Black Holes
A window into Strong (Quantum?) Gravity

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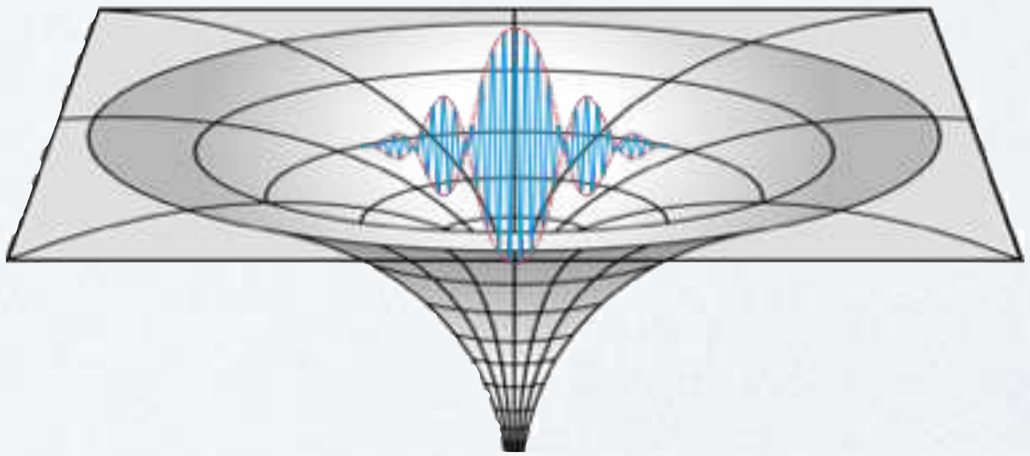

Plan of the talk

1. Gravitational collapse of quantum matter

2. Gravity: Status Quo

3. Horizon Quantum Mechanics*: Schwarzschild radius of quantum source

   - Single particle: horizon wave-function and the GUP

   - BEC black holes: fuzzy horizon and semiclassical limit

4. Summary and outlook

*Papers:
Entropy 17 (2015) 6893
IJMP D 25 (2016) 1630006
JCAP 09 (2015) 002
JHEP 05 (2015) 096
PLB *732* (2014) 105; *747* (2015) 68
PRD 90 (2014) 084040; 91 (2015) 124069
EPJC *74* (2014) 2685; *75* (2015) 160; *75* (2015) 445

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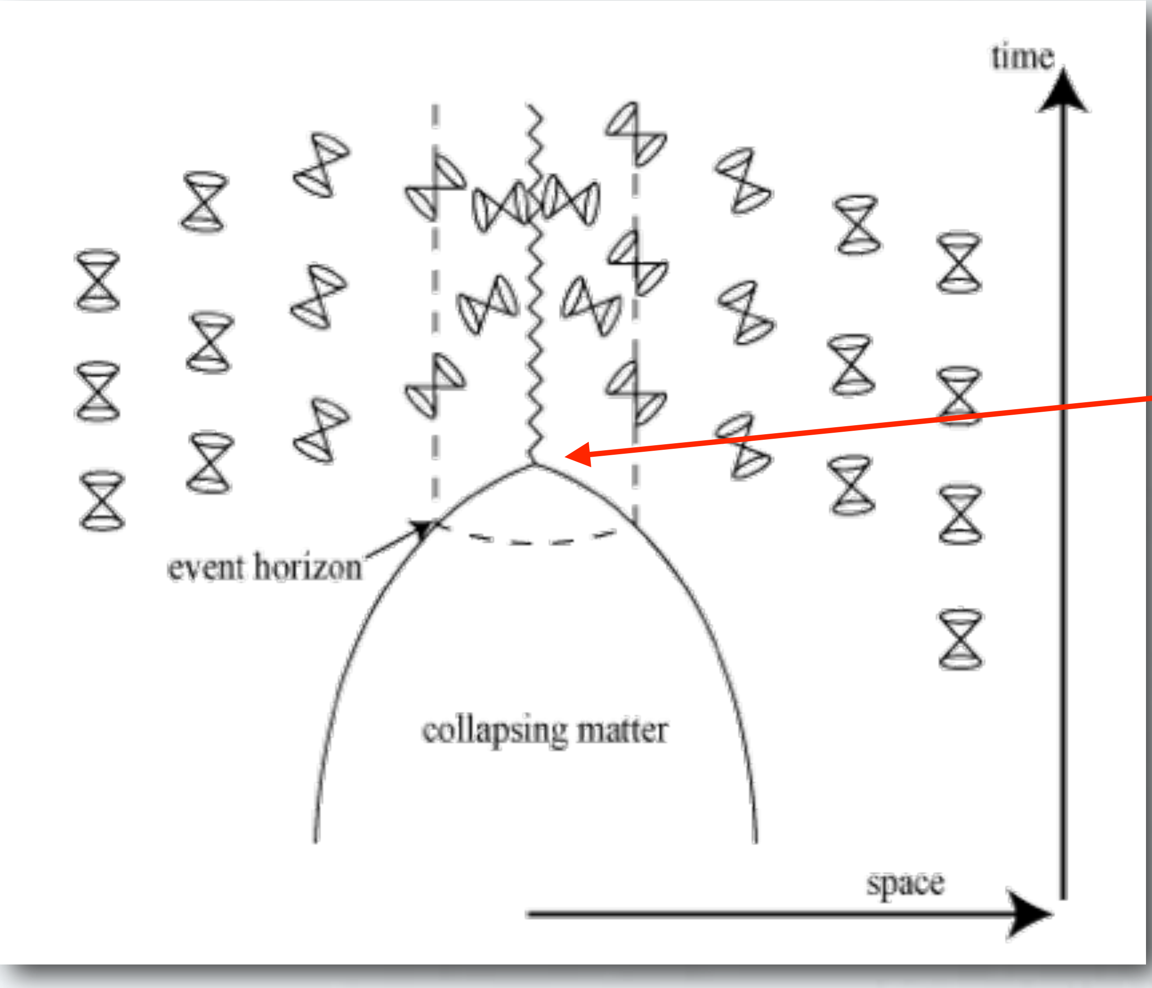
1) Gravitational collapse

Standard CLASSICAL picture = General Relativity

CLASSICAL matter and GEOMETRICAL space-time*



Central Singularity

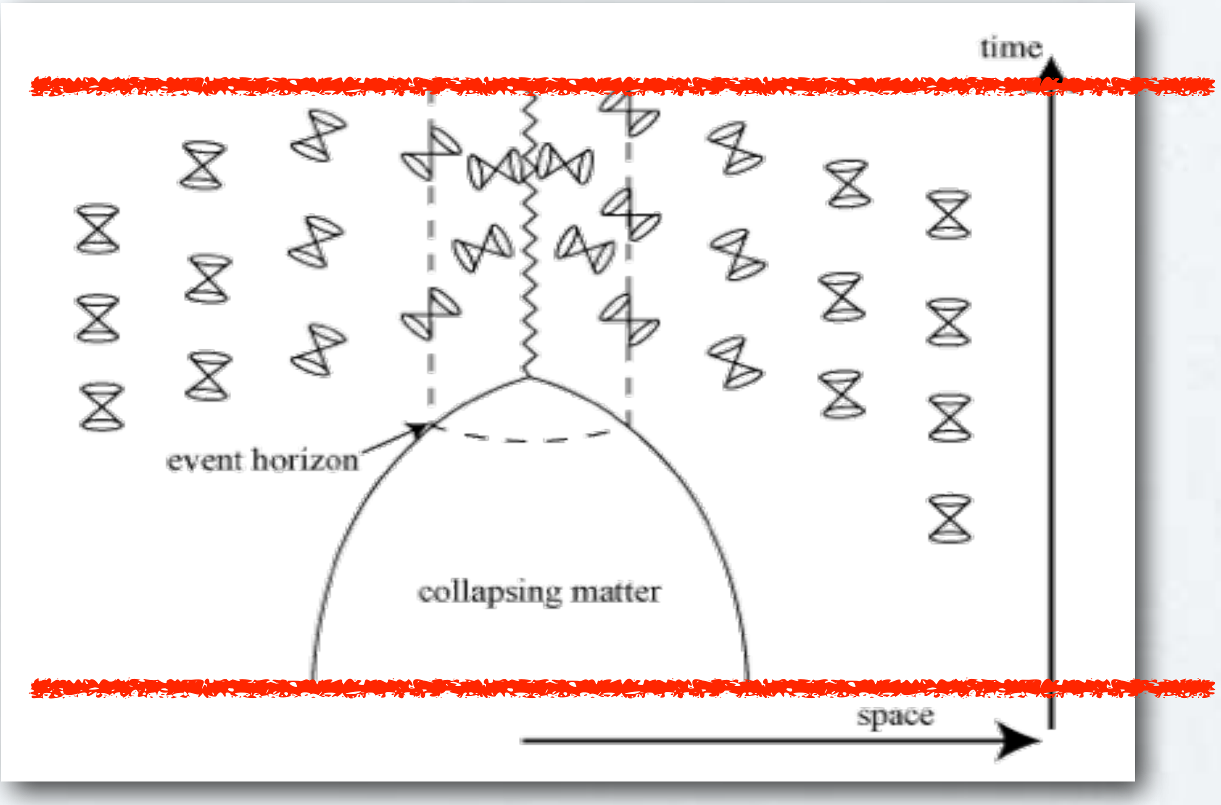
*Prototype background: $ds^2 = -\left(1 - \frac{2\,M}{r}\right) dt^2 + \left(1 - \frac{2\,M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2$

But matter is **QUANTUM**…!

1) Gravitational collapse

Standard SEMICLASSICAL picture = QFT on curved space-time

Background: CLASSICAL matter and GEOMETRICAL space-time
Foreground: QUANTUM particles



$$|0; t = +\infty\rangle$$

$$e^{-\frac{i}{\hbar}\int \hat{H}\, dt}$$

$$|0; t = -\infty\rangle$$

$$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$$

But world is **QUANTUM**...!(?)

1) Gravitational collapse

Perhaps we need **Quantum Gravity** … but what might it really be?

$$\sqrt{\hbar\, G_{\rm N}} = \ell_{\rm p} \simeq 10^{-35}\ {\rm m}$$

Numerology or…?

$$\sqrt{\hbar / G_{\rm N}} = M_{\rm p} \simeq 10^{19}\ {\rm GeV}$$

$$\boxed{\frac{\hbar}{M_{\rm p}} \simeq G_{\rm N}\, M_{\rm p}}$$

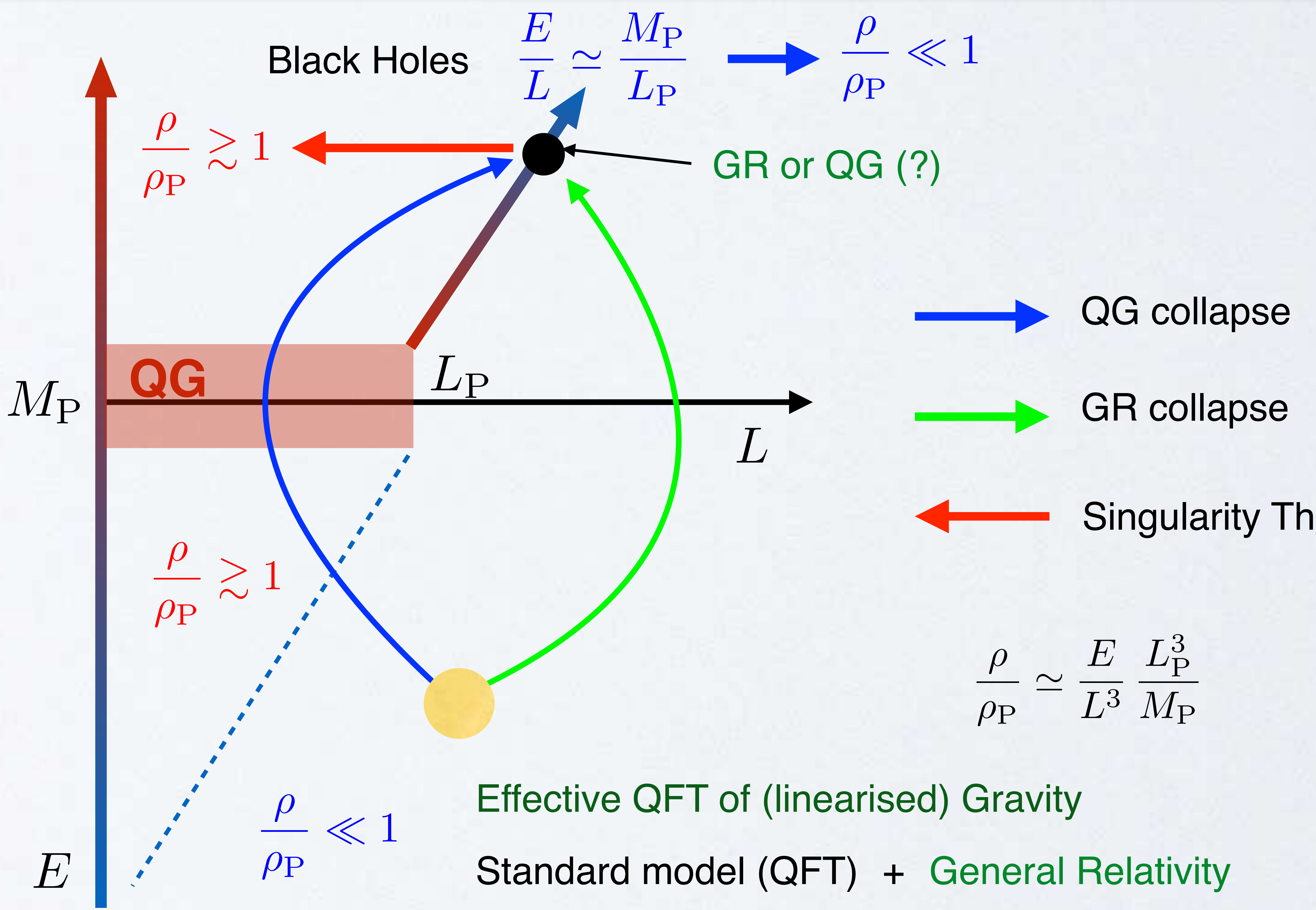
Q(FT) scale
(Compton length)

(Strong) Gravity scale
(Schwarzschild radius)

Quantum Gravity
(black holes)

$$\boxed{G_{\rm N} = \frac{\ell_{\rm p}}{m_{\rm p}}}$$

2) Gravity: Status Quo



2) Gravity: Status Quo

BH (Quantum Gravity) vs **Hadron** (QCD)

(Almost) no data!

Expectations:

Confines everything
in non-perturbative regime
(within causal horizon)

Asymptotically classical

$M \gg M_{\rm p}$

A lot of data!

Facts:

Confines quarks
in non-perturbative regime
(below $\Lambda_{\rm QCD} \simeq 220\,{\rm MeV}$)

Asymptotically free

$E \gg \Lambda_{\rm QCD}$

Employ effective Quantum (Field) Theory (~ chiral theory)

3) Horizon Quantum Mechanics

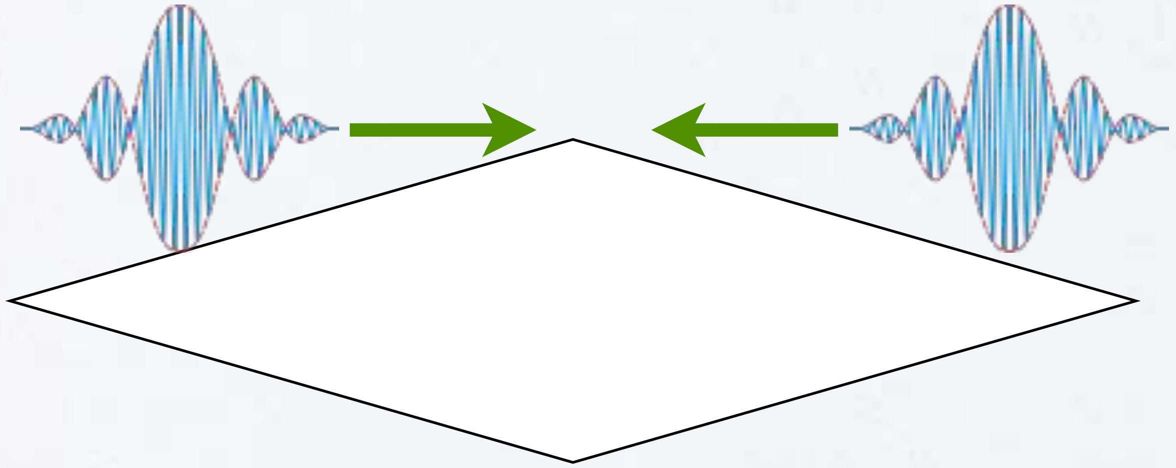
What is the Schwarzschild radius of QM states?

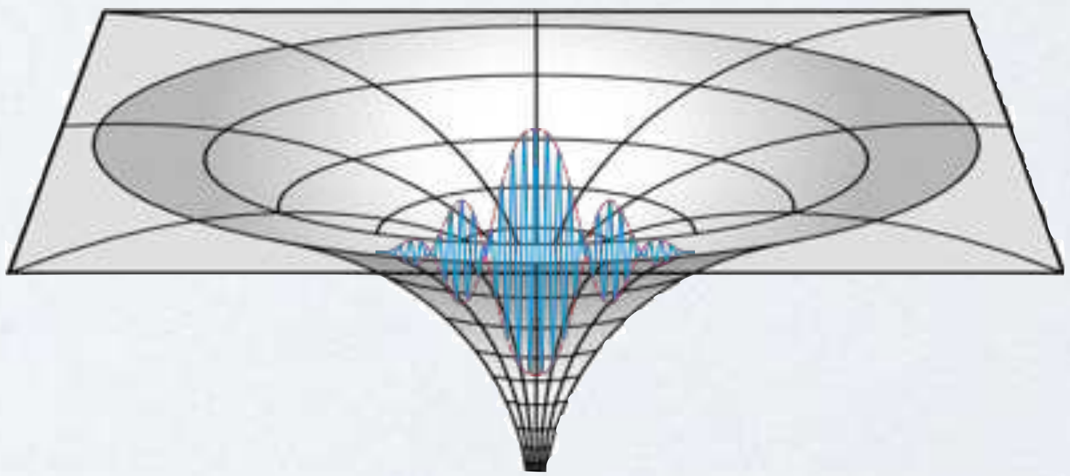
Many attempts at quantising "pure" black hole/horizon degrees of freedom (independently of source)

"Background field approach"

But **GR is non-linear:**

(non-perturbatively) quantise black hole/horizon **and** matter source!





3) Horizon Quantum Mechanics

Classical spherically symmetric system:

$$ds^2 = g_{tt}\, dt^2 + g_{rr}\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Einstein equation ("Hamiltonian constraint"):

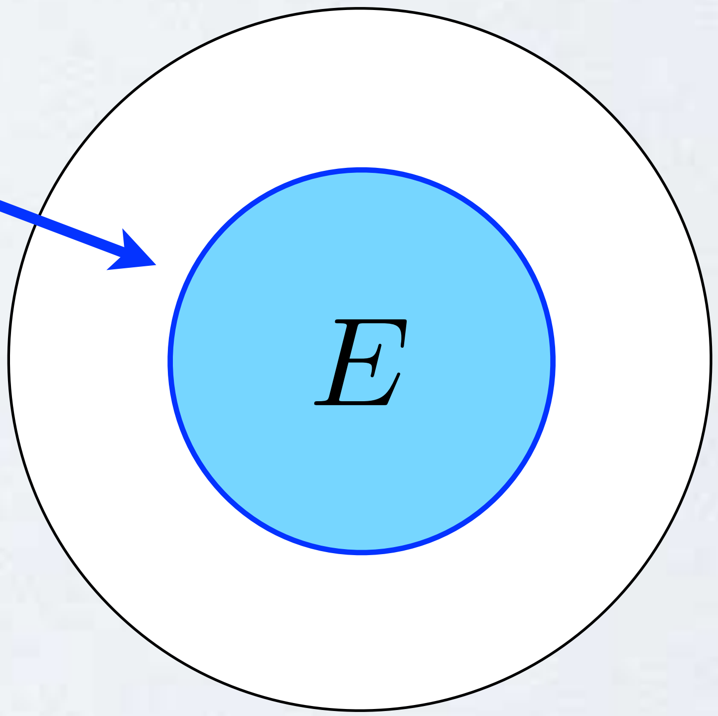
$$g^{rr} = 1 - \frac{2\, G_{\mathrm{N}}\, E(t,r)}{r}$$

Misner-Sharp mass

$$E = \frac{4\,\pi}{3} \int_0^r \rho(t,r')\, r^2\, dr'$$

$$G_{\mathrm{N}} = \frac{\ell_{\mathrm{p}}}{m_{\mathrm{p}}}$$

$$R_{\mathrm{H}} = 2\,\ell_{\mathrm{p}}\, \frac{E}{m_{\mathrm{p}}}$$



Sphere is a **trapping surface**
("escape velocity = c") if

$$4\,\pi\, R_{\mathrm{H}}^2 = 4\,\pi\, r^2$$

Schwarzschild radius

Areal radius

3) Horizon Quantum Mechanics

A) "Hamiltonian constraint" relates ADM mass to gravitational radius

$$|\Psi\rangle = \sum_{\alpha,\beta} C(E_\alpha, R_{\mathrm{H}\beta})\,|E_\alpha\rangle|R_{\mathrm{H},\beta}\rangle$$

$$\hat{H}\,|E_\alpha\rangle = E_\alpha\,|E_\alpha\rangle$$

$$\hat{R}_\mathrm{H}\,|R_{\mathrm{H}\alpha}\rangle = R_{\mathrm{H}\alpha}\,|R_{\mathrm{H}\alpha}\rangle$$

$$0 = \left(\hat{H} - \frac{m_\mathrm{p}}{2\,\ell_\mathrm{p}}\,\hat{R}_\mathrm{H}\right)|\Psi\rangle = \sum_{\alpha,\beta}\left(E_\alpha - \frac{m_\mathrm{p}}{2\,\ell_\mathrm{p}}\,R_{\mathrm{H}\beta}\right) C(E_\alpha, R_{\mathrm{H}\beta})\,|E_\alpha\rangle|R_{\mathrm{H}\beta}\rangle$$

$$C(E_\alpha, R_{\mathrm{H}\beta}) = C(E_\alpha, 2\,\ell_\mathrm{p}\,E_\alpha/m_\mathrm{p})\,\delta_{\alpha\beta}$$

3) Horizon Quantum Mechanics

B) trace out gravitational d.o.f. = source wave-function:

$$|\psi_{\rm S}\rangle = \sum_\gamma \langle R_{{\rm H}\gamma}| \sum_\alpha C(E_\alpha, 2\,\ell_{\rm p}\,E_\alpha/m_{\rm p})\,|E_\alpha\rangle |R_{{\rm H}\alpha}\rangle$$

$$= \sum_\gamma C_{\rm S}(E_\gamma)\,|E_\gamma\rangle$$

(spectral decomposition)

C) trace out source d.o.f. = horizon wave-function:

$$|\psi_{\rm H}\rangle = \sum_\gamma \langle E_\gamma| \sum_\alpha C_{\rm S}(E_\alpha)\,|E_\alpha\rangle |R_{{\rm H}\alpha}\rangle$$

$$= \sum_\gamma C_{\rm S}(m_{\rm p}\,R_{{\rm H}\gamma}/2\,\ell_{\rm p})\,|R_{{\rm H}\gamma}\rangle$$

D) local construction also available, see arXiv:1605.tomorrow

E) extensions to modified gravity theories (under way…)

3) Single Particle

Localised particle at rest:

Gaussian wave-function:

$$\boxed{\psi_{\rm S}(r) \simeq e^{-\frac{r^2}{2\,\ell^2}}}$$

$$\Big\downarrow \quad \Delta = \frac{\hbar}{\ell} \sim m$$

Energy spectrum: $\quad |\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$

$$\psi_{\rm S}(p) \simeq e^{-\frac{p^2}{2\,\Delta^2}}$$

$$\Big\downarrow \quad E^2 = p^2 + m^2$$

(flat space)

Horizon wave-function:

$$R_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$

$$\boxed{\psi_{\rm H}(r_{\rm H}) \simeq e^{-\frac{\ell^2\,r_{\rm H}^2}{8\,\ell_{\rm p}^4}}}$$

3) Single Particle

Probability density particle is inside its own Schwarzschild radius = horizon:

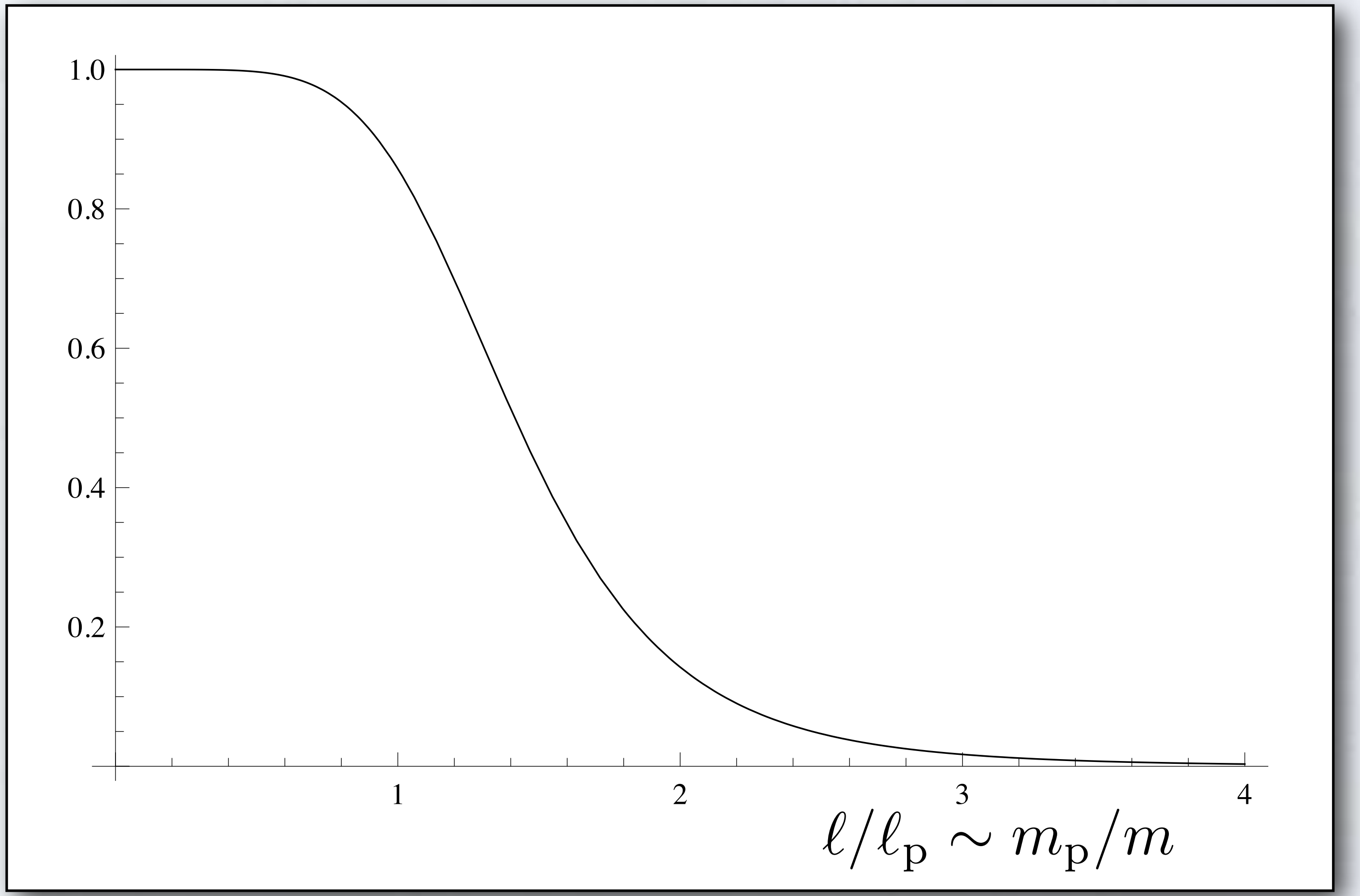
$$\mathcal{P}_<(r < r_\mathrm{H}) = P_\mathrm{S}(r < r_\mathrm{H})\,\mathcal{P}_\mathrm{H}(r_\mathrm{H})$$

$$P_\mathrm{S}(r < r_\mathrm{H}) = 4\,\pi \int_0^{r_\mathrm{H}} |\psi_\mathrm{S}(r)|^2\, r^2\, dr$$

$$\mathcal{P}_\mathrm{H}(r_\mathrm{H}) = 4\,\pi\, r_\mathrm{H}^2\, |\psi_\mathrm{H}(r_\mathrm{H})|^2$$

Probability particle is a Black Hole:

$$\boxed{P_\mathrm{BH} = \int_0^\infty \mathcal{P}_<(r < r_\mathrm{H})\, dr_\mathrm{H}}$$



$$M_\mathrm{BH} \gtrsim m_\mathrm{p}$$

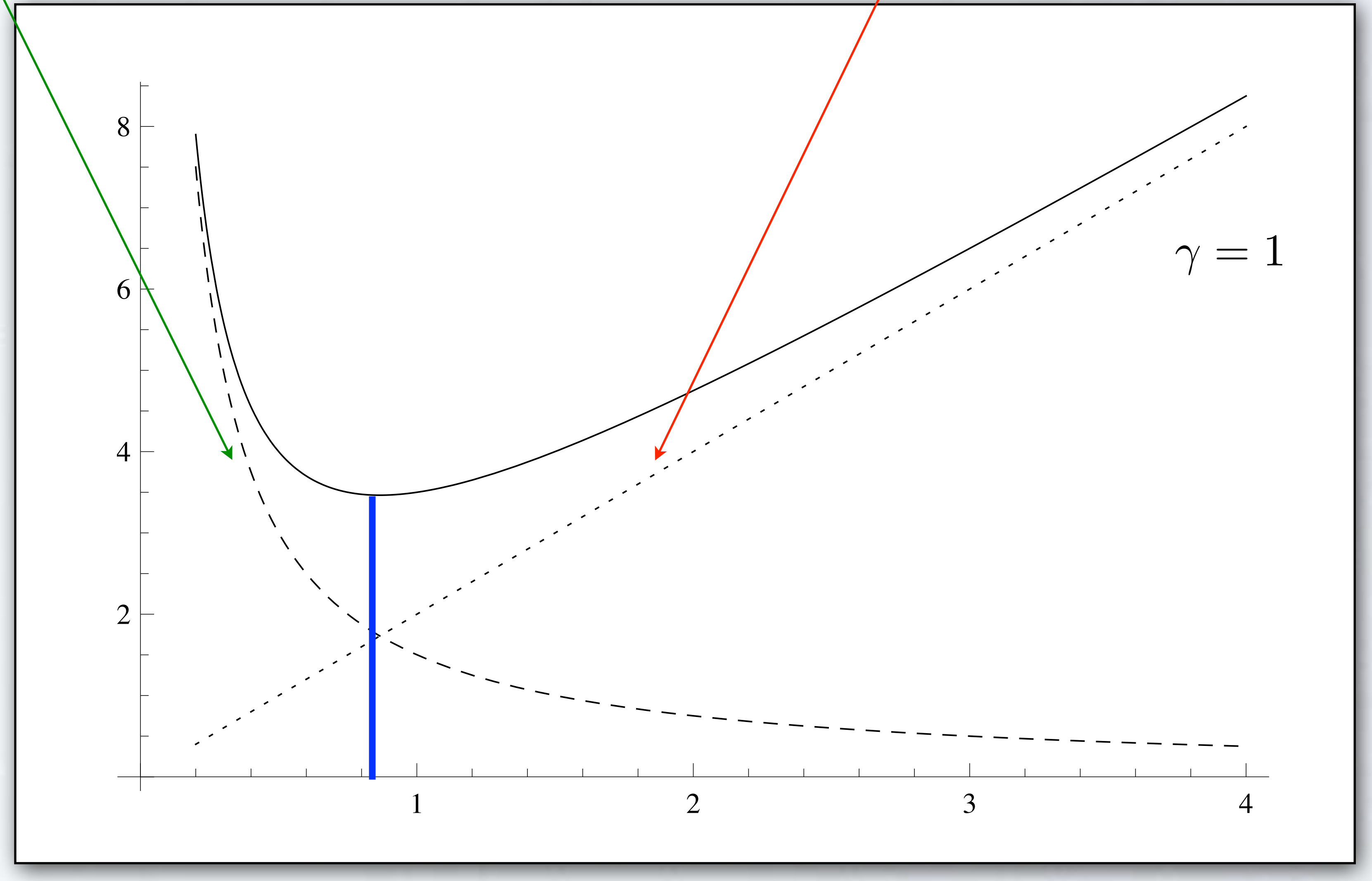
"Fuzzy" minimum mass

3) Single Particle

Two uncertainties: $\langle \Delta r^2 \rangle \sim \ell^2$ $\langle \Delta r_{\mathrm{H}}^2 \rangle \sim \dfrac{\ell_{\mathrm{p}}^2}{\ell^2}$

$$\Delta r = \sqrt{\langle \Delta r^2 \rangle} + \gamma \sqrt{\langle \Delta r_{\mathrm{H}}^2 \rangle}$$
$$= \ell_{\mathrm{p}} \frac{m_{\mathrm{p}}}{\Delta p} + 2\,\gamma\, \ell_{\mathrm{p}} \frac{\Delta p}{m_{\mathrm{p}}}$$

$$\langle \Delta p^2 \rangle \sim \frac{\ell_{\mathrm{p}}^2}{\ell^2}$$



Minimum measurable length

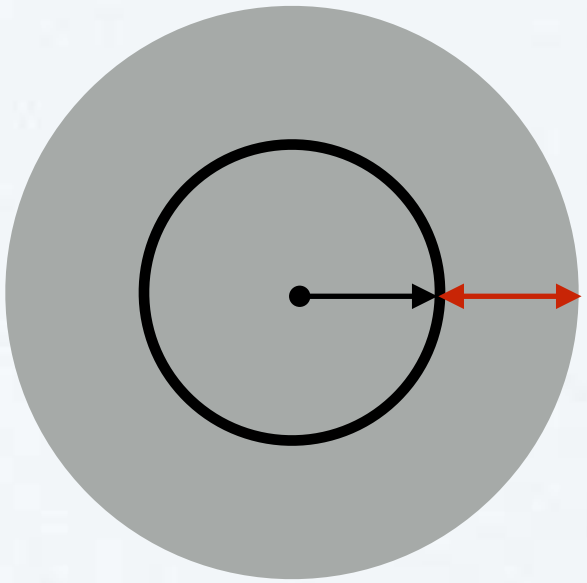
N.B. Uncertainty **derived** with standard canonical commutators: $[q, p] = i\,\hbar$
(gravity is more than kinematics...?)

3) Single Particle

Horizon uncertainty:

$$\Delta R_{\rm H} = \sqrt{\langle \hat{R}_{\rm H}^2 \rangle - \langle \hat{R}_{\rm H} \rangle^2} \simeq \ell_{\rm p}^2/\ell$$

$$\langle \hat{R}_{\rm H} \rangle \simeq \ell_{\rm p}^2/\ell$$



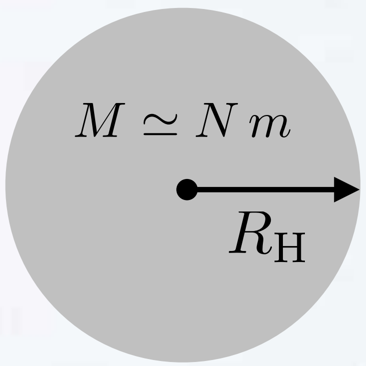
$$\boxed{\frac{\Delta R_{\rm H}}{\langle \hat{R}_{\rm H} \rangle} \simeq 1}$$

Semiclassical black holes cannot be made of a single particle!

3) BEC Black Hole

[Dvali & Gomez]

BH = self-sustained gravitons = BEC



$$U \simeq -m\,\frac{\ell_{\rm p}}{m_{\rm p}}\,\frac{M \sim N\,m}{r \sim \lambda_m \sim \frac{\ell_{\rm p}\,m_{\rm p}}{m}} \simeq -m\,N \left(\alpha \sim \frac{m^2}{m_{\rm p}^2}\right)$$

$$E \simeq m + U \simeq 0$$

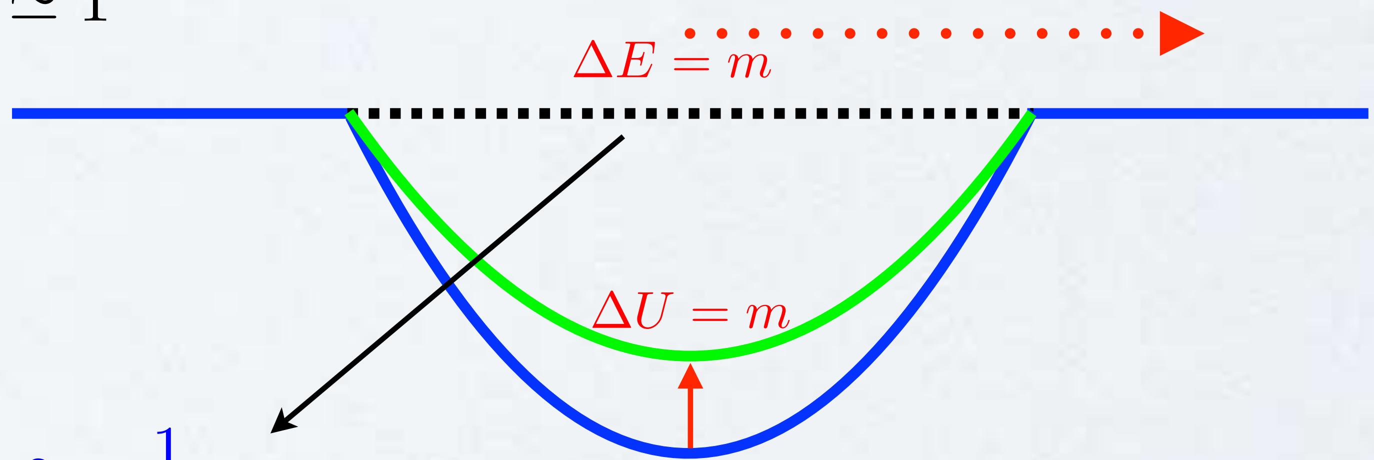
$$R_{\rm H} \simeq \sqrt{N}\,\ell_{\rm p}$$

$$m \simeq \frac{m_{\rm p}}{\sqrt{N}}$$

N>>1

$$N\,\alpha \simeq 1$$

Hawking quantum



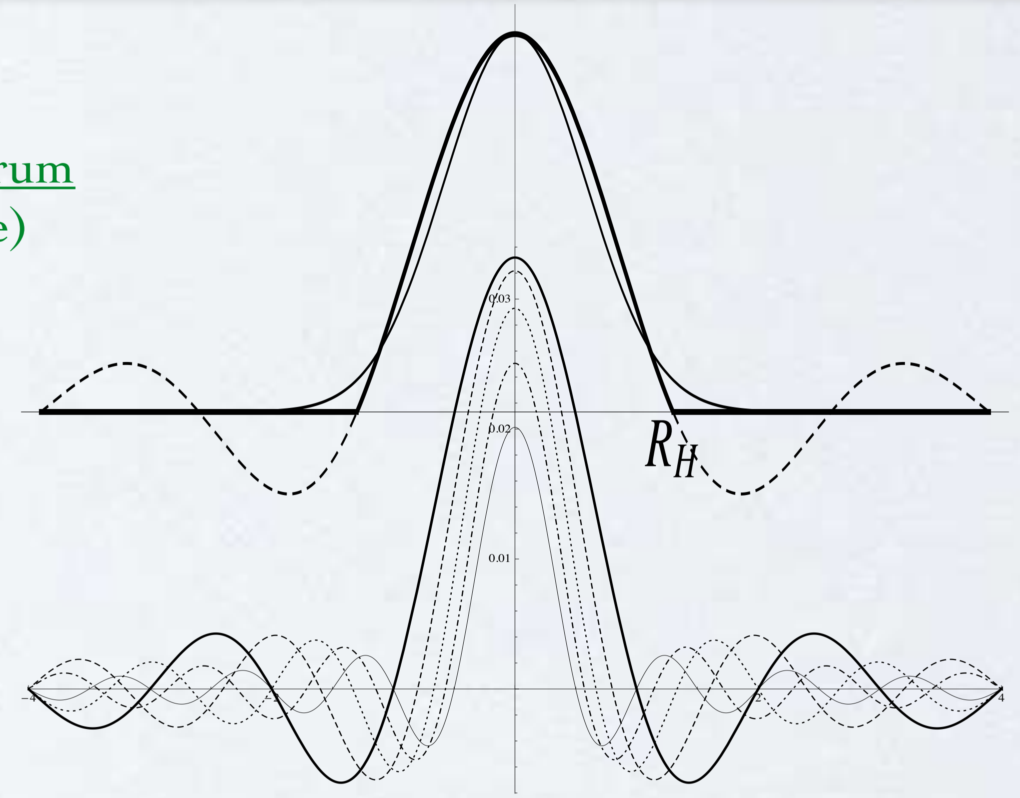
$$\Gamma \sim \frac{1}{N^2}\,N^2\,\frac{1}{\sqrt{N}\,\ell_{\rm p}}$$

$$\dot{M} \simeq m\,\frac{\dot{N}}{\sqrt{N}} \sim -m\,\frac{\Gamma}{\sqrt{N}} \sim -\frac{m_{\rm p}^3/\ell_{\rm p}}{M^2}$$

3) BEC Black Hole

BH with Planckian hair:

$$|\psi_{\rm S}^{(i)}\rangle \simeq |m\rangle + \gamma \int_m^\infty \frac{d\omega_i}{m^{3/2}} \frac{(\omega_i - m)}{\exp\left(\frac{\omega_i - m}{m}\right) - 1} |\omega_i\rangle$$



N-particle state:

$$|\psi_{\rm S}\rangle = \frac{1}{N!} \sum_{\{\sigma_i\}}^{N} \left[\bigotimes_{i=1}^{N} |\psi_{\rm S}^{(i)}\rangle \right]$$

$$C(E > M) \sim \frac{\gamma}{m^{3/2}} \frac{(E - M)}{\exp\left(\frac{E-M}{m}\right) - 1}$$

$$\boxed{\frac{\Delta r_{\rm H}}{\langle \hat{r}_{\rm H} \rangle} \sim \frac{\gamma}{N}}$$

4) Summary and outlook

1. Horizon Quantum Mechanics so far:
   - GUP for single particle
   - Semiclassical behaviour for radiating BEC black holes (end of evaporation)
   - Quantum Hoop conjecture (for 2 particles in 1+1D)
   - Minimum mass quantum black holes from dressed graviton propagator
   - Charged sources and Quantum Cosmic Censorship
   - Black holes in higher and lower dimensions
   - Horizon wave-function in cosmology
2. Modified Time Evolution
3. Generalise to **spinning** systems (~ Kerr)
4. Analyse (2-)particle collisions with angular momentum+spin
5. (Hope for) quantum description of gravitational collapse

Thank you!