Polarization at SuperB and new Novosibirsk code for the spin tracking

I.A.Koop, BINP, 630090 Novosibirsk, Russia

XVII SuperB workshop and Kick Off Meeting
La Biodola (Isola d'Elba, Italy), May 28 - June 1, 2011

Outline

- Longitudinal polarization of an electron beam at LER, main requirements, different possibilities
- Two 90^0 spin rotators scheme with restoration of the vertical direction of a spin in arcs
- A scheme with 3 Siberian Snakes, spaced by 120^0 arcs in between
- Analytical estimations, ASPIRRIN code results
- Spin tracking approach at Novosibirsk
- Conclusion

Polarization review at Annecy,2010

- U.Wienands, D.P.Barber - “Polarization update”, scheme with two 90^0 spin rotators
- Ken Moffeit - “Polarization at SuperB” (Physics request and a measurement)
- Cecile Rimbault – “Beam-beam depolarization. Spin tracking. GUINEA-PIG++ code.”
- N.Monseu – “Spin tracking. Zgoubi code.”
- Other workshops + SPIN-2010, Juelich.

Requirements to longitudinal polarization

High polarization degree demands: $\tau_p >> \tau_{beam} = 3.5$ min

Expected polarization from a gun: $P_{beam} = 90\%$

Mixing a fresh beam with an old one slightly dilutes P:

$$\tau_{beam} = 3\,\text{min}, \quad P_{gun} = +(-)90\%; \quad \tau_p = 22\,\text{min}, \quad P_{rad} = +7\%.$$

$$P = P_{gun}\frac{\tau_p}{\tau_{beam}+\tau_p} + P_{rad}\frac{\tau_{beam}}{\tau_{beam}+\tau_p} \quad \to \quad P = +78.6\% \text{ or } -76.7\%$$

Continuous polarization monitoring (bunch to bunch) using the Compton back-scattering technique.

To fight with systematics the polarization measurement accuracy needs to be better than: $|\Delta P/P| \le 1 \div 5 \cdot 10^{-3}$

Alter a sign of P from bunch to bunch, randomly!

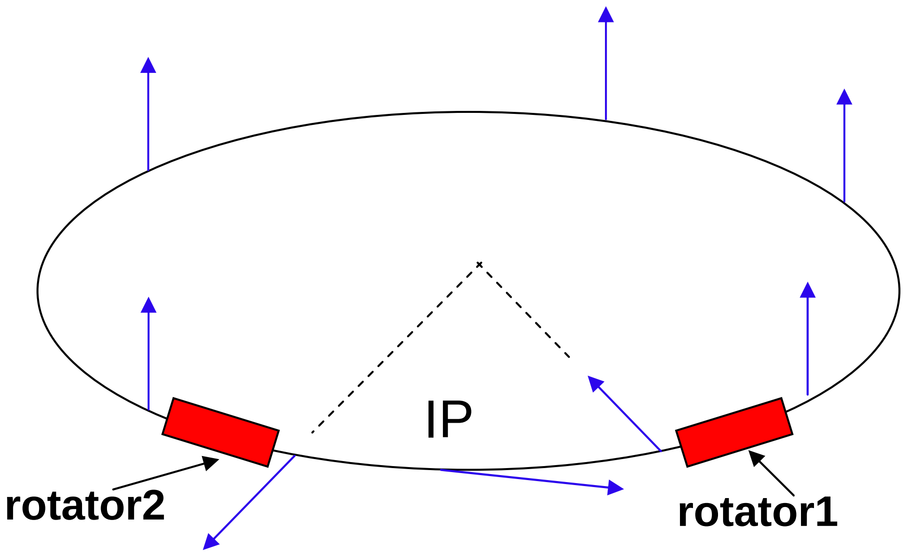
Alter sometimes the spin filling pattern in a train!

Two 90^0 spin rotators scheme at LER

Spin is directed longitudinally at IP at two specific energies.

It makes a half turn in the FF-arc when $E = 1.4$ GeV

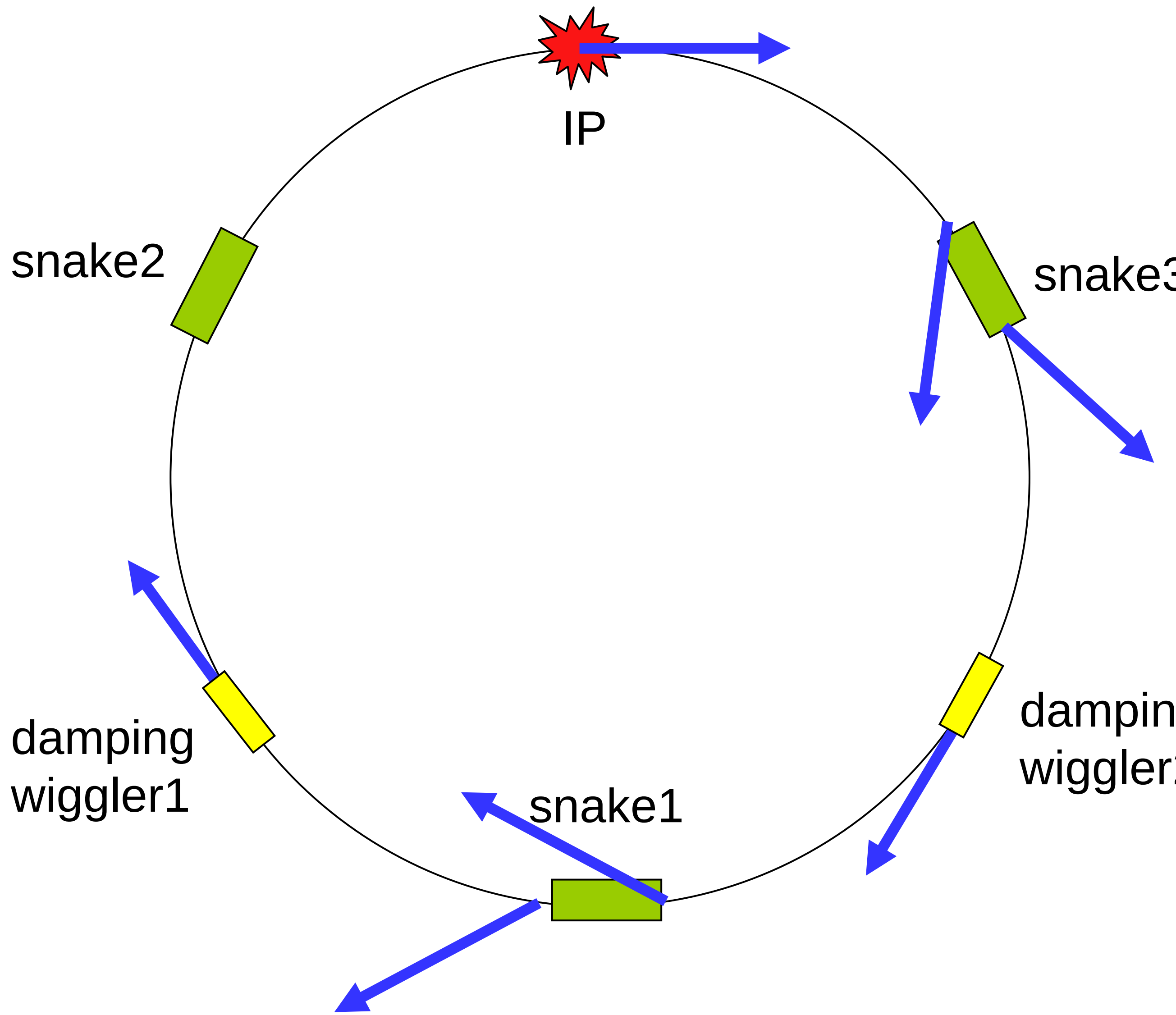
and it makes 1.5 turns at E=4.18 GeV (that's is nominal E)



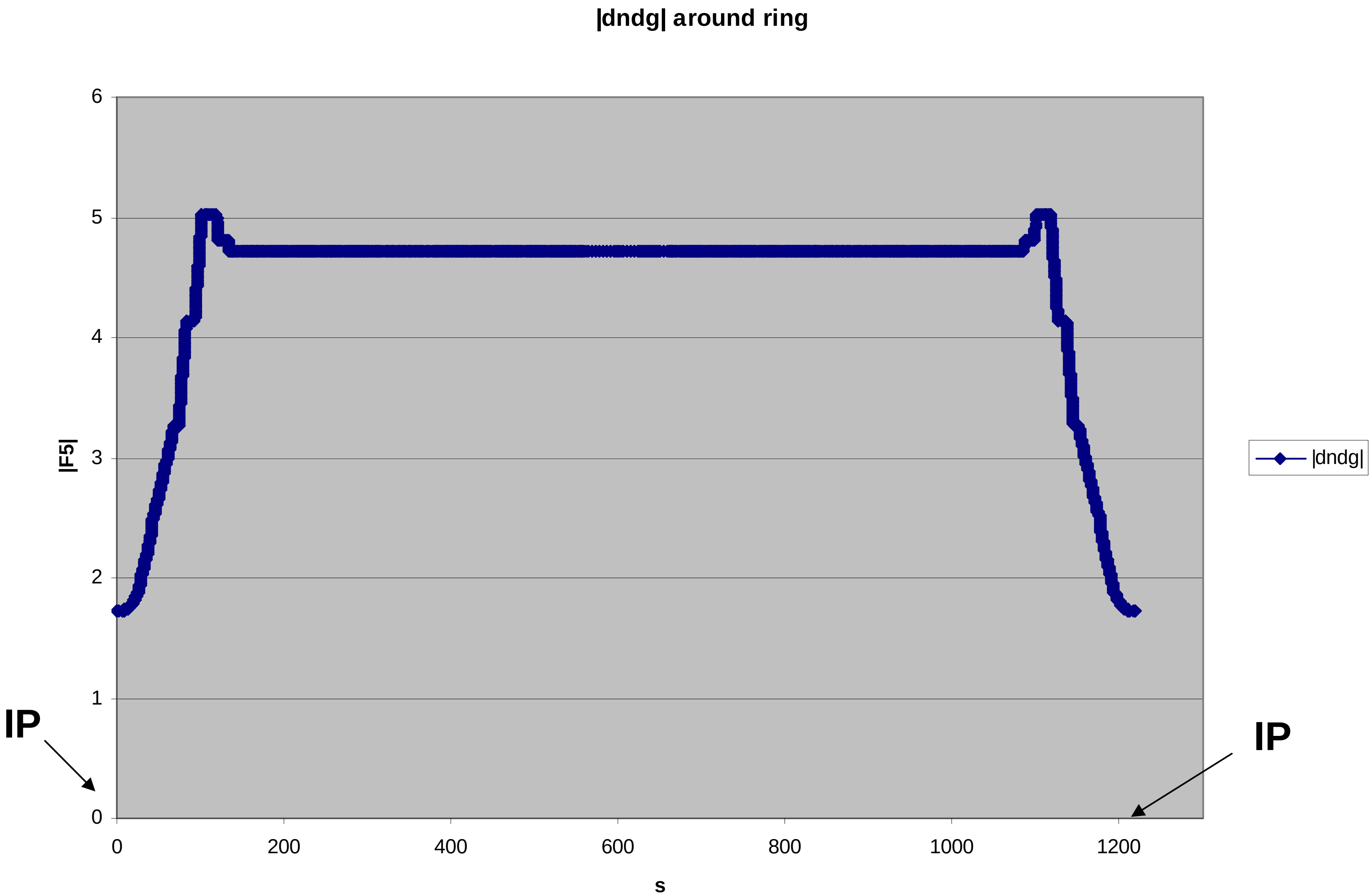
$\vec{d} = \gamma \dfrac{\partial \vec{n}}{\partial \gamma}$ - the spin – orbit coupling vector, $\left|\vec{d}\right| \sim \gamma$

$$\tau_p^{-1} = \frac{5\sqrt{3}}{8} \lambda_e r_e c \gamma^5 \left\langle \frac{1 - \frac{2}{9}(\vec{n}\vec{v})^2 + \frac{11}{18}\vec{d}^2}{|r|^3} \right\rangle \sim \begin{cases} \gamma^5 \text{ if } \left|\vec{d}\right| \le 1 \\ \gamma^7 \text{ if } \left|\vec{d}\right| \gg 1 \end{cases}$$

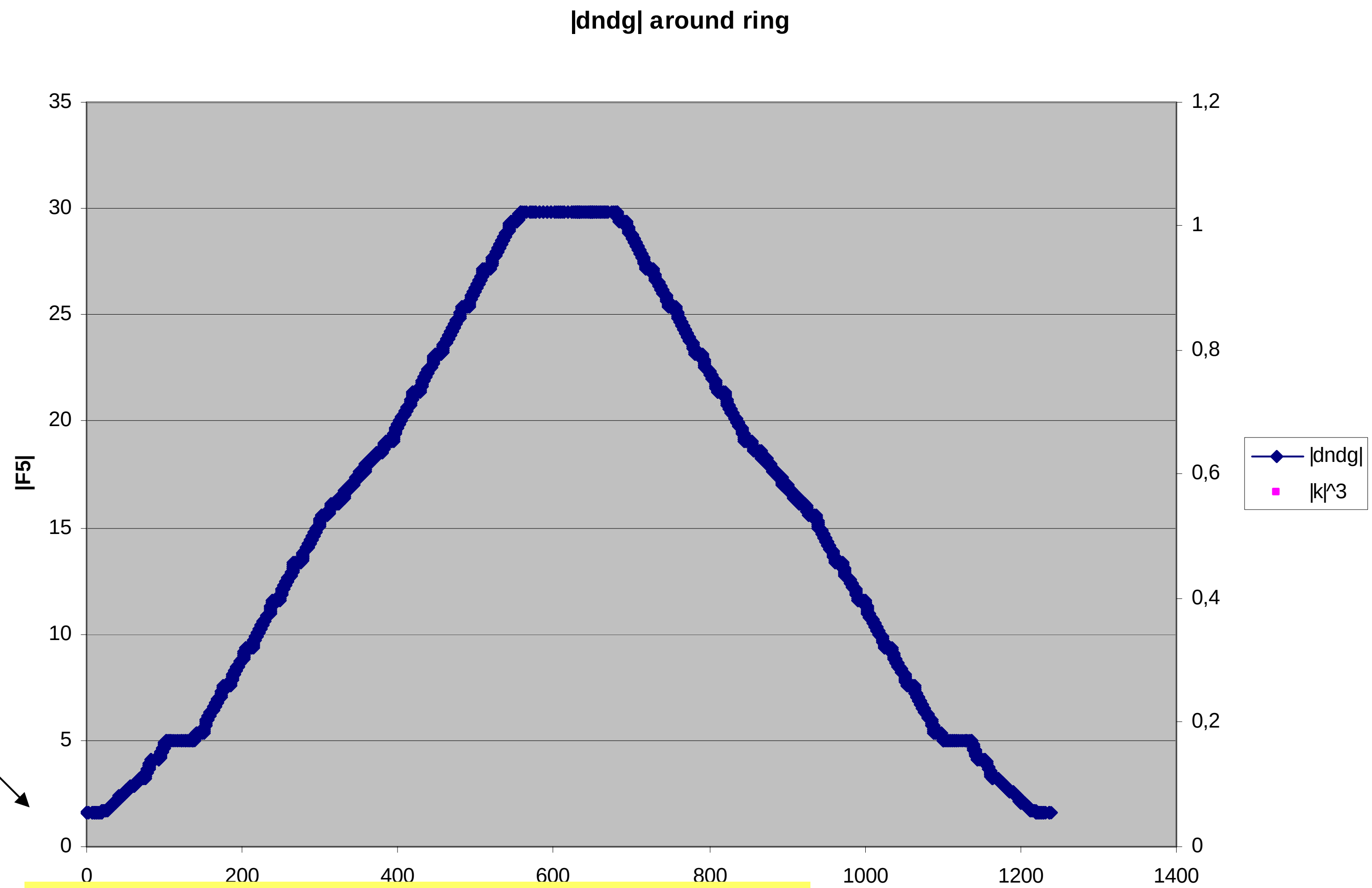
Polarization scheme with 3 snakes (arc=120^0 +2 damping wigglers in the arc's middle)



ASPIRRIN results for 90^0 option



ASPIRRIN, single snake option

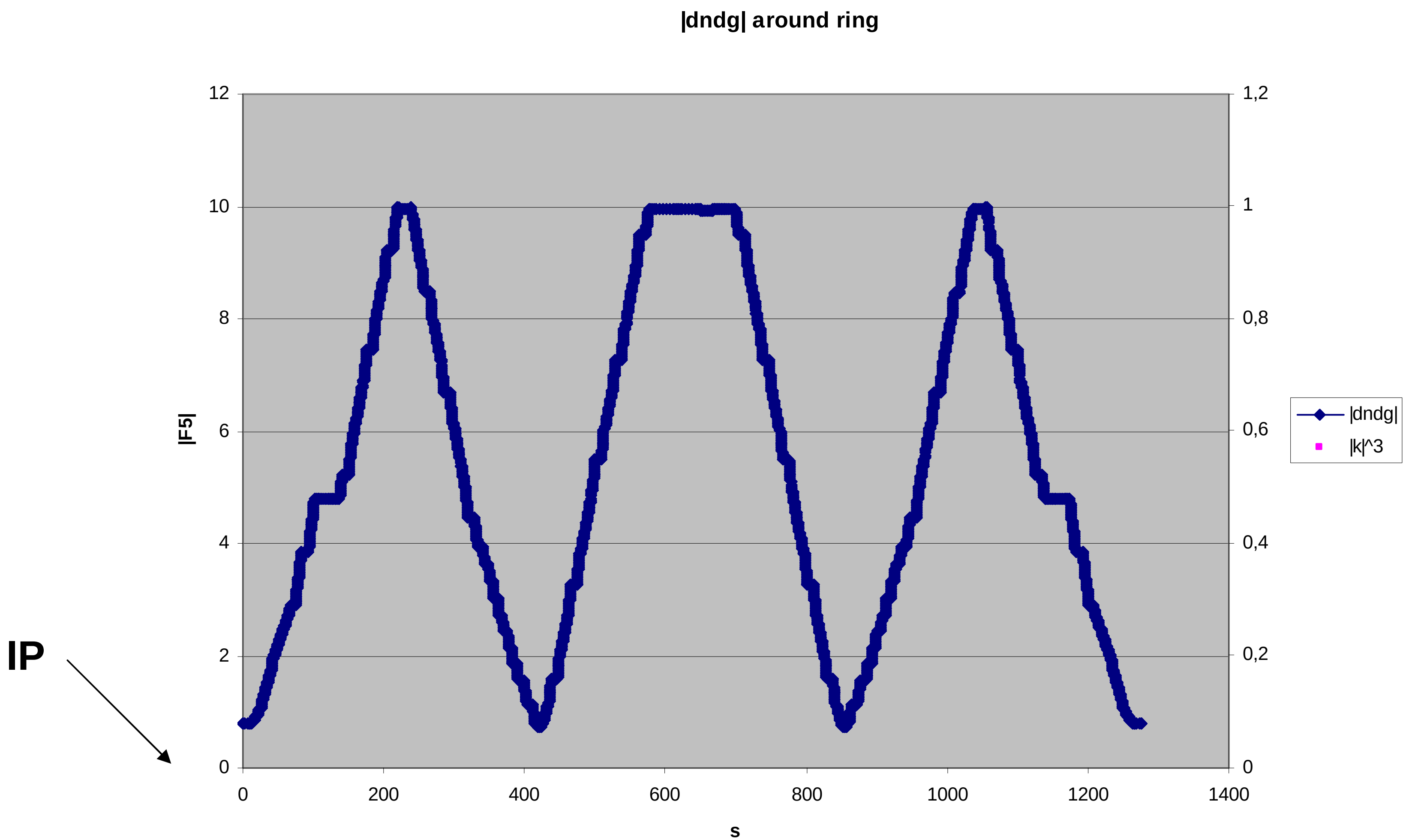


IP

$$\tau_p = 90\ s \quad \text{at} \quad E = 4.18\ \text{GeV}$$

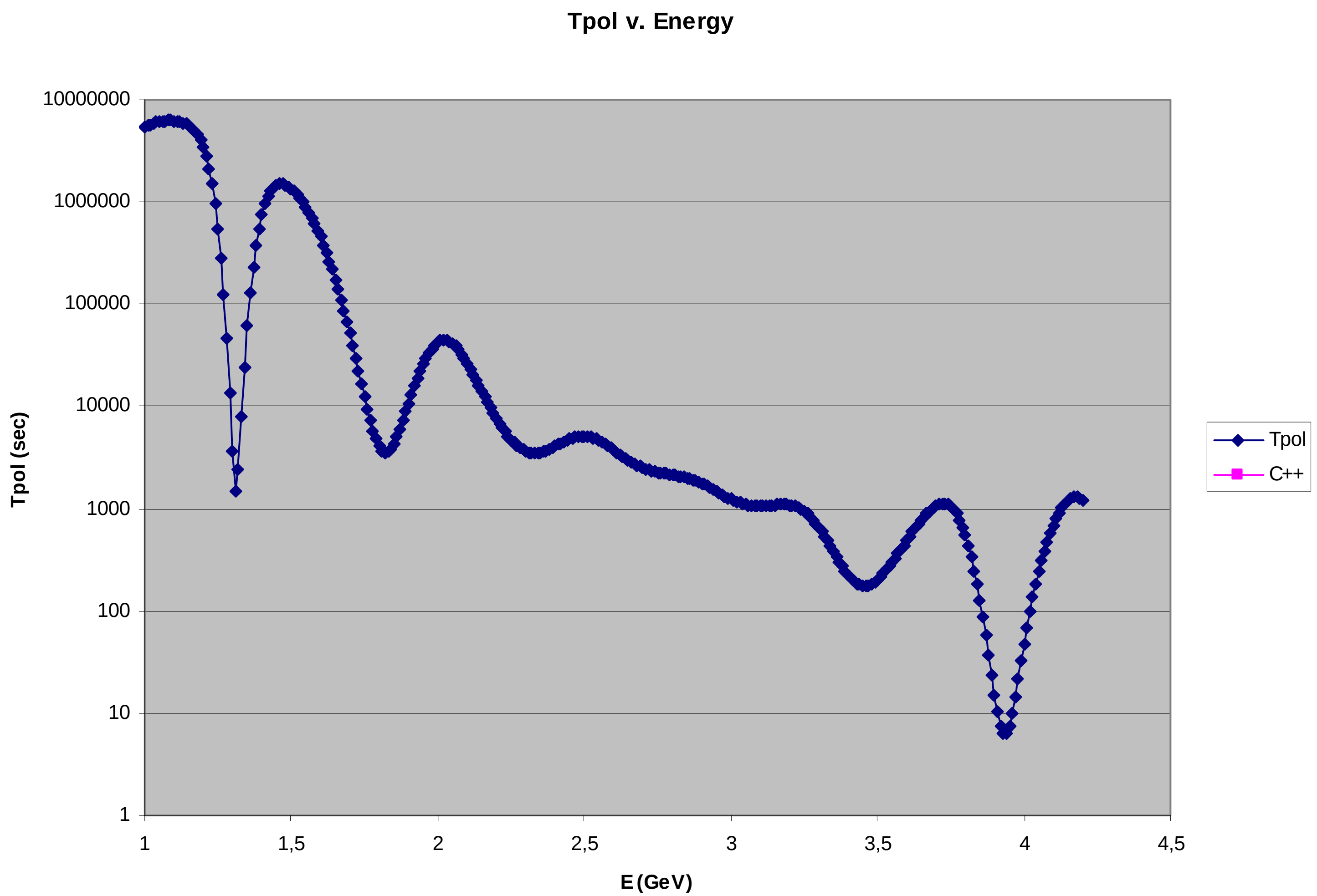
$$\tau_p = 181000\ s \quad \text{at} \quad E = 1.4\ \text{GeV}$$

ASPIRRIN, 3 snakes option



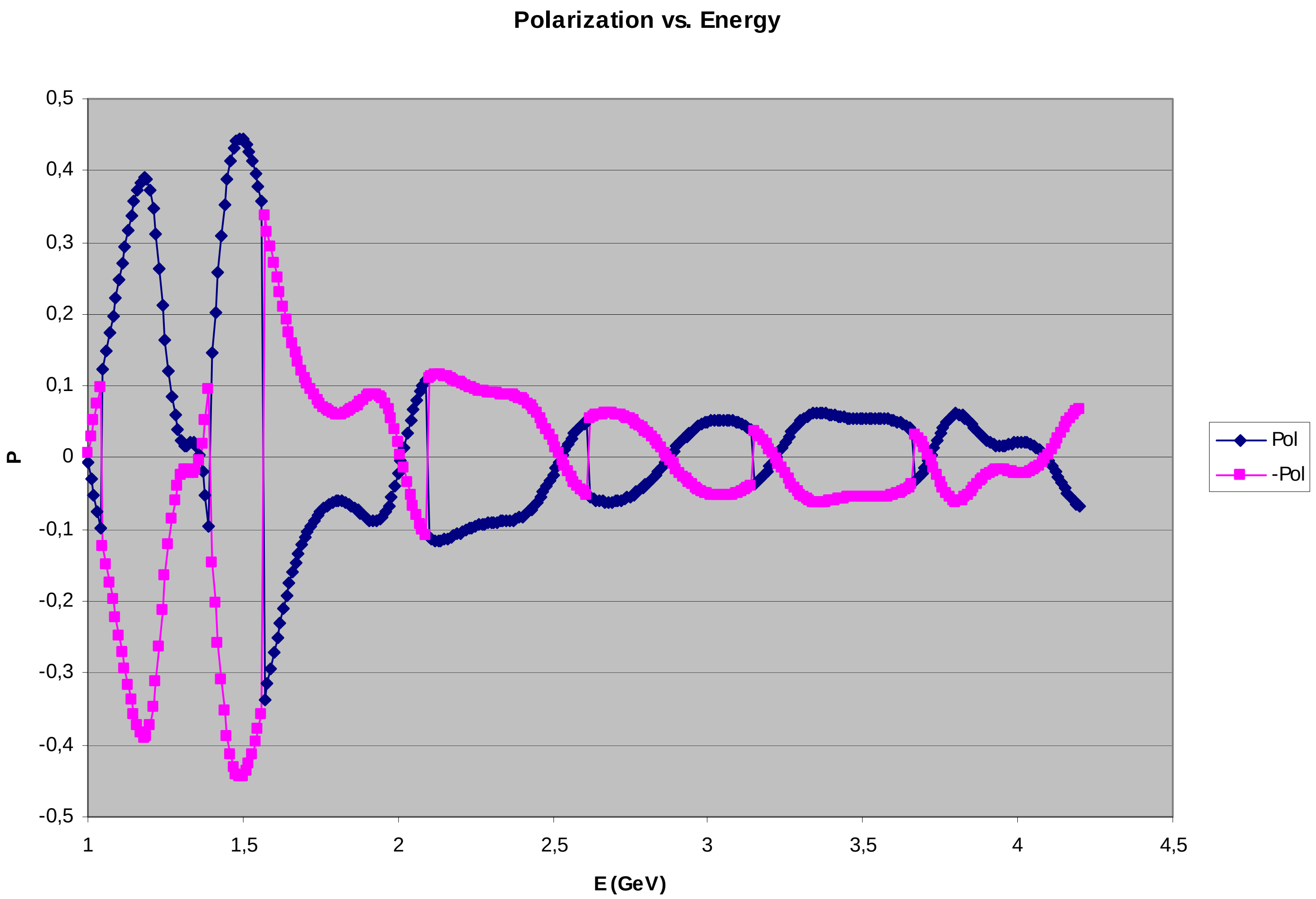
$$\tau_p = 780\ s, \quad P_{rad} = +2.7\% \ \text{ at } \ E = 4.18 \text{ GeV}$$

ASPIRRIN, Pol. Time, 90^0 rotators



$$\tau_p = 1300\ s, \quad P_{rad} = +7\% \ \text{ at } \ E = 4.18 \text{ GeV}$$

Selfpolarization, 90^0 option



Conclusion on polarization

- A scheme with two 90^0 spin rotators provides up to 80% of the longitudinal polarization in LER at 4.2 GeV.
- Single snake scheme is feasible at E<2 GeV
- 3 snakes option looks not favorable
- Tolerances on the quads gradient integrals and the solenoid field integrals are in a range of few percents

Spin tracking approach

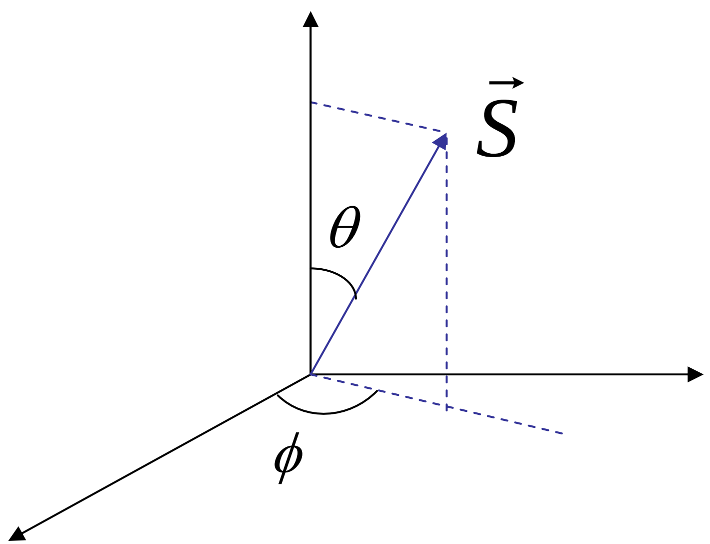
- Idea: To extend the existing particle tracking codes ACCELERATICUM (P.Piminov) and LIFETRAC (D.Shatilov) by an option of spin tracking
- Use SU(2) formalism for spin rotations, as, seems, more convenient tool for this task
- To do tracking as accurately as possible
- For the calculation of the rotation angles the spin perturbations will be accounted in linear approximation on orbital variables. But resulted rotations become not fully linear, due to axis direction and phi-angle of a rotation depend both on particle deviation from an equilibrium orbit.

SU(2) representation of rotations

Description of a spin direction by a spinor: $S_i = \chi^+ \sigma_i \chi$

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} \qquad \chi^+ = \begin{pmatrix} a^* & b^* \end{pmatrix} \qquad \chi^+\chi = |a|^2 + |b|^2 = 1$$

$$a = e^{-i\frac{\phi}{2}} \cos\frac{\theta}{2} \qquad b = e^{i\frac{\phi}{2}} \sin\frac{\theta}{2}$$



$$S_x = 2\,\mathrm{Re}(a^* b) = \sin\theta\cos\phi$$

$$S_y = 2\,\mathrm{Im}(a^* b) = \sin\theta\sin\phi$$

$$S_z = |a|^2 - |b|^2 = \cos\theta$$

Rotation: $\chi_2 = U\chi_1$

$$U = e^{-i\frac{\varphi}{2}(\vec{\sigma}\vec{n})} = \cos\frac{\varphi}{2} - i\left(\vec{\sigma}\vec{n}\right)\sin\frac{\varphi}{2}$$

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ i \end{pmatrix} \quad \rightarrow \theta = \phi = \frac{\pi}{2}$$

Spin motion perturbations

Early derived by Kondratenko, Derbenev, Chao. Also see:
Ptitsyn, Mane, Shatunov, Nucl. Instr. and Meth. A608 (2009) 225-233

$$w_x = (1+\nu_0)z'' + \left(\nu_0 - \frac{a}{\gamma_0}\right)K_x p_\sigma + (1+a)K_y x'$$

$$w_y = (1+a)\left(K_x' x + K_z' z + \Delta K_y - K_y p_\sigma\right) - (\nu_0 - a)\left(K_x p_x + K_z p_z\right)$$

$$w_z = -(1+\nu_0)x'' + \left(\nu_0 - \frac{a}{\gamma_0}\right)K_z p_\sigma + (1+a)K_y z'$$

Tracking through the straight elements, including a beam lens

Everywhere except of solenoids: $x' \equiv p_x \qquad z' \equiv p_z$

$$K_{y,z} = 0 \quad \rightarrow \begin{cases} w_x = (1+\nu_0) z'' \\ w_y = 0 \\ w_z = -(1+\nu_0) x'' \end{cases} \rightarrow \begin{cases} \varphi_x = (1+\nu_0)(p_{z2} - p_{z1}) \\ \varphi_y = 0 \\ \varphi_z = -(1+\nu_0)(p_{x2} - p_{x1}) \end{cases}$$

Angle φ and axis $\vec{n}$ of a spin rotation:

$$\varphi = \sqrt{\varphi_x{}^2 + \varphi_y{}^2 + \varphi_z{}^2}$$

$$\vec{n} = \left(\frac{\varphi_x}{\varphi}, \frac{\varphi_y}{\varphi}, \frac{\varphi_z}{\varphi} \right)$$

Tracking through the dipole edge

We shall account the edge focusing (for non-sector magnet)

and the longitudinal field component: $\int H_y d\theta = \pm H_z z \cdot \cos(\alpha)$

$$\begin{cases} \varphi_x = (1+\nu_0)\Delta p_z \\ \varphi_y = \pm(1+a)K_z z \cdot \cos(\alpha) \\ \varphi_z = -(1+\nu_0)\Delta p_x \end{cases}$$

(+) at the entrance and (-) at the exit

Here $\Delta p_x, \Delta p_z$ - transverse momentum kicks, α - edge angle

Tracking through a dipole main body

Rotation of the velocity and spin vectors projected onto a horizontal plane:

$$\phi = \phi_0 + x_1' - x_2' \quad \rightarrow \quad \varphi_{z,lab} = (1+\nu)\phi$$

$$\nu = \nu_0 \left(1 + \frac{\Delta\gamma}{\gamma_0} \right) = \nu_0 \left(1 + \left(1 - \frac{1}{\gamma_0{}^2} \right) p_\sigma \right)$$ - non equilibrium spin tune

Rotation of a spin around the vertical direction: $\varphi_y = -(\nu_0 - a)\phi_0 p_z$

Now subtract $\vec{\Omega}_V$, thus transforming to a frame where the spin angular frequency is constant: $\varphi_z = \varphi_{z,lab} - \phi = \nu\phi$

Effectively we rotate around the direction $\vec{n}$, which at the magnet exit is:

$$n_x = 0 \qquad n_y = \varphi_y / \varphi \qquad n_z = \varphi_z / \varphi \qquad \varphi = \sqrt{\varphi_y{}^2 + \varphi_z{}^2}$$

Finally we shall return to a frame of equilibrium particle, making rotation around the z-axis: $\Delta\varphi_z = \phi - \phi_0 = x_1' - x_2'$ - that's all,

except of the exit edge transformation, discussed at the previous slide

Tracking through the solenoid edges

Motion equations for the solenoid:

$$x' = p_x + \frac{1}{2}K_y z \qquad p_x{}' = \frac{1}{2}K_y\left(p_z - \frac{1}{2}K_y x\right)$$

$$z' = p_z - \frac{1}{2}K_y x \qquad p_z{}' = -\frac{1}{2}K_y\left(p_x + \frac{1}{2}K_y z\right)$$

Canonical momentums $p_{x,z}$ are continuous at the edge,

while kinetic momentums x', z' are jumping:

$$\Delta x' = \pm\frac{1}{2}K_y z \qquad \Delta z' = \mp\frac{1}{2}K_y x \quad \text{(entrance/exit edges)}$$

Spin perturbations:

$$\begin{cases} w_x = (1+\nu_0)z'' + (1+a)K_y x' \\ w_y = -(1+a)K_y p_\sigma \\ w_z = -(1+\nu_0)x'' + (1+a)K_y z' \end{cases} \rightarrow \begin{cases} \varphi_x = (1+\nu_0)\Delta z' = \mp\dfrac{1+\nu_0}{2}K_y x \\ \varphi_y = 0 \\ \varphi_z = -(1+\nu_0)\Delta x' = \mp\dfrac{1+\nu_0}{2}K_y z \end{cases}$$

Tracking through a solenoid itself

In the lab frame the spin angular frequency components are:

$$\Omega_x = -\omega_0 K_y a(\gamma_0 - 1)x'$$

$$\Omega_y = \omega_0 K_y (1+a)\left[1-\left(1-\frac{1}{\gamma_0{}^2}\right)p_\sigma\right]$$

$$\Omega_z = -\omega_0 K_y a(\gamma_0 - 1)z'$$

The propagation time: $t = \dfrac{\theta}{\omega_0}\left[1-\dfrac{1}{\gamma_0{}^2}p_\sigma\right]$

$$\vec{\Omega}_V = -\frac{q_0}{\gamma}\vec{H} \quad \to \quad \Omega_V = \omega_0 K_y\left[1-\left(1-\frac{1}{\gamma_0{}^2}\right)p_\sigma\right]$$

In the rotating with the velocity frame the spin rotations became:

$$\begin{cases} \varphi_x = -\phi_0 a(\gamma_0 - 1)x' \\ \varphi_y = t\left(\Omega_y - \Omega_V\right) = \phi_0 a\left(1 - p_\sigma\right) \\ \varphi_z = -\phi_0 a(\gamma_0 - 1)z' \end{cases} \qquad \begin{cases} n_x = \varphi_x / \varphi \\ n_y = \varphi_y / \varphi \\ n_z = \varphi_z / \varphi \end{cases} \qquad \varphi = \sqrt{\varphi_x{}^2 + \varphi_y{}^2 + \varphi_z{}^2}$$

Tracking through a solenoid, cont'd

Now let's transform back to a lab frame,

adding the velocity rotation angle:

$$\Delta\varphi_y = t\Omega_V = \phi_0\left(1 - p_\sigma\right)$$

That's all!

But, not forget to make the exit edge transformation,

described above!

Conclusion on tracking code

- The work is at the initial stage
- First results are expected at the fall 2011
- Then comparison with other tracking codes would be interesting to make