Parton Correlations in Multi-Parton Interactions

Sergio Scopetta

Dipartimento di Fisica e Geologia, Università di Perugia
and INFN, Sezione di Perugia, Italy

with

Matteo Rinaldi (Perugia), Marco Traini (Trento), Vicente Vento (Valencia)

Cortona, April, 20^{th} 2015

Outline

- Are double parton correlations (DPCs) relevant in double parton scattering (DPS) @LHC?
  Check within quark models
  Short summary of recent results

  M. Rinaldi, S.S. and V.Vento, PRD 87, 114021 (2013)

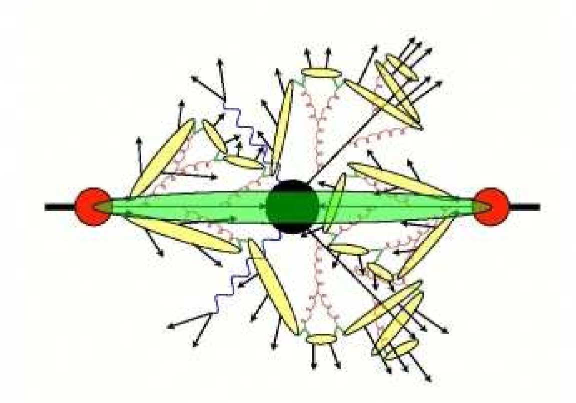
  M. Rinaldi, S.S., M. Traini and V.Vento, JHEP, 1412 (2014) 028

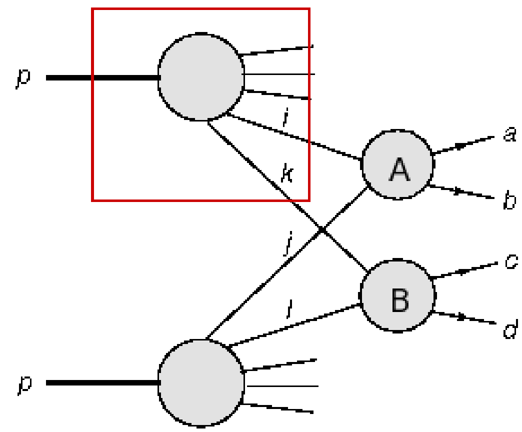
- Work in progress:
  - relation to available data: σ_{eff}
  - Can the 3D nucleon structure be accessed through MPI?
  - Are model calculations useful for that?

- What next?

DPS and Double Parton Distributions (dPDFs)

- In an LHC collision, hard MPI can occur:





- The DPS cross section is written **(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))**

$$
\begin{aligned}
\mathrm{d}\sigma &= \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(x_1,x_3,\mu_A)\hat{\sigma}_{kl}(x_2,x_4,\mu_B) \\
&\times \int \mathrm{d}^2\vec{r}_\perp\, F_{ik}(x_1,x_2,\vec{r}_\perp,\mu_A,\mu_B)F_{jl}(x_3,x_4,\vec{r}_\perp,\mu_A,\mu_B)
\end{aligned}
$$

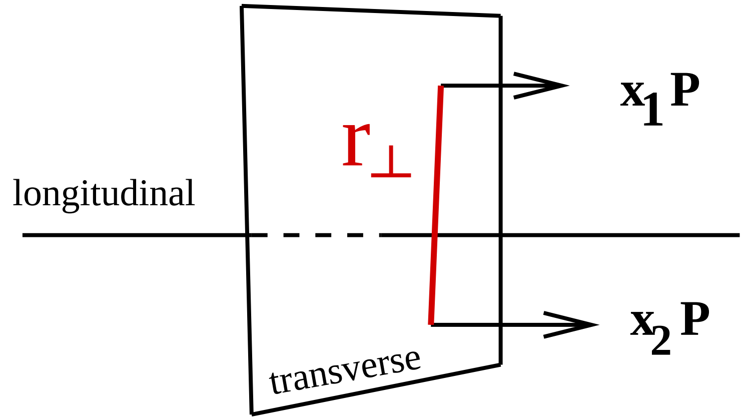
x_i = momentum fraction carried by the parton inside the hadron;
$\mu_{A,B}$ = momentum scale; $r_\perp$ = transverse distance between the two partons

- DPS: a background to be taken into account for fundamental studies;
for us: the dPDF $F_{ik}(x_1,x_2,\vec{r}_\perp,\mu_A,\mu_B)$ in one of the protons is very interesting!

Nucleon 3D structure from dPDFs?

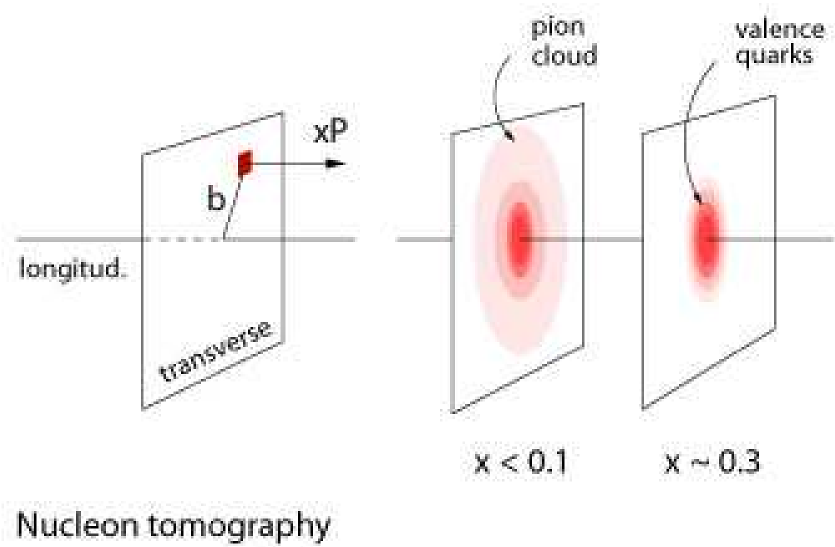
$F_{ij}(x_1, \mu_A, x_2, \mu_B, r_\perp)$ is a 2-body density

- It is dimensioned;
- Its F.T. wrt $r_\perp$, dimensionless, is NOT a density (sometimes it is called ${}_2GPD$)



Nucleon tomography proposed through GPDs (in Impact parameter space)

- $H(x, \xi = 0, b_\perp, \mu)$ is a 1-body density
- Its F.T. wrt $b_\perp$, dimensionless, is NOT a density (standard GPD)



F_{ij} is very interesting: 2-body quantities are always theoretically intriguing (their measurement, challenging). The difference between a 2-body quantity and the product of two 1-body quantities is a measurement of CORRELATIONS

dPDFs and correlations

F_{ij} is usually factorized as follows ($(x_1, x_2) - r_\perp$ factorization):

$$F_{ij}(x_1, x_2, \vec{r}_\perp, \mu_A, \mu_B) = F_{ij}(x_1, x_2, \mu) T(\vec{r}_\perp, \mu)$$

AND (x_1, x_2 factorization):

$$F_{ij}(x_1, x_2, \mu) = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF}}\, q_j(x_2, \mu)}\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

NO CORRELATION ANSATZ

This means that correlations between the quarks in the proton are neglected.

- Is this a safe approximation? Important for LHC fundamental studies (actually it is broken even by QCD evolution)
- Taking into account correlations in the analysys, can one understand better the proton structure through MPI observation?

Double Parton Correlations (DPCs)

- In principle, correlations are there

  At very low x, due to the large population of partons, the role of correlations may be less relevant **BUT** quantitative theoretical estimates are necessary.
  We are not alone in addressing this issue
  (**Calucci and Treleani (1999), Del Fabbro and Treleani (2002), Korotkikh and Snigirev (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012), Schweitzer, Strikman and Weiss (2013)...**)

- Difficult to study DPCs, non perturbative quantities, from first principles

- Our contribution: a quark model analysis as a possible useful tool

  Constituent Quark Models CQM, unlike bag models (treated in **Chang, Manohar, Waalewijn PRD 87 (2013) 3, 034009**)
  provide a proper correlated framework for DPCs studies

  They reproduce the gross features of experimental PDFs in the valence region (**BUT** present LHC date are at low x...)

  They are widely used to guide measurements of NP quantities (e.g., TMDs, GPDs)

Model calculations of PDFs

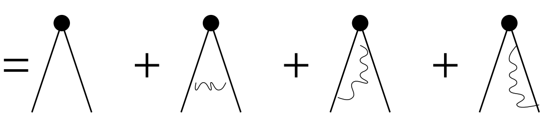
- In order to consistently compare data of twist-2 PDFs with the predictions of a CQM, one has to follow a 2-steps procedure:

  (firstly suggested by R.L. Jaffe and G.G. Ross, PLB 398 (1980) 313)

  1. evaluate in the model the twist-2 part of the corresponding observable, which has to be related to a low momentum scale, μ_o^2
  2. perform a perturbative QCD evolution to the DIS experimental scale, Q^2

$$f(x,\mu_o^2) \xrightarrow{R.G.E.,\ p.\ QCD} f(x,Q^2),\ DIS$$

Twist-2

$L.O. =$ 

$+\ N.L.O.\ \ (2\ \ loops)$

- *Caveat 1*: if there are only valence quarks, the scale has to be very low ($\mu_o \simeq$ 300 MeV according to NLO pQCD)
- *Caveat 2*: in the simplest CQM picture, all the gluons and sea quarks are perturbatively generated

dPDFs in a Light-Front approach

M. Rinaldi, S.S., M. Traini and V.Vento, JHEP, 1412 (2014) 028

To improve the approach of our first calculation (**M. Rinaldi, S.S. and V.Vento, PRD 87, 114021 (2013)**), we implemented Relativity using a Light-Front (LF) approach. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, (Dirac, 1949), one has:

- full Poincarè covariance
- fixed number of on-mass-shell constituents

Among the 3 possible forms of RHD, the LF (initial hypersurface: $x^+ = x_0 + x_3 = 0$; in standard, “Instant Form” QM: $x_0 = 0$). has several advantages. The most relevant here:

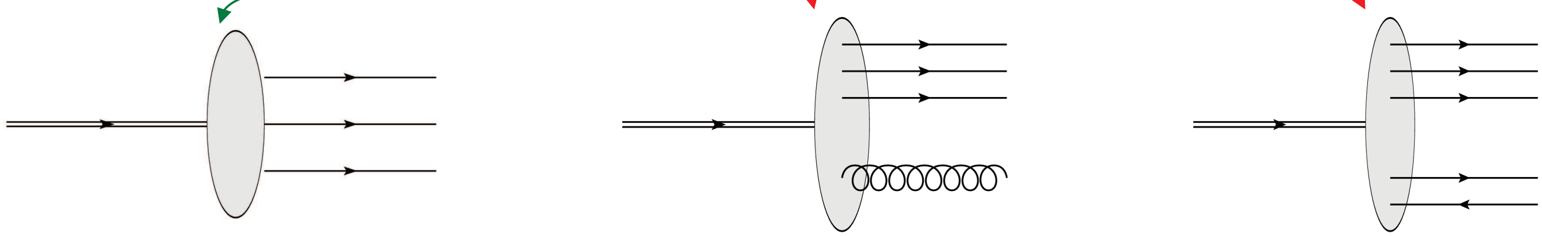
- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $P^+, \mathbf{P}_\perp$, iii) Rotation around z.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- in a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- The IMF description of DIS is easily included. Systematically applied to calculate FFs, PDFs, GPDs, TMDs...

A Light-Front wave function representation

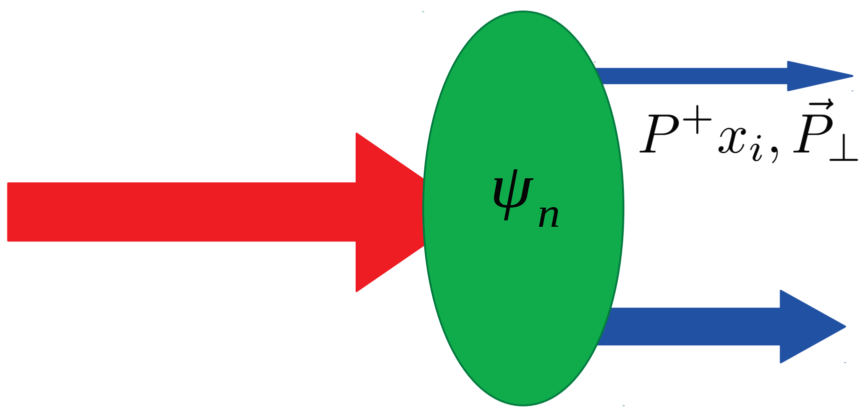
The proton wave function can be represented in the following way:

see *e.g.:* **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+\vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle + \psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots.$$



$p : P^+, \vec{P}_\perp$



$P^+ x_i, \vec{P}_\perp x_i, \vec{k}_{\perp i}$

$$\begin{cases} x_i = \dfrac{k_i^+}{P^+} \\ \displaystyle\sum_i^n x_i = 1 \\ \displaystyle\sum_i^n \vec{k}_{\perp i} = 0 \end{cases}$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i) \longleftrightarrow$ Invariant under LF boosts!

A Light-Front wave function representation

In our approach, it is possible to connect LF states to IF ones, through the Melosh Rotations $D^{1/2}_{\lambda\lambda'}(R_{il}(\vec{k}))$

(see, e.g., B.D. Keister, W.N. Polyzou, Adv. Nucl. Phys. 20, 225 (1991))

$$|\vec{k}_\perp, \lambda, \tau\rangle_{[l]} \propto \sqrt{2k_0} \sum_{\lambda'} D^{1/2}_{\lambda\lambda'}(R_{il}(\vec{k})) |\vec{k}_\perp, \lambda', \tau\rangle_{[i]} \;,$$

so that a relation between the LF $\psi^{[l]}_\lambda$ and the IF $\psi^{[i]}_\lambda$ is obtained

$$\begin{aligned} \psi^{[l]}_\lambda(\beta_1, \beta_2, \beta_3) \quad &\propto \quad \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right] \sum_{\mu_1\mu_2\mu_3} D^{1/2*}_{\mu_1\lambda_1}(R_{il}(\vec{k}_1)) D^{1/2*}_{\mu_2\lambda_2}(R_{il}(\vec{k}_2)) D^{1/2*}_{\mu_3\lambda_3}(R_{il}(\vec{k}_3)) \\ &\times \quad \psi^{[i]}_\lambda(\alpha_1, \alpha_2, \alpha_3) \end{aligned}$$

with $\beta_i = \{x_i, \vec{k}_{i\perp}, \lambda_i, \tau_i\}, \; \alpha_i = \{\vec{k}_i, \mu_i, \tau_i\}, \; \omega_i = k_{i0}, \; M_0 = \sum_i \omega_i = \sum_i \sqrt{m^2 + \vec{k}_i^2}$

Now this formalism can be used in the definition of the dPDF...

dPDFs in the Light-Front approach

quark-quark dPDFs are defined through Light-Cone quantized states and fields (see, e.g., **M. Diehl, D. Ostermeier, A. Schäfer JHEP 03 (2012) 089**). From that, extending a procedure used, e.g., in **Pasquini, Boffi and Traini NPB 649 (2003) 243** for GPDs, the LF dPDF is obtained, **in mom. space**, in the intrinsic frame $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$:

$$
\begin{aligned}
F_{12}(x_1, x_2, k_\perp) \quad &= \quad 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \, \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, k_\perp) \Phi(\{\vec{k}_i\}, -k_\perp) \\
&\times \quad \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)
\end{aligned}
$$

with

$$
\Phi(\{\vec{k}_i\}, k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)
$$

(**NB**: here $k_\perp$ is the momentum conjugated to $r_\perp$; it is a *relative* momentum, it is **NOT** the argument of TMDs. Sorry for a possibly confusing notation)

$$
\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3) = D^{1/2*}(R_{il}(\vec{k}_1)) D^{1/2*}(R_{il}(\vec{k}_2)) D^{1/2*}(R_{il}(\vec{k}_3)) \psi^{[i]}(\vec{k}_1, \vec{k}_2, \vec{k}_3)
$$

Now one always gets the correct support: $x_1 + x_2 > 1 \rightarrow F_{12}(x_1, x_2, k_\perp) = 0$

A model is needed for the usual IF wave function $\psi^{[i]}$

Example: dPDFs in a LF Hyper-central CQM

Hyper-central CQMs have a long tradition (see, i.e., **Giannini and Santopinto, arXiv:1501.03722**). We use here the relativistic version developed in **Faccioli, Traini, Vento NPA656,400 (1999)**. The proton w.f.

$$\psi^{[i]} = \frac{1}{\pi\sqrt{\pi}} \Psi(k_\xi) \times SU(6)_{spin-isospin}$$

is solution of the mass equation

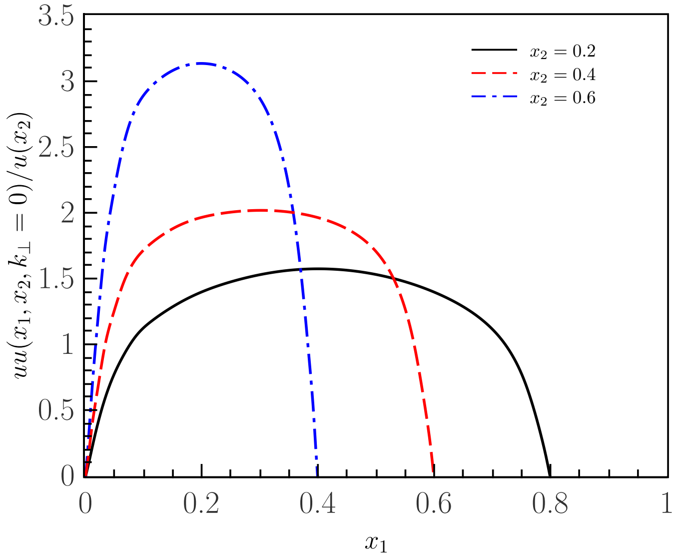
$$(M_0 + V)\Psi(k_\xi) \equiv \left(\sum_{i=1}^{3} \sqrt{m^2 + \vec{k}_i^2} - \frac{\tau}{\xi} + \kappa_l \xi \right) \Psi(k_\xi) = M\Psi(k_\xi)$$

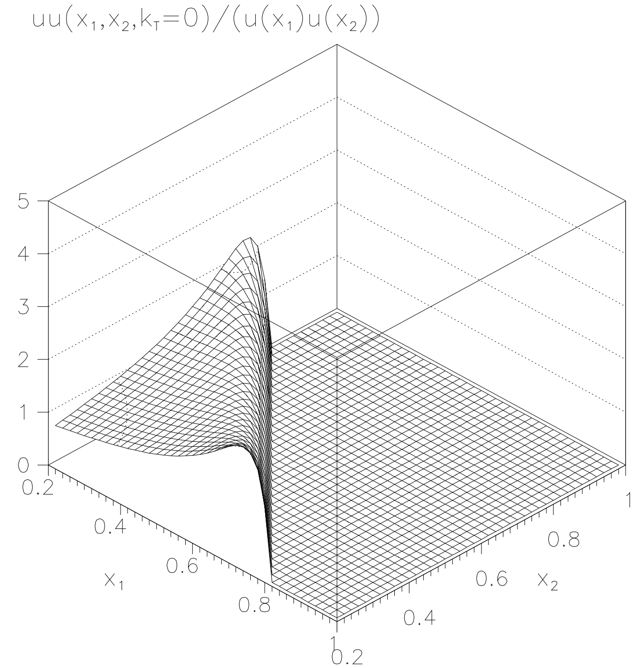
with $k_\xi = \sqrt{2(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)}$, $\quad \tau = 3.30 \quad \kappa_l = 1.80$ fm^{-2} and

$$\Psi(k_\xi) = \sum_{\nu=0}^{16} c_\nu \frac{(-1)^\nu}{\alpha^3} \left[\frac{2\nu!}{(\nu+2)!} \right]^{1/2} e^{-k_\xi^2/(2\alpha^2)} \sum_{m=0}^{\nu} \frac{(-1)^m}{m!} \frac{(\nu+2)}{(\nu-m)!(m+2)!} \left(\frac{k_\xi^2}{\alpha^2} \right)^m$$

The parameters have been chosen to reproduce the light baryon spectrum. Successful despite its simplicity. Used in several calculations of PDFs and GPDs (see **Pasquini, Boffi and Traini NPB 649 (2003) 243... Traini PRD (2014) 3 034021)**

Results: $x_1 - x_2$-factorization





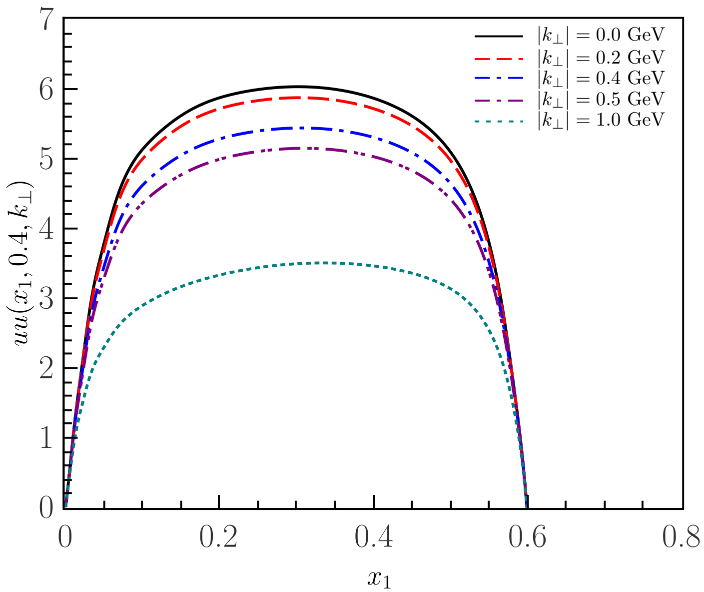
LEFT: The ratio $uu(x_1, x_2, k_\perp = 0)/u(x_2)$, for three different values of x_2.
There should be no x_2 dependence in the ratio if the $x_1 - x_2$-factorization were realized;

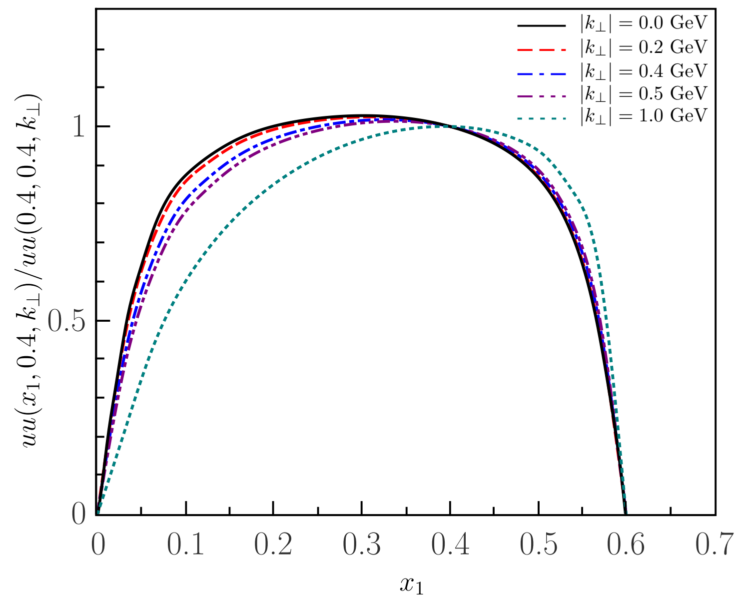
RIGHT: The ratio $uu(x_1, x_2, k_\perp = 0)/(u(x_2)u(x_1))$,.
It should be 1 everywhere if the $x_1 - x_2$-factorization were realized;

- The $x_1 - x_2$-factorization is badly violated in the valence region
- Already found in bag model and NR calculations

Model independent feature

Results: $(x_1, x_2) - k_T$ factorization





LEFT: General trend increasing $k_\perp$

RIGHT: The ratio $uu(x_1, x_2 = 0.4, k_\perp)/uu(x_1 = 0.4, x_2 = 0.4, k_\perp)$,
for different $k_\perp$ values: there should be no $k_\perp$ dependence if factorization worked

- mildly violated as in the NR model (and in the Bag);

Model independent lesson

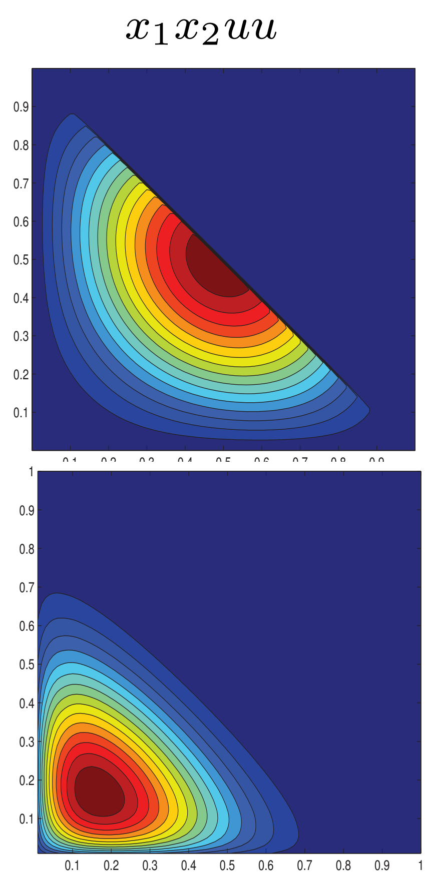
We have a fully correlated model with correct symmetries and dynamical (non-factorized) $k_\perp$ dependence. So far, at the model scale μ_0^2...

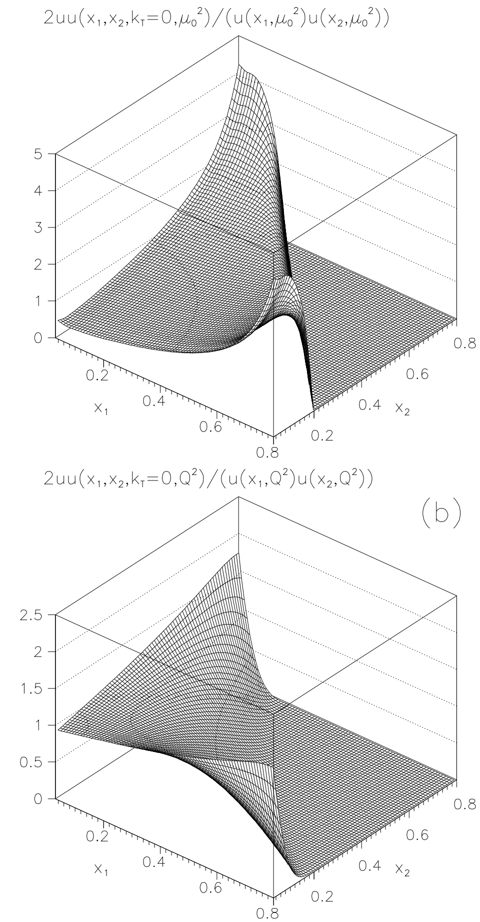
pQCD evolution of the LF dPDFs

dPDFs evolution known (**Kirshner et al., 1982**). We solved the evolution equations, in their inhomogenous part, in the valence (non-singlet) sector, by inversion of double Mellin transforms. We have taken only one momentum scale, $\mu_A = \mu_B$

$$\mu_o^2 = 0.1 \text{ GeV}^2$$

$$Q^2 = 10 \text{ GeV}^2$$





Strong correlations in the valence region also at Q^2. Weaker at low x. We are working in the singlet sector... Comparison with existing data?

The effective Cross Section σ_{eff}

Defined through: $\boxed{\sigma^{pp}_{double} = \frac{m}{2}\frac{\sigma_A^{pp'}\sigma_B^{pp'}}{\sigma_{eff}}}$ σ_{eff} is the only measured quantity

$$\sigma_A^{pp'}(x_1, x_1') = single\, scattering = \sum_{i,k} q_i^p(x_1) q_k^{p'}(x_{1'})\hat{\sigma}_{ik}^A(x_1, x_1')\ ,$$

$$\begin{aligned}\sigma^{pp}_{double}(x_1, x_1', x_2, x_2') \quad &= \quad \frac{m}{2}\sum_{i,j,k,l}\int D_{ij}(x_1, x_2; \mathbf{k}_\perp)\hat{\sigma}_{ik}^A(x_1, x_1') \\ \times \quad &\hat{\sigma}_{jl}^B(x_2, x_2') D_{kl}(x_1', x_2'; -\mathbf{k}_\perp)\frac{d\mathbf{k}_\perp}{(2\pi)^2}\ .\end{aligned}$$

If (reasonable!): $\hat{\sigma}_{ij}(x, x') = C_{ij}\bar{\sigma}(x, x')$ with $C_{gg} : C_{qg} : C_{qq} = 1 : (4/9) : (4/9)^2$ then

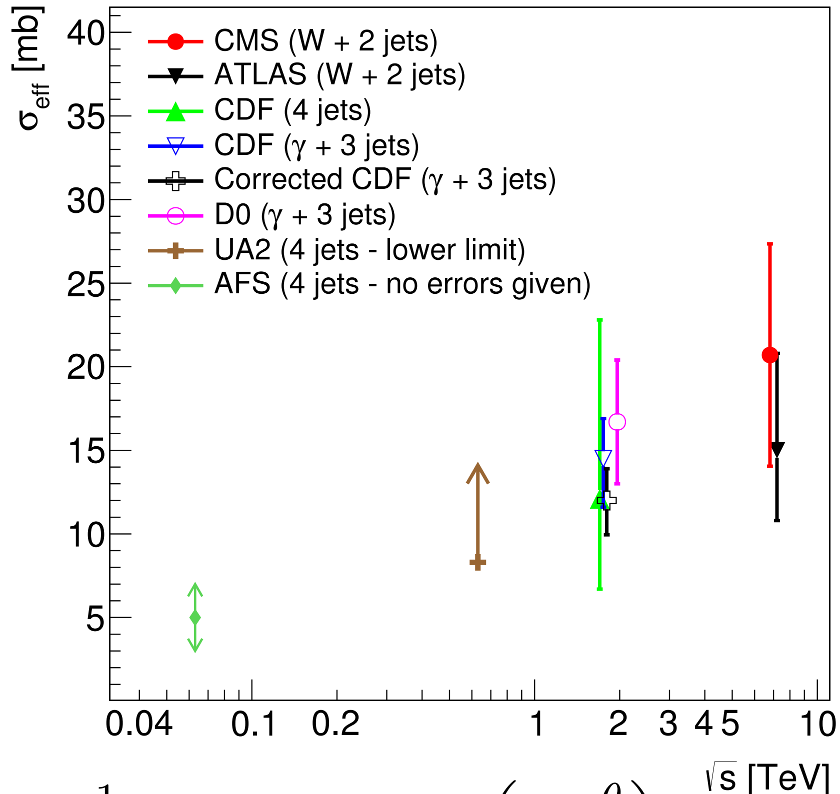
$$\sigma_{eff}(x_1, x_1', x_2, x_2') \quad = \quad \frac{\sum_{i,k,j,l} q_i(x_1) q_k(x_{1'}) q_j(x_2) q_l(x_{2'}) C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ik} C_{jl}\int D_{ij}(x_1, x_2; \mathbf{k}_\perp) D_{kl}(x_1', x_2'; -\mathbf{k}_\perp)\frac{d\mathbf{k}_\perp}{(2\pi)^2}}$$

BUT $\longrightarrow if\ D_{ij} = q\,q f(\mathbf{k}_\perp) \longrightarrow \sigma_{eff} = \frac{1}{\int f^2(\mathbf{k}_\perp)\frac{d\mathbf{k}_\perp}{(2\pi)^2}} = \frac{1}{\int \tilde{f}^2(\mathbf{r}_\perp) d\mathbf{r}_\perp}$

No $x-$dependence, no scale dependence... No correlations

σ_{eff}: experimental situation

- Difficult extraction;
  σ_{pp}^{double} not measured...
  see talks later today and tomorrow
- Older data at lower $\sqrt{S}$
- “constant” (large errorbars)
- Different ranges in x_i accessed in different experiments.



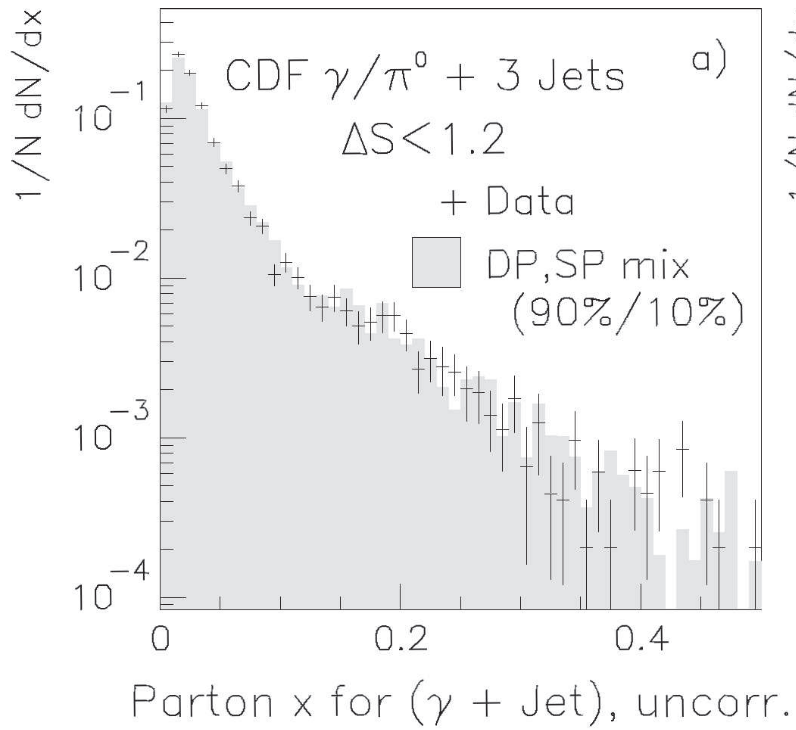
Kinematics:

$$x_{1,2} = \sqrt{\tau} e^{\pm y} \quad \tau = x_1 x_2 = \frac{s}{S} \quad y = \frac{1}{2} \ln \frac{x_1}{x_2} \simeq \eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

High x for hard jets (heavy particles detected, large partonic s)
For example: AFS, $y \simeq 0$, $x_1 = x_2$ in $[0.2, 0.3]$
CDF: x_1, x_2, x_1', x_2' in $[0.02, 0.4]$

Valence region included...

σ_{eff}: x dependence (?)



CDF, F. Abe et al.

PRD 56, 3811 (1997)

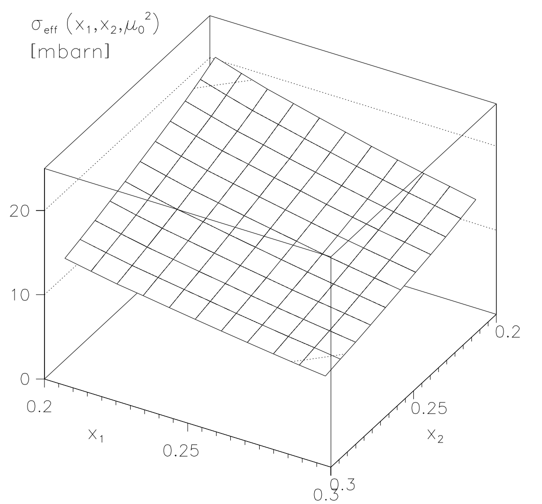
Shaded area: Montecarlo $without$ correlations in x

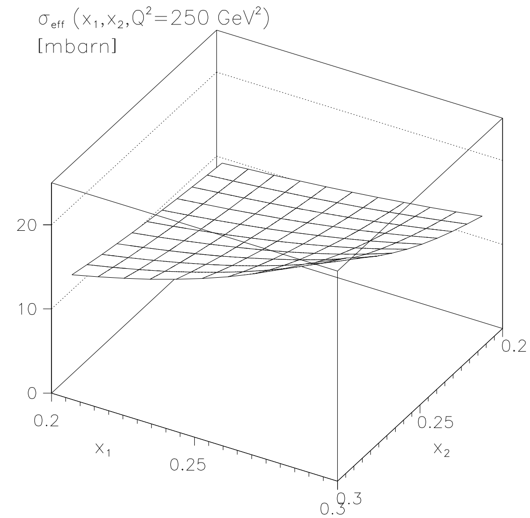
Data well described (?) for x_1, x_2, x_1', x_2' in $[0.02, 0.4]$ (also in the valence region...)

May be not enough accuracy for high x? No x dependence?

Actually, our understanding is that, in the valence region, x dependence has to be seen. Let the model guide us...

σ_{eff}: x dep. from model calculations (*preliminary*)





- Shown at $y \simeq 0$, $x_i = x_i'$ in $[0.2, 0.3]$, at μ_0^2 (left) and at Q^2=250 GeV^2 (AFS kin.) Not constant at all. A factor of 2 easily found. **Expected!** Remember:

$$\sigma_{eff}(x_1, x_2) \quad = \quad \frac{\sum_{i,k,j,l} q_i(x_1) q_k(x_1) q_j(x_2) q_l(x_2) C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ik} C_{jl} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp) D_{kl}(x_1, x_2; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}$$

  Numerator and denominator decrease with x with different velocity (one must have $x_1 + x_2 > 1 \to D_{12}(x_1, x_2, k_\perp) = 0$). **Model independent result!**

- It ranges between 10 and 20 mbarn... But taking the global average in $x_{1,2}$, using $\sigma_{eff} = 1/\int f^2(\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}$, looking for an x and scale independent value, we get 10.9 mbarn... Encouraging agreement! (but this is a model dependent result...)

x dependence of σ_{eff} and 3D proton structure

- **The x dependence of σ_{eff} could give information on the 3D nucleon structure**
- **Our model calculation shows that, in the valence region at least, either such a dependence is found or something is not well posed in the definition of σ_{eff}**
- **Other Authors, with different arguments, reach similar conclusions**
  (M. Diehl, talk at DIS 2013; Calucci Treleani 1999; Frankfurt, Strikman, Weiss 2003, 2004)

Conclusions

A LF-CQM calculation of DPCs has been presented:

* relevant dynamical correlations in both longitudinal and transverse momentum are found; NS evolution performed
* Work in progress towards lower x: modelling the quark sea at the model scale and Singlet evolution
* Work in progress: analysis of σ_{eff}.

Questions (and answers):

* Are model calculations useful?
  Yes, they can guide the intuition, also in this framework
* Is the 3D nucleon structure accessible through MPI?
  May be
* In this sense, σ_{eff} measurements in “narrow” x ($\longrightarrow$ rapidity) regions could be very useful.
  Are these measurements possible?

What next?

- May be nucleon tomography is possible with MPI
- Certainly difficult
  (not easy with electromagnetic interactions either...
  In any case, the information is different, and complementary)
- Worth to try
- We (in this room) have the expertise to face the problem

Backup: dPDF, formally

$$
\begin{aligned}
F_{q_1 q_2}(x_1, x_2, \mathbf{z}_\perp) \quad &= \quad -8\pi M^2 \int \frac{\mathrm{d}z_1^+}{4\pi} \frac{\mathrm{d}z_2^+}{4\pi} \frac{\mathrm{d}z_3^+}{4\pi} e^{-ix_1 M z_1^+/2} \, e^{-ix_2 M z_2^+/2} \, e^{ix_1 M z_3^+/2} \\
&\times \quad \langle P, \mathbf{p} = \mathbf{0} | \left[\bar{q}_1 \left(z_1^+ \frac{\bar{n}}{2} + z_\perp \right) \frac{\not{\bar{n}}}{2} \right]_c \\
&\times \quad \left[\bar{q}_2 \left(z_2^+ \frac{\bar{n}}{2} \right) \frac{\not{\bar{n}}}{2} \right]_d q_{1,c} \left(z_3^+ \frac{\bar{n}}{2} + z_\perp \right) q_{2,d}(0) | P, \mathbf{p} = \mathbf{0} \rangle \, . \qquad (1)
\end{aligned}
$$