Collective Excitations in QCD Plasma

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Outline

1. Review of relativistic chiral hydrodynamics
2. Chiral Magnetic wave & Chiral vortical wave
3. Hydrodynamic excitations in chiral hydro
4. Collective modes from kinetic-theory
5. Outlook

Hydrodynamics

- Consider a locally equilibrated system where thermodynamic variables are well-defined in small patches ($\sim \frac{1}{T}$)
- Hydrodynamics describes the dynamics of a thermal system slightly deviated from its global thermal equilibrium.

$$\boxed{\text{locally equilibrated system}} \xrightarrow{\text{Hydrodynamics}} \boxed{\text{global-equilibrium}}$$

$$u_\mu(x),\ T(x),\ \mu(x)$$

$$\rho = \frac{1}{Z} e^{\beta u_\mu P^\mu + \beta \mu N}$$

hydrodynamic variables = local thermodynamic functions

- Effectively only few degrees of freedom matter in Hydro-regime.

$$H = H_0 + \varepsilon H_1$$

Heisenberg's equation

$$\frac{d}{dt} Q = i \left[H_0, Q \right] + i \varepsilon \left[H_I, Q \right]$$

$$\frac{d}{dt} Q = i \overset{0}{\cancel{\left[H_0, Q \right]}} + i \varepsilon \left[H_I, Q \right] \approx O(\varepsilon)$$

- Conserved quantities are slowly varying
- So the non-conserved quantities reach to their equilibrium values faster than conserved quantities
- The only relevant physical quantities in the hydrodynamic regime are **conserved quantities**.

Hydrodynamic equations = Conservation laws

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$\partial_\mu J^\mu = 0$$ (Non-anomalous current)

$$\partial_\mu J^\mu = C E^\mu B_\mu$$ (Anomalous current)

- To solve these equations we need the relations between currents and hydro-variables: Constitutive relations

- Since Hydro variables are slowly varying functions we may write derivative expansions for currents

$$T^{\mu\nu} = T^{\mu\nu}{}_{(0)} + T^{\mu\nu}{}_{(1)} + \ldots$$

- Constitutive relations

Ideal fluid = globally boosted of thermodynamic

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Ideal fluid
$$\begin{cases} T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P \eta^{\mu\nu} \\ J^\mu = n u^\mu \end{cases}$$

Viscose fluid
$$\begin{cases} T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P \eta^{\mu\nu} + \tau^{\mu\nu} \\ J^\mu = n u^\mu + \nu^\mu \end{cases}$$

First derivative corrections

Non-anomalous Fluid

- In non-anomalous fluids the first order derivative corrections are given by some dissipative transport coefficients

$$\tau^{\mu\nu} = -2\,\eta\,\sigma^{\mu\nu} - \zeta\,\theta\,P^{\mu\nu}$$

Shear- viscosity

Bulk- viscosity

$$\nu^{\mu} = -\sigma T\,\partial^{\mu}\left(\frac{\mu}{T}\right) + \sigma E^{\mu}$$

conductivity

$$E^{\mu} = F^{\mu\nu}u_{\nu}$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\frac{\nabla_{\alpha}u_{\beta} + \nabla_{\beta}u_{\alpha}}{2} - \frac{\nabla.u}{3}\eta_{\alpha\beta}\right)$$

$$\theta = \nabla.u$$

$$P^{\mu\nu} = u^{\mu}u^{\nu} + \eta^{\mu\nu}$$

Chiral-Fluid

$$J^\mu = nu^\mu - \sigma T\partial^\mu\left(\frac{\mu}{T}\right) + \xi\omega^\mu + \xi_B B^\mu$$

$$B^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu \partial_\alpha u_\beta$$

New odd-parity terms related to quantum anomaly
anomalous transports

- Positivity of entropy production completely fixes anomalous coefficients. $\partial_\mu s^\mu \geq 0$

$$\xi = C\mu^2\left(1 - \frac{2}{3}\frac{n\mu}{\varepsilon + P}\right) + DT^2\left(1 - \frac{2n\mu}{\varepsilon + P}\right),$$

$$\xi_B = C\mu\left(1 - \frac{1}{2}\frac{n\mu}{\varepsilon + P}\right) - \frac{D}{2}\frac{nT^2}{\varepsilon + P}$$

Landau frame

- D.T.Son and P.Surowka, arXiv:0906.5044
- Erdmenger et al. arXiv:0809.2488
- Banerjee et al. arXiv:0809.2596

$C = \frac{1}{4\pi^2}$, Chiral anomaly

$D = \frac{1}{12}$, Gauge-gravitational anomaly

Fluid with U(1) ×U(1) global currents – QCD

Y.Neiman and Y.Oz, arXiv:1011.5107
K.~Landsteiner et al. arXiv:1103.5006
A. V. Sadofyev and M. V. Isachenkov. arXiv:1010.1550

$$\partial_\mu J^\mu = 0, \qquad \partial_\mu J_5^\mu = C E^\mu B_\mu$$

$$J^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu, \qquad J_5^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_{B5} B^\mu$$

$$\xi = C(\mu\mu_5 - \frac{n\mu_5}{3(e+p)}(3\mu^2 + \mu_5^2)) - D\frac{n\mu_5}{e+p}T^2$$

$$\xi_5 = C(\mu^2 + \mu_5^2 - \frac{2n_5\mu_5}{3(e+p)}(3\mu^2 + \mu_5^2)) + \frac{D}{2}(1 - \frac{2n_5\mu_5}{e+p})T^2$$

$$\xi_B = C\mu_5(1 - \frac{n\mu}{e+p})$$

$$\xi_{5B} = C\mu(1 - \frac{n_5\mu_5}{e+p})$$

$$C = \frac{1}{2\pi^2}, \qquad D = \frac{1}{6},$$

Chiral anomaly

Gauge-gravitational anomaly

Chiral Magnetic Wave

- Consider a chiral-fluid (zero-chemical potential) in an external magnetic field

$$\begin{cases} \partial_t n + \vec{\partial}.\vec{j} = 0 \\ \partial_t n_5 + \vec{\partial}.\vec{j}_5 = 0 \end{cases} \qquad \begin{cases} \vec{j} = C\,\vec{B}\,\mu_5 & \text{Chiral Magnetic Effect} \\ \vec{j}_5 = C\,\vec{B}\,\mu & \text{Chiral Separation Effect} \end{cases}$$

Due to CME and CSE Conserved equations are coupled to each other.

Kharzeev, Yee, arXiv:1012.6026

$$\chi = \frac{n}{\mu} = \frac{n_5}{\mu_5} \qquad \begin{cases} \partial_t \mu + \frac{C\,B}{\chi}\partial_z \mu_5 = 0 \\ \partial_t \mu_5 + \frac{C\,B}{\chi}\partial_z \mu = 0 \end{cases} \Rightarrow \partial_t^2 \mu - \left(\frac{C\,B}{\chi}\right)^2 \partial_z^2 \mu = 0 \Rightarrow \vec{V}_{\mathrm{CMW}} = \pm\frac{C}{\chi}\vec{B} = \pm\frac{1}{2\pi^2\chi}\vec{B}$$

susceptibility

Lattice:

Müller, Schlichting and Sharma. **arXiv:1606.00342**

Buividovich and Valgushev, **arXiv:1611.05294**

Chiral Vortical Wave

- Rotating Chiral fluid with constant vorticity.

$$\vec{V}_{\mathrm{CVW}} = \pm \frac{\mu C}{\chi} \vec{\Omega}$$

- In contrast to the CMW, CVW does not propagate at zero vector charge density.

Jiang, Huang, Liao. arXiv:1504.03201

Full Spectrum of Chiral Fluid Excitations

- In all of these studies the CMW and CVW are computed by considering just the fluctuations of vector and axial charge densities.
- neglecting the energy and momentum fluctuations.
- This is good approximation in high temperature or density regime.

M. Stephanov, H. U. Yee and Y. Yin, Phys. Rev. D 91, no. 12, 125014 (2015).

Our Goal:

Studding the effects of energy and momentum fluctuations on chiral waves. Finding the full spectrum of chiral fluid excitations.

N. Abbasi, D.AllahBakhshi, A.D, F.Taghavi arXiv:1612.08614

Chiral fluid in an External magnetic field

- The equilibrium state of the system is specified with

$$T = \text{const.} \qquad \mu = \text{const.} \qquad \mu_5 = \text{const.} \qquad u^{\mu} = (1, \vec{0}).$$

$$\vec{B} = \text{const.}$$

- We choose the hydro variables to be as follows

$$\phi_a = (T, \pi_i, \mu, \mu_5), \quad i = 1, 2, 3. \quad a = 1, 2, 3, 4, 5, 6.$$

$$\pi_i = (e + p) v_i$$

In total we have six hydro variables and so six hydro modes

- Linearizing hydro equations around equilibrium in Fourier space

$$M_{ab}(\vec{k},\omega)\,\delta\phi_b(\vec{k},\omega) = 0$$

$$\begin{bmatrix} -i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\ i\alpha_1 v_s^2 k^i & -i\omega\delta^i_j - i\frac{\xi}{2\bar{w}}\left(\boldsymbol{B}\cdot\boldsymbol{k}\delta^i_j - B_j k^i\right) - \frac{\bar{n}}{\bar{w}}\epsilon^i{}_{jl}B^l & i\alpha_2 v_s^2 k^i & i\alpha_3 v_s^2 k^i \\ -i\beta_1\omega + \left(\frac{\partial\xi_B}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}}{\bar{w}} ik_j - \frac{\xi_B}{\bar{w}} i\omega B_j & -i\beta_2\omega + \left(\frac{\partial\xi_B}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\beta_3\omega + \left(\frac{\partial\xi_B}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \\ -i\gamma_1\omega + \left(\frac{\partial\xi_{5B}}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}_5}{\bar{w}} ik_j - \frac{\xi_{5B}}{\bar{w}} i\omega B_j & -i\gamma_2\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\gamma_3\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{bmatrix}$$

- This matrix is not block-diagonal in general
- All fluctuations are coupled to each other.
- hydrodynamic waves might be a coherent excitation of all hydro-variables

$$\alpha_1 = \frac{\partial\epsilon}{\partial T}, \quad \alpha_2 = \frac{\partial\epsilon}{\partial\mu}, \quad \alpha_3 = \frac{\partial\epsilon}{\partial\mu_5}$$
$$\beta_1 = \frac{\partial n}{\partial T}, \quad \beta_2 = \frac{\partial n}{\partial\mu}, \quad \beta_3 = \frac{\partial n}{\partial\mu_5}$$
$$\gamma_1 = \frac{\partial n_5}{\partial T}, \quad \gamma_2 = \frac{\partial n_5}{\partial\mu}, \quad \gamma_3 = \frac{\partial n_5}{\partial\mu_5}$$

Susceptibility matrix

Hydrodynamic Modes

- Finding the roots of the determinants of the matrix M, perturbatively order by order, in derivative expansion (powers of wave-number k).

6 - Modes

2 modes are propagations of T, μ, μ_5 → Chiral Magnetic-Heat Waves

Counterpart of CMW

4 modes are propagations of T, μ, μ_5, $\boldsymbol{\pi_i}$ →

Sound waves

Chiral Alfven Waves

Chiral Magnetic-Heat Waves

$$\omega_{1,2}(k) = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \mathcal{A}_2\mathcal{E}}}{\mathcal{E}}\,\boldsymbol{B}.\boldsymbol{k}$$

$$\mathcal{A}_1 = \frac{\mathcal{C}}{2}\left(\alpha_{[3}\beta_{1]} + \alpha_{[2}\gamma_{1]} + \frac{2\mu\mu_5}{w}\mathcal{E} + \mathcal{A}_0\right)$$

$$\mathcal{A}_2 = \mathcal{C}^2\alpha_1\left(1 - \frac{n\mu + n_5\mu_5}{w}\right) + \mathcal{C}^2\frac{\mu\mu_5}{w}\left(\alpha_{[3}\beta_{1]} + \alpha_{[2}\gamma_{1]} + \frac{\mu\mu_5}{w}\mathcal{E} + \mathcal{A}_0\right)$$

$$\mathcal{A}_0 = +\frac{n\mu}{w}\alpha_{[1}\gamma_{2]} + \frac{n_5\mu_5}{w}\alpha_{[1}\beta_{3]} - \frac{n\mu_5}{w}\alpha_{[1}\gamma_{3]} - \frac{n_5\mu}{w}\alpha_{[1}\beta_{2]}.$$

$$\mathcal{E} = -\epsilon^{ijk}\alpha_i\beta_j\gamma_k \qquad (\epsilon^{123} = 1)$$

$$A_{[i}B_{j]} = A_iB_j - A_jB_i$$

- In contrast to zero-chemical potential case, two modes propagates with different velocities.

- Zero chiral chemical potential $\mu_5 = 0$

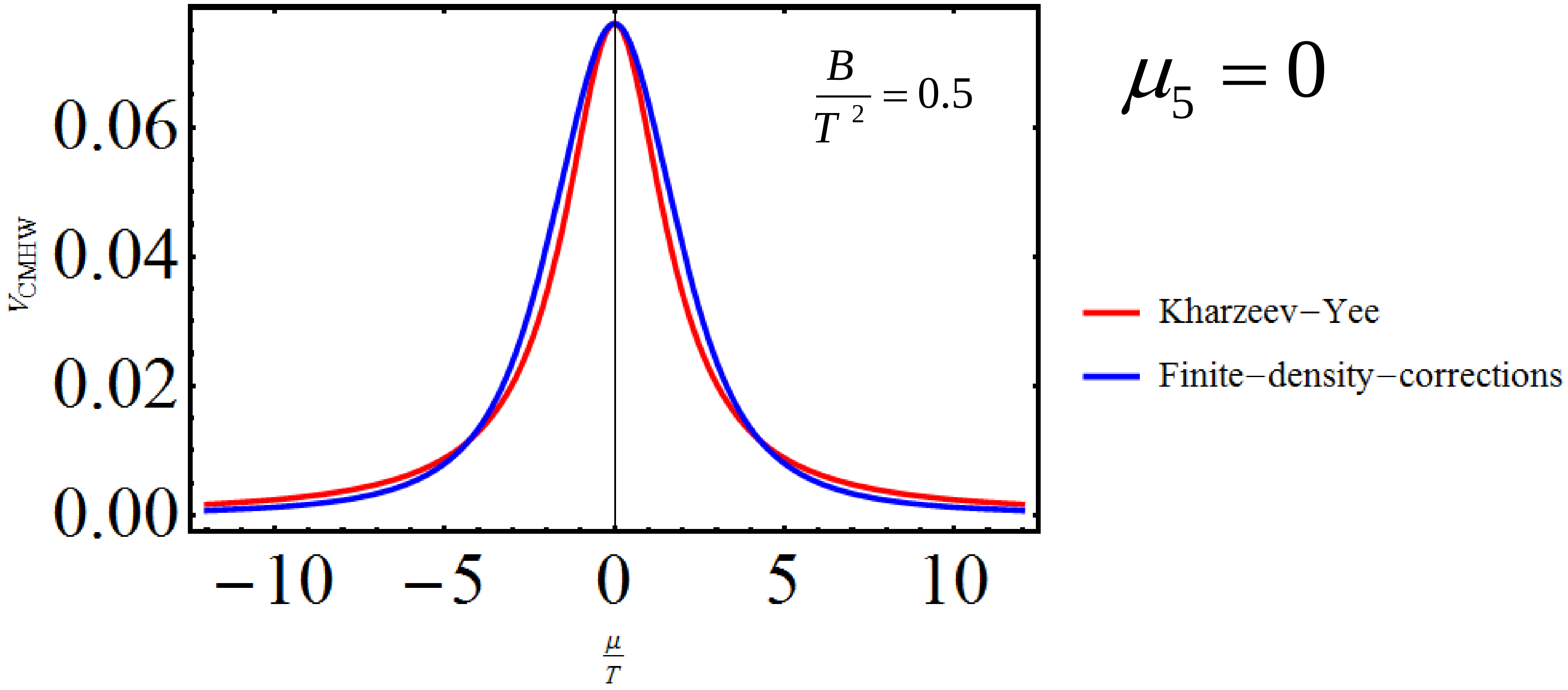
$$\omega_{1,2} = \pm \underbrace{\frac{Bk}{2\pi^2\chi}} \frac{1 - \frac{\mu n}{w}}{\sqrt{1 - \frac{\mu n}{w} - \frac{n}{\chi w}\left(\frac{n}{c_s^2} - \chi\mu\right)}}$$

D. Kharzeev and H.-Y. Yee, 2011

- Two modes propagates with the same velocity

- Using the free-fermion equation of state

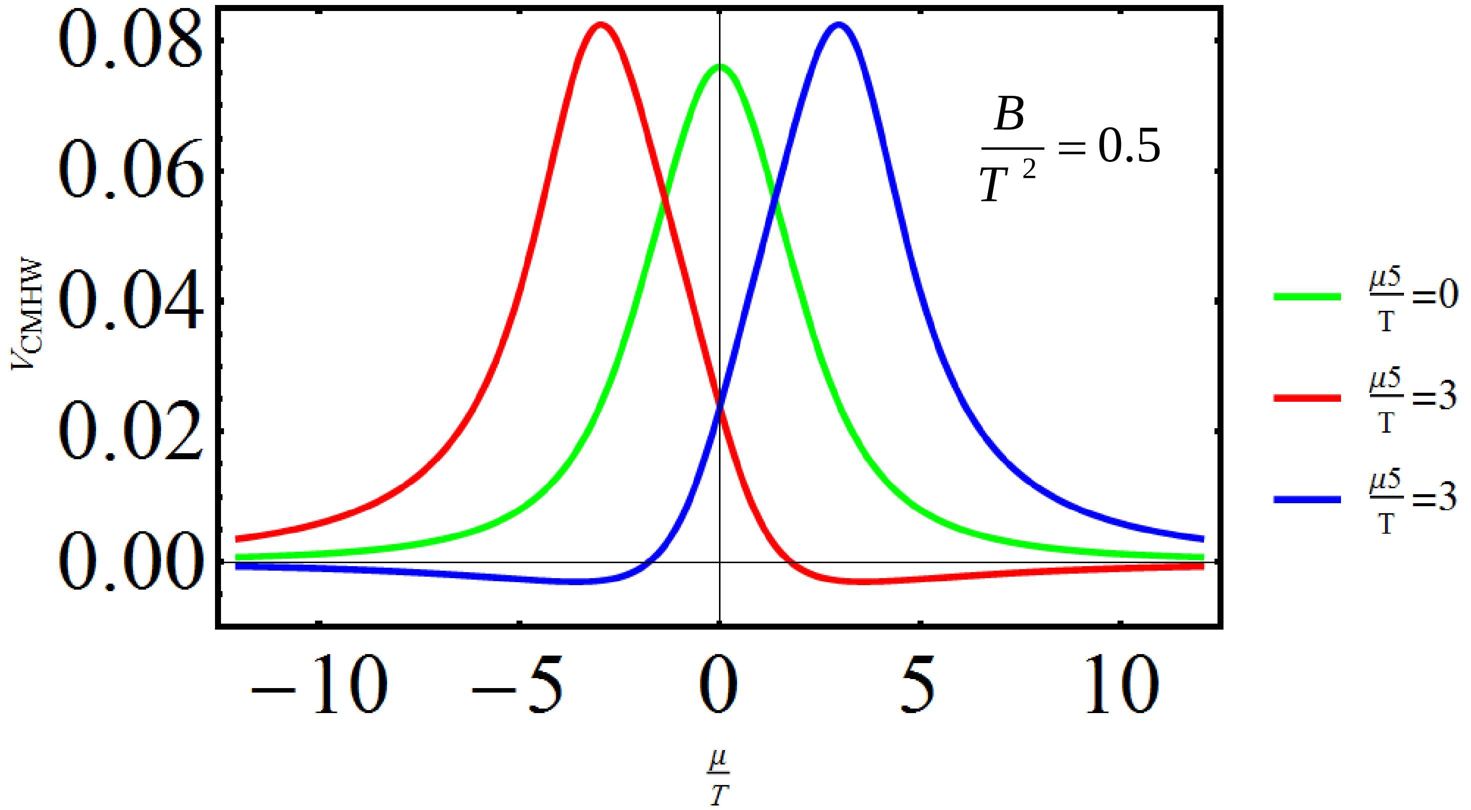
$$\epsilon = 3p = \frac{7\pi^2}{60}T^4 + \frac{1}{2}(\mu^2 + \mu_5^2)T^2 + \frac{1}{4\pi^2}(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4).$$



$\mu_5 = 0$

- Corrections are small at zero chiral chemical potential

- Finite chiral density: we observe a significant difference



- At finite chiral density two CMWs propagates with different velocities.

- each of the fast and slow waves reaches to its maximum velocity when or $\mu = \mu_5$ or $\mu = -\mu_5$

Sound-Alfven Waves

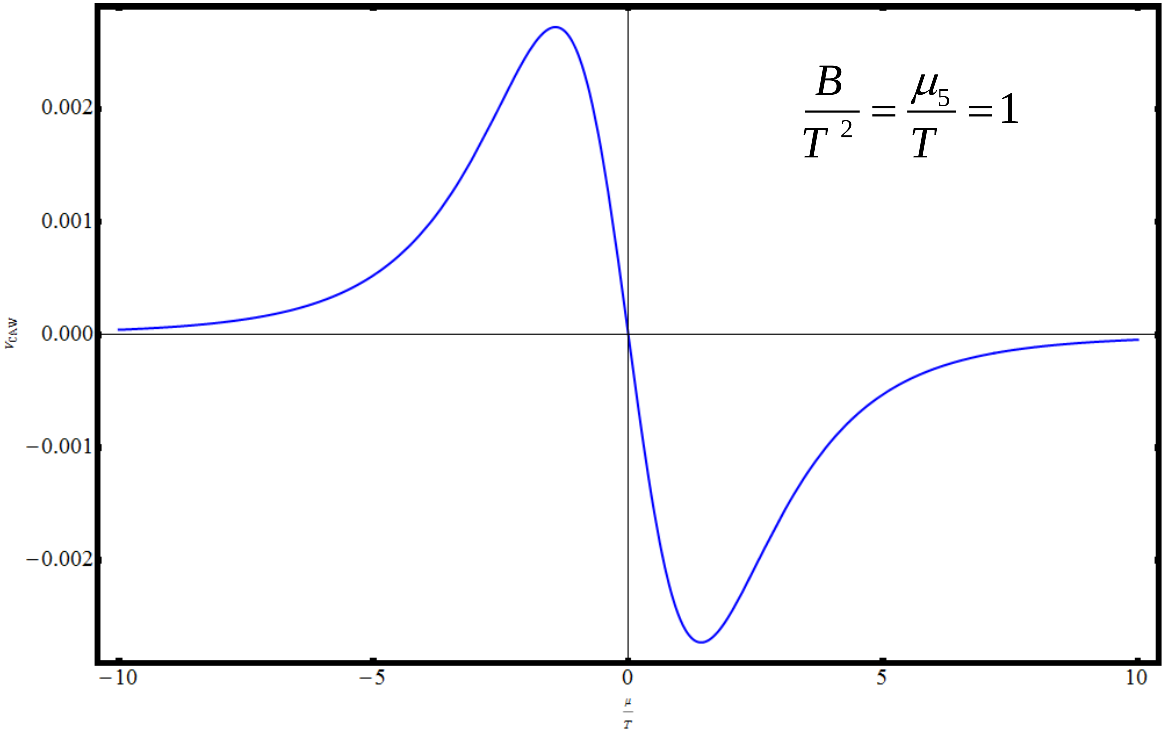
$$\boldsymbol{B} \parallel \boldsymbol{k}$$

$$v_{3,4} = \pm c_s$$ → Sound waves

$$v_{5,6} = -\frac{\xi}{w} B$$ → Chiral Alfven waves

N. Yamamoto, **arXiv:1505.05444**

chiral fluid with single chirality



$$\boldsymbol{B} \perp \boldsymbol{k}$$

$$v_{3,4} = \pm \frac{c_s}{\sqrt{1 + \frac{\Omega_L^2}{c_s^2 k^2}}}$$

$$\Omega_L = \frac{nB}{w}.$$

Larmore frequency

No-anomaly effect

Rotating Chiral Fluid

- Equilibrium state

$T = \text{const.} \quad \mu = \text{const.} \quad \mu_5 = \text{const.}$

$\omega^{\mu} = (0, \vec{\Omega})$

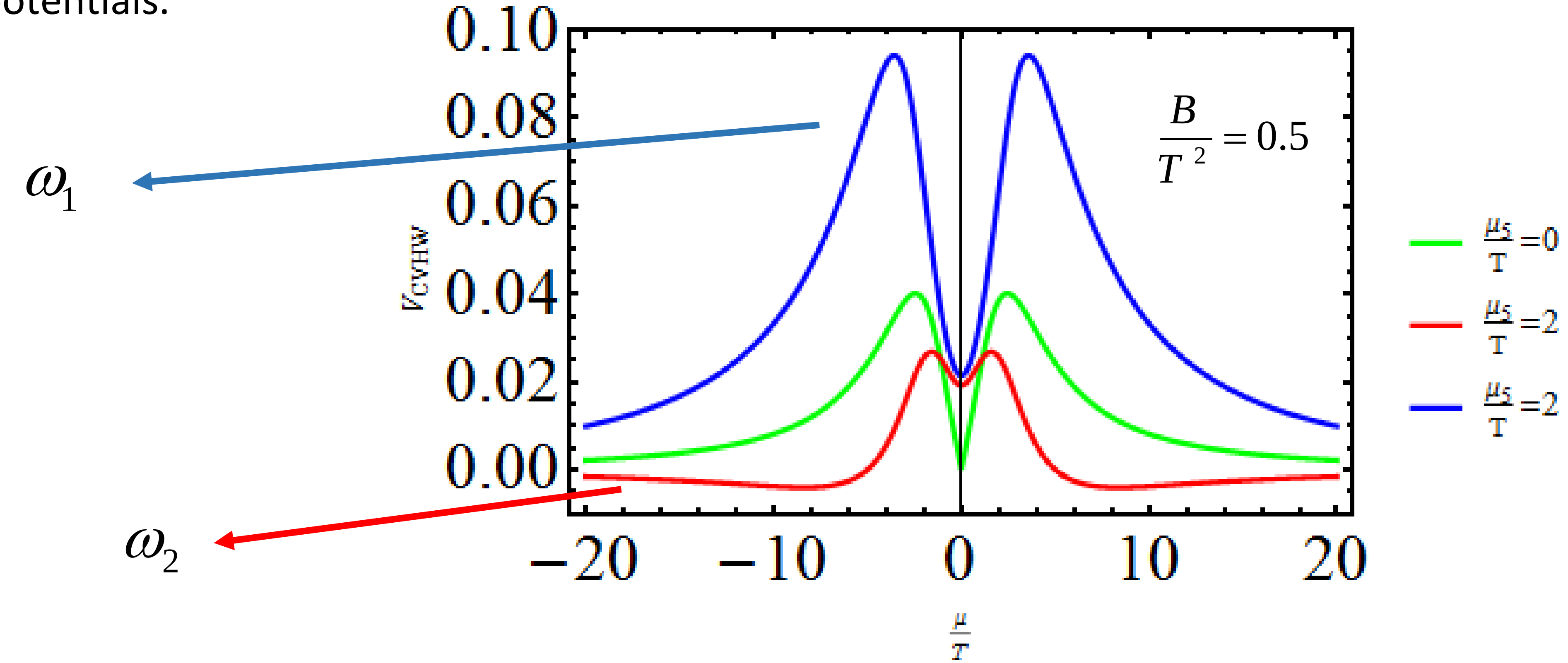
$$M_{ab}^{\Omega}(\boldsymbol{k},\omega)\delta\phi_a(\boldsymbol{k},\omega) = 0$$

$$\begin{bmatrix} -i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\ i\alpha_1 v_s^2 k^i & -i\omega\delta^i_j - \epsilon^i{}_{jl}\Omega^l & i\alpha_2 v_s^2 k^i & i\alpha_3 v_s^2 k^i \\ -i\beta_1\omega + \left(\frac{\partial\xi}{\partial T}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} & \frac{\bar{n}}{\bar{w}} ik_j - \frac{2\xi}{\bar{w}} i\omega\Omega_j & -i\beta_2\omega + \left(\frac{\partial\xi}{\partial\mu}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} & -i\beta_3\omega + \left(\frac{\partial\xi}{\partial\mu_5}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ -i\gamma_1\omega + \left(\frac{\partial\xi_5}{\partial T}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} & \frac{\bar{n}_5}{\bar{w}} ik_j - \frac{2\xi_5}{\bar{w}} i\omega\Omega_j & -i\gamma_2\omega + \left(\frac{\partial\xi_5}{\partial\mu}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} & -i\gamma_3\omega + \left(\frac{\partial\xi_5}{\partial\mu_5}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \end{bmatrix}$$

Chiral Vortical Heat Waves

$$\omega_{1,2}(k) = -\frac{\mathcal{A}_3 \pm \sqrt{\mathcal{A}_3^2 - \mathcal{E}\mathcal{A}_4}}{\mathcal{E}}\ \boldsymbol{\Omega}.\boldsymbol{k}$$

- These two modes carry the perturbations of temperature together with the vector and axial chemical potentials.



Sound-Sector

$\boldsymbol{\Omega} \parallel \boldsymbol{k}$

$$\omega_{3,4} = \pm\, c_s k$$
$$\omega_{5,6} = \pm\, \Omega.$$

$\boldsymbol{\Omega} \perp \boldsymbol{k}$

$$v_{3,4} = \pm \frac{c_s}{\sqrt{1 + \frac{\Omega^2}{c_s^2 k^2}}}$$

No-Anomaly effect in contrast to plasma in magnetic field

Rotating Chiral Fluid Coupled to Magnetic Field

$$u^\mu = \Big(1,\ \boldsymbol{\Omega} \times \boldsymbol{x}\Big)$$

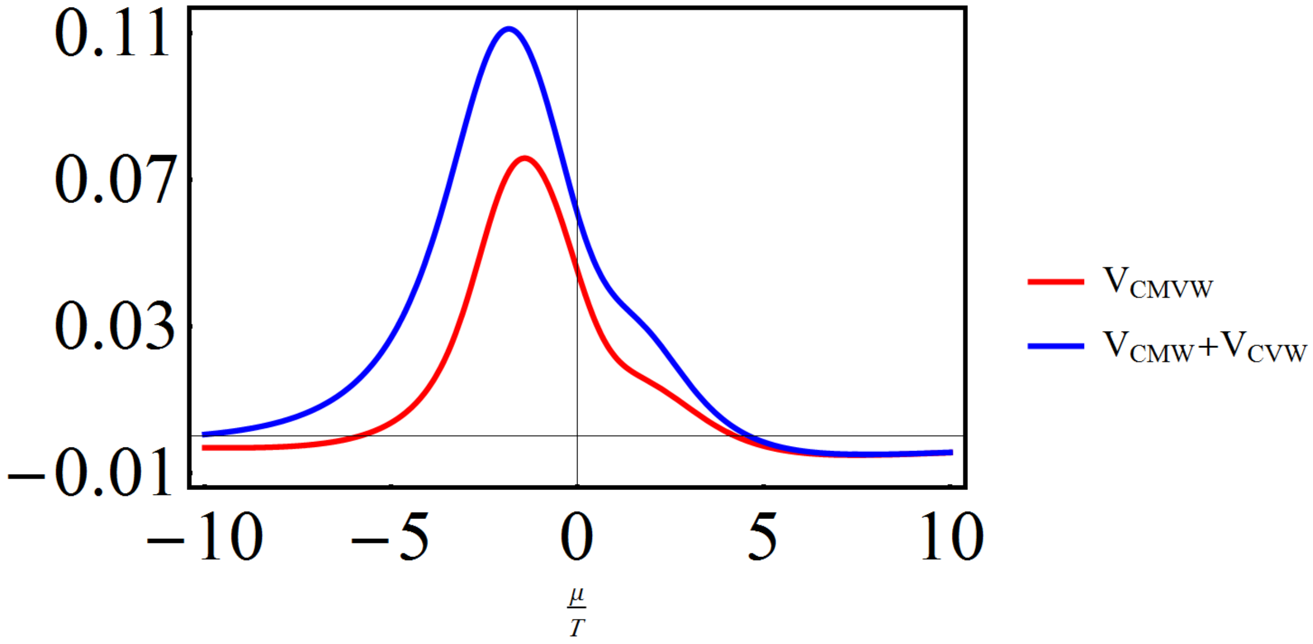
$$T = Const.,\ \ \mu = Const.,\ \ \mu_5 = Const.$$

$$\boldsymbol{B} = Const.$$

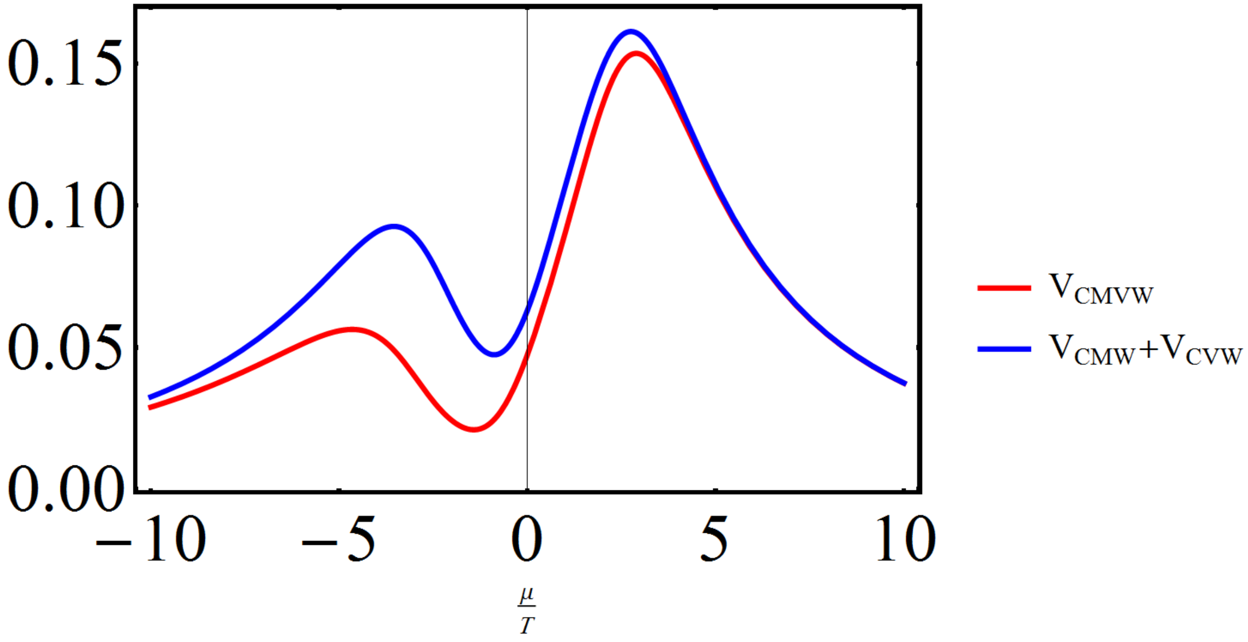
$$\omega_{1,2} = -\frac{1}{\mathcal{E}}\Big(\mathcal{A}_1\,\boldsymbol{B}.\boldsymbol{k} + \mathcal{A}_3\,\boldsymbol{\Omega}.\boldsymbol{k}\Big)$$
$$\pm\frac{1}{\mathcal{E}}\sqrt{\Big(\mathcal{A}_1\,\boldsymbol{B}.\boldsymbol{k} + \mathcal{A}_3\,\boldsymbol{\Omega}.\boldsymbol{k}\Big)^2 - \mathcal{E}\,\Big(\mathcal{A}_1(\boldsymbol{B}.\boldsymbol{k})^2 + \mathcal{A}_5\boldsymbol{B}.\boldsymbol{k}\ \boldsymbol{\Omega}.\boldsymbol{k} + \ \mathcal{A}_4\ (\boldsymbol{\Omega}.\boldsymbol{k})^2\Big)}$$

$\Downarrow$

Chiral Magnetic Vortical Waves



Mode with plus sign



Mode with minus sign

Collective Modes from Chiral Kinetic Theory

- Kinetic theory is a framework to study the systems including weakly interacting particles under the assumption of rare collisions.

- All the information of the system is characterized by a distribution function.
- The dynamics of distribution function is governed by Boltzmann equation.

$$\frac{\partial f}{\partial t} + \dot{\vec{x}}.\frac{\partial f}{\partial \vec{x}} + \dot{\vec{p}}.\frac{\partial f}{\partial \vec{p}} = \mathcal{C}[f].$$

Boltzmann equation

- equations of motion of the chiral particles

$$\sqrt{G}\dot{\vec{x}} = \hat{p} + a\ \vec{E}\times\vec{b} + a^2\vec{B}(\vec{b}.\hat{p}),$$

$$\sqrt{G}\dot{\vec{p}} = \vec{E} + a\ \hat{p}\times\vec{B} + a^2\vec{b}(\vec{E}.\vec{B}),$$

D. T. Son and N. Yamamoto. **arXiv:1203.2697**
M. Stephanov and Y. Yin, **arXiv:1207.0747**

$$\vec{b} = \vec{\nabla}_p \times \vec{a_p} = \frac{\vec{p}}{2|\vec{p}|^3}, \quad \sqrt{G} \equiv 1 + a^2\vec{B}.\hat{b}. \quad \vec{a}_p = \frac{\vec{p}}{2|p|^2}.$$

Berry curvature

- Chiral magnetic wave from chiral kinetic theory in high temperature – high density plasma

M. Stephanov, H. U. Yee and Y. Yin, Phys. Rev. D 91, no. 12, 125014 (2015).

- Equilibrium distribution function

$$f_{L,R}(\vec{p}) = \frac{1}{e^{\beta(p^\mu.u_\mu - \mu_{L,R})} + 1}$$

- Linearizing Boltzmann equation around the equilibrium

$$\delta f_{\pm R} = \left([p.u \mp \mu_R]\frac{\delta\beta}{\beta} \mp \delta\mu_R + p_i \delta u^i \right) \frac{\partial f_\pm^R(p)}{\partial p}$$
$$\delta f_{\pm L} = \left([p.u \mp \mu_L]\frac{\delta\beta}{\beta} \mp \delta\mu_L + p_i \delta u^i \right) \frac{\partial f_\pm^L(p)}{\partial p}.$$

N.Abbasi, A.D, F. Taghinavaz, **arXiv:……**

$$\sum_{b=1}^{6} \mathcal{M}_{ab}(\nu, \vec{k})\delta X_b(\nu, \vec{k}) = 0, \quad a = 1, \ldots, 6.$$

- CMW from CKT

$$v_{\mathrm{CMW}}^{\mathrm{K}} = \frac{3}{2\pi^2}\frac{B}{T^2} - \frac{81}{14\pi^4}\frac{\mu}{T}\frac{\mu_5}{T}\frac{B}{T^2} + \mathcal{O}(\frac{\mu^2}{T^2}, \frac{\mu_5^2}{T^2})$$

- Comparing with hydro prediction

$$v_{\mathrm{CMW}}^{\mathrm{H}} = \frac{3}{2\pi^2}\frac{B}{T^2} - \frac{9}{\pi^4}\frac{\mu}{T}\frac{\mu_5}{T}\frac{B}{T^2} + \mathcal{O}(\frac{\mu^2}{T^2}, \frac{\mu_5^2}{T^2})$$

$$v_{\mathrm{CMW}}^{\mathrm{K}} - v_{\mathrm{CMW}}^{\mathrm{H}} = \frac{45}{14\pi^4}\frac{\mu}{T}\frac{\mu_5}{T}\frac{B}{T^2} + \mathcal{O}(\frac{\mu^2}{T^2}, \frac{\mu_5^2}{T^2})$$

!?

The Hydro and Kinetic theory results do not coincide with each other

Chiral Kinetic theory $\longrightarrow$ Hydrodynamic in the thermodynamic frame

$$T^{\mu\nu}_{Lab} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} + \sigma_{\epsilon,B}(u^\mu B^\nu + u^\nu B^\mu) + \sigma_{\epsilon,\omega}(u^\mu \omega^\nu + u^\nu \omega^\mu)$$

Velocity transformation $\Big\downarrow$

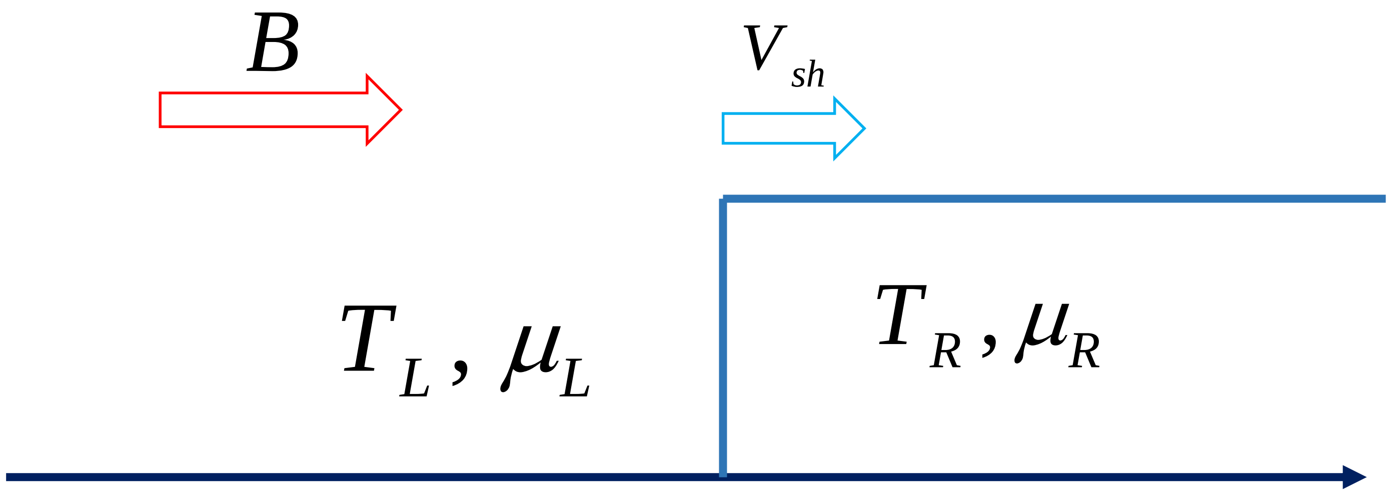
$$\delta u^\mu = -\frac{1}{\epsilon + p}\left(\sigma_{\epsilon,B} B^\mu + \sigma_{\epsilon,\omega}\omega^\mu\right)$$

$$T^{\mu\nu}_{Landau} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$\begin{aligned} v^{\mathrm{K}}_{\mathrm{CMW}} - v^{\mathrm{H}}_{\mathrm{CMW}} = \frac{1}{\epsilon + p}\sigma_{\epsilon,B} B &= \frac{45\mu\mu_5 B}{2\left(15\left(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4\right) + 7\pi^4 T^4 + 30\pi^2 T^2\left(\mu^2 + \mu_5^2\right)\right)} \\ &= \frac{45}{14\pi^4}\frac{\mu}{T}\frac{\mu_5}{T}\frac{B}{T^2} + \mathcal{O}\left(\frac{\mu^2}{T^2}, \frac{\mu_5^2}{T^2}\right) \end{aligned}$$

Hydro Modes are frame dependent!

Non-Linear Modes in Chiral Plasma
Shock-Waves



Jump Conditions

$$v^n_{sh} = \frac{J^x_L - J^x_R}{J^0_L - J^0_R}$$

$$v^E_{sh} = \frac{T^{xt}_L - T^{xt}_R}{T^{tt}_L - T^{tt}_R}$$

$$v^{P_x}_{sh} = \frac{T^{xx}_L - T^{xx}_R}{T^{tx}_L - T^{tx}_R}$$

For special case that $V_L = V_R = 0$, we have

$$v_{sh}^{n} = \frac{\sigma^{\mathcal{B}}(\mu_L, T_L) - \sigma^{\mathcal{B}}(\mu_R, T_R)}{n(\mu_L, T_L) - n(\mu_R, T_R)} B$$

$$v_{sh}^{E} = \frac{\sigma_{\epsilon}^{\mathcal{B}}(\mu_L, T_L) - \sigma_{\epsilon}^{\mathcal{B}}(\mu_R, T_R)}{\epsilon(\mu_L, T_L) - \epsilon(\mu_R, T_R)} B$$

$$v_{sh}^{P_x} = \frac{p(\mu_L, T_L) - p(\mu_R, T_R)}{\left(\sigma_{\epsilon}^{\mathcal{B}}(\mu_L, T_L) - \sigma_{\epsilon}^{\mathcal{B}}(\mu_R, T_R)\right) B}$$

$$v_{sh}^{n} = v_{sh}^{E}$$

$$v_{sh}^{E} = v_{sh}^{P_x}$$

Taub Equations

$$15\left(\mu_L + \mu_R\right)\left(\mu_L - \mu_R\right)^4 + 10\pi^2\left(\mu_L - \mu_R\right)^2\left(T_L^2\left(5\mu_L + \mu_R\right) + T_R^2\left(\mu_L + 5\mu_R\right)\right) + \pi^4\left(T_L^2 - T_R^2\right)\left(T_L^2\left(11\mu_L - \mu_R\right) + T_R^2\left(21\mu_L - 11\mu_R\right)\right) = 0$$

$$45B^2\left(\mu_L - \mu_R\right)\left(3\left(\mu_L - \mu_R\right)\left(\mu_L + \mu_R\right) + \pi^2\left(T_L - T_R\right)\left(T_L + T_R\right)\right) - 2\left(\mu_L^3 + \pi^2\left(\mu_L T_L^2 - \mu_R T_R^2\right) - \mu_R^3\right)\left(15\left(\mu_L^4 - \mu_R^4\right) + 30\pi^2\left(\mu_L^2 T_L^2 - \mu_R^2 T_R^2\right) + 7\pi^4\left(T_L^4 - T_R^4\right)\right) = 0$$

$$v_{sh} = \frac{5\left(3\left(\mu_L^2 - \mu_R^2\right) + \pi^2\left(T_L^2 - T_R^2\right)\right)}{15\left(\mu_L^4 - \mu_R^4\right) + 30\pi^2\left(\mu_L^2 T_L^2 - \mu_R^2 T_R^2\right) + 7\pi^4\left(T_L^4 - T_R^4\right)} B$$

$$v_{sh} = \frac{3B}{2\left(\mu_L^2 + \pi^2 T_L^2\right)}.$$

higher density – lower velocity

N.Abbasi, M. Chernodub, D. Allahbakhshi, A.D, F. Taghavi: arXiv:.....

Shock waves in rotating chiral fluid

S.Sen and N.Yamamoto, arXiv:1609.07030

Conclusion and outlook

- We have studied the spectrum of conformal - non-dissipative chiral fluid
- In general there are six hydro-modes.
- Sound mode mixes with chiral modes.
- Finding the spectrum with dissipative effects.
- Taking into account the back-reaction on electromagnetic fields, chiral magneto-hydrodynamics.