CONFINEMENT, MONOPOLES AND GAUGE INVARIANCE

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PLAN OF THE TALK

- INTRODUCTION :

  CONFINEMENT OF COLOR AND MONOPOLES

  MONOPOLES IN N.A.G.T.

- MONOPOLES AND GAUGE INVARIANCE

- MONOPOLES ON LATTICE

- CONCLUDING REMARKS AND OUTLOOK.

CONFINEMENT OF COLOR AND MONOPOLES- 1

QUARKS STRICTLY CONFINED IN NATURE

- $\frac{n_q}{n_p} \leq 10^{-27}$ EXPECT $\approx 10^{-12}$
- $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$ EXPECT $O(1)$

NATURAL EXPLANATION :
$n_q = 0$, $\sigma_q = 0$ PROTECTED BY A SYMMETRY.

DECONFINING TRANSITION A CHANGE OF SYMMETRY.

CONFINEMENT OF COLOR AND MONOPOLES-2

- GAUGE INVARIANCE CAN NOT BE BROKEN

$$U_G(\vec{x}) = U(\vec{x})U_B(\vec{x})$$

$$U(\vec{x}) = 1 \text{ on } S_2 \qquad\qquad U_B(\vec{x}) = 1 \text{ except on } S_2.$$

  S_2 THE 2-DIMENSIONAL SURFACE AT SPATIAL INFINITY.

- D.O.F. INVOLVED LIVE ON THE SURFACE AT SPATIAL INFINITY (TOPOLOGY).

- TOPOLOGICAL EXCITATIONS IN (3+1)d MONOPOLES $[S_2 \rightarrow SU(2)]$, IN (2+1)d VORTICES $[S_1 \rightarrow U(1)]$

- CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM ['tHooft , Mandelstam 1975].

- MONOPOLES: SOLITONS OF THE $SU(2)$ HIGGS MODEL WITH Φ IN THE ADJOINT REPRESENTATION. ['t HOOFT 74, POLYAKOV 74]

$$L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}D_\mu\vec{\Phi}D_\mu\vec{\Phi} - \frac{m^2}{2}\vec{\Phi}\vec{\Phi} - \frac{\lambda}{4}(\vec{\Phi}\vec{\Phi})^2$$

$$\Phi^a = \rho\frac{x^a}{|\vec{x}|}H(\xi), \quad H(\xi)_{\xi\to\infty} \to 1, \quad H(0) = 0$$

$$A_0^a = 0, \; A_i^a = -\frac{1}{gx^2}\epsilon_{iab}x_b[1 - K(\xi)], \quad K(\infty) = 0, \; K(0) = 1$$

$$\xi = g\rho x, \qquad \rho \equiv \sqrt{(-\tfrac{m^2}{\lambda})}\ .$$

- $\Phi^a{}_{\vec{x}\to 0} \approx constant.x^a \quad \Phi^a{}_{x\to\infty} = \rho\frac{x^a}{x} \quad Q^{mag} = \frac{1}{g}H(\infty)$

  MAPPING $S_2 \to O(3)/U(1)$. MAGNETIC CHARGE $\equiv$ WINDING NUMBER.

MONOPOLES IN NON ABELIAN GAUGE THEORIES-2

- UNITARY GAUGE: $\Phi \equiv \Phi^a \sigma_a$ DIAGONAL $\rightarrow$ $\Phi = |\vec{\Phi}|\sigma_3$.
  ON S_2 $\Phi = \rho\sigma_3$. (ABELIAN PROJECTION)

- ABELIAN MAGNETIC FIELD IN UNITARY GAUGE

$$B^3_k \equiv \frac{1}{2}\epsilon_{kij} F^3_{ij} \qquad F^3_{ij} = \partial_i A^3_j - \partial_j A^3_i = F_{ij}$$

$$F_{ij} \equiv \hat{\Phi}[\vec{G}_{ij} - \frac{1}{g} D_i\hat{\Phi} \wedge D_j\hat{\Phi}] \quad [\hat{\Phi} \equiv \frac{\vec{\Phi}}{|\vec{\Phi}|}] \quad (t' HOOFT \;\; TENSOR)$$

$$\vec{B}^3 \approx_{r\to\infty} \frac{1}{g}[\frac{\vec{r}}{r^3} + Dirac - string] \qquad (\text{ MONOPOLE })$$

- DIRAC STRING CAN BE PUT ALONG +z-AXIS.

MONOPOLES -PROJECTION INDEPENDENCE-1

- CONSIDER THE QUANTITY

$$M = \oint_c \hat{\Phi}\vec{A}_i dx_i .$$

c A CLOSED PATH ON S_2 ENCIRCLING $+$ z axis.

1) $M =$ TOTAL MAGNETIC CHARGE

2) M GAUGE INVARIANT AND ABELIAN PROJECTION INDEPENDENT

PROOF of 1) BY STOKES THEOREM

$$M = \int_S d\vec{\sigma}.\vec{\nabla} \wedge (\hat{\Phi}^a \vec{A}^a) = \int_S d\vec{\sigma}\vec{B}$$

PROOF OF 2) :

$$M' = \oint_c \hat{\Phi}' \vec{A}'_i dx_i = \oint_c (\hat{\Phi}\vec{A}_i + \frac{i}{g}\hat{\Phi}\partial_i R^\dagger) dx_i \quad [\ \vec{\Phi}' = R\vec{\Phi}]$$

GLOBAL TRANSFORMATION: $M = M'$

MONOPOLES : PROJECTION INDEPENDENCE-2

- FOR GAUGE GROUPS LARGER THAN $SU(2)$ MONOPOLES CAN EXIST IN ALL $SU(2)$ SUBGROUPS.
- IN QCD NO FUNDAMENTAL HIGGS FIELD. EFFECTIVE HIGGS FIELD NEEDED TO DEFINE MONOPOLES [$S_2 \to SU(2)/U(1)$]. E.G. $A_4 = iA_0$ IN THE GAUGE $\partial_0 A_0 = 0$ [JULIA, ZEE '75].
- PROOF OF PROJECTION INDEPENDENCE STAYS UNCHANGED BEING BASED ON SYMMETRY.
- MAGNETIC CHARGE IS PROJECTION INVARIANT.

  $\longrightarrow$ IF AN OPERATOR μ CREATES A MONOPOLE IN ANY ABELIAN PROJECTIONS IT CREATES A MONOPOLE IN ALL OF THEM.

MONOPOLES ON LATTICE.

LATTICE A TOOL TO EXPLORE LARGE DISTANCES IN QCD.

TWO MAIN STRATEGIES TO STUDY CONFINEMENT:

- A) DEFINE A MONOPOLE CREATION OPERATOR, μ .
  $\langle\mu\rangle \equiv \langle 0|\mu|0\rangle$ ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY.
  [PISA, BARI]
- B) DETECT MONOPOLES IN LATTICE CONFIGURATIONS AND DETERMINE THEIR POTENTIAL. A MINIMUM AT NON ZERO FIELD SIGNALS SUPERCONDUCTIVITY.
  [ITEP, BERLIN, KANAZAWA]
- IN BOTH CASES CHOOSE THE $U(1)$ SUBGROUP IN WHICH MONOPOLES LIVE (ABELIAN PROJECTION)

- DEFINITION OF μ [A.D.G. et al '95 $\rightarrow$]

$$\mu(\vec{x},t) = \exp\left(i\int d^3y \vec{E}^3_\perp(\vec{y},t).\vec{A}^0_\perp(\vec{y}-\vec{x})\right)$$

$\vec{A}^0_\perp(\vec{y}-\vec{x})$: THE FIELD IN $\vec{y}$ PRODUCED BY A MONOPOLE SITTING in $\vec{x}$ (IN TRANSVERSE GAUGE).

$$\exp(ipa)|x\rangle = |x+a\rangle \;\rightarrow\; \mu(\vec{x},t)|\vec{A}^3_\perp(\vec{z},t)\rangle = |\vec{A}^3_\perp(\vec{z},t)+\vec{A}^0_\perp(\vec{z}-\vec{x})\rangle$$

- $$\langle\mu(\beta)\rangle = \langle 0|\mu 0\rangle = \frac{Z\left(\beta(S+\Delta S)\right)}{Z\left(\beta S\right)} \qquad \beta \equiv \frac{2N}{g^2}$$

$$\rho(\beta) \equiv \frac{\partial ln(\langle\mu(\beta)\rangle)}{\partial\beta} = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{S+\Delta S}$$

$$\langle\mu(\beta)\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right) \qquad \mu(0) = 1$$

- CREATING A MONOPOLE IS ABELIAN PROJECTION INDEPENDENT OPERATION $\rightarrow$ CREATE IT ALONG FORMAL 3-AXIS IN COLOUR SPACE USED IN THE UPDATING PROCEDURE, AS DONE IN [PISA , BARI].
- 2) AN INFRARED PROBLEM FOUND IN THE ORIGINAL DEFINITION OF ORDER PARAMETER [COSSU et al '07]. SOLVED IN [BONATI et al '12] :

$$\langle 0|\mu 0\rangle \rightarrow \mu' = \frac{\langle 0|\mu 0\rangle}{\sqrt{(\langle 0|0\rangle\langle \mu 0|\mu 0\rangle)}}$$

A RIGOROUS TEST OF DUAL SUPERCONDUCTIVITY.

- $SU(2)$ PURE GAUGE $\mu' \neq 0$ IN CONFINED $\mu' = 0$ IN DECONFINED. CORRECT SCALING AT THE TRANSITION AND CLUSTER PROPERTY. [BONATI et al '12] HIGHER GROUPS AND QUARKS IN PROGRESS.

MONOPOLES ON LATTICE-APPROACH B) -1

- DIFFICULT TO DETECT A (MAGNETIC) CHARGE IN A (DUAL) SUPERCONDUCTOR [SHIELDING] .
- IN *QCD* NO FUNDAMENTAL HIGGS FIELD, EXCEPT $\vec{A}_4$ IN THE GAUGE $\partial_4 \vec{A}_4 = 0$ [JULIA, ZEE '75; PRASAD , SOMMERFELD '73, BOGOMONLYI '76]
- EFFECTIVE HIGGS FIELD $\Phi(\vec{A}_\mu)$:
  1) $\Phi \in$ ADJOINT REPRESENTATION
  2) HAVE A ZERO AT THE LOCATION $\vec{x}_0$ OF THE MONOPOLE $\Phi^a \approx constant.(x - x_0)^a$
  3) BEHAVE AS $\Phi^a = \rho \hat{x}^a$ ON THE SPHERE S_2 AT SPATIAL INFINITY
  4) $A_i^a = -\frac{1}{gx^2}\epsilon_{iab}x_b[1 - K(\xi)] \quad K(\infty) = 0, \quad K(0) = 1$
- ANY (SCALAR) OPERATOR $\vec{O}$ IN THE ADJOINT REPRESENTATION CAN ACT AS AN EFFECTIVE HIGGS. MONOPOLES SIT AT ITS ZEROES. ['t HOOFT '81].

- THE ARGUMENT : IN THE UNITARY GAUGE NEAR A ZERO $\vec{x}_0$ $\langle O^a(\vec{x})\rangle \propto \delta_{a3} r$ WITH $r = |\vec{x} - \vec{x}_0|$, AS THE HIGGS IN THE MONOPOLE SOLUTION.
- A NECESSARY BUT NOT GENERALLY SUFFICIENT CONDITION TO HAVE A MONOPOLE!
- MONOPOLES IN A GIVEN ABELIAN PROJECTION DETECTED ON LATTICE BY LOOKING AT THE DIRAC STRING AS AN EXCESS OF PHASE THROUGH A PLAQUETTE. [DEGRAND, TOUSSAINT '80]

$$\vec{B}^3 \approx_{r\to\infty} \frac{1}{g}[\frac{\vec{r}}{r^3} + Dirac - string]$$

EXISTENCE OF MONOPOLE PROJECTION DEPENDENT!

$\longrightarrow$ LATTICE ARTEFACTS.

REDUCE THEM GOING TO LARGE $\beta \equiv \frac{2N}{g^2}$.
- FOR $U(1)$ GAUGE GROUP APPROACH A) AND B) WORK BOTH AND GIVE CONSISTENT RESULTS . [A.D.G., PAFFUTI '97, POLIKARPOV et AL '91]

CONCLUDING REMARKS AND OUTLOOK.

- DUAL SUPERCONDUCTIVITY A MECHANISM FOR CONFINEMENT BASED ON SYMMETRY. DECONFINEMENT A PHASE TRANSITION NOT A CROSSOVER $\rightarrow$ NO CRITICAL END POINT IN THE PHASE DIAGRAM AT NON-ZERO μ.
- CREATION OF A MONOPOLE A GAUGE INVARIANT FACT: $\rightarrow$ DEFINE IN A SIMPLE WAY AN ORDER PARAMETER AND TEST THE MECHANISM.
- ALTERNATIVE DEFINITION BASED ON THE EFFECTIVE POTENTIAL FROM LATTICE MONOPOLES PLAGUED BY ARTEFACTS TO BE REMOVED.
- ARGUMENTS LIKE MONOPOLE DOMINANCE OR BASED ON ELIMINATION OF MONOPOLES INCONCLUSIVE FOR CONFINEMENT.