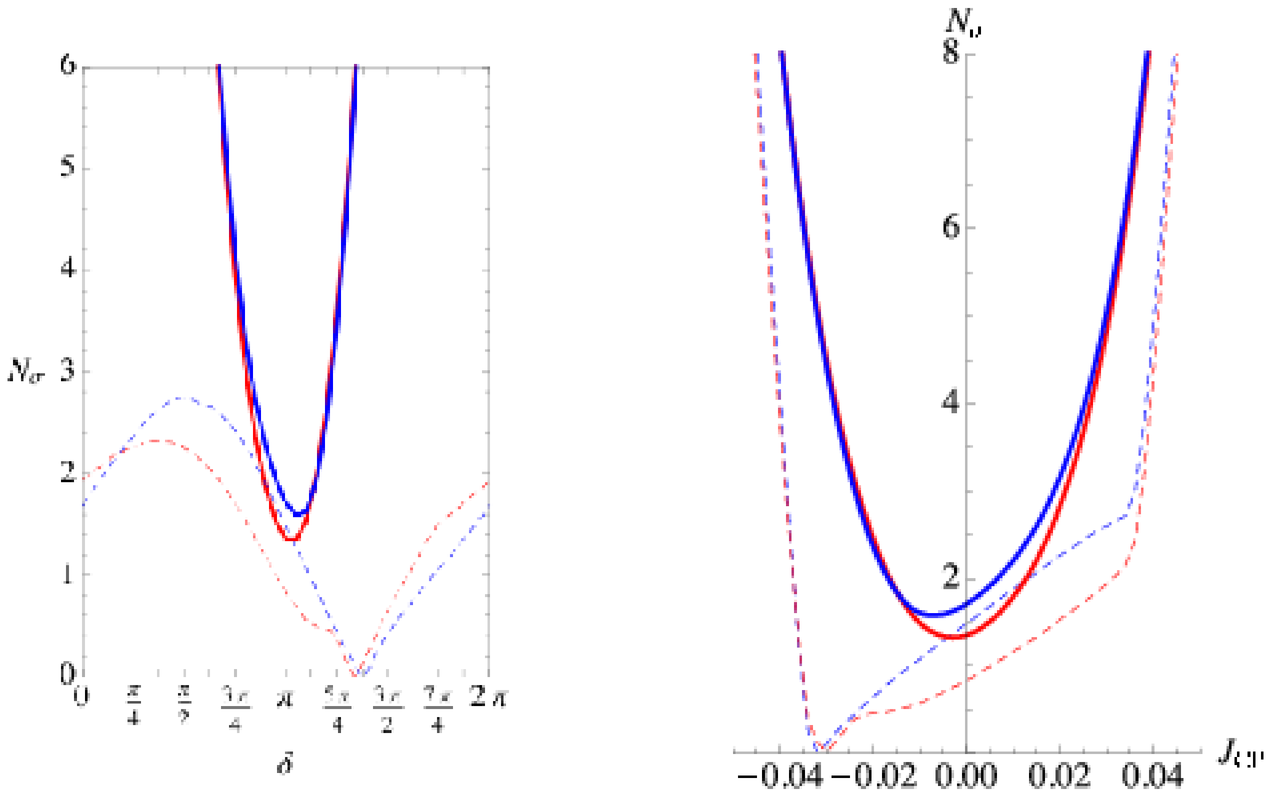
Dashed lines - Fogli et al., solid lines - our analysis.

$J_{CP} \neq 0$ **at** $\sim 5\sigma$; **b.f.v.:** $J_{CP} \cong -0.034$, **NO,IO;**

at 3σ, **NO:** $0.032 \lesssim J_{CP} \lesssim 0.036$ **or** $-0.038 \lesssim J_{CP} \lesssim -0.028$;

at 3σ, **IO:** $0.027 \lesssim J_{CP} \lesssim 0.037$ **or** $-0.039 \lesssim J_{CP} \lesssim -0.024$.

Standard Ordering - BM, LC



N_σ as a function of δ, J_{CP}. Blue lines - NO, red lines - IO.

Dashed lines - Fogli et al., solid lines - our analysis.

NO, IH b.f.v.: $J_{CP} \cong 0$; NO, IO, at 3σ: $-0.026 \lesssim J_{CP} \lesssim 0.022$.

Determining the $\nu-$Mass Hierarchy (sgn($\Delta m^2_{\rm atm}$))

- **Reactor $\bar{\nu}_e$ Oscillations in vacuum (JUNO, RENO50).**

- **Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (HK, ORCA, PINGU (IceCube), INO).**

- **LBL $\nu-$oscillation experiments (T2K, NOνA; LBNO, LBNE, $\nu-$factory); designed to search also for CP violation.**

- ^{3}H β**-decay Experiments (sensitivity to 5×10^{-2} eV) (NH vs IH).**

- $(\beta\beta)_{0\nu}$**–Decay Experiments; ν_j- Majorana particles (NH vs IH).**

- **Cosmology: $\sum_j m_j$ (NH vs IH).**

- **Atomic Physics Experiments: RENP.**

Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\mathrm{NO}}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \tfrac{1}{2}\,\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\mathrm{A}}\,L}{2\,E_\nu}\right) - \tfrac{1}{2}\cos^4\theta_{13}\,\sin^2 2\theta_{\odot}\,\left(1 - \cos\frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\,\sin^2\theta_{\odot}\,\sin\frac{\Delta m^2_{\odot}\,L}{4\,E_\nu}\,\sin\left(\frac{\Delta m^2_{\mathrm{A}}\,L}{2\,E_\nu} - \frac{\Delta m^2_{\odot}\,L}{4\,E_\nu}\right)\,,$$

$$P_{\mathrm{IO}}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \tfrac{1}{2}\,\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\mathrm{A}}\,L}{2\,E_\nu}\right) - \tfrac{1}{2}\cos^4\theta_{13}\,\sin^2 2\theta_{\odot}\,\left(1 - \cos\frac{\Delta m^2_{\odot}\,L}{2\,E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\,\cos^2\theta_{\odot}\,\sin\frac{\Delta m^2_{\odot}\,L}{4\,E_\nu}\,\sin\left(\frac{\Delta m^2_{\mathrm{A}}\,L}{2\,E_\nu} - \frac{\Delta m^2_{\odot}\,L}{4\,E_\nu}\right)\,,$$

$\theta_{\odot} = \theta_{12}\,, \Delta m^2_{\odot} = \Delta m^2_{21} > 0\,;\ \ \sin^2\theta_{12} \leq 0.36$ at 3σ;

$\Delta m^2_{\mathrm{A}} = \Delta m^2_{31} > 0\,,$ NO spectrum ,

$\Delta m^2_{\mathrm{A}} = \Delta m^2_{23} > 0\,,$ IO spectrum

M. Piai, S.T.P., hep-ph/0112074;

The reactor $\bar{\nu}_e$ detected via

$$\bar{\nu}_e + p \to e^+ + n\,.$$

The visible energy of the detected e^+:

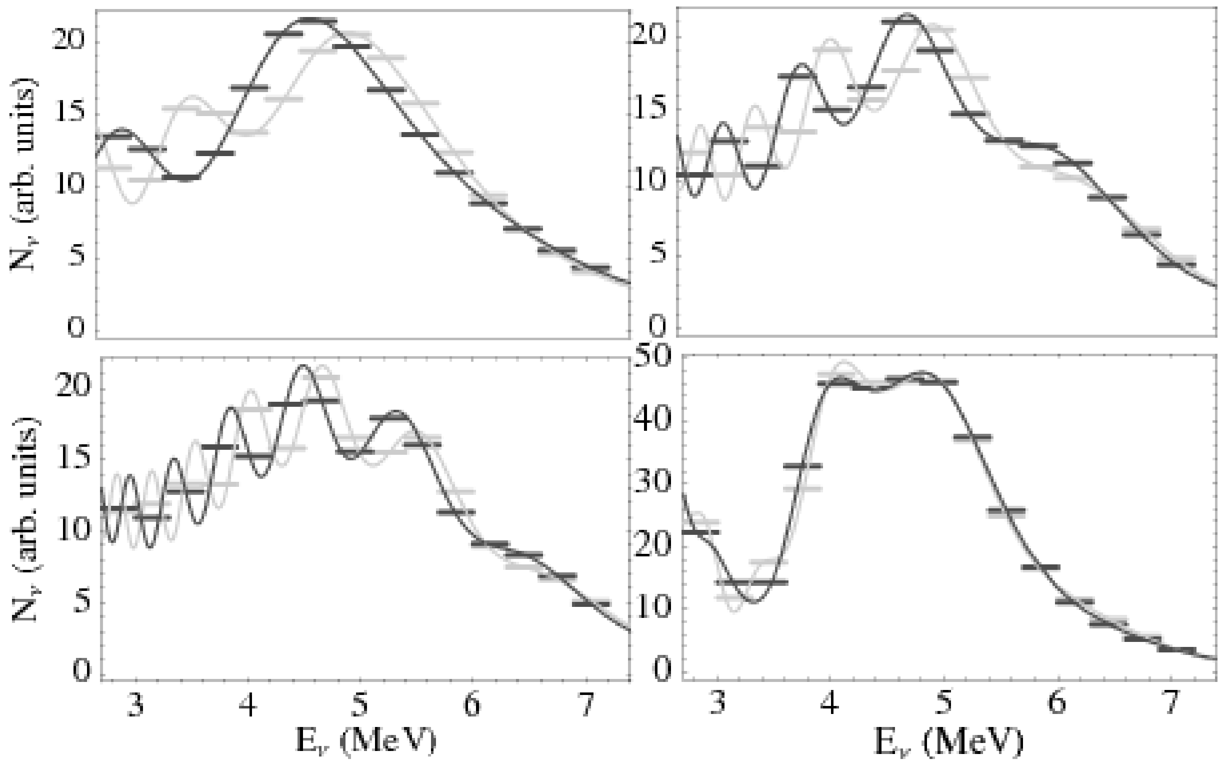
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8\ \text{MeV}\,.$$

The measured event rate spectrum vs. L/E_m:

$$N(L/E_m) = \int R(E, E_m)\Phi(E)\sigma(\bar{\nu}_e p \to e^+ n; E) P_{\bar{e}\bar{e}}^{NO(IO)} dE\,.$$

$$|P_{\text{NO}}(\bar{\nu}_e \to \bar{\nu}_e) - P_{\text{IO}}(\bar{\nu}_e \to \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$\cos 2\theta_{12} \cong 0.38\,;\ \ 3\sigma:\ \ \cos 2\theta_{12} \geq 0.28\,;\qquad \sin^2 2\theta_{13} \cong 0.09\,.$



M. Piai, S.T.P., 2001

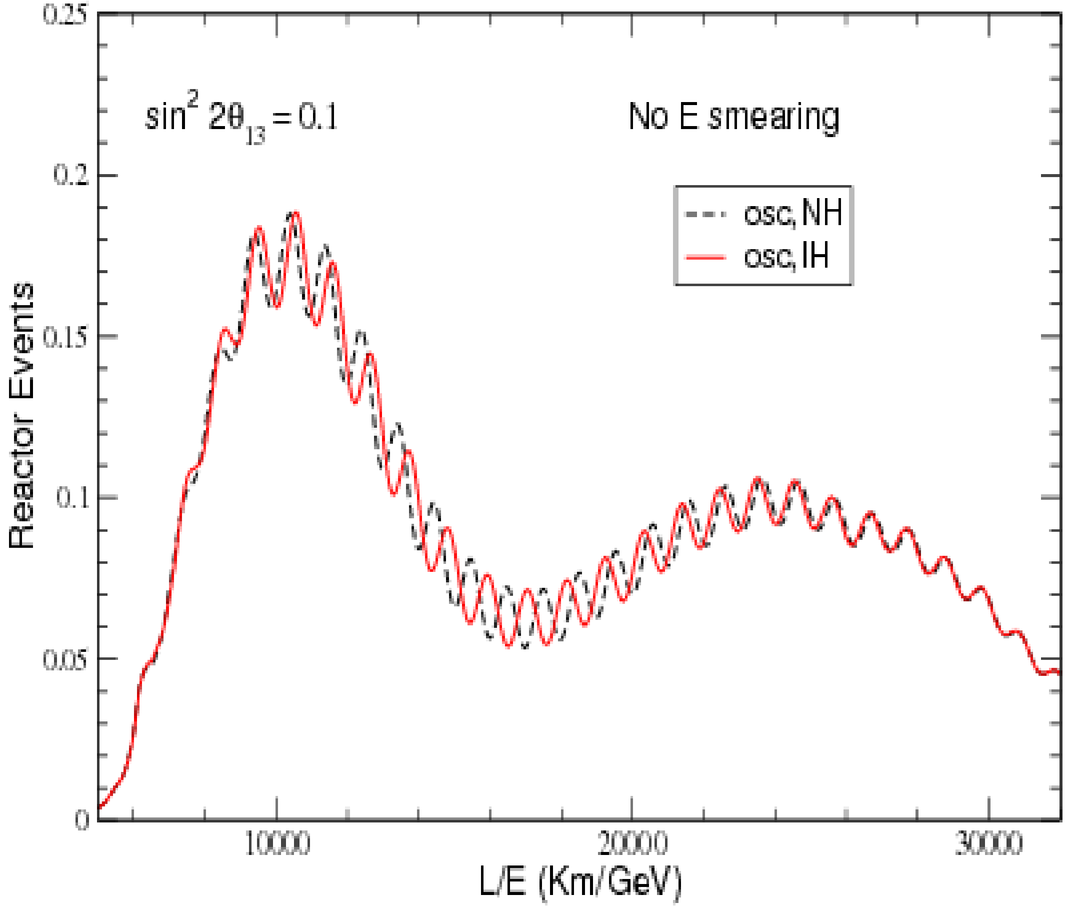
$\sin^2\theta_{13} = 0.05$, $\Delta m^2_{21} = 2 \times 10^{-4}$ eV^2; $\Delta m^2_{\rm A} = 1.3;\ 2.5;\ 3.5 \times 10^{-3}$ eV^2

$L = 20$ km, $\Delta E_\nu = 0.3$ MeV.

$\Delta m^2_{21} = 2 \times 10^{-4}$ eV^2: $L = 20$ km;

$\Delta m^2_{21} = 7.6 \times 10^{-5}$ eV^2: $L \cong 53$ km.

NO – light grey; IO – dark grey

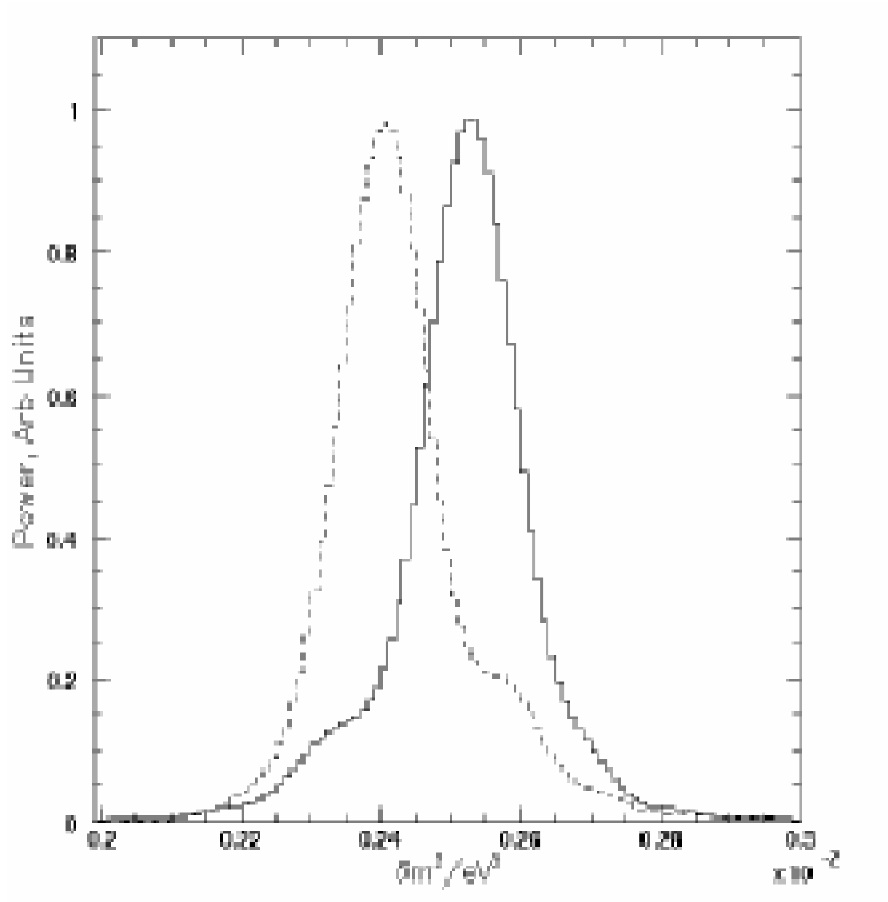


P. Ghoshal, S.T.P., arXiv:1011.1646

Fourier Analysis:

$$\text{NO}: \quad \cos^2\theta_{12}\,\sin^2\Delta + \sin^2\theta_{12}\,\sin^2(\Delta - \Delta_{21})\,,$$
$$\text{IO}: \quad \sin^2\theta_{12}\,\sin^2\Delta + \cos^2\theta_{12}\,\sin^2(\Delta - \Delta_{21})\,,$$

$$\Delta \equiv \Delta_{31}(NO) = |\Delta_{32}(IO)|\,;$$
$$\sin^2\theta_{12} \cong 0.31\,, \quad \cos^2\theta_{12} \cong 0.69\,.$$



J.Learned et al., 2007

Very challenging; requires:

- energy resolution $\sigma/E_{\rm vis} \lesssim 3\%/\sqrt{E_{\rm vis}}$;

- relatively small energy scale uncertainty;

- relatively large statistics ($\sim$ (300 − 1000) kT GW yr);

- relatively small systematic errors;

- subtle optimisations (distance, number of bins, effects of "interfering distant" reactors).

Two experiments planned with $L \cong 50$ km: Juno (20 kT, approved), RENO50 (18 kT). Can measure also $\sin^2\theta_{12}$, Δm^2_{21} and $|\Delta m^2_{31}|$ with remarkably high precision. Can be used for detection of Geo, solar, SN neutrinos as well.

Atmospheric Neutrino Experiments on sgn(Δm^2_{31})

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s^2_{23}\, P_{2\nu}\,,\, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c^2_{23}\, P_{2\nu}\,,$$

$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s^4_{23}\ P_{2\nu} - 2c^2_{23}s^2_{23}\ \left[1 - Re\ (e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\right]\ ,$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{31}, \theta_{13}; E, \theta_n; N_e)$: 2-$\nu$ $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,
$\nu'_\tau = s_{23}\,\nu_\mu + c_{23}\,\nu_\tau$; $\Delta m^2_{21} \ll |\Delta m^2_{31(32)}|$, $E_\nu \gtrsim 2$ GeV;

κ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2-ν amplitude.

NO: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced, $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

IO: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced, $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$−suppressed

No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime): $[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the backgrpound of electrons, protons and neutrnos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \to \nu_e) \cong \sin^2\theta_{23}\, \sin^2 2\theta_{13}^m\, \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$\sin^2 2\theta_{13}^m$, ΔM_{31}^2 depend on the matter potential
$V_{eff} = \sqrt{2}\, G_F\, N_e$,

For antineutrinos V_{eff} has the opposite sign:
$V_{eff} = -\sqrt{2}\, G_F, N_e$.

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
$\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
$\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ −suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta_{13}},$$

$$\cos 2\theta_{13}^m = \frac{1-N_e/N_e^{res}}{\sqrt{(1-\frac{N_e}{N_e^{res}})^2+\tan^2 2\theta_{13}}},$$

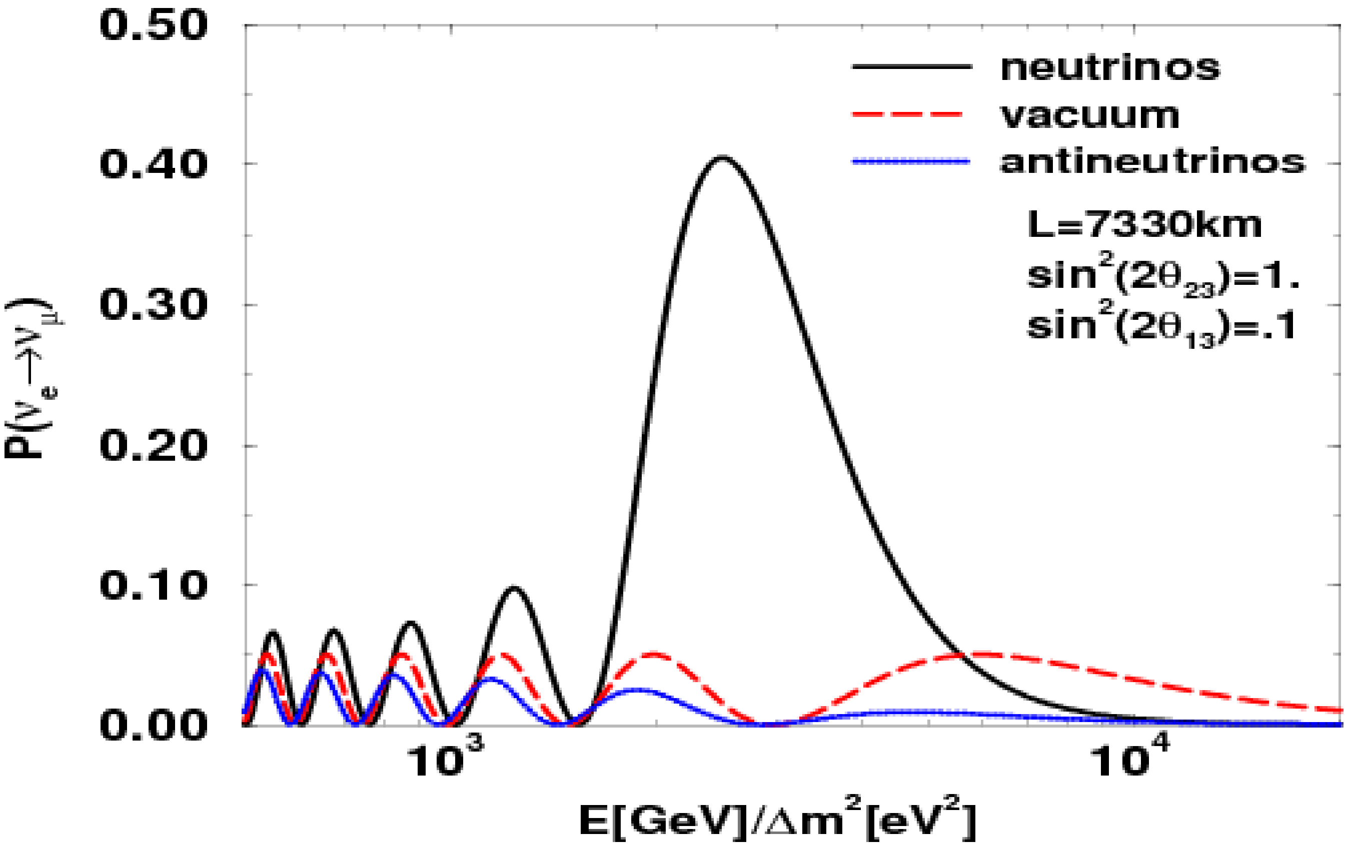
$$N_e^{res} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F} \cong$$

$$6.56 \times 10^6 \, \frac{\Delta m^2[\mathrm{eV}^2]}{E[\mathrm{MeV}]} \cos 2\theta \, \mathrm{cm}^{-3} \;\; \mathrm{N_A}\,,$$

$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E}\left((1-\frac{N_e}{N_e^{res}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13}\right)^{\frac{1}{2}}$$

For $\bar{\nu}_\mu \to \bar{\nu}_e$: $N_e \to (-N_e)$.

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



$\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $E^{res} = 6.25$ GeV; $P^{3\nu} = \sin^2\theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}$;
$N_e^{res} \cong 2.3$ cm^{-3} N$_A$; $L_m^{res} = L^\nu / \sin 2\theta_{13} \cong 6250/0.32$ km; $2\pi L/L_m \cong 0.75\pi (\neq \pi)$.

HyperKamiokande (10SK), IceCube-PINGU, ANTARES-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-, no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

ANTARES in Mediteranian sea: ORCA

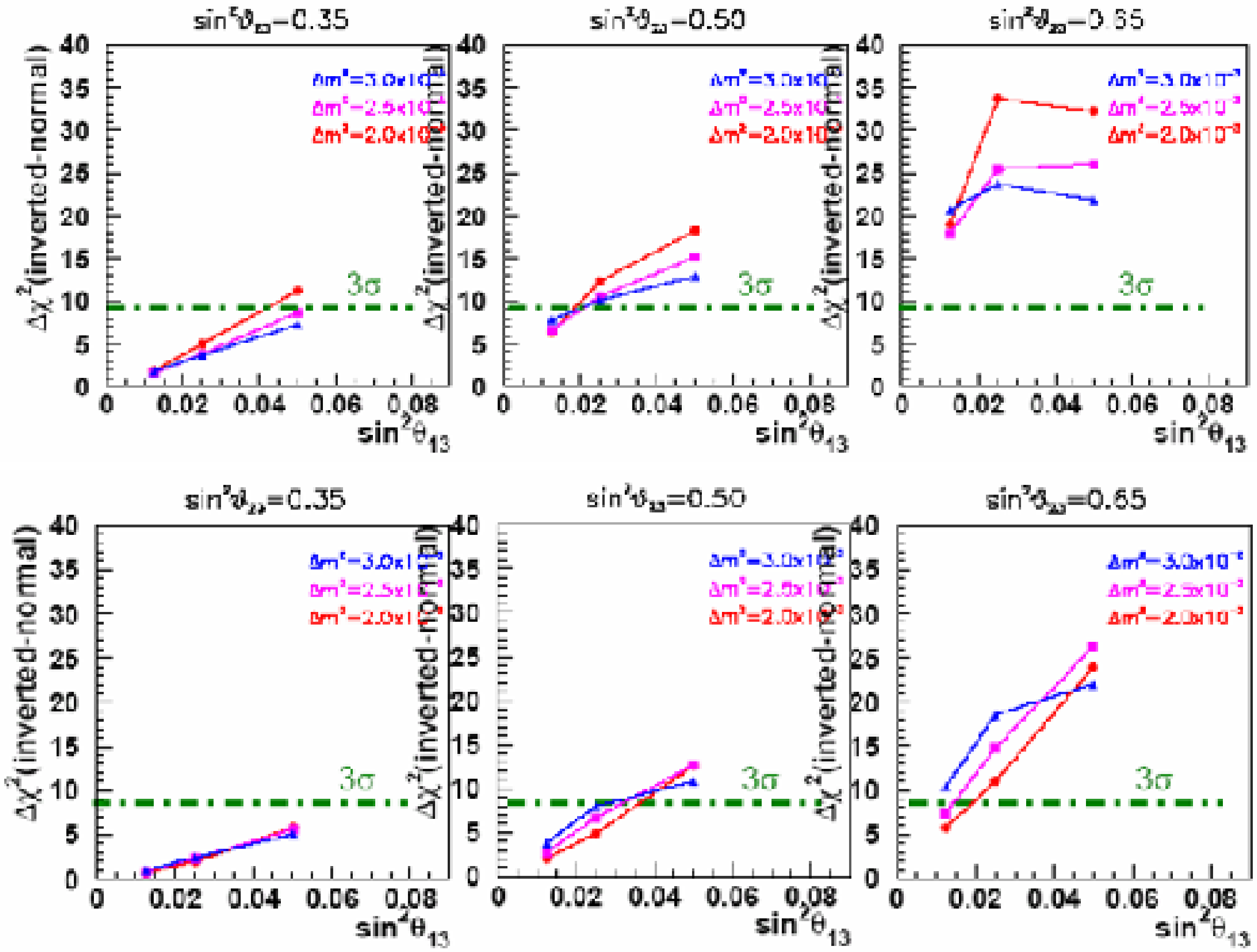
Water-Cerenkov detector: Hyper Kamiokande (10SK)

Sensitivity depends critically on θ_{23}, the "true" hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

$$P(\nu_\mu \to \nu_e) \cong \sin^2\theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

No charge identification (SK, HK, PINGU, ORCA); event rate (DIS regime):
$[2\sigma(\nu_l + N \to l^- + X) + \sigma(\bar{\nu}_l + N \to l^+ + X)]/3$

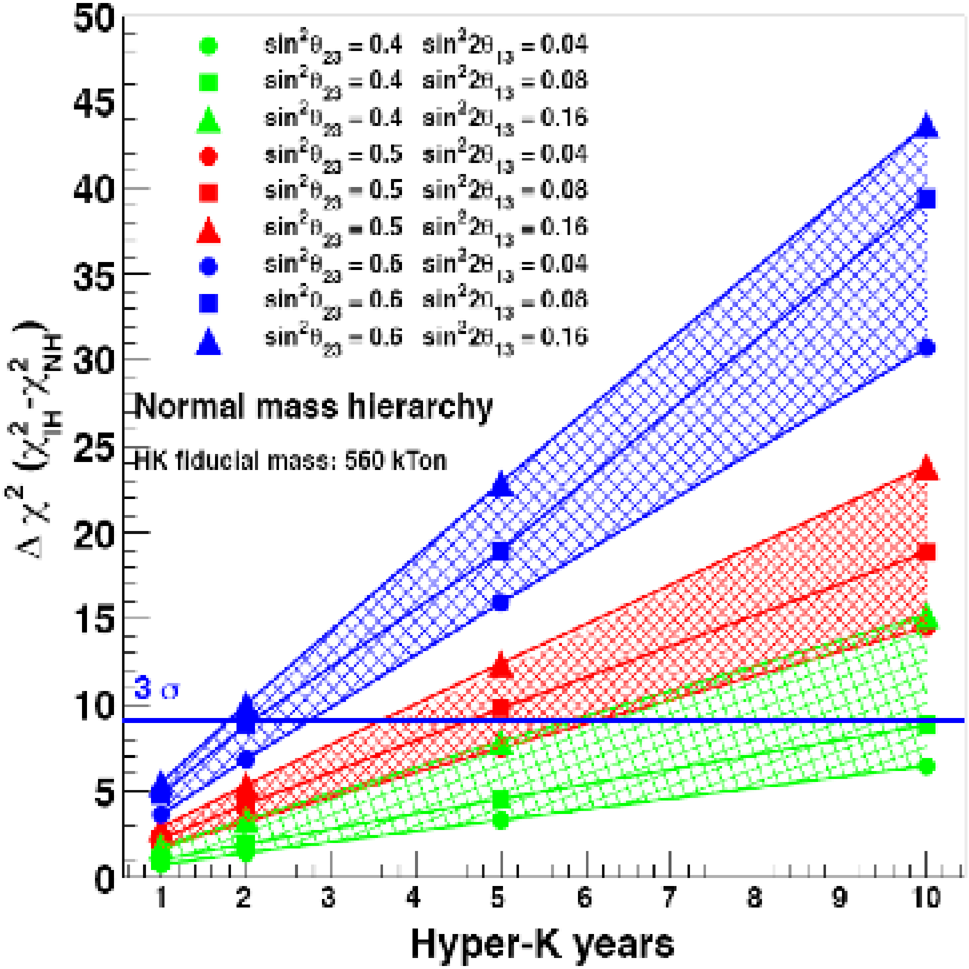


Water-Cerenkov detector, 1.8 MTy (HK = 10SK)

Critical dependence on θ_{23}, "true hierarchy".

T. Kajita et al., 2004

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003



Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data. θ_{23} and θ_{13} are assumed to be known as indicated in the figure.

K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

Future LBL Neutrino Oscillation Experiments on sgn(Δm^2_{31}) (the Hierarchy) and CP Violation

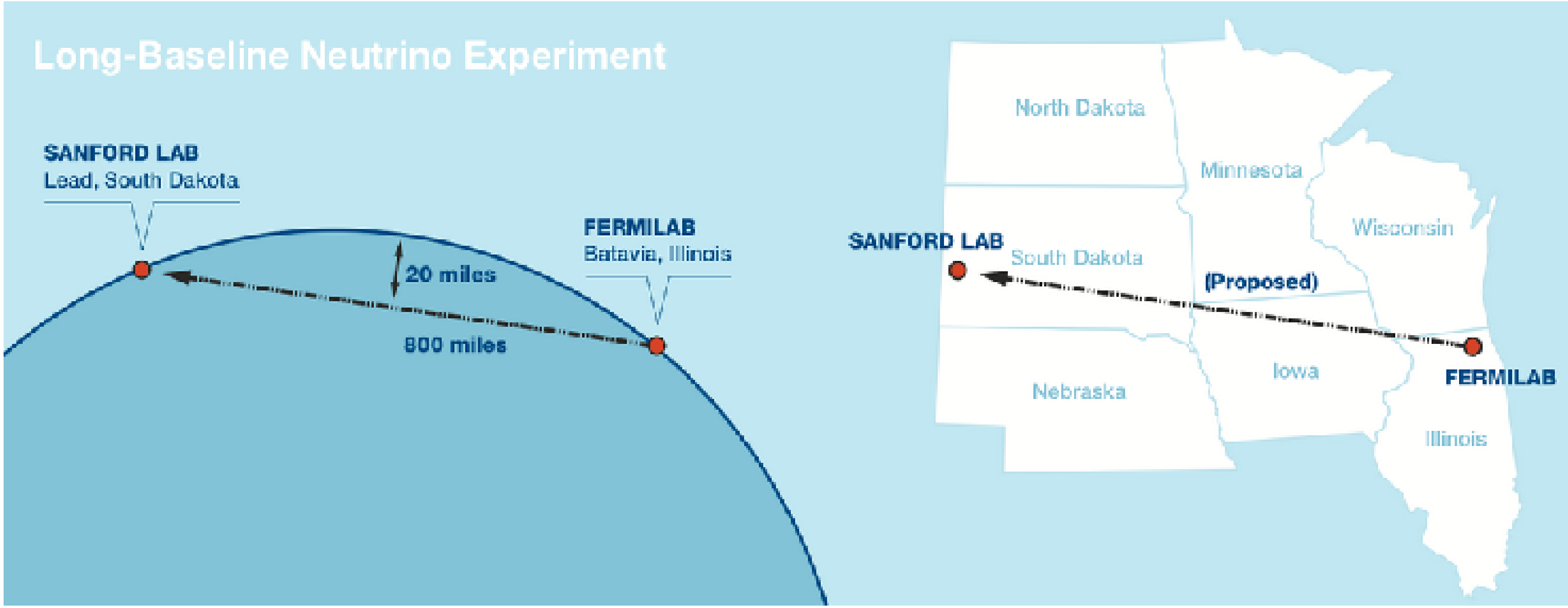
LBL Oscillation Experiments NOνA, LBNE, LBNO

NOνA: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

LBNF: Fermilab-DUSEL, $L = 1290$ km, 700 KW wide band ν beam (first and second osc. maxima at $E = 2.4$ GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with 15% to 30% PMT covarage, or multiple 17 kt fiducial volume LAr detecors; plans to run 5 years with ν_μ and 5 years with $\bar{\nu}_\mu$; 202X (?)

LBNO: CERN-Pyhasalmi, $L = 2290$ km, wide band ν_μ 1.6 MW super beam (first and second osc. maxima at $E \cong 4$ GeV and 1.5 GeV); 440 kt Water Cherenkov, or 100 kt LAr, or 50 kt liquid scintillator detector; 202Y (?)


Ontario
NOvA
Soudan
Minnesota
Wisconsin
735 kilometers
Iowa
Fermilab
Illinois



Long-Baseline Neutrino Experiment
SANFORD LAB
Lead, South Dakota
FERMILAB
Batavia, Illinois
20 miles
800 miles
North Dakota
Minnesota
Wisconsin
SANFORD LAB
South Dakota
(Proposed)
Nebraska
Iowa
FERMILAB
Illinois


Pyhäsalmi
Finland
Norway
Oslo
Stockholm
Helsinki
St Petersburg
Tallinn
Estonia
Latvia
Lithuania
2300km
North Sea
Denmark
United Kingdom
DESY
Berlin
Netherlands
Poland
Moscow
Protvino
Belarus
RAL
Belgium
Germany
Prague
Paris
Czech Rep
Kyiv
Kharkiv
Munich
Vienna
Slovakia
Ukraine
Austria
Moldova
Zaporozhy
Hungary
France
Bay of Biscay
Zagreb
Romania
CERN
Croatia
Bucharest
Serbia
Black Sea
Bulgaria
Istanbul
Ankara
Greece
Turkey
Athens
2100 km from RAL, 1500 km from DESY, and 1160 km from Protvino.

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

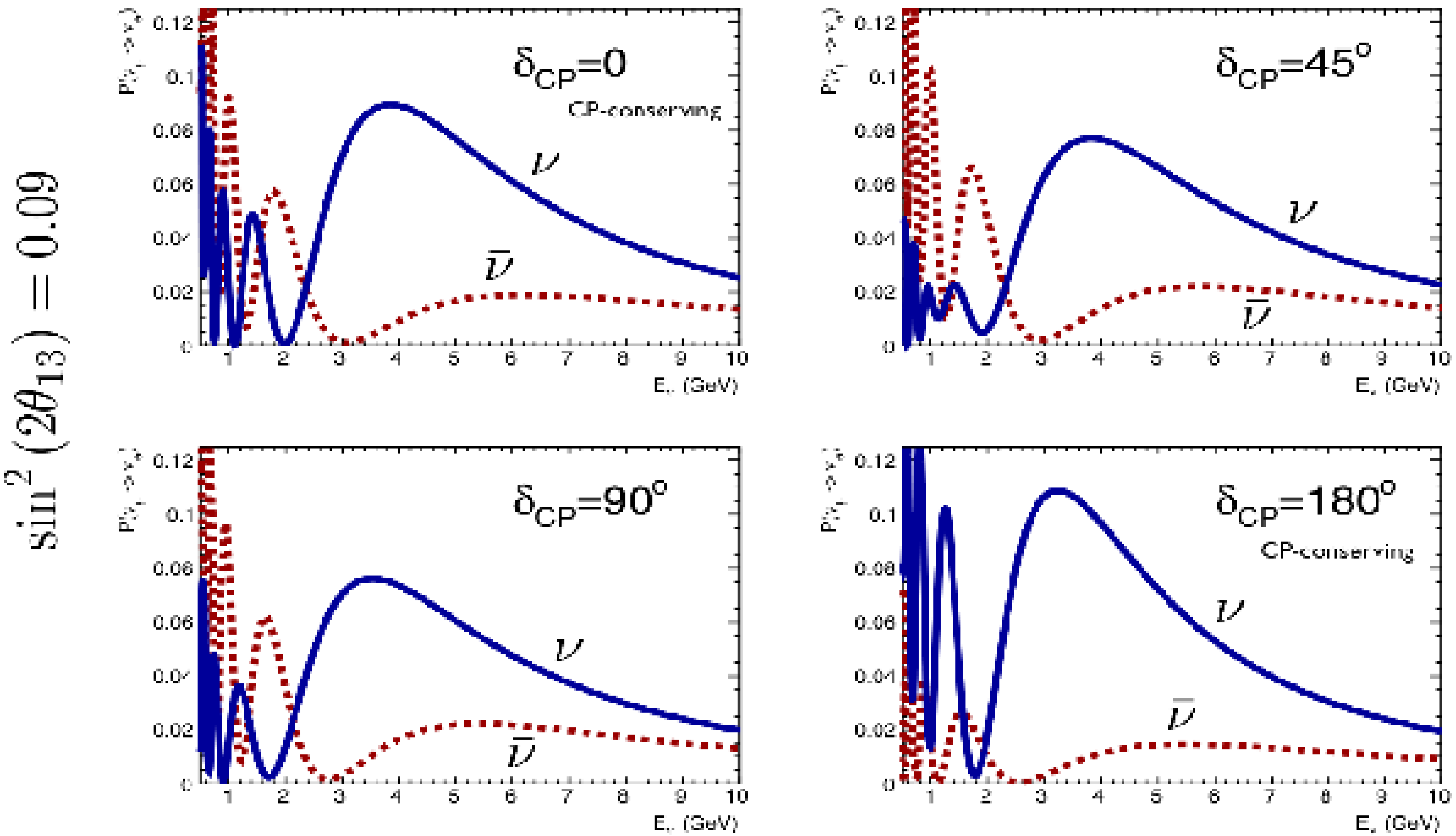
$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\;(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E},\;\; A = \sqrt{2}G_{\mathrm{F}}N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$: $\delta,\;\; A \rightarrow (-\delta),\;\; (-A)$

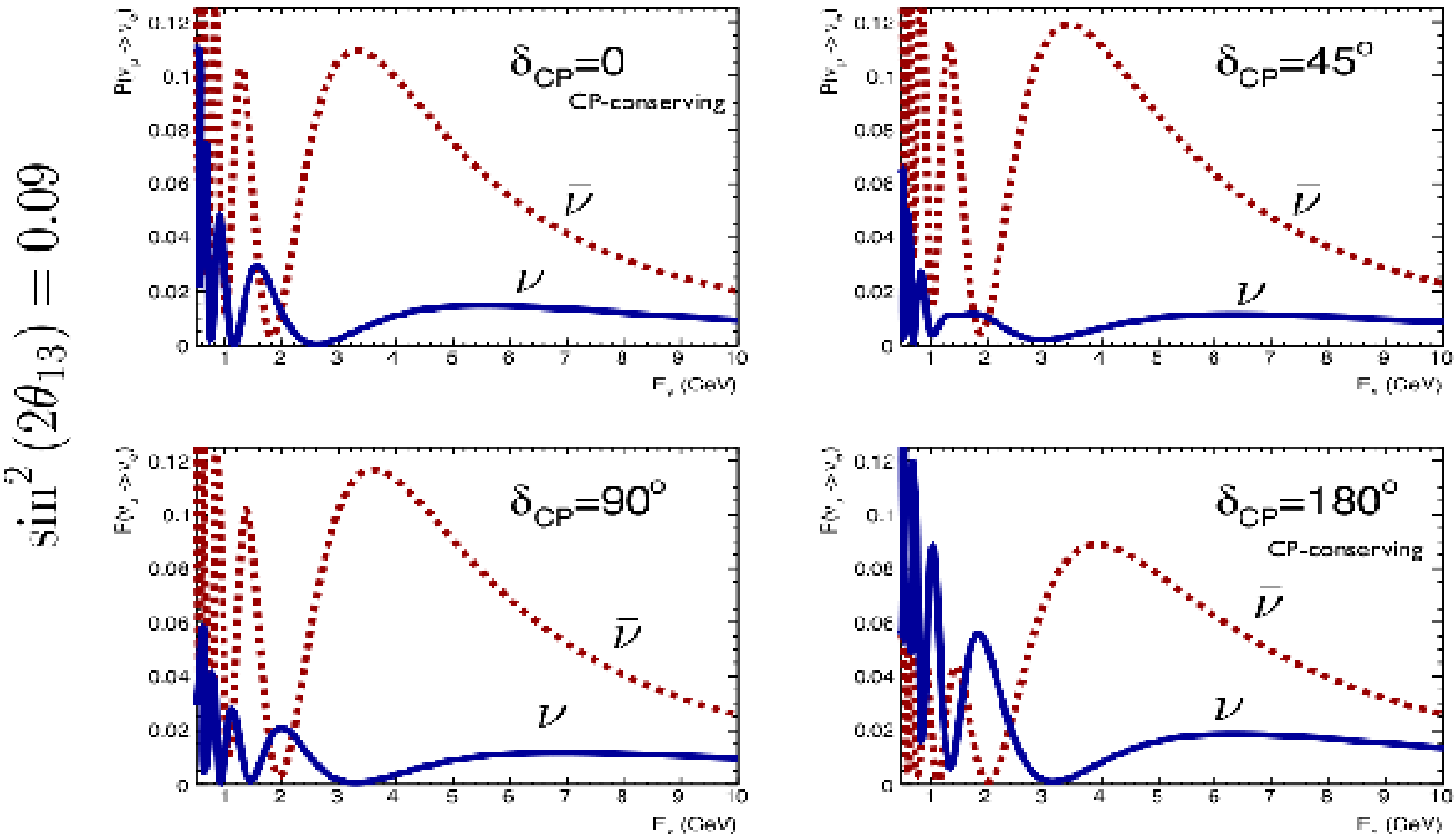
CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★Normal mass hierarchy L=2300 km



CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Inverted mass hierarchy L=2300 km



LBNF, for example, could achieve the determination of the mass hierarchy at 3σ in less then a year.

LBNF could also have very good sensitivity to CP-violation with a 60% coverage at 3σ in the allowed range of values of $\sin^2 2\theta_{13}$, for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

Determining the Nature of Massive Neutrinos

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l,l' = e,\mu,\tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP\ :\ \ P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j.

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

If $\nu_j -$ Majorana particles, $U_{\rm PMNS}$ contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana physical CPV phases

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- are not sensitive to the nature of ν_j,

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \ ({\rm A,Z}) \rightarrow \mu^+ + \ ({\rm A,Z}-2)$$

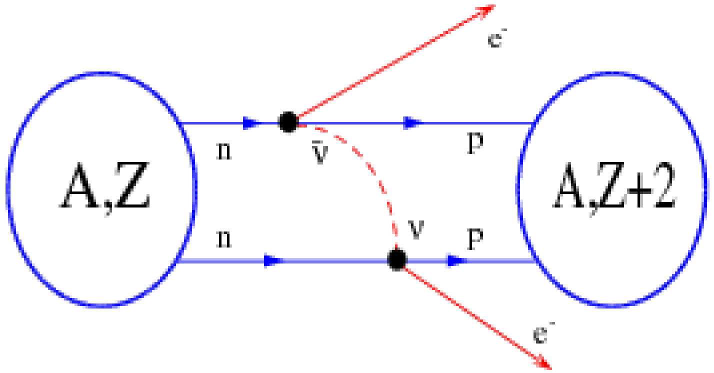
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$({\rm A,Z}) \rightarrow ({\rm A,Z}+2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

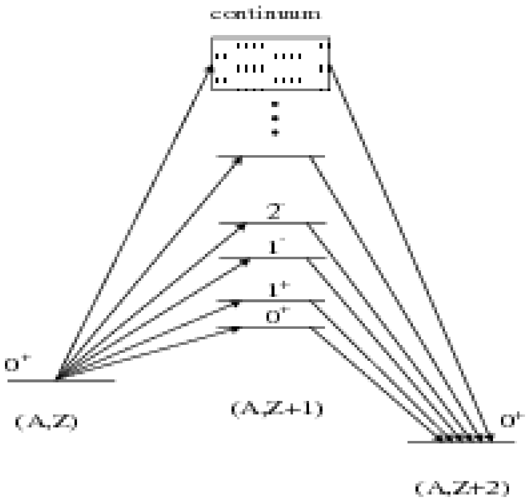
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \to uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$A(\beta\beta)_{0\nu} \sim < m >$ M(A,Z), M(A,Z) - NME,

$$\left| < m > \right| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 \ e^{i\alpha_{21}} + m_3 |U_{e3}|^2 \ e^{i\alpha_{31}} \right|$$

$$= \left| m_1 \ c_{12}^2 \ c_{13}^2 + m_2 \ s_{12}^2 \ c_{13}^2 \ e^{i\alpha_{21}} + m_3 \ s_{13}^2 \ e^{i\alpha_{31}} \right|, \quad \theta_{12} \equiv \theta_{\odot}, \ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi$, $\alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$\left|<m>\right| \cong \left|\sqrt{\Delta m^2_{\odot}}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta_M}\right|\,,\ m_1 \ll m_2 \ll m_3\ \text{(NH)},$$

$$\left|<m>\right| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\ m_3 < (\ll) m_1 < m_2\ \text{(IH)},$$

$$\left|<m>\right| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|\,,\ m_{1,2,3} \cong m \gtrsim 0.10\ \text{eV}\ \ \text{(QD)},$$

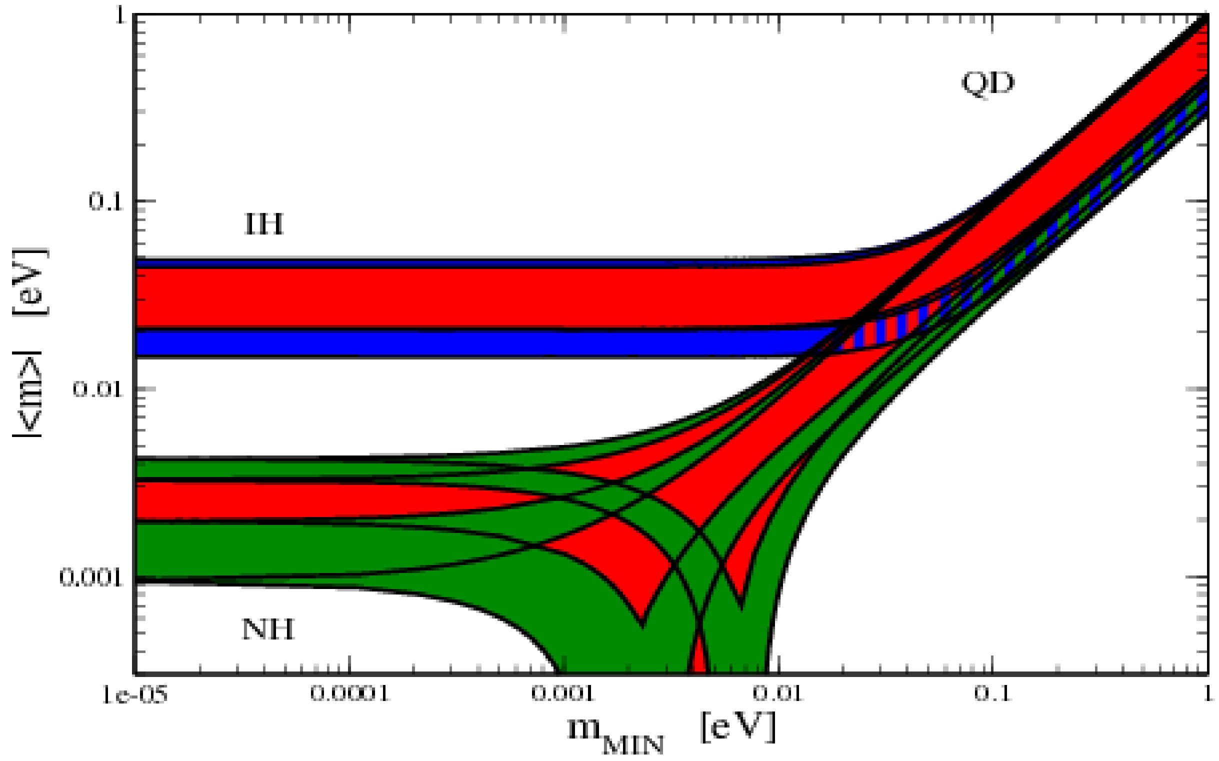
$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$\left|<m>\right| \lesssim 5 \times 10^{-3}$ eV, NH;

$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013$ eV $\lesssim \left|<m>\right| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055$ eV, IH;

$m\cos 2\theta_{12} \lesssim \left|<m>\right| \lesssim m$, $m \gtrsim 0.10$ eV, QD .



S. Pascoli, PDG, 2012

$\sin^2\theta_{13} = 0.0236 \pm 0.0042$; $\delta = 0$.

$1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$, $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$.

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|<m>|)$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}m{>}| = (0.1 - 0.9)$ eV (99.73% C.L.); b.f.v.: $|{<}m{>}| = 0.33$ eV.

IGEX ^{76}Ge: $|{<}m{>}| < (0.33 - 1.35)$ eV (90% C.L.).

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|{<}m{>}| <$(0.45–0.96) eV, $|{<}m{>}| <$(0.18–0.64) eV (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$T(^{76}\mathrm{Ge}) = 2.23^{+0.44}_{0.31} \times 10^{25}\ \mathrm{yr\ at\ 90\%\ C.L.}$$

Results from 2012-2013:

$$T(^{136}\mathrm{Xe}) > 1.6 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ EXO}$$

$$T(^{136}\mathrm{Xe}) > 1.9 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ KamLAND - Zen}$$

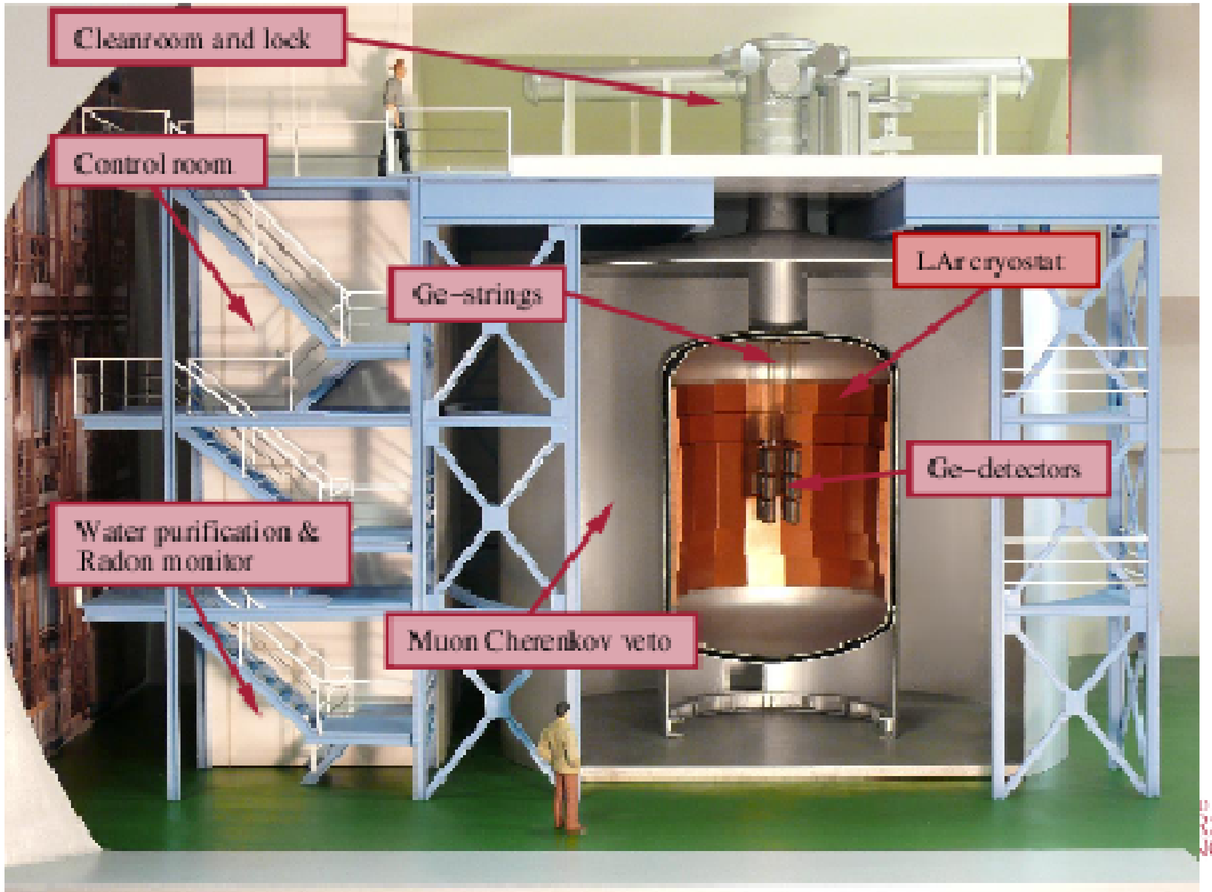
$$T(^{76}\mathrm{Ge}) > 2.1 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ GERDA.}$$

$$T(^{76}\mathrm{Ge}) > 3.0 \times 10^{25}\mathrm{yr\ at\ 90\%\ C.L.,\ GERDA + IGEX + HdM.}$$

Large number of experiments: $|<m>|$ $\sim$ (0.01-0.05) eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
KamLAND-ZEN - ^{136}Xe;
EXO - ^{136}Xe;
SNO+ - ^{130}Te;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca;
SuperNEMO - ^{82}Se,...;
MAJORANA - ^{76}Ge;
COBRA - ^{116}Cd;
MOON - ^{100}Mo.

GERDA: Experimental Setup



Majorana CPV Phases and $|\!<\!m\!>\!|$

CPV can be established provided

– $|\!<\!m\!>\!|$ measured with $\Delta \lesssim 15\%$;

– Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

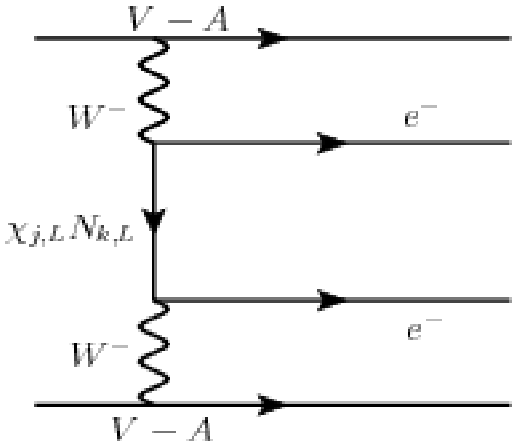
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay



Light Majorana Neutrino Exchange

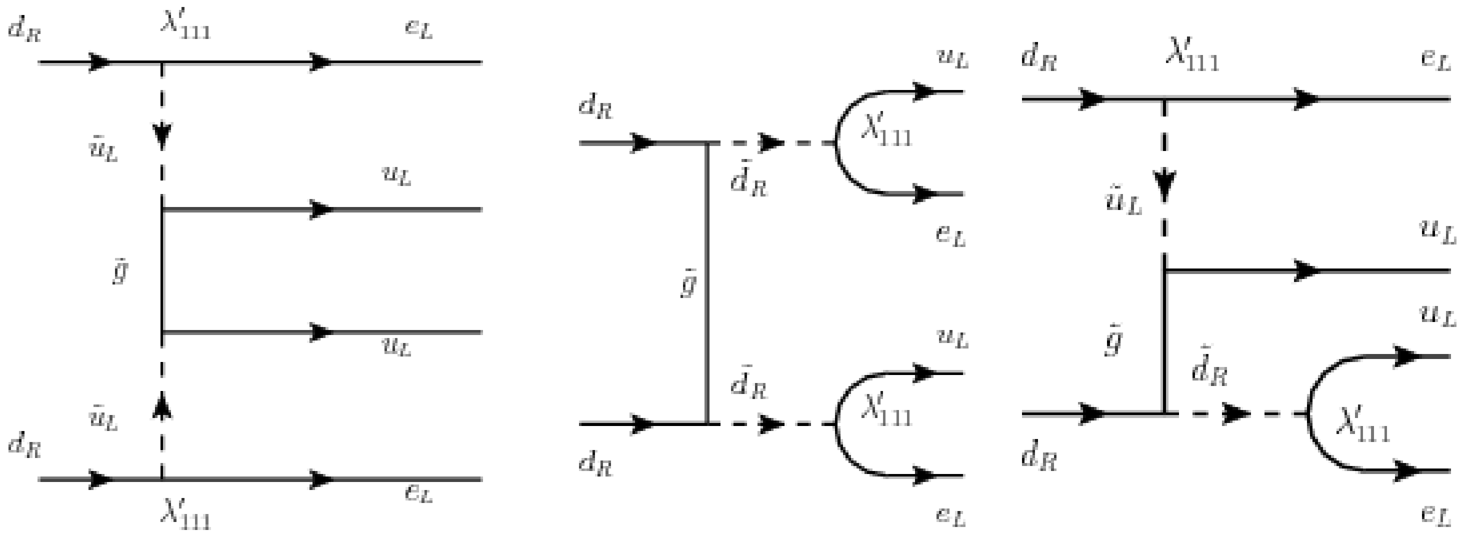
$$\eta_\nu = \frac{<m>}{m_e}\,.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim 10$ GeV:

$\eta_N^L \;=\; \Sigma_k^{heavy}\; U_{ek}^2 \frac{m_p}{M_k}$, m_p - proton mass, U_{ek} - CPV.

SUSY Models with R-Parity Non-conservation



$$\mathcal{L}_{R_p} = \lambda'_{111}\Big[(\bar{u}_L\ \bar{d}_L)\begin{pmatrix} e^c_R \\ -\nu^c_{eR}\end{pmatrix}\tilde{d}_R + (\bar{e}_L\ \bar{\nu}_{eL})d_R\begin{pmatrix}\tilde{u}^*_L \\ -\tilde{d}^*_L\end{pmatrix}$$
$$+ (\bar{u}_L\ \bar{d}_L)d_R\begin{pmatrix}\tilde{e}^*_L \\ -\tilde{\nu}^*_{eL}\end{pmatrix}\Big] + h.c.$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$-decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, "Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nuclei with Largely Different Nuclear Matrix Elements", (arXiv:1212.1331, JHEP **1302** (2013) 025.

Earlier studies include:
A. Halprin, S.T.P., S.P. Rosen, "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335).

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on ${}^3\mathrm{H} \to {}^3\,\mathrm{He} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
UNITED KINGDOM
IRELAND
Dublin
Belfast
Leeds
Manchester
Liverpool
Birmingham
Cardiff
London
Celtic Sea
Irish Sea
Amsterdam
Rotterdam
NETH.
Brussels
Lille
BEL.
Guernsey (U.K.)
Jersey (U.K.)
Paris
Nantes
Bay of Biscay
FRANCE
Bordeaux
Lyon
Toulouse
Marseille
Andorra
MONACO
DENMARK
Copenhagen
Malmö
Bornholm
Kaliningrad
RUSSIA
Hamburg
Bremen
Berlin
Essen
Cologne
Bonn
Leipzig
GERMANY
Frankfurt am Main
Luxembourg
LUX.
Stuttgart
Munich
Strasbourg
Zurich
Bern
Geneva
Vaduz
LIECH.
Milan
Turin
Genoa
Venice
Florence
Ligurian Sea
Poznań
Gdańsk
POLAND
Łódź
Warsaw
Wrocław
Kraków
Prague
CZECH REPUBLIC
Brno
SLOVAKIA
Bratislava
Vienna
AUSTRIA
Budapest
HUNGARY
Ljubljana
SLOVENIA
Zagreb
CROATIA
BOSNIA AND HERZEGOVINA
Sarajevo
Belgrade
Serbia
Vilnius
Minsk
Mahilyow
BELARUS
Hrodna
Homyel'
Brest
Kiev
Rivne
L'viv
UKRAINE
Chernivtsi
Mykolayiv
Chişinău
Iaşi
MOLDOVA
Odesa
Cluj-Napoca
ROMANIA
Bucharest
Constanţa
Black Sea
Varna
BULGARIA
Sofia
Skopje
Tirana
Istanbul
Bursa
Thessaloniki
TURKEY
GREECE
Athens
Aegean Sea
Izmir
Rhodes
Crete
Ionian Sea
Naples
Tyrrhenian Sea
Sardinia
Cagliari
Palermo
Sicily
Malta
Valletta
Tunis
TUNISIA
Algiers
ALGERIA
Oran
Melilla (SPAIN)
Ceuta (SPAIN)
Rabat
Casablanca
MOROCCO
Strait of Gibraltar
Alboran Sea
Gibraltar
Malaga
Sevilla
SPAIN
Madrid
Valencia
Barcelona
Bilbao
Porto
Lisbon
PORTUGAL
Balearic Sea
BALEARIC ISLANDS
Mediterranean Sea
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \le 25$) + ACT (large $l \ge 2500$) CMB data + ΛCDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

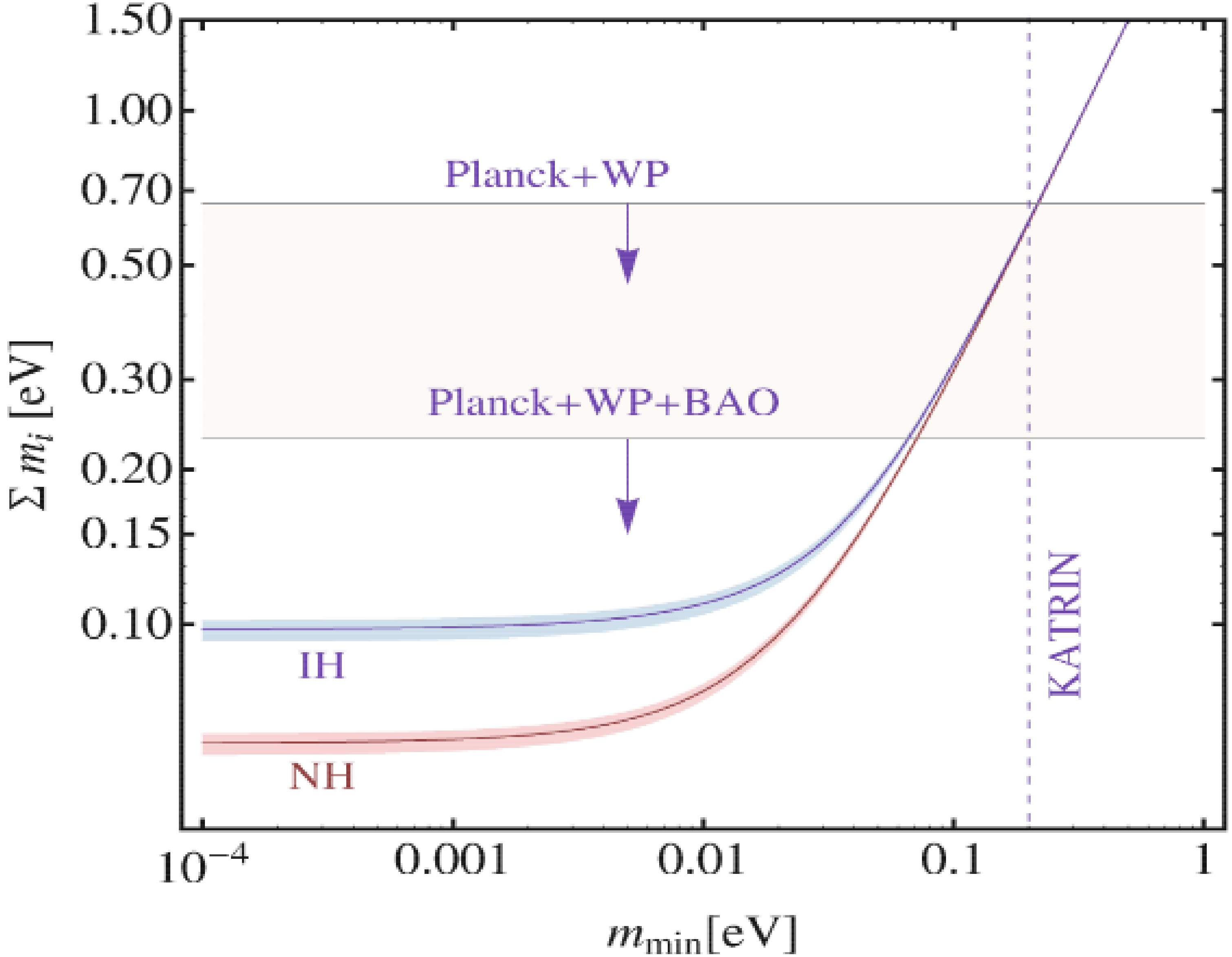
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP, and Planck experiments, and/or data from future EUCLID experiment, might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04) \text{ eV}.$$

NH: $\sum_j m_j \le 0.05$ eV (3σ);

IH: $\sum_j m_j \ge 0.10$ eV (3σ).

Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q, \ q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986; GUT's: M. Yoshimura, 1978.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\mathrm{lep}}(x) = \mathcal{L}_{\mathrm{CC}}(x) + \mathcal{L}_{\mathrm{Y}}(x) + \mathcal{L}_{\mathrm{M}}^{\mathrm{N}}(x)\,,$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{CC}} &= -\,\frac{g}{\sqrt{2}}\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,\\
\mathcal{L}_{\mathrm{Y}}(x) &= \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,\\
\mathcal{L}_{\mathrm{M}}^{\mathrm{N}}(x) &= -\,\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.
\end{aligned}$$

ψ_{lL} - LH doublet, $\psi_{lL}^T = (\nu_{lL}\ \ l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \mathrm{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\mathrm{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\ \ v = 174\ \text{GeV},\ \ v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\,\lambda^T\,M_R^{-1}\,\lambda = U^*_{\mathrm{PMNS}}\,m_\nu^{\mathrm{diag}}\,U^\dagger_{\mathrm{PMNS}}\ .$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\mathrm{PMNS}})^\dagger/v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

$$m_\nu \simeq v^2 \ \lambda^T M_R^{-1} \lambda = U^*_{\mathsf{PMNS}} \, m_\nu^{\mathsf{diag}} \, U^\dagger_{\mathsf{PMNS}} \, ,$$

$$\lambda \equiv Y_\nu$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\ R\ \sqrt{D_\nu}\ (U_{\text{PMNS}})^\dagger / v_u$, all at M_R;

$$R\text{-complex},\ \ R^T R = 1.$$

J.A. Casas and A. Ibarra, 2001

$D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

Models: R - CP conserving ($SU(5) \times T'$); CPV parameters in R determined by the CPV phases in U (class of A_4 models).
Texture zeros in Y_ν: CPV parameters in R and U - related.

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j,\;\; C(\overline{N}_k)^T = N_k,\;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\text{CP}}\, N_j(x)\, U^{\dagger}_{\text{CP}} &= \eta^{NCP}_j\, \gamma_0\, N_j(x')\,, \quad \eta^{NCP}_j = i\rho^N_j = \pm i\,, \\ U_{\text{CP}}\, \nu_k(x)\, U^{\dagger}_{\text{CP}} &= \eta^{\nu CP}_k\, \gamma_0\, \nu_k(x')\,, \quad \eta^{\nu CP}_k = i\rho^\nu_k = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda^*_{jl} = \lambda_{jl}\, (\eta^{NCP}_j)^*\, \eta^l\, \eta^{H*}\,, \quad j = 1,2,3,\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i,\;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\begin{aligned} \lambda^*_{jl} &= \lambda_{jl}\, \rho^N_j\,,\; \rho^N_j = \pm 1\,, \\ U^*_{lj} &= U_{lj}\, \rho^\nu_j\,,\; \rho^\nu_j = \pm 1\,, \\ R^*_{jk} &= R_{jk}\, \rho^N_j\, \rho^\nu_k, \quad j,k = 1,2,3,\; l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$\begin{aligned} P_{jkml} &\equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,,\; k \neq m\,, \\ CP: \quad P^*_{jkml} &= P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \text{Im}(P_{jkml}) = 0\,. \end{aligned}$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,,\ k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at "high" E.

Baryon Asymmetry

$$Y_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}\,, \qquad \text{CMB}$$

Sakharov conditions for a dynamical generation of $Y_B \neq 0$ in the Early Universe

- B **number non-conservation.**
- **Violation of C and CP symmetries.**
- **Deviation from thermal equilibrium.**

Leptogenesis

• The heavy Majorana neutrinos N_i are in equilibrium in the Early Universe as far as the processes which produce and destroy them are efficient.

• When $T < M_1$, N_1 drops out of equilibrium as it cannot be produced efficiently anymore.

• If $\Gamma(N_1 \to \Phi^- \ell^+) \neq \Gamma(N_1 \to \Phi^+ \ell^-)$, a lepton asymmetry will be generated.

• Wash-out processes, like $\Phi^+ + \ell^- \to N_1$, $\quad \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc. tend to erase the asymmetry. Under the condition of non-equilibrium, they are less efficient than the direct processes in which the lepton asymmetry is created. The final resullt is a net (non-zero) lepton asymmetry.

• This lepton asymmetry is then converted into a baryon asymmetry by $(B+L)$ violating but $(B-L)$ conserving sphaleron processes which exist within the SM (at $T \gtrsim M_{\rm EWSB}$).

S. Fukugita, T. Yanagida, 1986.

In order to compute Y_B:

1. calculate the CP-asymmetry:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

2. solve the Boltzmann (or similar) equation to account for the wash-out of the asymmetry:

$$Y_{\mathsf{L}} = \kappa\,\varepsilon$$

where $\kappa = \kappa(\widetilde{m})$ is the "efficiency factor", $\widetilde{m}$ is the "the wash-out mass parameter" - determines the rate of wash-out processes;

3. the lepton asymmetry is converted into a baryon asymmetry:

$$Y_{\mathsf{B}} = \frac{c_s}{g_*}\kappa\varepsilon$$

Baryon number violation in the SM

Instanton and Sphaleron processes

SU(2) instantons lead to (leading order) to effective 12 fermion $(B+L)$ nonconserving, but $(B-L)$ conserving, interactions:

$O(B+L) = \Pi_i q_{Li} q_{Li} q_{Li} l_{Li}$

These would induce $\Delta B = \Delta L = 3$ processes:

$$u_L + d_L + c_L + s_L + t_L + b_L + \nu_{eL} + \nu_{\mu L} + \nu_{\tau L} \to \bar{d}_R + \bar{b}_R + \bar{s}_R$$

However, at T = 0 the probability of such processes is $\Gamma/V \sim e^{-4\pi/\alpha} \sim 10^{-165}$.

't Hooft, 1976

At finite T, the transitions proceed via thermal fluctuations (over the barrier) with an unsuppressed probability (due to sphaleron (static) configurations - saddle "points" of the field energy of the $SU(2)$ gauge - Higgs field system):

$$\Gamma/V \sim \alpha^4 T^4.$$

Kuzmin, Rubakov, Shaposhnikov, 1985;
Arnold et al., 1987 and 1997.

Sphaleron processes are efficient (in the case of interest) at

$$T_{\text{EW}} \sim 100 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Can generate $B \neq 0$, $L \neq 0$ at $T_{\text{EW}} < T (< 10^{12}$ GeV) from $(B-L)_0 \neq 0$ (with $(B-L) = const.$).

Harvey, Turner, 1990

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma:\ \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2}\ \ \varepsilon\, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ– efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

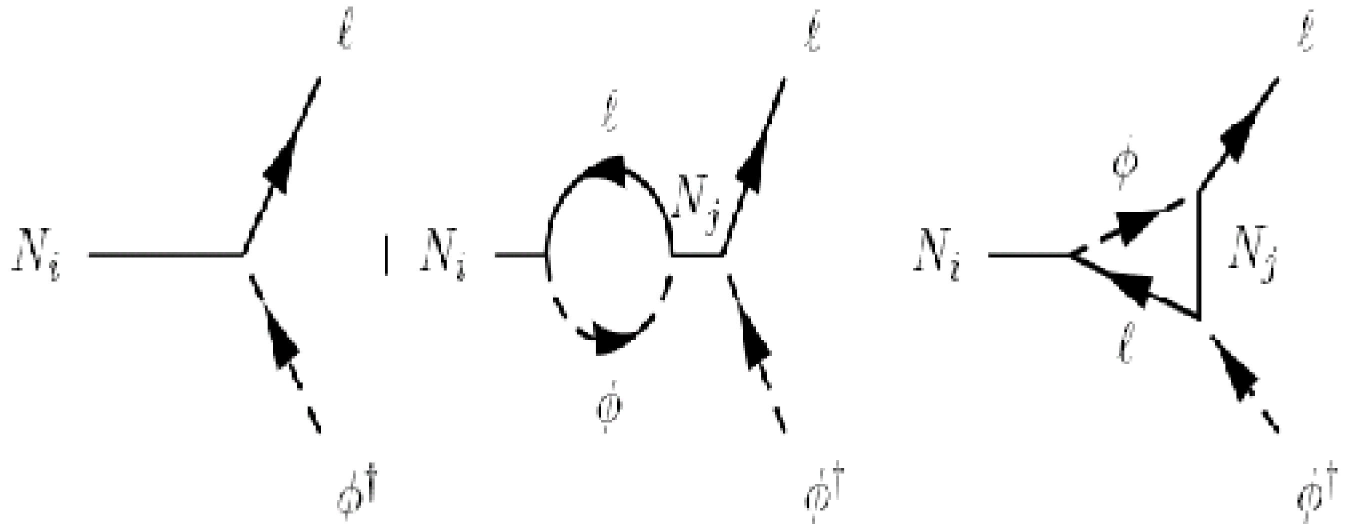
$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004


N_i
ℓ
$\phi^\dagger$
N_i
ℓ
ϕ
N_j
ℓ
$\phi^\dagger$
N_i
ϕ
ℓ
N_j
ℓ
$\phi^\dagger$

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} = \left| \sum_k R_{1k} \, m_k^{1/2} \, U_{lk}^* \right|^2, \quad l = e, \mu, \tau \, .$$

The "one-flavor" approximation - $\mathbf{Y}_{e,\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l \; H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \, |R_{1k}|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \, |R_{1k}|^2 \, .$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \to (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \to N_1$;

$\tau_L^- + \Phi^0 \to \tau_R^-$, $\quad \tau_L^- + \tau_L^+ \to N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^{9}$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 \, (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} \;\; = \;\; \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2 , \quad l = e, \mu, \tau \, .$$

The baryon asymmetry is

$$Y_B \;\; \simeq \;\; -\frac{12}{37 g_*} \left(\epsilon_2 \, \eta \left(\frac{417}{589} \, \widetilde{m_2} \right) + \epsilon_\tau \, \eta \left(\frac{390}{589} \, \widetilde{m_\tau} \right) \right) ,$$

$$\eta \left(\widetilde{m_l} \right) \;\; \simeq \;\; \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3} \, \mathrm{eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \, \mathrm{eV}}{\widetilde{m_l}} \right)^{-1.16} \right)^{-1} .$$

$Y_B = -(12/37) \, (Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, R_{1j}R_{1k} = \pm \left|R_{1j}R_{1k}\right|, \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, \left|R_{1j} R_{1k}\right| \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, R_{1j}R_{1k} = \pm i\left|R_{1j}R_{1k}\right|
\end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, $\mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right) \right)$$

$m_1 \ll m_2 \ll m_3$, $M_1 \ll M_{2,3}$; $R_{12}R_{13}$ – real; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi$, $\delta = 0$: $\mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0$, CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0 \ (2\pi)$, $\beta_{23} = \pi \ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \ \cong 2.8 \times 10^{-13} \, |\sin\delta| \left(\frac{s_{13}}{0.2}\right) \left(\frac{M_1}{10^9 \ \text{GeV}}\right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.11 \,, \qquad \sin\theta_{13} \gtrsim 0.11 \,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0 \ (2\pi)$, $\beta_{23} = 0 \ (\pi)$:

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.09 \,, \qquad \sin\theta_{13} \gtrsim 0.09 \,; \qquad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

The requirement $\sin\theta_{13} \gtrsim 0.09$ (0.11) - compatible with the Daya Bay result: $\sin\theta_{13} \cong 0.15$.

$|\sin\theta_{13}\sin\delta| \gtrsim 0.11$ implies $|\sin\delta| \gtrsim 0.7$ - compatible with $\delta \cong 3\pi/2$.

$\sin\theta_{13} \cong 0.15$ and $\delta \cong 3\pi/2$ imply relatively large (observable) CPV effects in neutrino oscillations: $J_{CP} \cong -3.5 \times 10^{-2}$.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}}\right) \left(\frac{M_1}{10^9 \text{ GeV}}\right) .$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m^2_{\odot}/||\cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

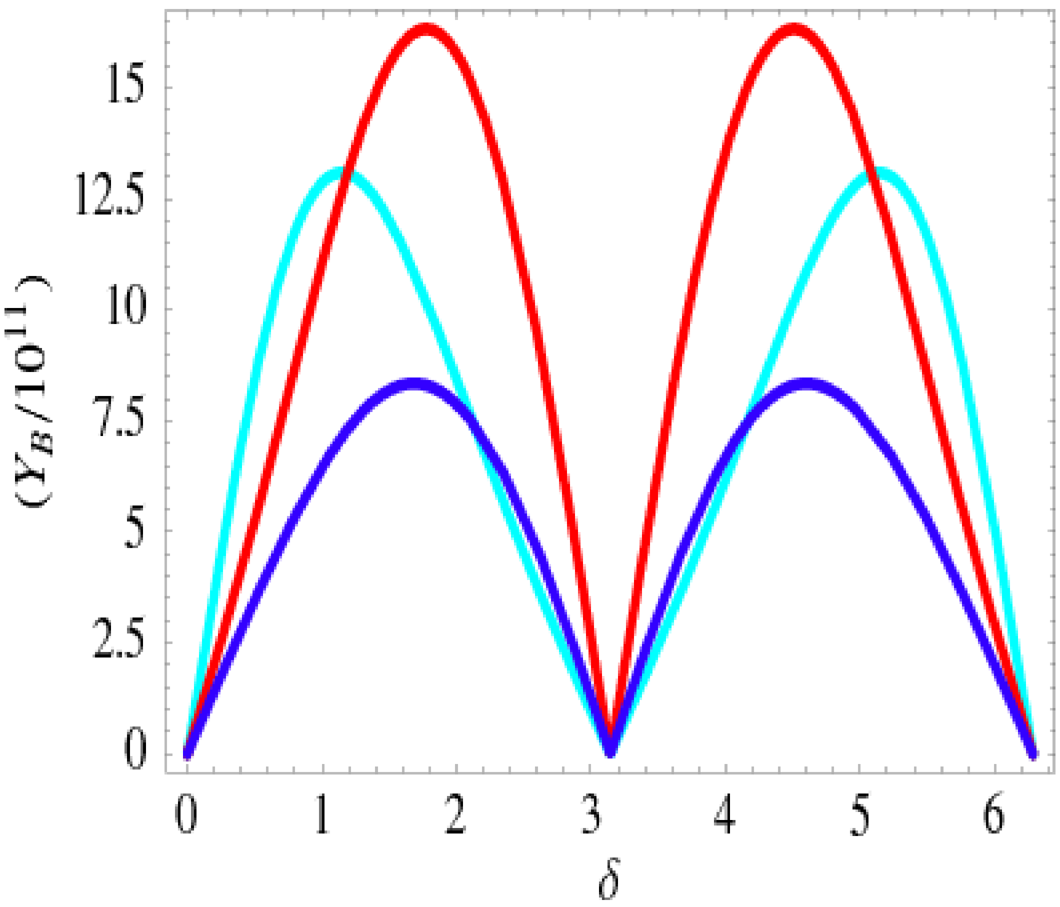
$$|Y_B| \cong 8.1 \times 10^{-12}\, |s_{13}\, \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\, \sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,.$$

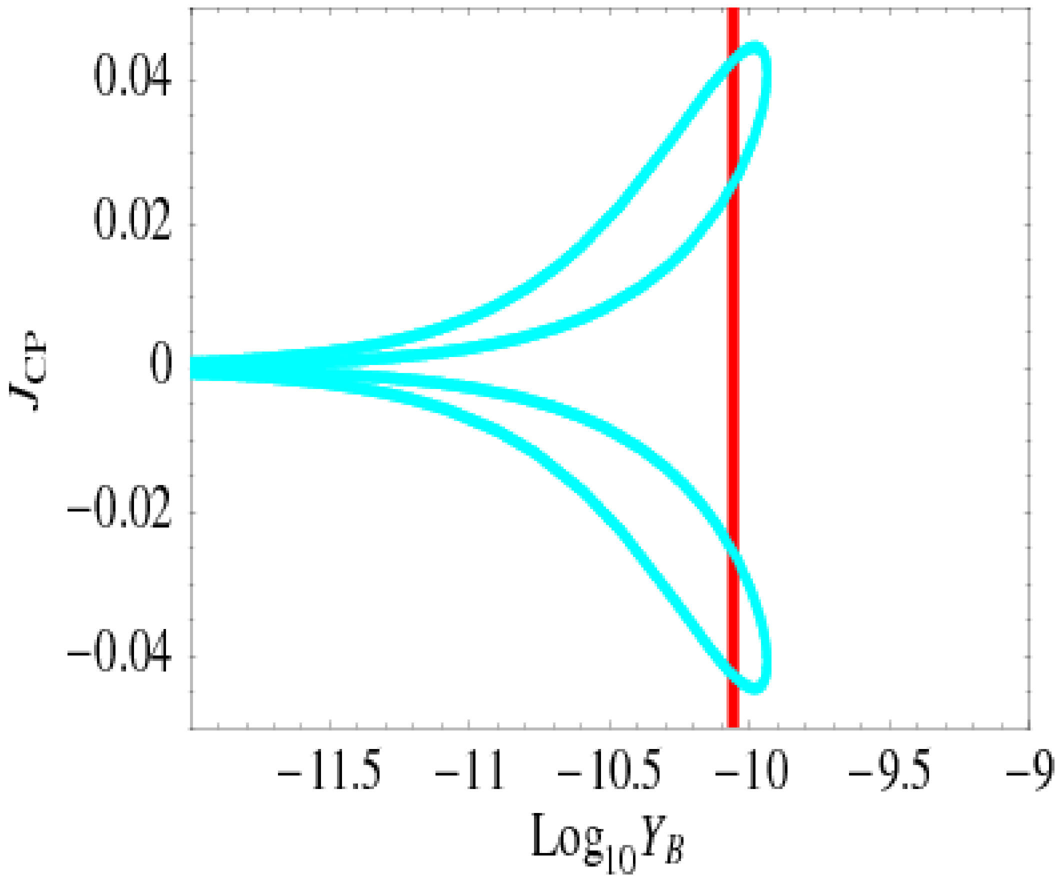
The lower limit corresponds to

$$|J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$



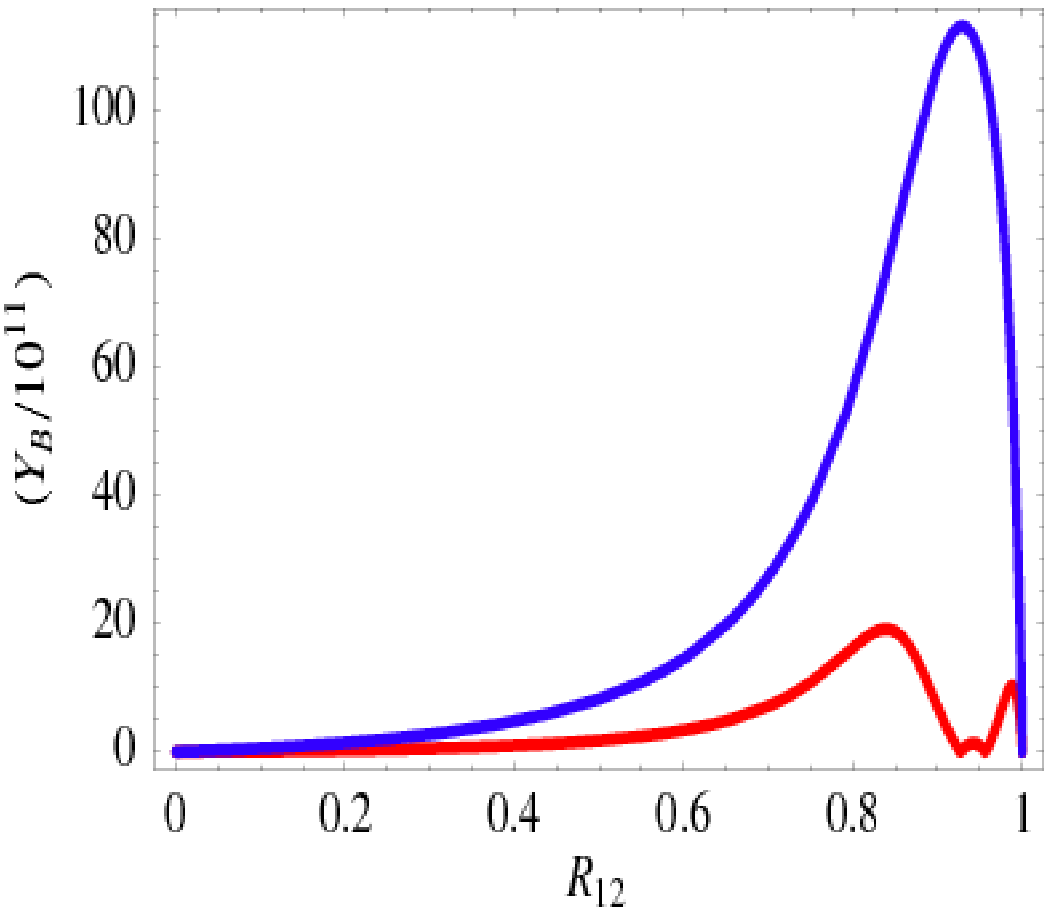
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

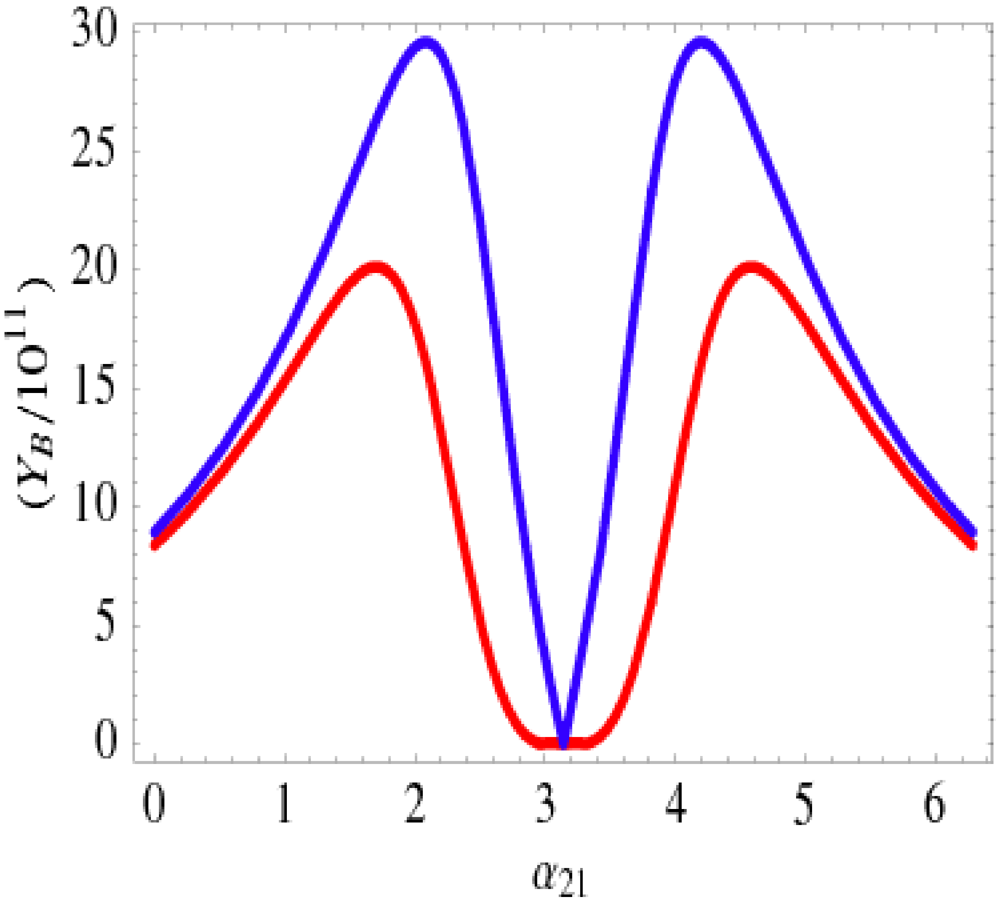


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0$ (2π);
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$ (-1) ($\beta_{23} = 0$ (π), $\kappa' = +1$);
The red region denotes the 2σ allowed range of $Y_{\rm B}$.

S. Pascoli, S.T.P., A. Riotto, 2006.

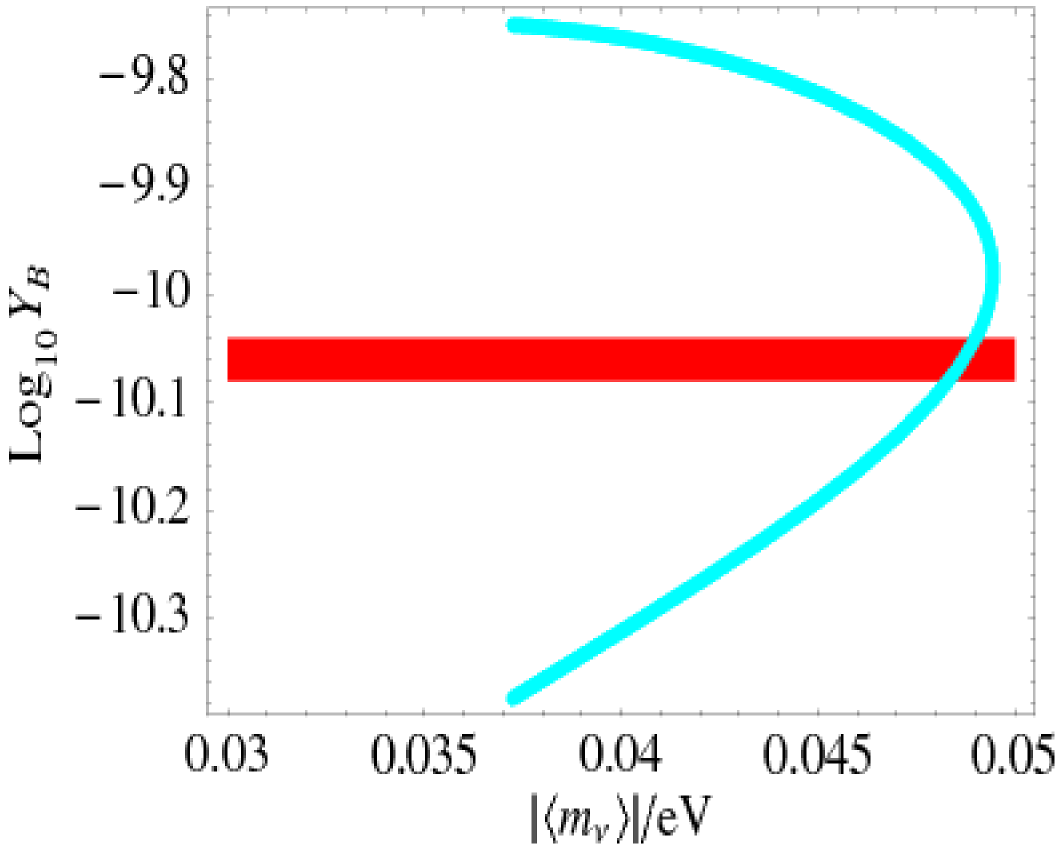


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign $(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

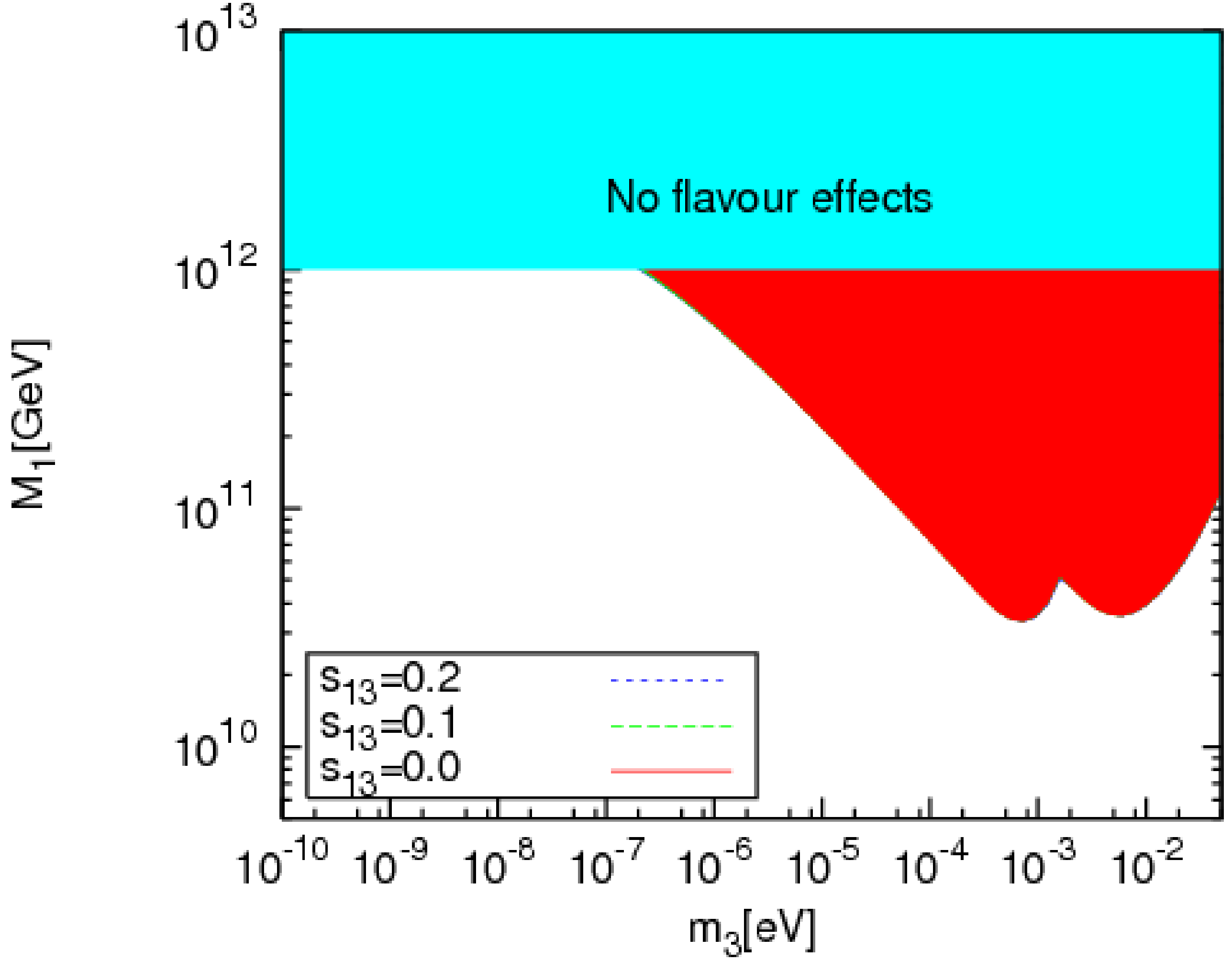
$$|\sin\theta_{13}\sin\delta|,\ \sin\theta_{13} \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{\mathrm{CP}}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 <\ (\ll)\ m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0;$

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0\times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6\times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5\times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Complex R: $\varepsilon_{1l} \neq 0$, CPV from U and R

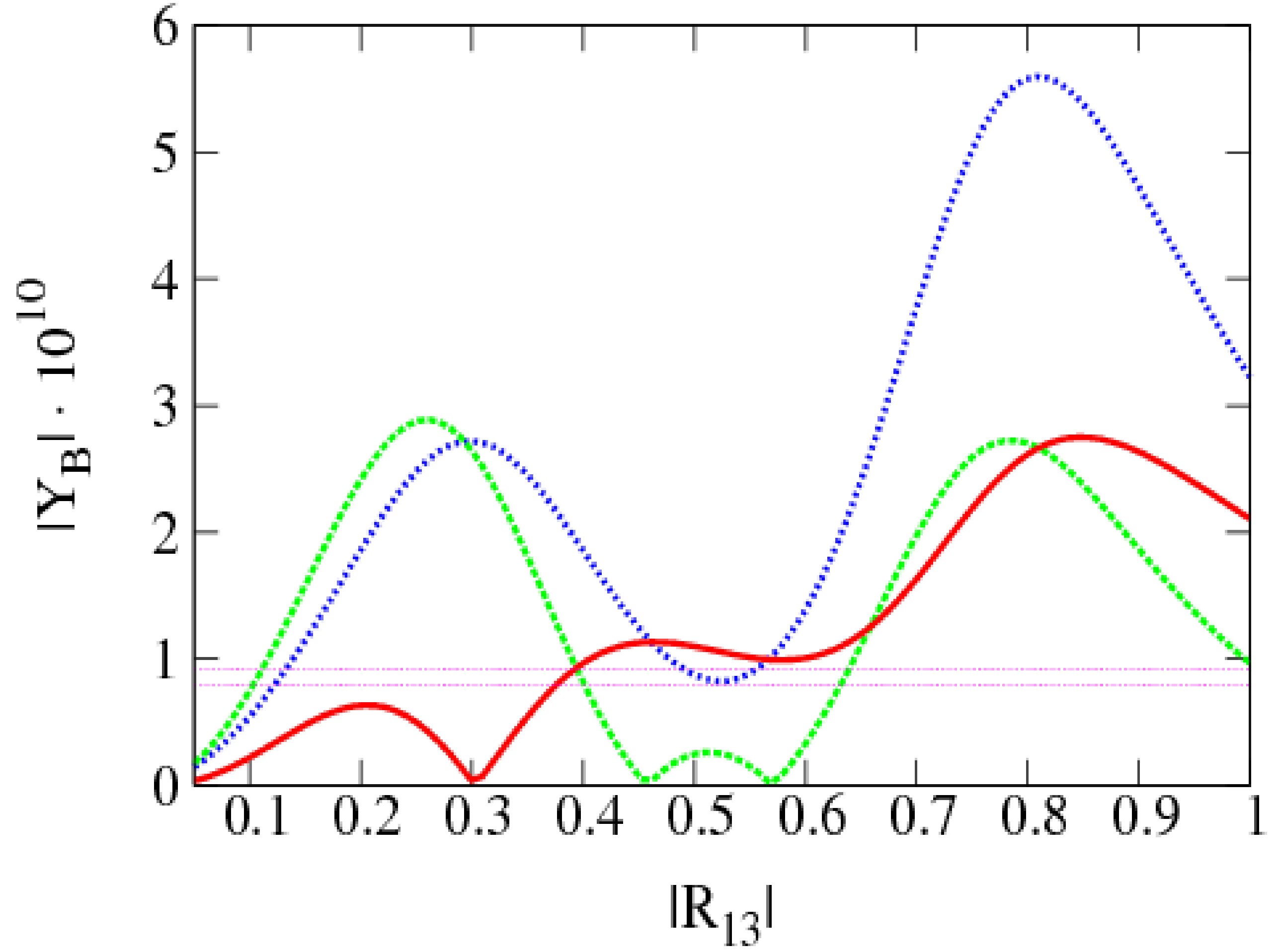
$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$R_{12}^2 + R_{13}^2 = |R_{12}|^2 e^{i2\varphi_{12}} + |R_{13}|^2 e^{i2\varphi_{13}} = 1\,,$

$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0:\ \mathrm{sgn}(\sin 2\varphi_{12}) = -\mathrm{sgn}(\sin 2\varphi_{13})\,.$

$\cos 2\varphi_{12} = \frac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}\,, \quad \sin 2\varphi_{12} = \pm\sqrt{1-\cos^2 2\varphi_{12}}\,,$

$\cos 2\varphi_{13} = \frac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}\,, \quad \sin 2\varphi_{13} = \mp\sqrt{1-\cos^2 2\varphi_{13}}\,.$



$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0.2$, $\delta = 0$, $\sin^2\theta_{23} = 0.64$, $|R_{12}| \cong 1$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line)

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis (contd.)

E. Interesting case: CPV due to the Majorana phases in U_{PMNS} and the R-phases

$m_3 \ll m_1 < m_2$ (IH), $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $\text{Im}(R_{13}^2) = 0$.

$R_{11}^2 + R_{12}^2 + R_{13}^2 = 1$:

$|R_{11}|^2 e^{i2\varphi_{11}} + R_{12}^2 e^{i2\varphi_{12}} + R_{13}^2 = 1$,

$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0$.

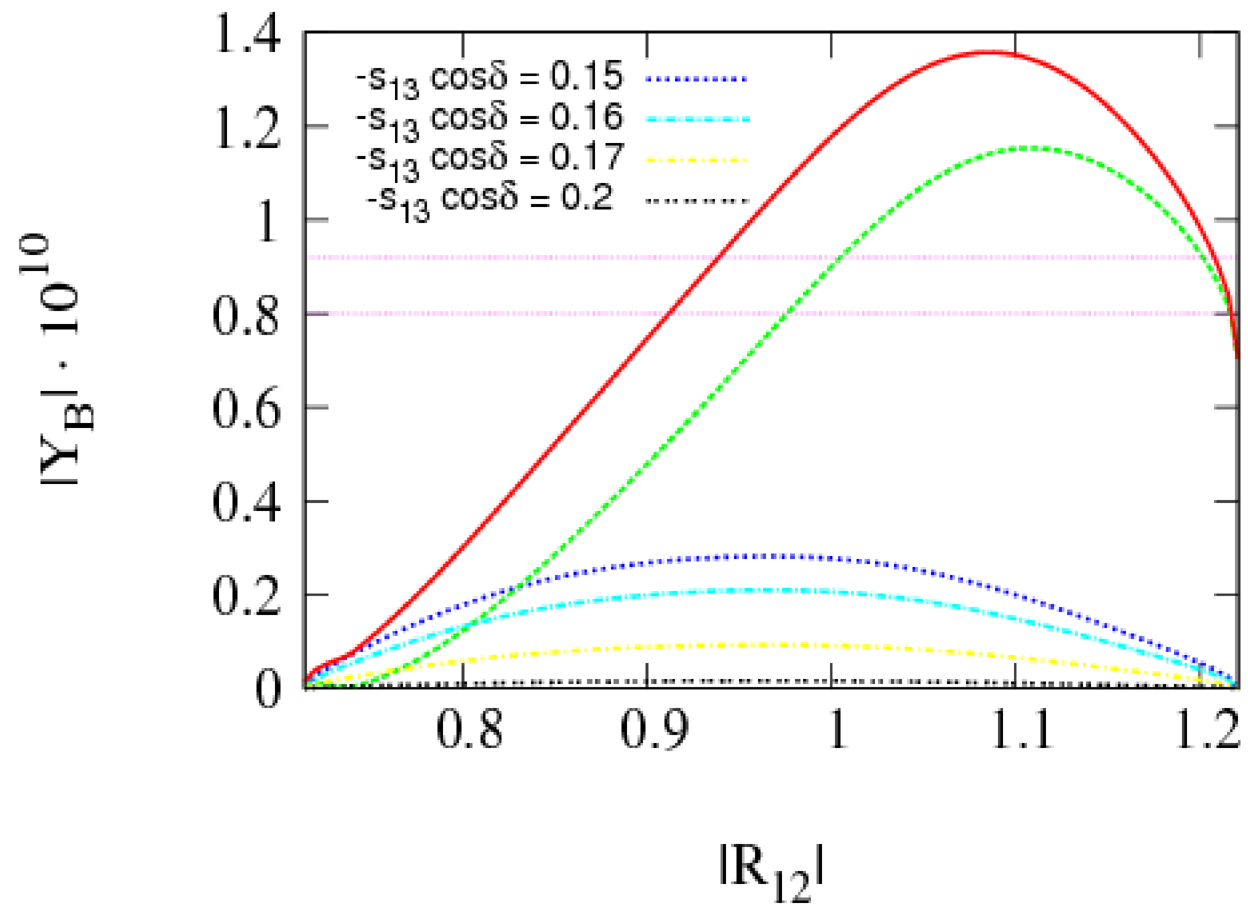
$|Y_B^0 A_{\text{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11}) \left(|U_{\tau 1}|^2 - |U_{\tau 2}|^2\right)$ - can be suppressed:

$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2) s_{23}^2 - 4 s_{12} c_{12} s_{23} c_{23} s_{13} \cos\delta \cong -0.20 - 0.92\, s_{13} \cos\delta$.

$\sin^2\theta_{12} = 0.3$, $\sin^2\theta_{23} = 0.5$: $(-\sin\theta_{13} \cos\delta) \gtrsim 0.15$

($\sin^2\theta_{12} = 0.38$, $\sin^2\theta_{23} = 0.36$: $0.06 \lesssim (-\sin\theta_{13} \cos\delta) \lesssim 0.12$)

E. Molinaro, S.T.P., 2008, 2010.



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV,
$\alpha_{21} = \pi/2$, $(-s_{13}\cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\rm HE}|$ (R CPV, blue), $|Y_B^0 A_{\rm MIX}|$ (U CPV, green), total $|Y_B|$ (red line).

E. Molinaro, S.T.P., 2008

The preceding results: for

$|R_{13}|^2|\sin(2\tilde{\varphi}_{13})| << \min(|R_{11,12}|^2|\sin(2\tilde{\varphi}_{11,12})|.$

Results for arbitrary complex R_{13}:

the "high energy" contribution to the BAU is subdominat (or strongly suppressed) for, e.g., $M_1 = 10^{11}$ GeV and arbitrary $\arg(R_{13}) \equiv \tilde{\varphi}_{13}$ if

- for $|R_{11}| < 0.5$, $|R_{13}|$ satisfies $|R_{13}| \lesssim |R_{11}|$;
- for $0.5 \lesssim |R_{11}| < 1$ we have $|R_{13}| < 0.5$;
- and if for $|R_{11}| > 1$ we have $|R_{13}| < |R_{11}|/2$.

In each of these cases we can have successful leptogenesis due to the contribution to the baryon asymmetry associated with the Majorana CP violating phase(s) in the neutrino mixing matrix.

Conclusions

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

Dirac and Majorana CPV may have the same source.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

The see-saw mechanism provides a link between the ν-mass generation and the baryon asymmetry of the Universe (BAU).

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Low energy leptonic CPV can be directly related to the existence of BAU.

These results underline further the importance of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.

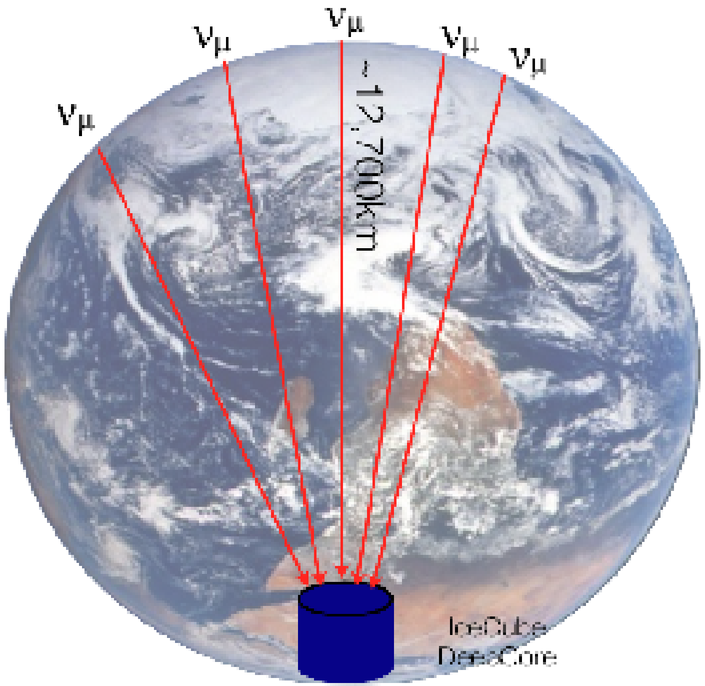
Conclusions (Contd.)

We are at the beginning of the Road...

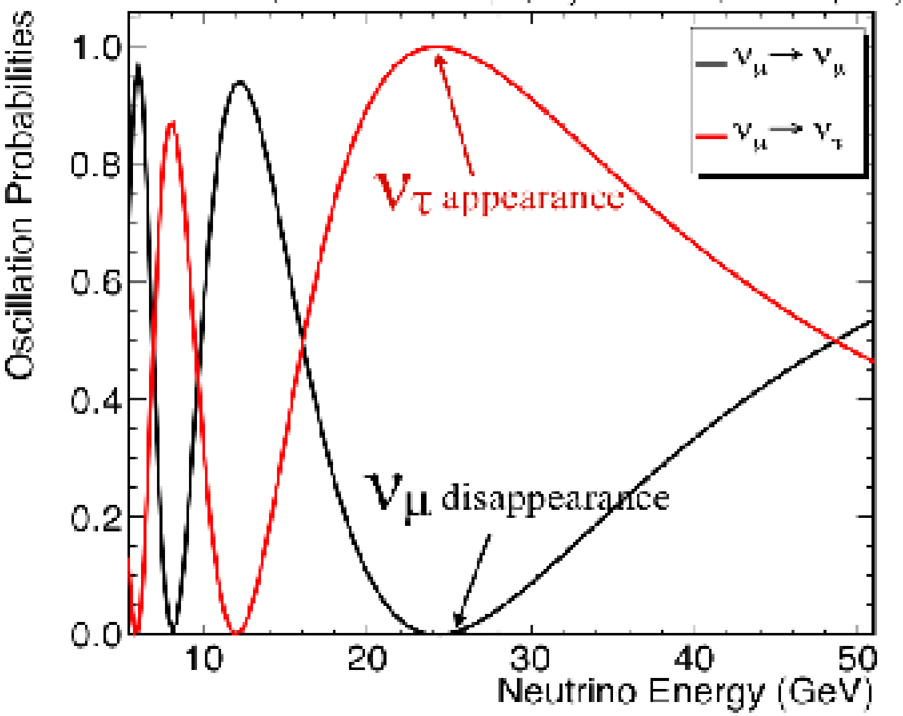
The future of neutrino physics is bright.

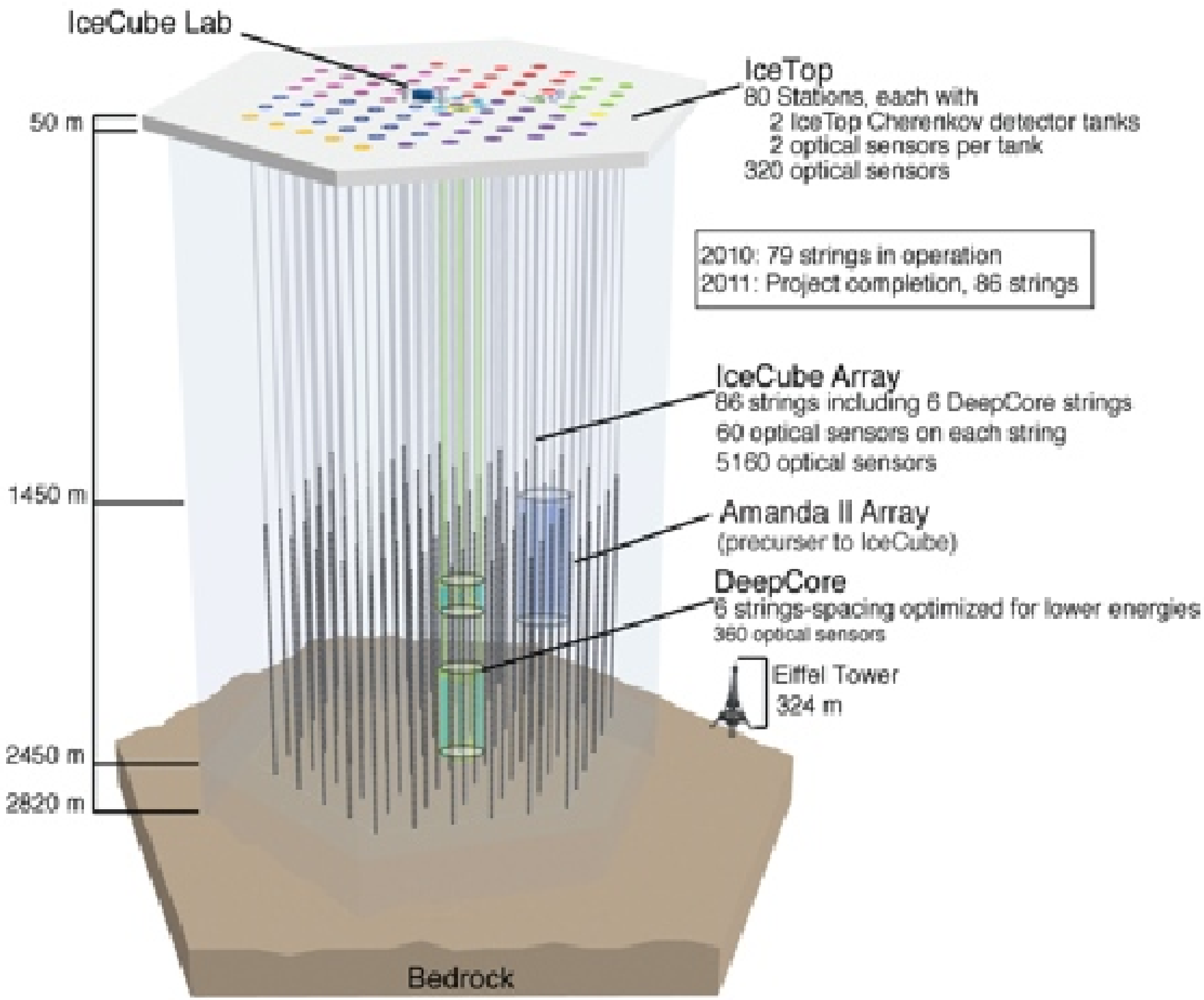
Supporting Slides

- Northern Hemisphere ν_μ oscillating over one earth radii produces ν_μ (ν_τ) oscillation minimum(maximum) at ~25 GeV
  - Covers all possible terrestrial baselines
  - “Beam” is free and never turns off

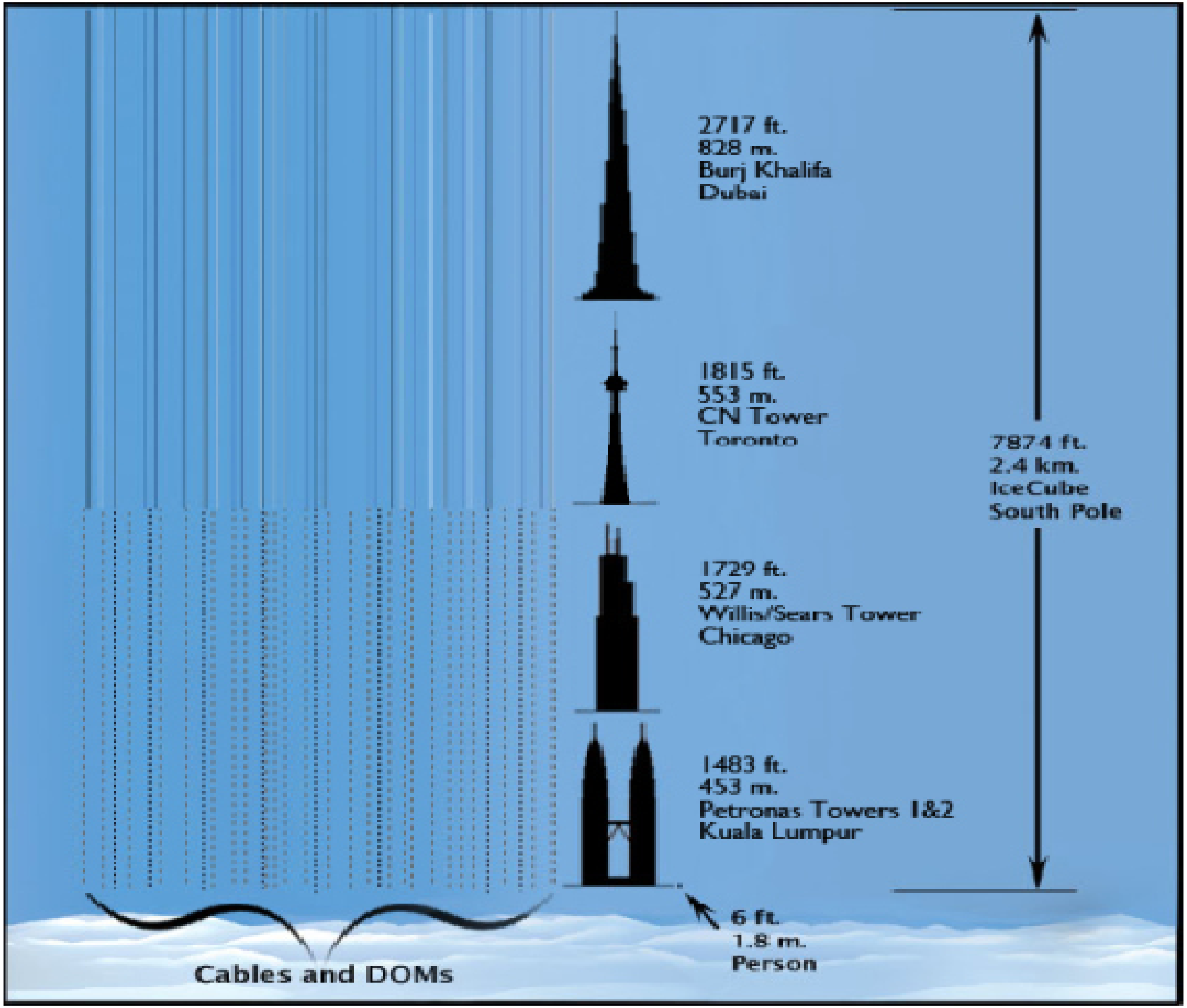


Mena, Mocioiu & Razzaque, *Phys. Rev.* D**78**, 093003 (2008)





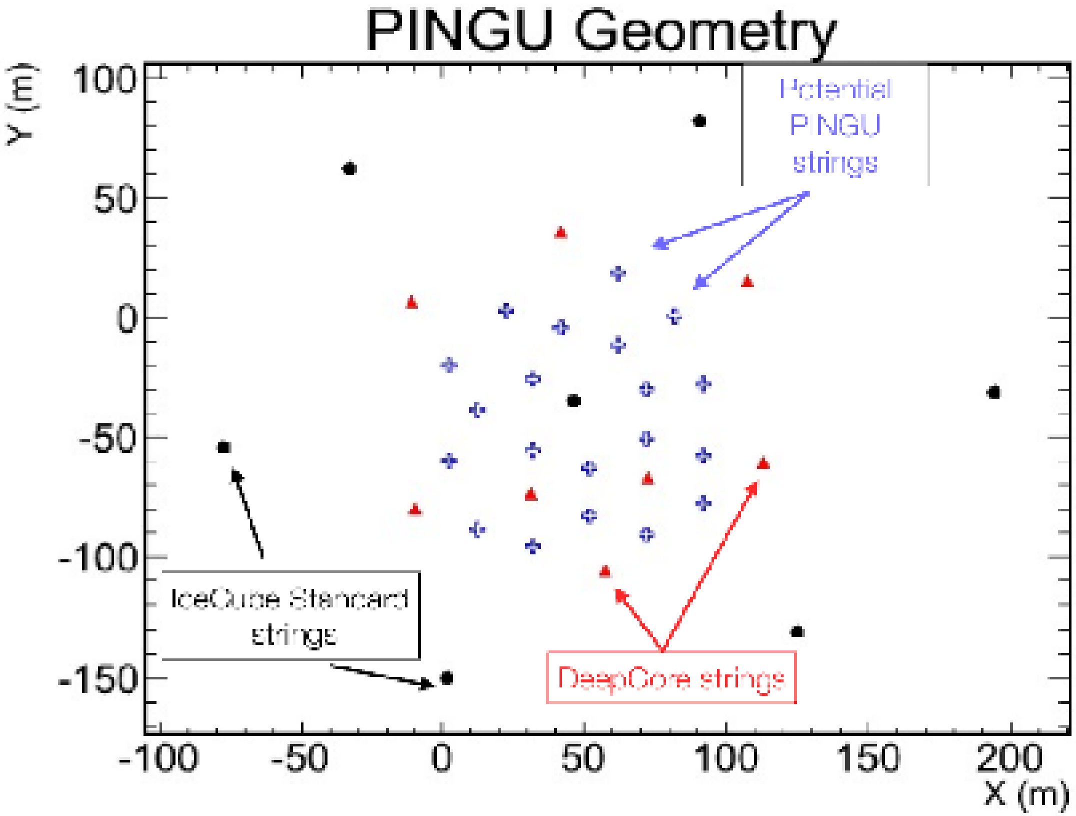
IceCube Lab
IceTop
80 Stations, each with
2 IceTop Cherenkov detector tanks
2 optical sensors per tank
320 optical sensors
2010: 79 strings in operation
2011: Project completion, 86 strings
IceCube Array
86 strings including 6 DeepCore strings
60 optical sensors on each string
5160 optical sensors
Amanda II Array
(precurser to IceCube)
DeepCore
6 strings-spacing optimized for lower energies
360 optical sensors
Eiffel Tower
324 m
50 m
1450 m
2450 m
2820 m
Bedrock



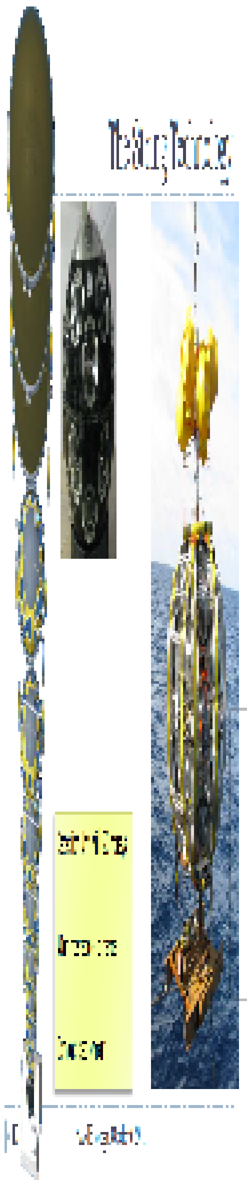
2717 ft.
828 m.
Burj Khalifa
Dubai
1815 ft.
553 m.
CN Tower
Toronto
1729 ft.
527 m.
Willis/Sears Tower
Chicago
1483 ft.
453 m.
Petronas Towers 1&2
Kuala Lumpur
7874 ft.
2.4 km.
IceCube
South Pole
6 ft.
1.8 m.
Person
Cables and DOMs

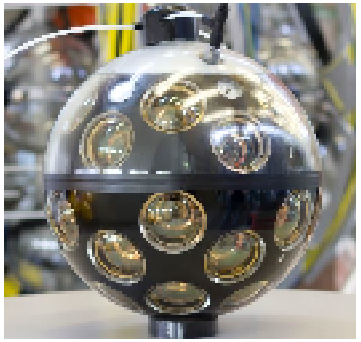
PINGU: Possible Geometry

- ~20 strings within DeepCore volume w/ short string-string spacing
  - IC-IC: 125m
  - DC-DC: ~80m
  - PINGU-PINGU: <= 26m
- Shorter DOM-DOM spacing
  - IC-IC: 17m
  - DC-DC: 7m
  - PINGU-PINGU: <= 5m
- R & D for future water/ice cerenkov

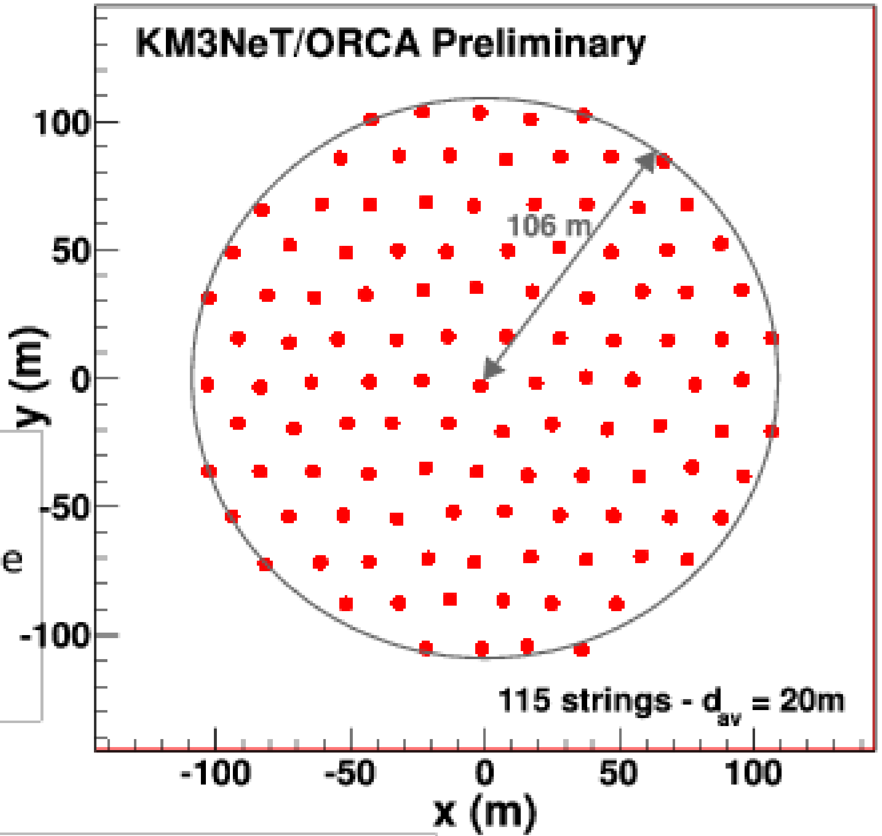


he ORCA proposed detector



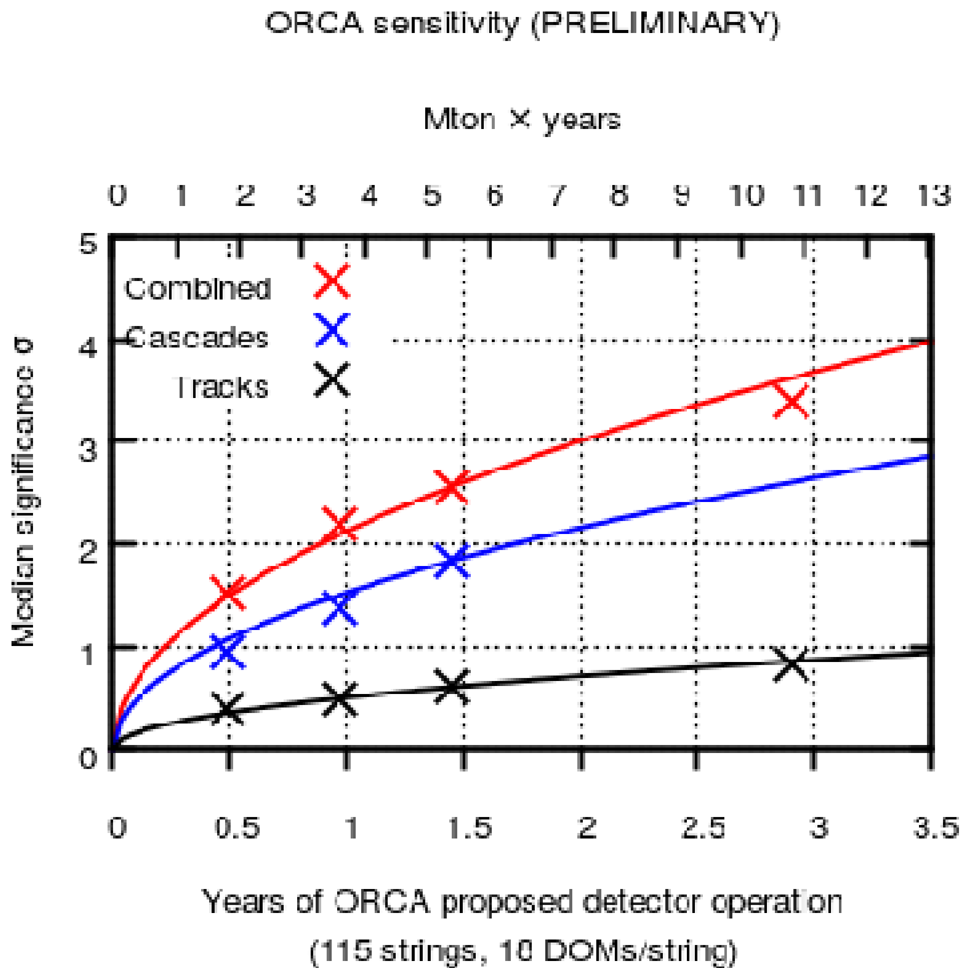


Multi-PMT DOM
31 small PMTs
Almost uniform coverage
Photon counting
Direction of photon
All electronics inside



115 strings (building block) of 18 DOMs each
Estimated cost 40M€

Sensitivity to the NMH



40 M€ investment

The Nature of Massive Neutrinos III:

The Seesaw Mechanisms of Neutrino Mass Generation

M_ν **from the See-Saw Mechanism**

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;

T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explain the smallness of $\nu-$masses.

- Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH νs' (heavy).

Type II seesaw mechanism: $\mathbf{H(x)}$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\mathbf{T(x)}$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$-decay, LFV processes, etc.) and New Physics at LHC.

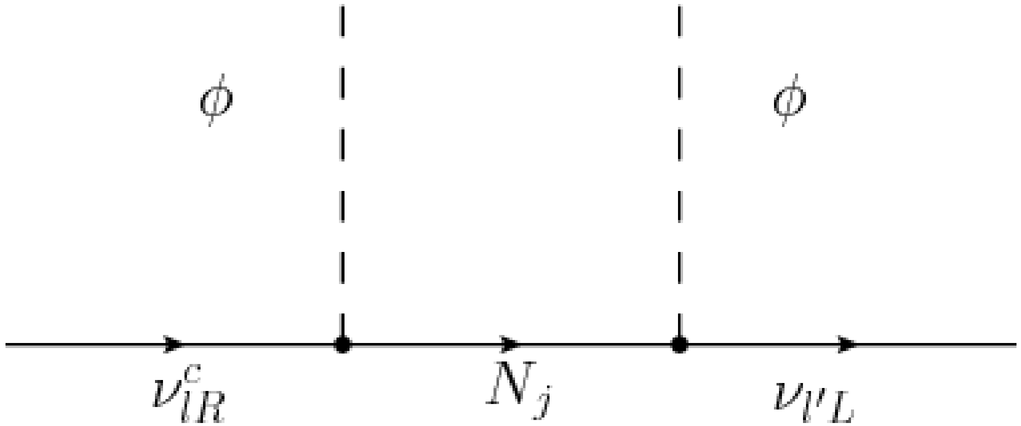
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^{\nu}/\sqrt{2}| << |M^{RR}|$.
- Diagonalising M^{RR}: N_j - heavy Majorana neutrinos, $M_j \sim$TeV; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large M_j, Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \cong v_u^2\,(Y^\nu)^T\,M_j^{-1}\,Y^\nu = U^*_{\mathrm{PMNS}}\,m_\nu^{\mathrm{diag}}\,U^\dagger_{\mathrm{PMNS}}\,.$$

$v_u Y^\nu = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.



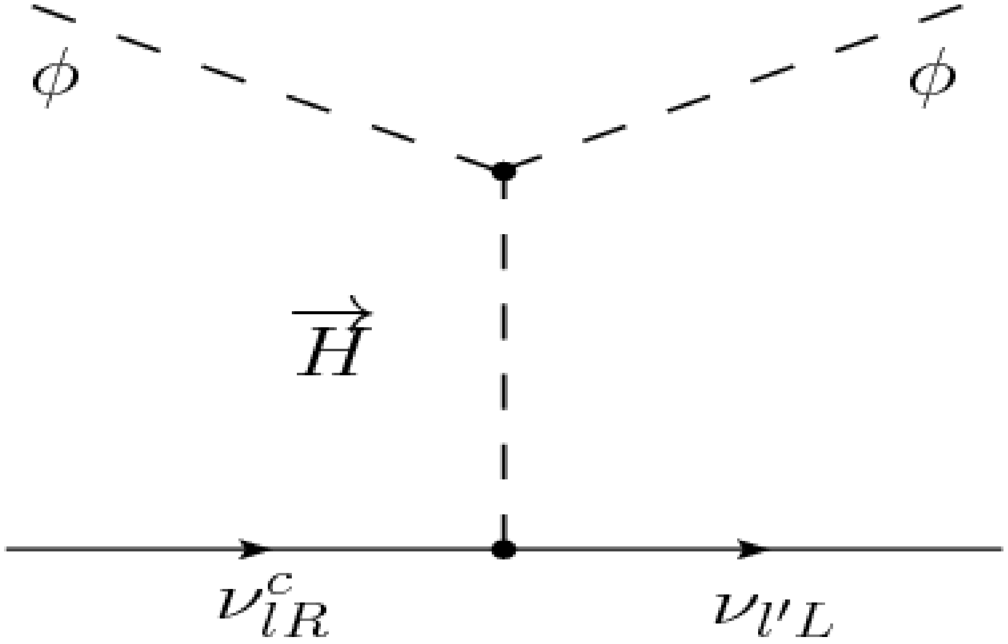
- $\nu_{l'R}(x)$: Majorana mass term at "high scale" ($\sim$TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)

$$\mathcal{L}^{\nu}_{M}(x) = + \, \frac{1}{2} \, \nu^{\top}_{l'R}(x) \; C^{-1} \; (M^{RR})^{\dagger}_{l'l} \; \nu_{lR}(x) + h.c. = -\frac{1}{2} \sum_{j} \bar{N}_j \, M_j \, N_j \; ,$$

- Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:

$$\begin{aligned} \mathcal{L}_{\mathrm{Y}}(x) &= \bar{Y}^{\nu}_{l'l} \, \overline{\nu_{l'R}}(x) \, \Phi^{T}(x) \, (i\tau_2) \, \psi_{lL}(x) + \text{h.c.}\,, \\ &= Y^{\nu}_{jl} \, \overline{N_{jR}}(x) \, \Phi^{T}(x) \, (i\tau_2) \, \psi_{lL}(x) + \text{h.c.}\,, \\ M_D &= \frac{v}{\sqrt{2}} Y^{\nu}, \quad v = 246 \text{ GeV}. \end{aligned}$$

Type II Seesaw Mechanism

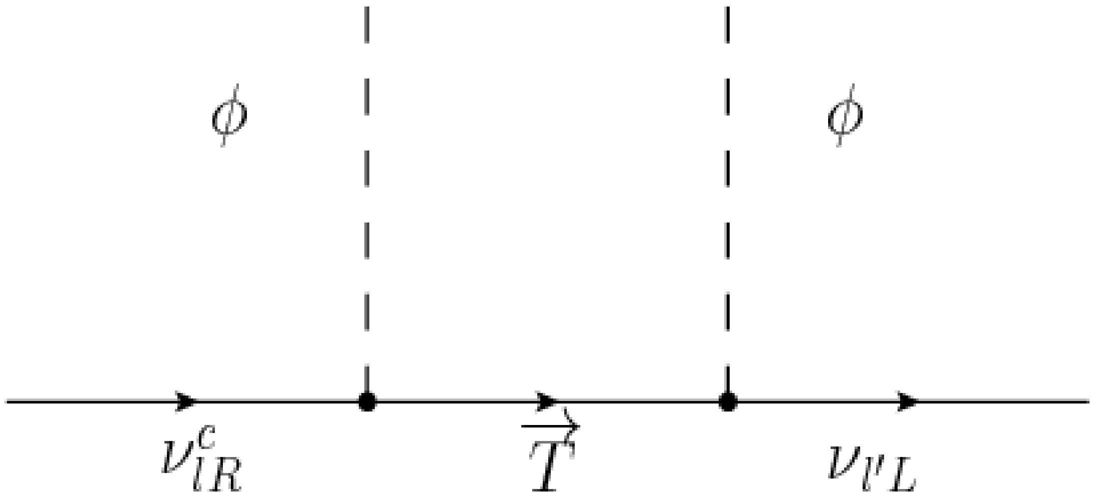


Due to I. Girardi

$$M_\nu \cong h\, v^2\, M_H^{-1} = U^*_{\text{PMNS}}\, m_\nu^{\text{diag}}\, U^\dagger_{\text{PMNS}}\,.$$

$h \sim 10^{-2}$, $v = 246$ GeV, $M_H \sim 10^{12}$ GeV: $M_\nu \sim 0.6$ eV.

Type III Seesaw Mechanism



$$M_\nu \cong v^2 \; (Y_T)^T \, M_T^{-1} \, Y_T = U^*_{\text{PMNS}} \; m_\nu^{\text{diag}} \; U^\dagger_{\text{PMNS}} \; .$$

$v\,Y_T \sim 1$ GeV, $M_T \sim 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.

TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$m_\nu \simeq -\, M_D \hat{M}_N^{-1} M_D^T$, $\hat{M} = \mathrm{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000)$ GeV.

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell}\,\gamma_\alpha\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^\alpha + \text{h.c.}\,, \quad (RV)_{\ell k} \equiv U_{\ell 3+k}\,,$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w}\,\overline{\nu_{\ell L}}\,\gamma_\alpha\,(RV)_{\ell k}\,N_{kL}\,Z^\alpha + \text{h.c.}$$

- **All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:**
  $M_2 = M_1(1+z)$, $0 < z \ll 1$.

- **Only NH and IH ν mass spectra possible:** $\min(m_j) = 0$.

• **Requirements:** $|(RV)_{\ell k}|$ **"sizable"**
+ reproducing correctly the neutrino oscillation data:

$$
\begin{aligned}
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_3}{m_2+m_3}\left|U_{\ell 3}+i\sqrt{m_2/m_3}U_{\ell 2}\right|^2, \quad \text{NH}\,,\\
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_2}{m_1+m_2}\left|U_{\ell 2}+i\sqrt{m_1/m_2}U_{\ell 1}\right|^2 \cong \frac{1}{4}\frac{y^2 v^2}{M_1^2}\left|U_{\ell 2}+iU_{\ell 1}\right|^2, \text{IH}\,,\\
(RV)_{\ell 2} &= \pm i\,(RV)_{\ell 1}\sqrt{\frac{M_1}{M_2}}\,,\ \ell = e,\mu,\tau\,,
\end{aligned}
$$

y- **the maximum eigenvalue of** Y^ν**,** $v_u \simeq 174$ **GeV.**

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

4 parameters: M**,** z**,** y **and a phase** ω**.**

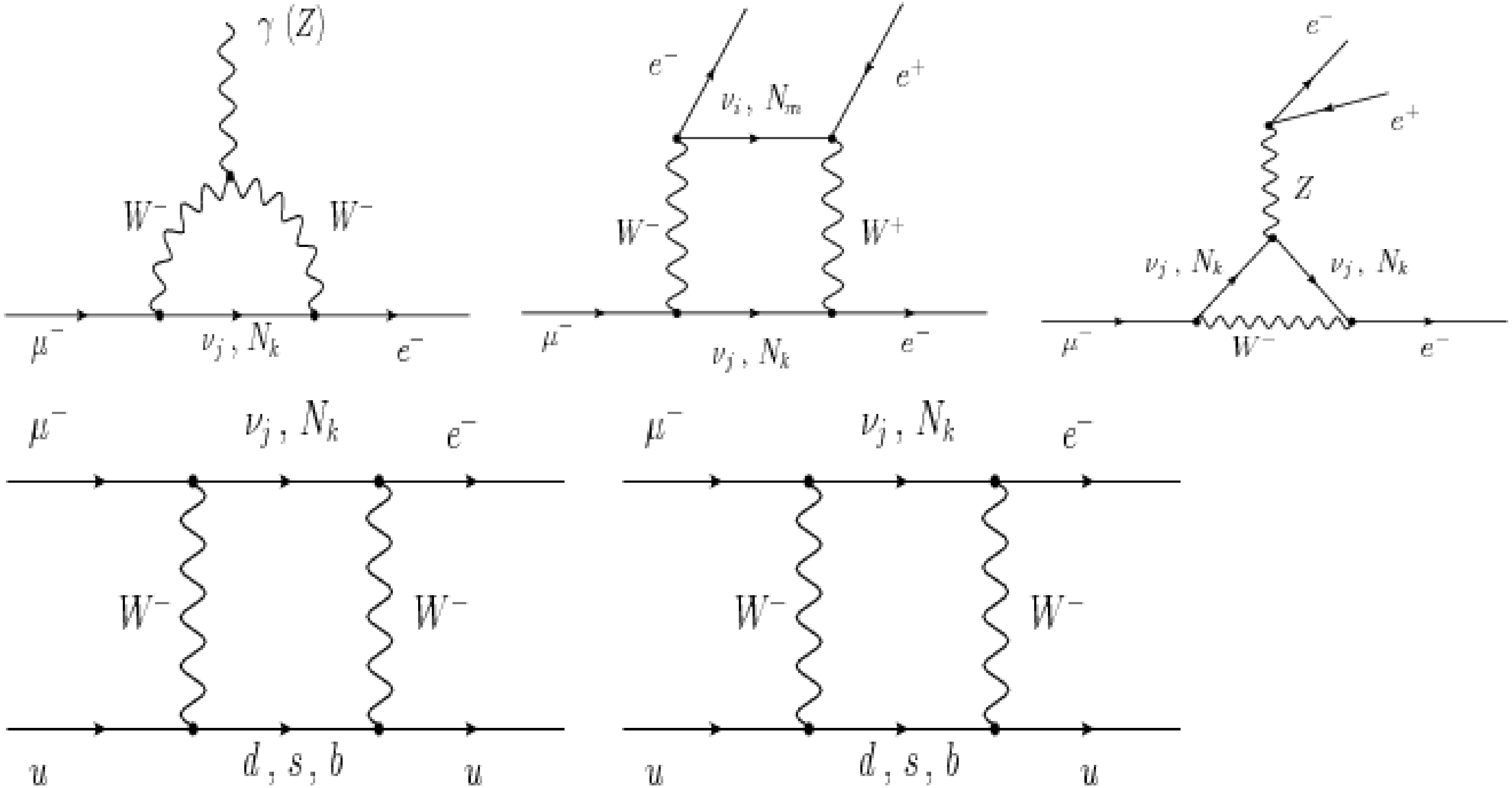
Low energy data:

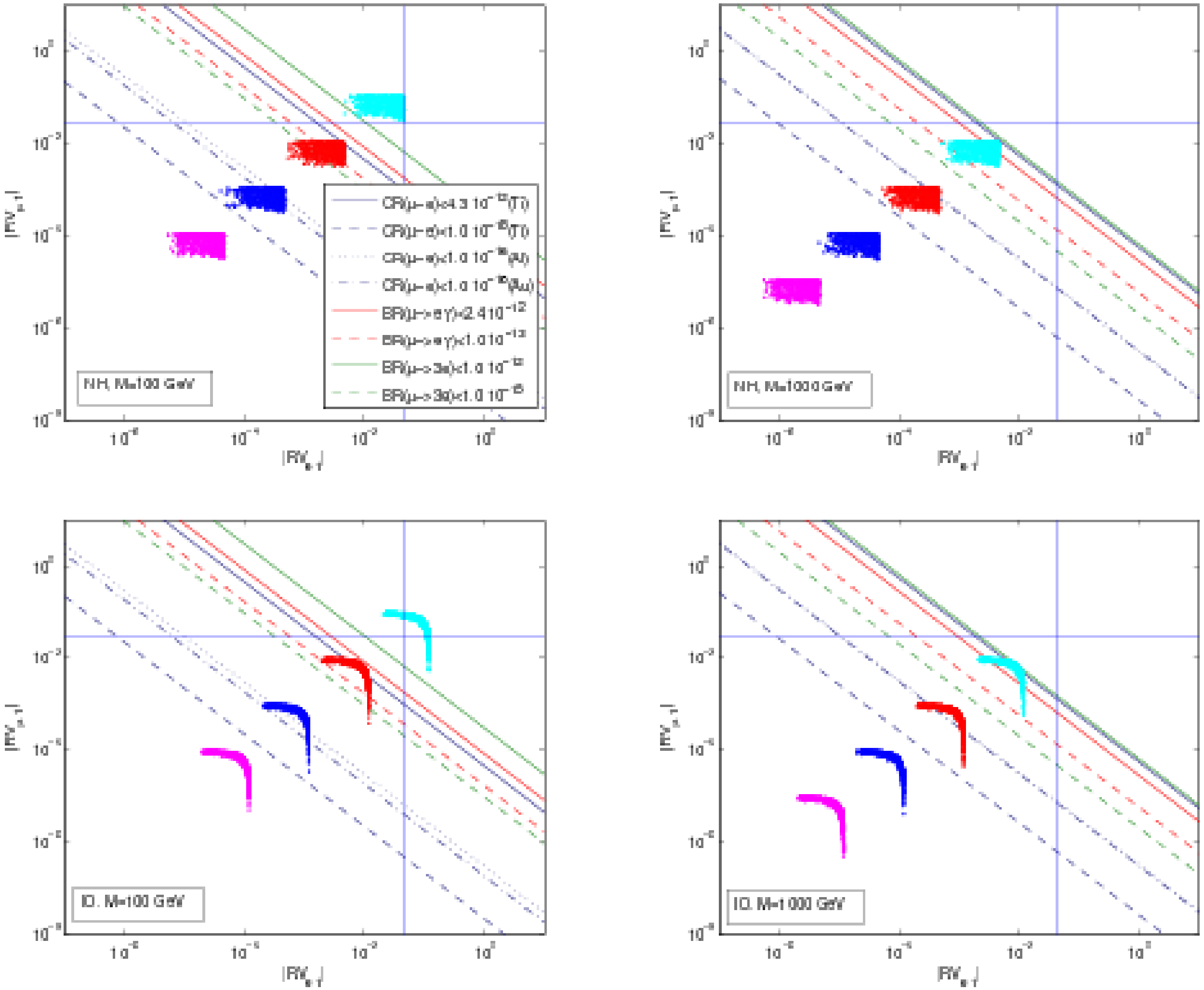
$$
\begin{aligned}
|(RV)_{e1}|^2 &\lesssim 2\times 10^{-3}\,,\\
|(RV)_{\mu 1}|^2 &\lesssim 0.8\times 10^{-3}\,,\\
|(RV)_{\tau 1}|^2 &\lesssim 2.6\times 10^{-3}\,.
\end{aligned}
$$

S. Antusch et al., 2008

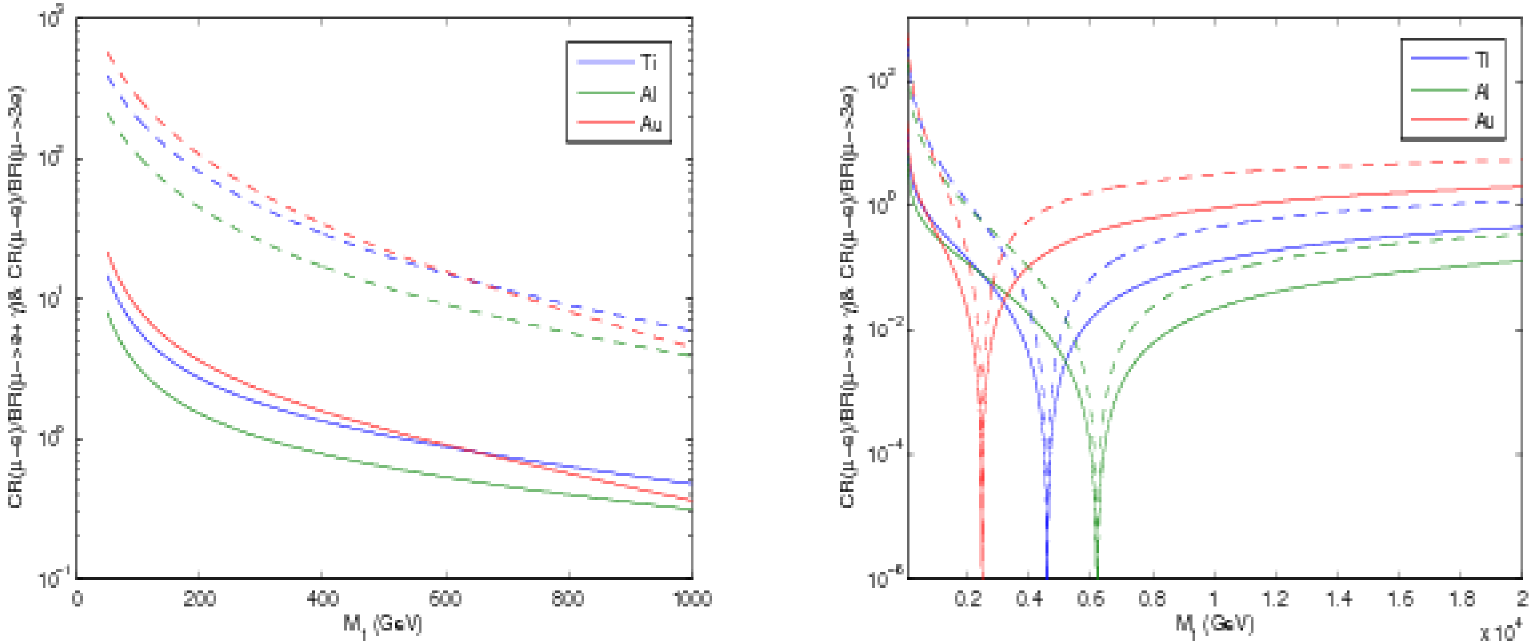
Observation of $N_{1,2}$ **at LHC - problematic.**

LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- + \mathcal{N} \to e^- + \mathcal{N}$: can proceed with exchange of virtual N_j:





Current limits and potential sensitivity to $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for $M_1 =$ 100 (1000) GeV and, $i)$ $y = 0.0001$ (magenta pts), $ii)$ $y = 0.001$ (blue pts), $iii)$ $y = 0.01$ (red pts) and $iv)$ $y = 0.1$ (cyan pts).



The ratio of the $\mu - e$ relative conversion rate and the branching ratio of the i) $\mu \to e\gamma$ decay (solid lines), ii) $\mu \to 3e$ decay (dashed lines), versus the type I see-saw mass scale M_1, for three different nuclei: $^{48}_{22}$Ti (blue lines), $^{27}_{13}$Al (green lines) and $^{197}_{79}$Au (red lines).

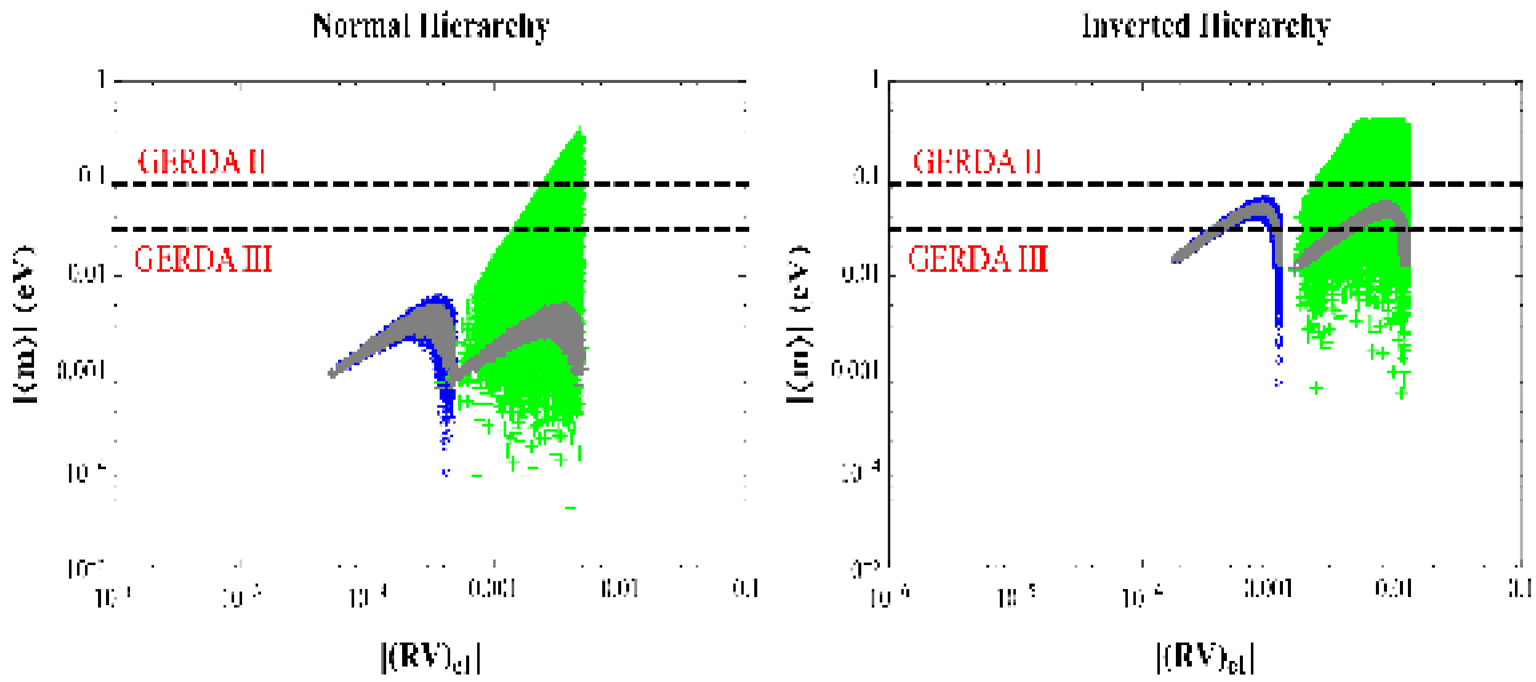
The exchange of virtual N_j gives a contribution to $|{<}m{>}|$:

$$|{<}m{>}| \; \cong \left| \sum_i (U_{PMNS})^2_{ei}\, m_i - \sum_k \, f(A, M_k)\, (RV)^2_{ek} \, \frac{(0.9 \;\; \text{GeV})^2}{M_k} \right| ,$$

$$f(A, M_k) \cong f(A) \, .$$

For, e.g., ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te and ^{136}Xe, the function $f(A)$ takes the values $f(A) \cong 0.033,\ 0.079,\ 0.073,\ 0.085$ and 0.068, respectively.

- **The Predictions for $|{<}m{>}|$ can be modified considerably.**



$|{<}m{>}|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and $i)$ $y = 0.001$ (blue), $ii)$ $y = 0.01$ (green). The gray markers correspond to $|{<}m{>}^{\rm std}| = |\sum_i (U_{PMNS})^2_{ei} m_i|$.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011