Multiparton Interactions - Overview.

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NPQCD 2015, Cortona, Italy, 20th April 2015

Outline

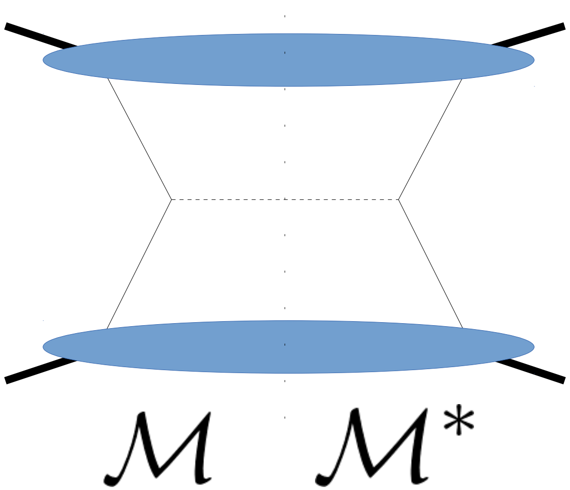
I will briefly review the theory description of multiple interactions (MPI) and double parton scattering (DPS), plus some recent developments.

- Why do we often ignore DPS/MPI? When should we take it into account?
- Theoretical expression for the DPS cross section in terms of two-parton distributions (2pGPDs). Approximations made leading to Pythia/Herwig models of MPI, and DPS 'pocket formula'.
- Effects recently studied by theory community in context of DPS, that are not in the Monte Carlo MPI models
  - Parton pair generation via perturbative splitting. Will discuss graphs in which parton pairs from one or both protons are perturbatively generated.
  - Interference and correlation effects in spin, colour, flavour.

Why/when do we ignore MPI?

Protons contain large numbers of QCD partons → in each LHC pp collision, it is likely that there will be several parton-parton interactions (MPI).

Consider production of some particle A (A = Z, W, H, new physics, etc.). Typically we do not concern ourselves with MPI when calculating cross sections for this process:



Total cross section:

$$\sigma = \hat{\sigma}_{ij\to A}(\hat{s} = x_A x_B s) \otimes f_i(x_A) \otimes f_j(x_B)$$

Parton distribution functions (PDFs)

Differential transverse momentum:

$$W^{\mu\nu} \propto C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}\left(x_A, \mathbf{b}_T; \zeta_A\right) \tilde{f}\left(x_B, \mathbf{b}_T; \zeta_B\right)$$

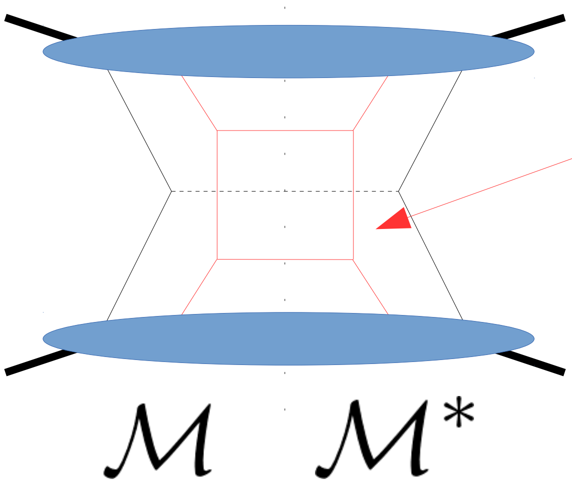
Transverse momentum dependent PDFs (TMDs)

The PDFs and TMDs are single parton distributions

Why/when do we ignore MPI?

Protons contain large numbers of QCD partons → in each LHC pp collision, it is likely that there will be several parton-parton interactions (MPI).

Consider production of some particle A (A = Z, W, H, new physics, etc.). Typically we do not concern ourselves with MPI when calculating cross sections for this process:



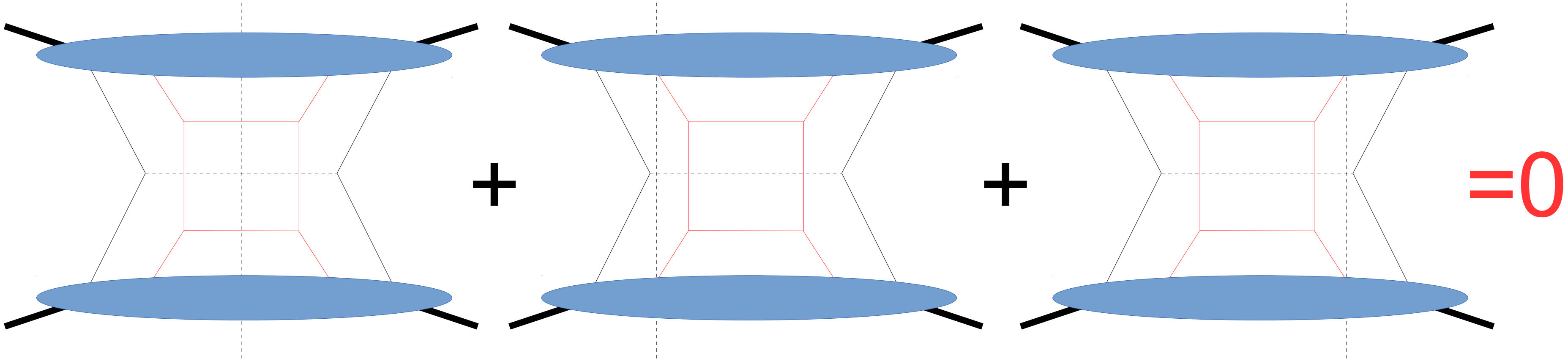
Q. Why do we not also need to calculate this process with an additional scattering (and indeed processes with arbitrary extra scatterings) to obtain the V production cross section?

Why/when do we ignore MPI?

A. Unitarity!

When we say cross section for production of A, what we really mean is inclusive cross section: pp → A + X.

X can be anything, we sum over all possibilities for X.

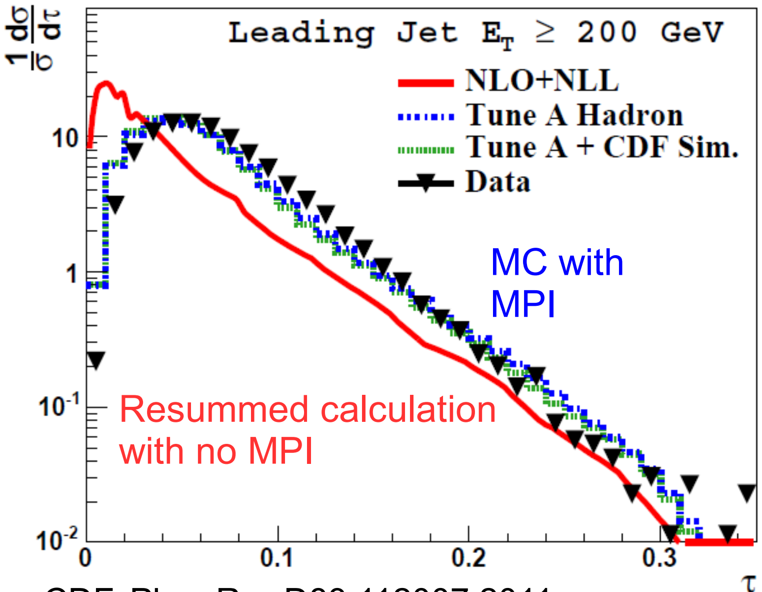


MPI

Absorptive processes

MPI sensitive observables

If you are not sufficiently inclusive on X, then you can become sensitive to additional scatters.



CDF, Phys.Rev.D83:112007,2011

Good example of such an observable is transverse thrust:

$$T_\perp \equiv \max_{\vec{n}_T} \frac{\sum_{i=1}^{n} |\vec{q}_{\perp,i} \cdot \vec{n}_T|}{\sum_{i=1}^{n} |\vec{q}_{\perp,i}|} \qquad \tau \equiv 1 - T_\perp$$

Additional uncorrelated soft scatters make event more spherical and raise τ – observable sensitive to MPI.

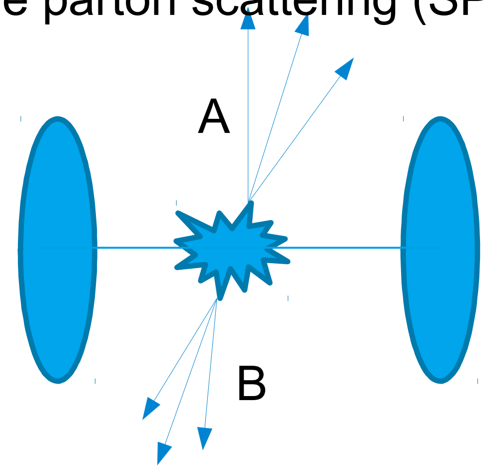
Soft interactions in this context are referred to as Underlying Event (UE). Can also just measure soft interactions in absence of a hard interaction – Minimum Bias (MB).

Double Parton Scattering

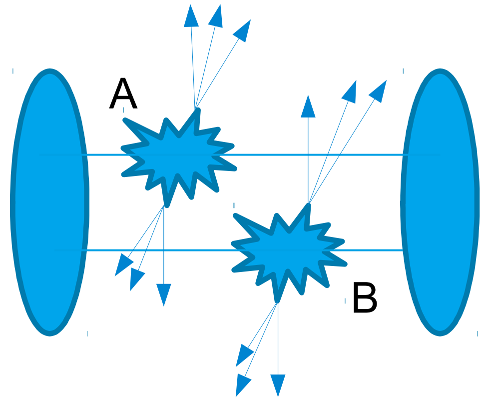
Also can think of observables in which we are sensitive to one (or more) hard additional scatters.

Consider production of two (sets of) hard objects A and B, with associated scales Q_A and Q_B, $p + p \rightarrow A + B + X$. Expect the cancellation of additional scatters producing particles in X still to go through, but now AB can be produced in two ways:

Single parton scattering (SPS)



Double parton scattering (SPS)



Double Parton Scattering

In terms of the total cross section, the DPS mechanism is power suppressed with respect to SPS:

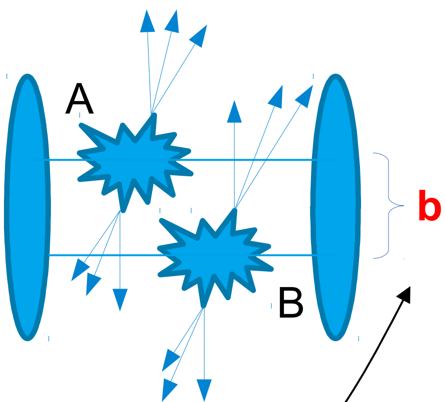
$$\sigma_{DPS}/\sigma_{SPS} \sim \Lambda^2/Q^2$$

However:

- DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants (same sign WW, H+W production).
- DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.
- DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller x values where there is a larger density of partons.
- DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton

Total Cross Section for DPS

How can we describe MPI theoretically? Let's look at DPS:



Assuming the factorisation of the hard processes A and B, the total DPS cross section may be written as:

$$\sigma_D^{(A,B)} = \underbrace{\frac{m}{2}}_{\text{Symmetry factor}} \sum_{i,j,k,l} \int \overbrace{\Gamma_h^{ik}(x_1, x_2, \mathbf{b}; Q_A, Q_B)\Gamma_h^{jl}(x_1', x_2', \mathbf{b}; Q_A, Q_B)}^{\text{Two-parton generalised PDF (2pGPD)}}$$

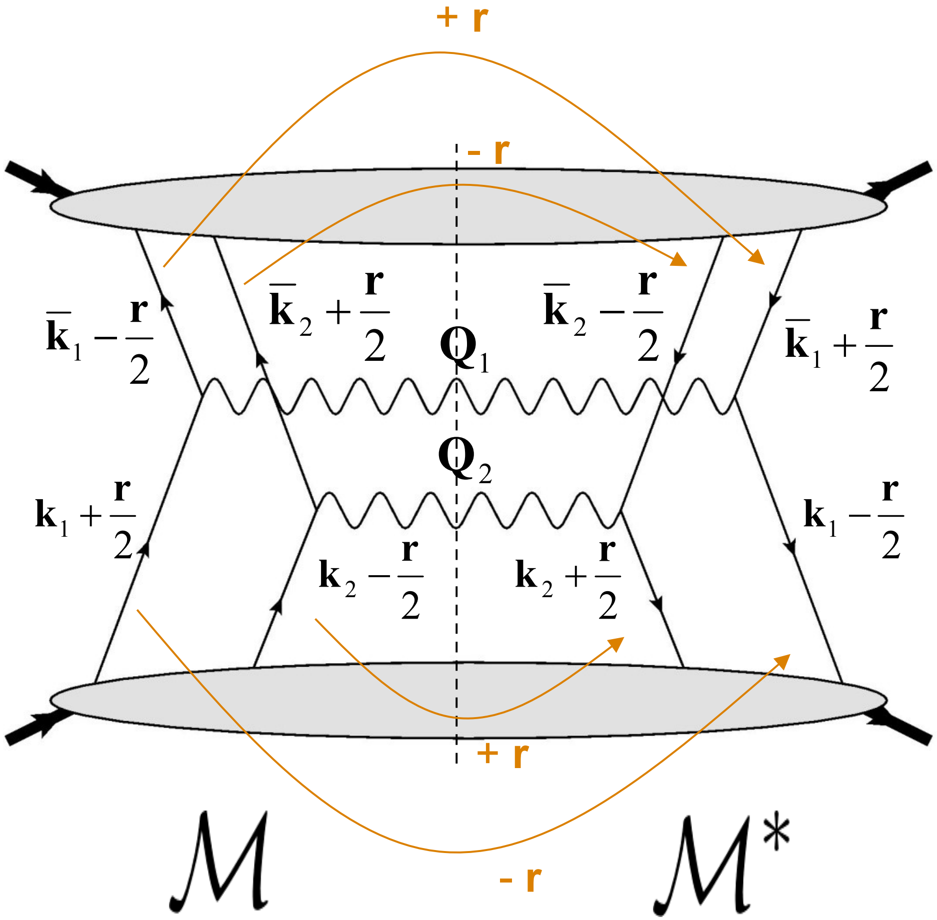
$$\times \underbrace{\hat{\sigma}_{ij}^{A}(x_1, x_1')\hat{\sigma}_{kl}^{B}(x_2, x_2')}_{\text{Parton level cross sections}} \mathrm{d}x_1\, \mathrm{d}x_1' \mathrm{d}x_2\, \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

Paver, Treleani, Nuovo Cim. A70 (1982) 215.
Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

b = separation in transverse space between the two partons

In this formula the two 2pGPDs are integrated over a common **b** – cannot express DPS cross section in terms of parton distributions independently integrated over their impact parameter arguments, as in single scattering case.

DPS – transverse momentum picture



Key point: transverse momentum of partons does not have to be equal in amplitude and conjugate!

Most general transverse momentum configuration of partons entering hard scatters

r = momentum imbalance of a parton line between amplitude and conjugate

$$\sigma = \int \frac{d^2\mathbf{r}}{(2\pi)^2} D_h^{p_1 p_2}(x_1, x_2, \mathbf{r}) D_h^{p_3 p_4}(x_1, x_2, -\mathbf{r})$$

Fourier transform of **b**-space 2pGPD wrt **b**

Simplifying assumptions for DPS cross section

If one ignores correlations between partons in the proton:

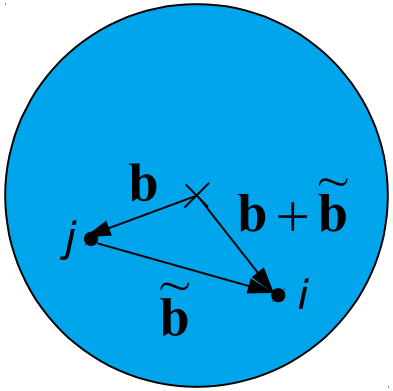
$$D_p^{ij}(x_1,x_2;\mathbf{b}) = \int d^2\tilde{\mathbf{b}}\, D_p^i(x_1;\tilde{\mathbf{b}}+\mathbf{b})\, D_p^j(x_2;\tilde{\mathbf{b}})$$

Impact parameter dependent PDFs

or equivalently

$$D_p^{ij}(x_1,x_2;\mathbf{\Delta}) \approx D_p^i(x_1;\mathbf{\Delta})\, D_p^j(x_2;-\mathbf{\Delta})$$

GPD



Common ‘lore’: approximately valid at low *x*, due to the large population of partons at such x values.

Further approximation that is often made: $D_p^i(x_1;\tilde{\mathbf{b}}) = D_p^i(x_1)\, F(\tilde{\mathbf{b}})$

$$\Longrightarrow D_p^{ij}(x_1,x_2;\mathbf{b}) = D_p^i(x_1) D_p^j(x_1) \int d^2\tilde{\mathbf{b}}\, F(\tilde{\mathbf{b}}+\mathbf{b})\, F(\tilde{\mathbf{b}})$$

$$\Longrightarrow \sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{eff}}$$

DPS 'pocket formula'. This is often used in phenomenological analyses and experimental studies of DPS

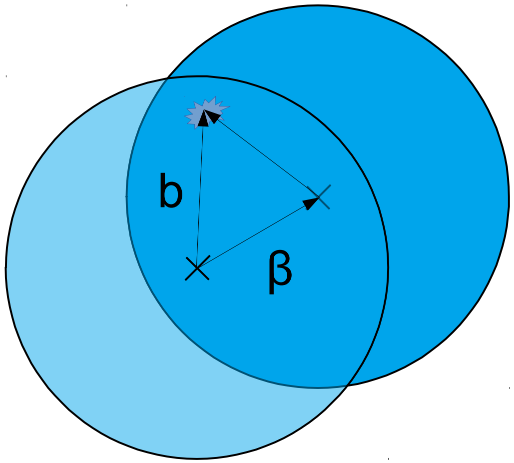
How MPI are modelled in MC Event Generators

DPS cross section with independent scatters:

$$\sigma_D = \int \left(\int \frac{1}{2!} D(x)F(\mathbf{b})\hat{\sigma}(x,x')D(x')F(\mathbf{b}-\beta)\mathbf{d^2bdxdx'} \right)^2 d^2\beta$$

Straightforward generalisation to N parton scatters:

$$\sigma_N = \int \left(\int \frac{1}{N!} D(x)F(\mathbf{b})\hat{\sigma}(x,x')D(x')F(\mathbf{b}-\beta)\mathbf{d^2bdxdx'} \right)^N d^2\beta$$



e.g. Calucci, Treleani, arXiv:0809.4217

This is the inclusive N parton scattering cross section – i.e. N + anything, and if there are M scatters in an event we count this ${}^{M}C_{N}$ times.

Easy to see that the above probability distributions can be generated from the following Poisson probability distribution:

$$P_N(\beta) \equiv \frac{(\sigma_S \mathcal{F}(\beta))^N}{N!} e^{-\sigma_S \mathcal{F}(\beta)} \qquad \mathcal{F}(\beta) = \int F(b)F(b-\beta)d^2\beta$$

Sjöstrand, van Zijl, Phys.Rev. D36 (1987) 2019
Amettler, Treleani Int.J.Mod.Phys. A3 (1988) 521-530
Capella, Tran Thanh Van, Kwiecinski, Phys.Rev.Lett. 58 (1987) 2015
Butterworth, Forshaw, Seymour Z.Phys.C72:637-646

How MPI are modelled in MC Event Generators

$$P_N(\beta) \equiv \frac{(\sigma_S \mathcal{F}(\beta))^N}{N!} e^{-\sigma_S \mathcal{F}(\beta)}$$

$$\mathcal{F}(\beta) = \int F(b) F(\beta - b) d^2\beta$$

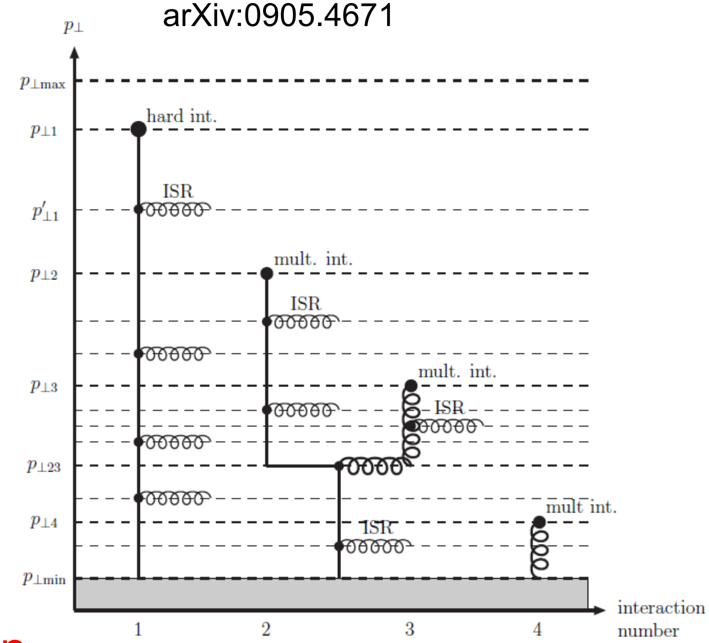
This Poissonian/eikonal model is the starting point for many MC models of MPI:

MPI model in HERWIG ≈ the Poissonian model, with hard and soft components.

Bahr, Butterworth, Gieseke, Seymour, arXiv:0905.4671

The Pythia model: MPI interleaved with ISR. Some account taken of momentum and flavour constraints. Option for an x-dependent proton size.

Sjostrand, Skands, Eur.Phys.J. C39 (2005), JHEP 0403 (2004) 053
Corke, Sjostrand JHEP 1105 (2011) 009



SHERPA: Underlying event model based on Khoze-Martin-Ryskin model (SHRiMPS). MPI generated by cutting Pomerons – some BFKL effects included.

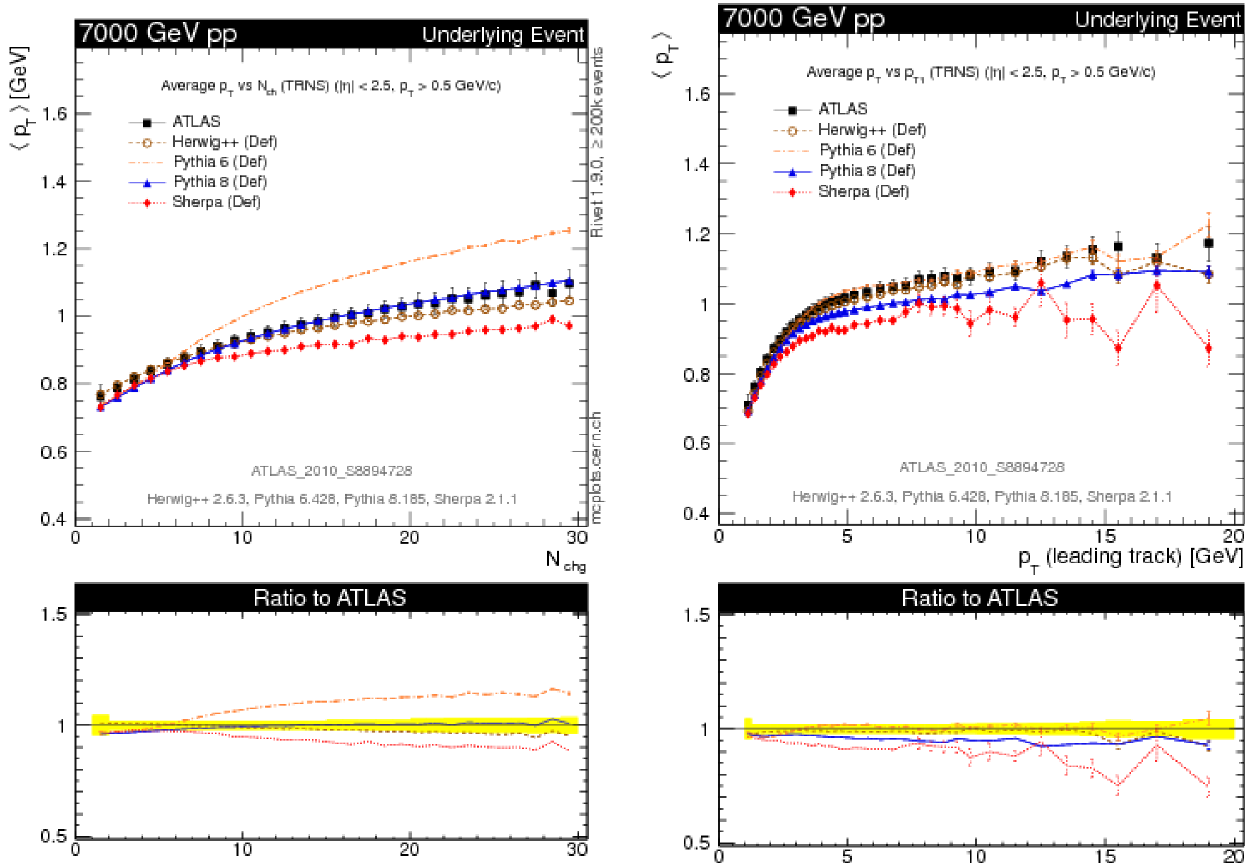
Martin *et al*., PoS QNP2012 (2012) 017

All models have some degree of colour reconnection.

MC Event Generator MPI models vs. data

MC MPI models do a pretty good job of describing underlying event and minimum bias events, but:

- Some tuning required
- Always some distributions that could be described better



What effects are missing from the simplest description of DPS, and MC models of MPI?

Parton splitting effects

Two possibilities for how a parton pair in the proton could have arisen:

1) Pair generated already at the perturbative level:

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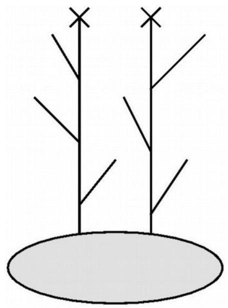

2) Pair generated by a 1→2 perturbative splitting:

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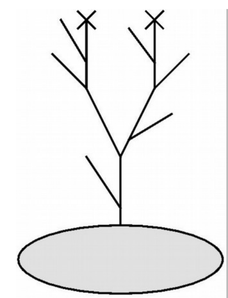
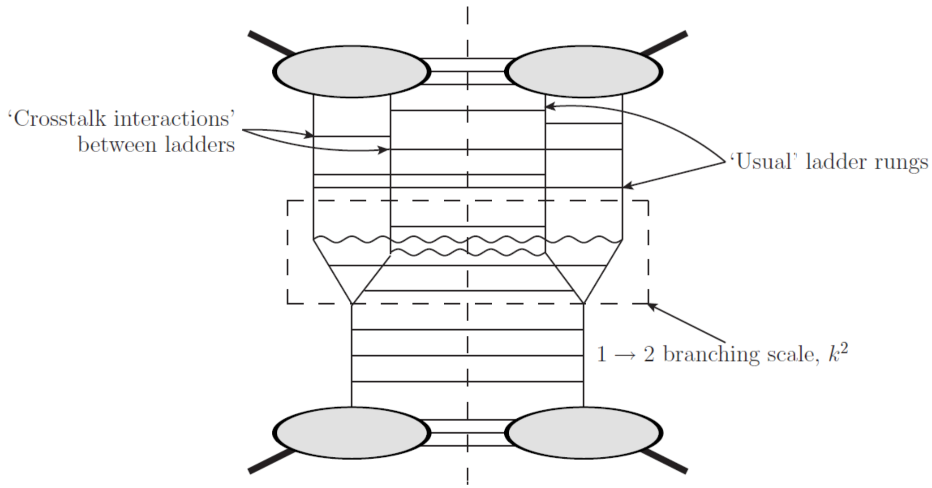


These two processes correspond to very different distributions in impact parameter space:

$$\Gamma(x_1, x_2, b) \simeq D(x_1)D(x_2)G(b) \qquad \Gamma(x_1, x_2, b) \propto \alpha_s \frac{D(x_1+x_2)}{x_1+x_2} P\left(\frac{x_1}{x_1+x_2}\right)\frac{1}{b^2}$$

Parton splitting and radiation can occur at all scales – in general these effects will break x_1-x_2-**b** factorisation in the 2pGPD.

Perturbative splitting in one proton – 2v1 graphs



'2v1' Graphs in which a perturbative splitting occurs in only one proton have been extensively studied – established that such graphs can contribute to DPS cross section, and LL evolution effects worked out.

BDFS, Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
JG, JHEP 1301 (2013) 042

- Geometrical '$1/\sigma_{eff}$' prefactor for these graphs is twice as big as 2v2 graphs with no 1→2 splitting.
- Numerical studies imply 2v1 cross section is sizeable ($\sigma_{2v1}/\sigma_{2v1}$ ~ 0.3-1.5 depending on scale and x values), but gives differential cross sections very similar to 2v2. JG, Maciula, Szczurek Phys. Rev. D 90 (2014) 054017, BDFS, Eur.Phys.J. C74 (2014) 2926
- This mechanism has been investigated in the context of Pythia 8 using a reweighting procedure – good fit to hard + soft MPI observables, although no conclusive discrimination between 2v1 model and default model yet.

Blok, Gunnellini, arXiv:1503.08246

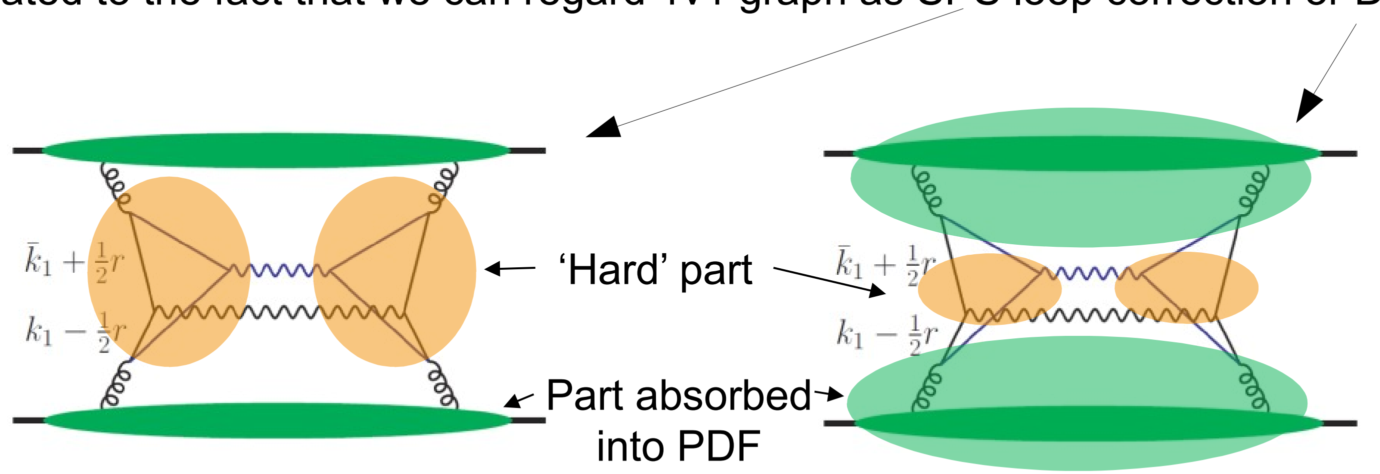
‘1v1’ or ‘Double Perturbative Splitting’ Diagrams

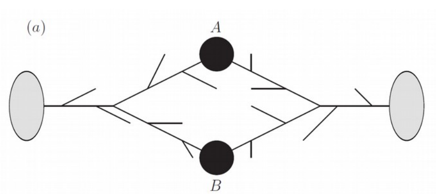
What about '1v1' graphs in which we have a perturbative splitting in both protons?

Trying to calculate this graph in a naive way using the DPS framework yields problematic quadratic divergences!

$$\int \frac{d^2b}{b^4} = ?$$

This is related to the fact that we can regard 1v1 graph as SPS loop correction or DPS





There is no natural power suppressed ($\propto \frac{\Lambda^2}{Q^4}\left[\alpha_s \log\left(\frac{Q^2}{\Lambda^2}\right)\right]^n$) part of the 1v1 graph that we can separate off as DPS ➔ regard all of these graphs as SPS?

JG and Stirling, JHEP 1106 048 (2011) & arXiv:1202.3056
Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.
BDFS, Eur.Phys.J. C72 (2012) 1963

Total Cross Section for DPS

Advantage: we avoid double counting between DPS and SPS!

Potentially concerning implication:

The cross section can no longer be written as parton level cross sections convolved with overall 2pGPD factors for each hadron.

Original expression written down on slide 4

2v2 1v2 2v1

$$\sigma^{DPS} \propto \cancel{\int d^2\mathbf{b}\, \Gamma_a(\mathbf{b})\Gamma_b(\mathbf{b})} \to \int d^2\mathbf{b}\, \Gamma_{a,NP}(\mathbf{b})\Gamma_{b,NP}(\mathbf{b}) + D_{a,P}\Gamma_{b,NP}(\mathbf{b}=\mathbf{0}) + \Gamma_{a,NP}(\mathbf{b}=\mathbf{0})D_{b,P}$$

BDFS, Eur.Phys.J. C72 (2012) 1963
Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201

$$(A+B)^2 \neq A^2 + AB + BA$$

There can be no concept of the 2pGPD for an individual hadron, with an associated operator definition and evolution equation. Appropriate hadronic operators in DPS would have to involve both hadrons at once!

BDFS, Eur.Phys.J. C74 (2014) 2926
Manohar and Waalewijn (Phys.Lett. B713 (2012) 196–201

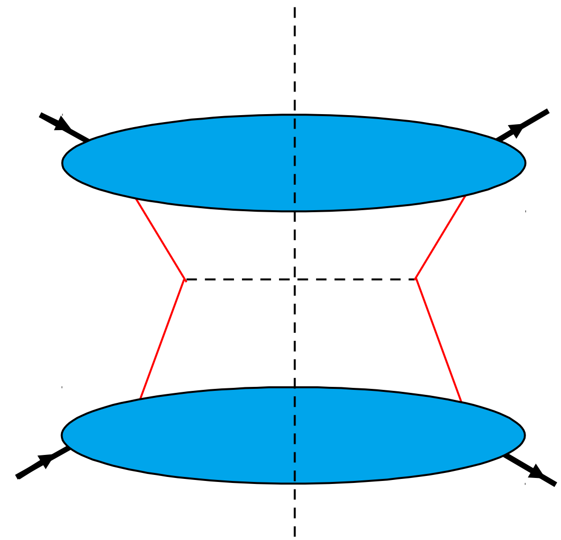
Interference contributions to proton-proton DPS

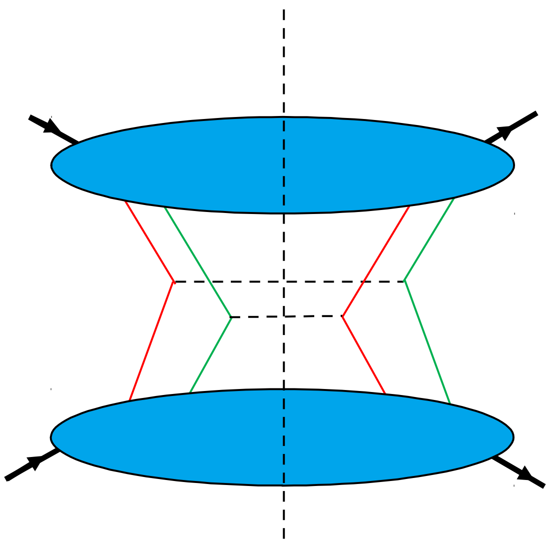
Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

SPS: One parton per proton ‘leaves’, interacts and ‘returns’.

To reform proton, parton must return with same quantum numbers.

No interference contributions to SPS cross section.





Here we have two partons per proton interacting.

Interference contributions to total cross section in which quantum numbers are swapped between parton legs. Complementary swap is required in other proton.

Can get interference contributions in colour, spin, flavour, and quark number.

Correlated parton contributions to DPS

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g.

$$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

For all of these distributions, positivity bounds analogous to the Soffer bound for single PDFs have been derived for LO distributions:

Diehl, Kasemets JHEP 1305 (2013) 150
Kasemets, Mulders Phys.Rev. D91 (2015) 014015

One example for spin case: $F_{qq} - F_{\Delta q \Delta q} \geq 2|F_{\delta q \delta q}|$

Transverse spin correlation

Based on the probability interpretation of certain combinations of LO 2pGPDs

Spin correlations and DPS

Model calculations with 3-quark wavefunctions suggest a large degree of spin correlation for large x → see Sergio's talk.

Manohar, Waalewijn, Chang, Phys. Rev. D 87, 034009 (2013)
Rinaldi, Scopetta, Traini, Vento, JHEP 1412 (2014) 028

What about the small x region?

Common 'lore' – two-chain evolution will tend to quickly wash out spin correlations. Two low *x* partons are most likely connected by a long branching chain extending down to low Q^2 → unlikely to be correlated closely in spin.

Assumption has been tested by Diehl, Kasemets, Keane

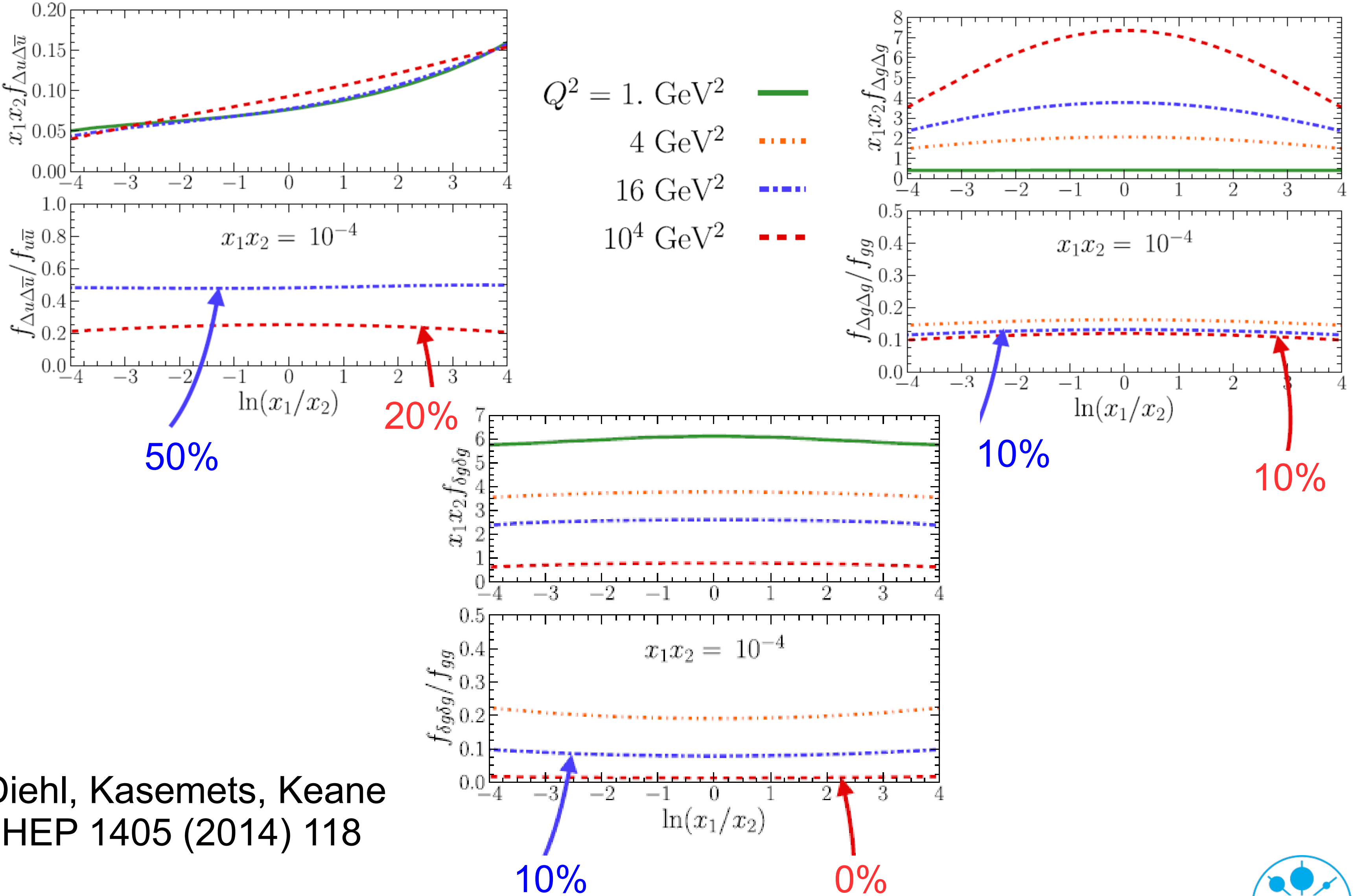
Diehl, Kasemets, Keane
JHEP 1405 (2014) 118

At Q = 1 GeV: Unpolarised 2pGPD = MSTW PDF x MSTW PDF x Gaussian in **b**
Polarised 2pGPDs = 2pGPDs saturating positivity bound (maximum polarisation)

(other scenarios are also tested → this represents a quite optimistic one for polarisation)

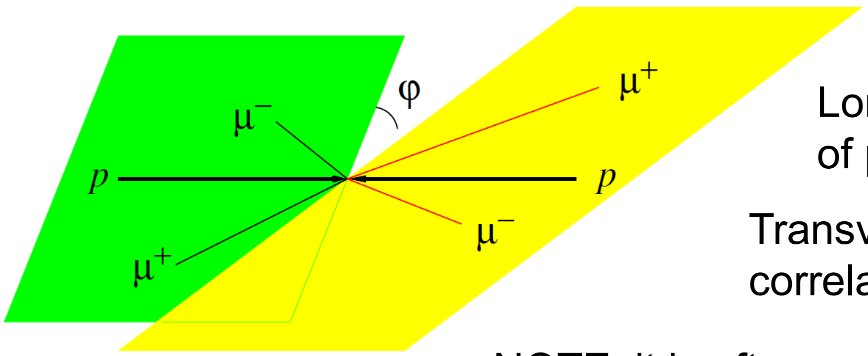
Now inputs evolved to higher scale using independent two-chain evolution – what happens to spin polarised vs unpolarised 2pGPDs?

Spin correlations under evolution



Diehl, Kasemets, Keane
JHEP 1405 (2014) 118

Spin correlations and DPS

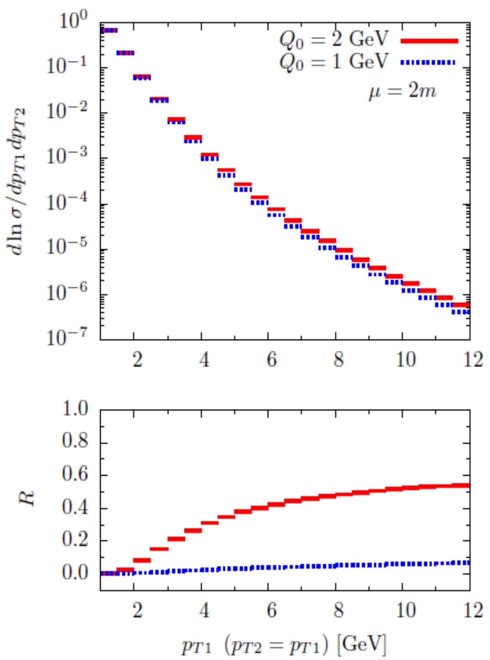


Study of spin correlations in Double Drell Yan producing lepton pairs:

Kasemets, Diehl
JHEP 1301 (2013) 121

Longitudinal spin correlations change overall rate of process and distribution in lepton rapidities

Transverse spin correlations cause azimuthal correlations between lepton planes



NOTE: it is often assumed that DPS produces two sets of final state particles that are completely uncorrelated in the transverse plane.

Study of spin correlations in double open charm production, including evolution effects:

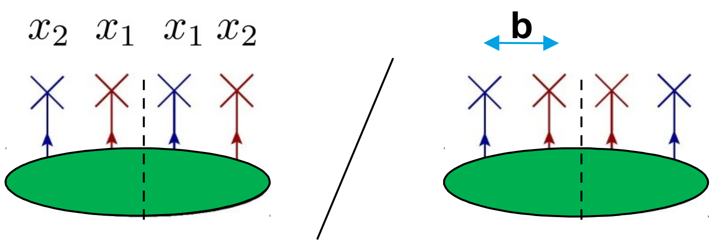
Echevarria, Kasemets, Mulders, Pisano, JHEP 1504 (2015) 034

For many distributions, polarisation effects are significant, but have same shape as unpolarised contribution – important exception is double differential distribution in charm quark p_Ts.

Sudakov Suppression of Colour Interference Distributions

For the 2pGPD with finite **b**, every distribution which does not have the partons with the same lightcone mtm fractions paired up into colour singlets is Sudakov suppressed:

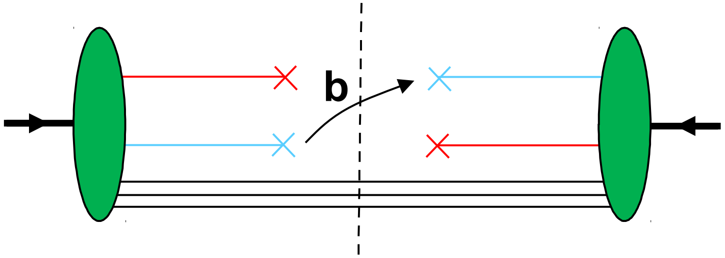
Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012) 089)
Manohar and Waalewijn, Phys.Rev. D85 (2012) 114009



$$\sim \exp\left(\frac{\alpha_s}{2\pi} \underbrace{\left(C_R^I - C_V^I\right)}_{<\,0} \ln^2\left(\mathbf{b}^2 Q^2\right)\right)$$

Physical explanation: Movement of colour by large transverse distance **b** in hadron between amplitude and conjugate.

Manohar and Waalewijn,
Phys.Rev. D85 (2012) 114009

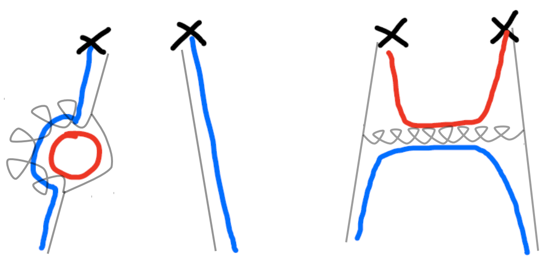


Sudakov Suppression of Colour Interference Distributions

At level of diagrams: Noncancellation of soft divergences in real and virtual diagrams in colour interference distributions.

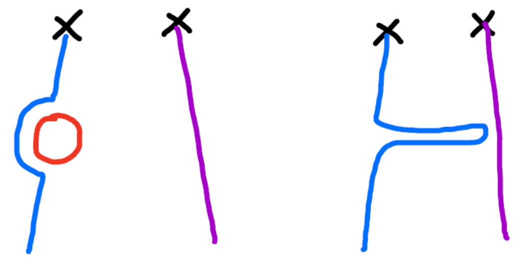
Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622

Colour singlet



$C_V = C_F$ $\quad$ $C_R = C_F$

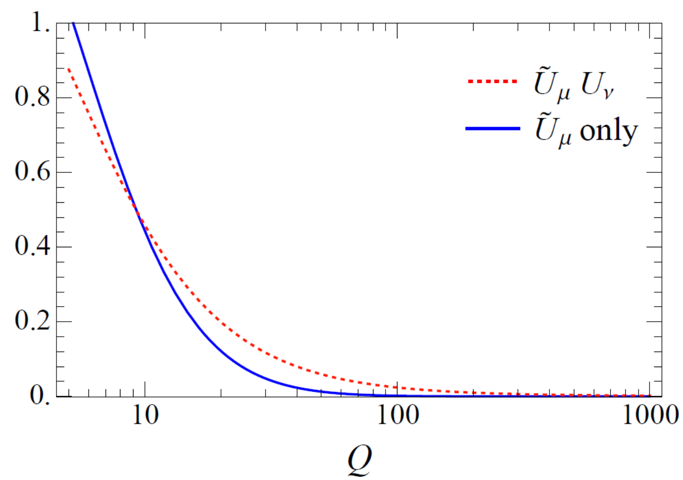
Colour interference



$C_V = C_F$ $\quad$ $C_R = -\dfrac{1}{2N}$

Numerical evaluation of Sudakov factor including single logarithmic terms:

[Lower cutoff in Sudakov factor taken to be Λ = 1.4 GeV]



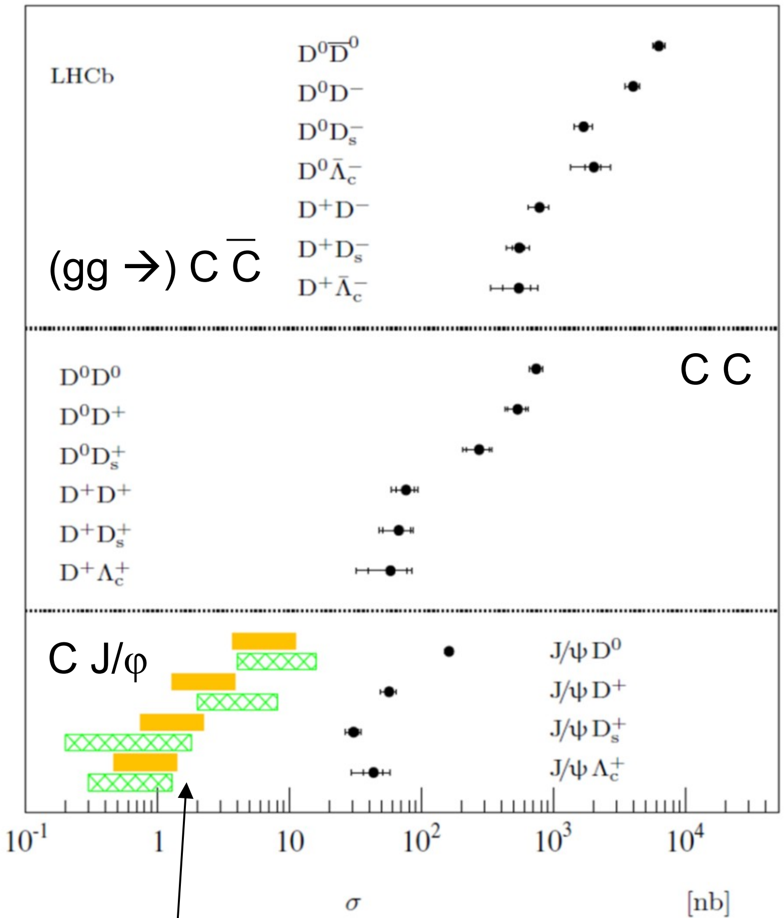
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

Summary

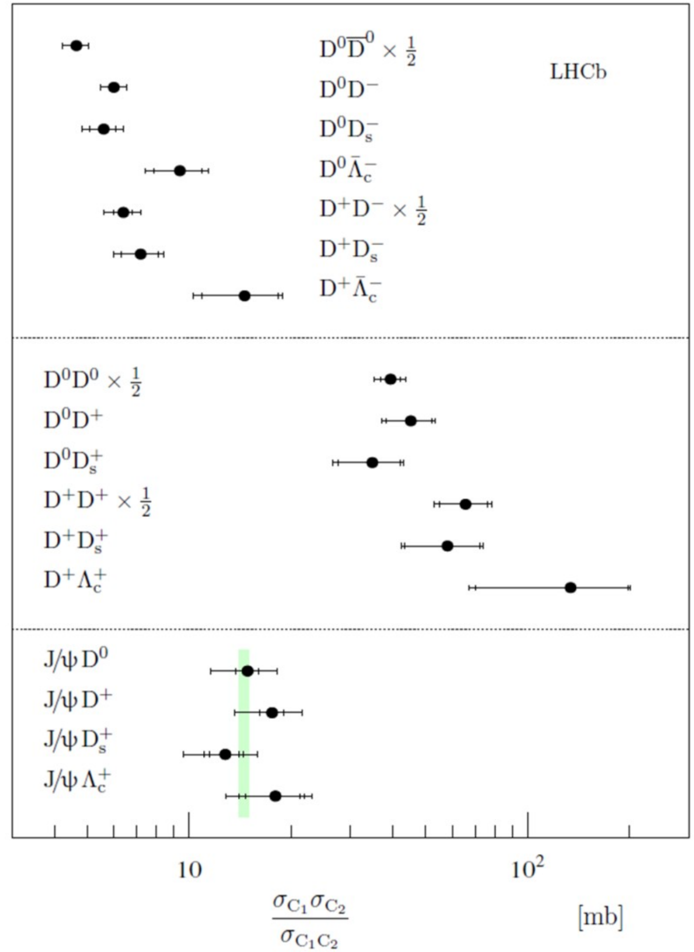
- Various observables can be defined at the LHC that are sensitive to soft or hard MPI. Process with one extra interaction, DPS, is interesting as a signal, and as a background to rare processes.
- Herwig/Pythia models of MPI – essentially uncorrelated additional scatters, with some improvements.
- Parton splitting effects in DPS: 'Single splitting' contribution extensively studied, of comparable size to nonsplitting contribution. 'Double splitting' contribution has overlap with SPS – treat as pure SPS?
- There are interference and correlated parton contributions to DPS in colour, flavour and spin space.
- Spin effects not necessarily negligible, and can change both normalisation and shapes of differential DPS cross sections.
- Colour interference contributions to DPS are Sudakov suppressed.

Backup Slides

Experimental Measurements of DPS



SPS predictions



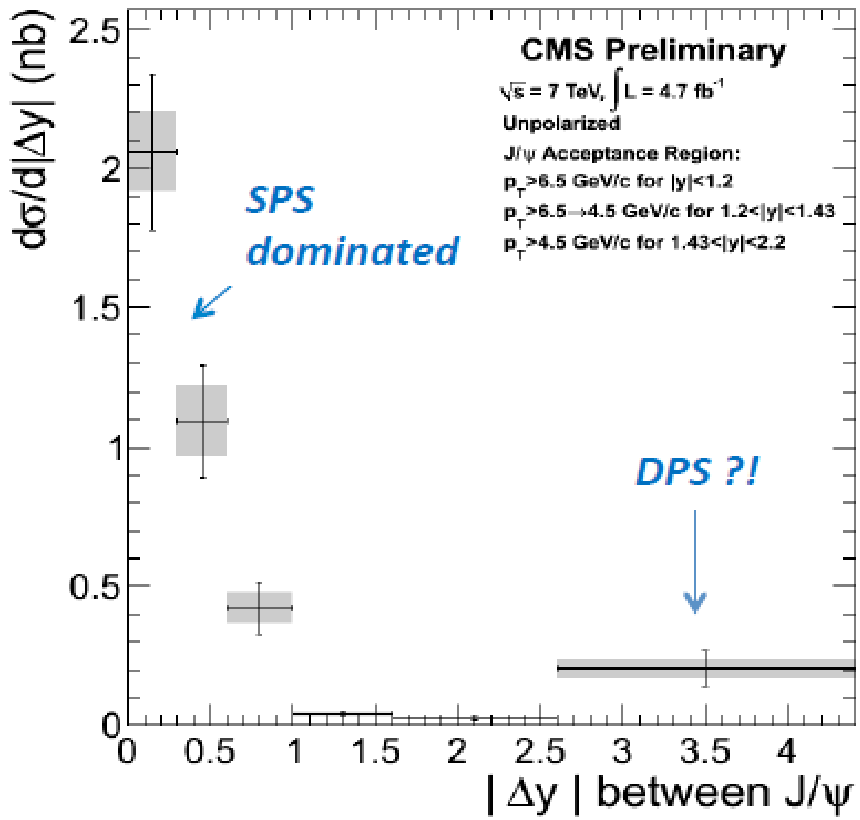
JHEP 06(2012) 141

Differential XS % Absolute Rapidity Difference

nb

|Δy|	dσ/d|Δy|	Stat. Err.	Syst. Err.
0-0.3	2.06	0.143	0.251
0.3-0.6	1.09	0.125	0.156
0.6-1	0.421	0.057	0.077
1-1.6	0.040	0.006	0.006
1.6-2.6	0.025	0.005	0.005
2.6-4.4	0.205	0.033	0.058

$\|y\| < 1.2$	pT > 6.5 GeV/c
$1.2 < \|y\| < 1.43$	pT > 6.5→4.5 GeV/c
$1.43 < \|y\| < 2.2$	pT > 4.5 GeV/c



Evidence for excess at |Δy| > 2.6

$$\frac{d\sigma(pp \to 2J/\psi + X)}{d|\Delta y|} = \sum_i \frac{N_i}{L \cdot BF(J/\psi \to \mu^+\mu^-)^2 \cdot \Delta|\Delta y|}$$

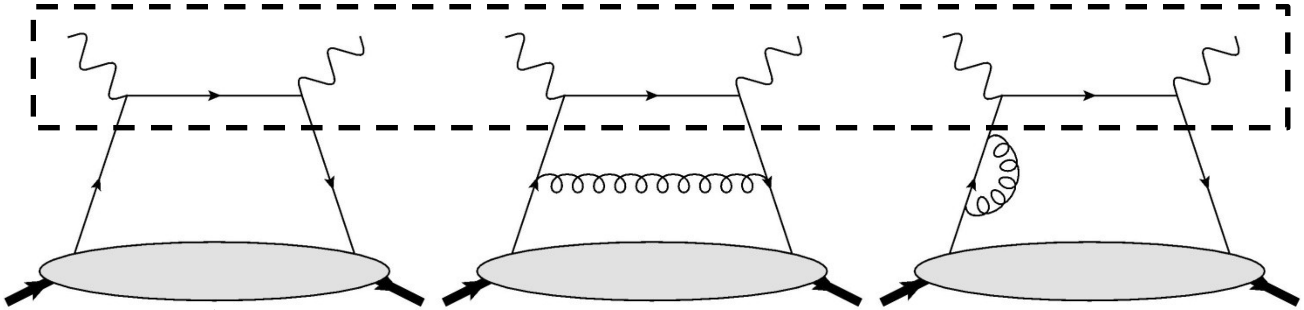
Sudakov Suppression of Colour Interference Distributions

Artru and Mekhfi, Phys.Rev. D37 (1988) 2618–2622
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

Illustrate using just one ladder (DIS):

$$\hat{\sigma}(x)$$

$$\otimes$$



$$f\left(x,\Lambda^2\right)$$

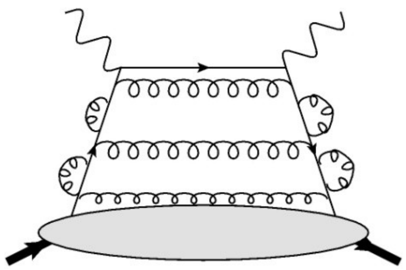
Ellis, Stirling, Webber, Chapter 5

$$+\frac{\alpha_s}{2\pi}C_R\ln\left(\frac{Q^2}{\Lambda^2}\right)\int_0^{1-\Lambda^2/Q^2}\frac{dx'}{x'}\frac{1+x'^2}{1-x'}f\left(\frac{x}{x'},\Lambda^2\right) \qquad -\frac{\alpha_s}{2\pi}C_V\ln\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right)\int_0^{1-\Lambda^2/Q^2}dx'\frac{1+x'^2}{1-x'}$$

$$+\frac{\alpha_s}{\pi}C_R\ln^2\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right) \qquad -\frac{\alpha_s}{\pi}C_V\ln^2\left(\frac{Q^2}{\Lambda^2}\right)f\left(x,\Lambda^2\right)$$

Sudakov Suppression of Colour Interference/Correlation Distributions

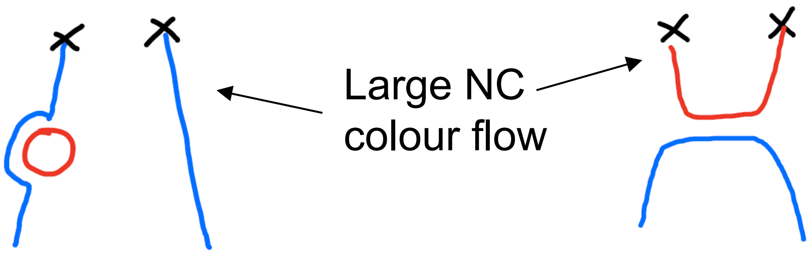
Resum arbitrary number of real & virtual emissions to double log order:



Sudakov factor

$$\sigma = \hat{\sigma}(x) \otimes f(x, \Lambda^2) \exp\left(\frac{\alpha_s}{\pi} (C_R - C_V) \ln^2 \left(\frac{Q^2}{\Lambda^2} \right) \right)$$

Quark legs are in colour singlet:



$C_V = C_F$

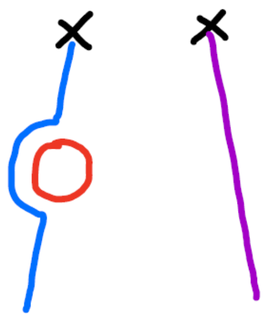
$C_R = C_F$

$C_R = C_V$

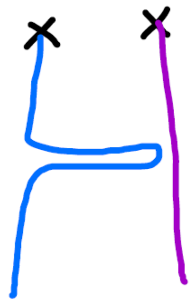
There is no Sudakov suppression!

Sudakov Suppression of Colour Interference/Correlation Distributions

Quark legs are in colour octet (as occurs in colour interference/correlation distributions):



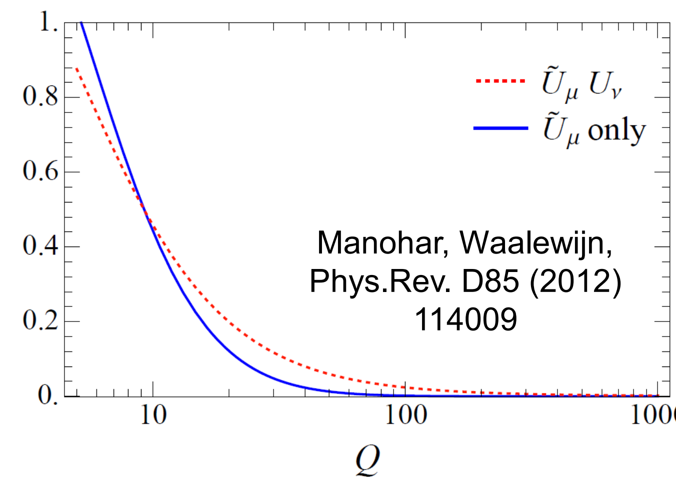
$$C_V = C_F$$



$$C_R = \left(C_F - \frac{1}{2} C_A \right) = -\frac{1}{2N}$$

$$C_R + C_V < 0$$

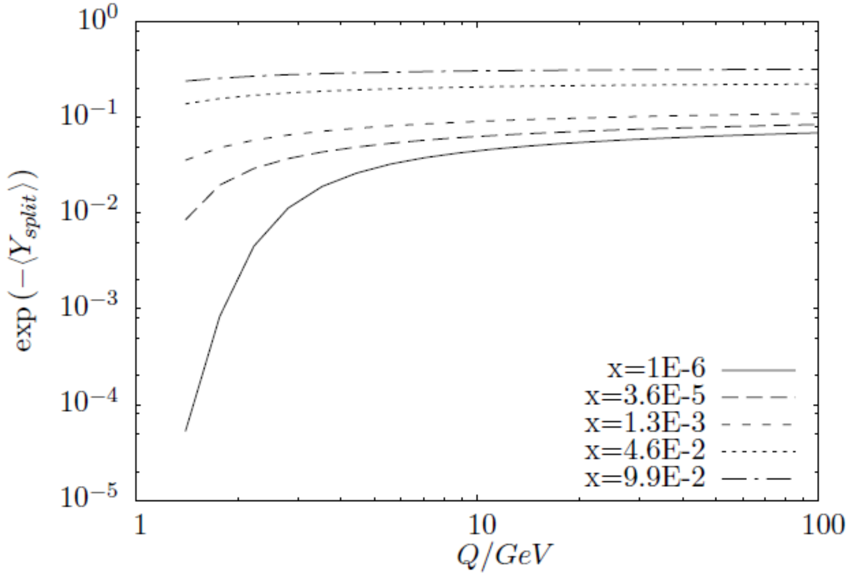
➔ Sudakov suppression:

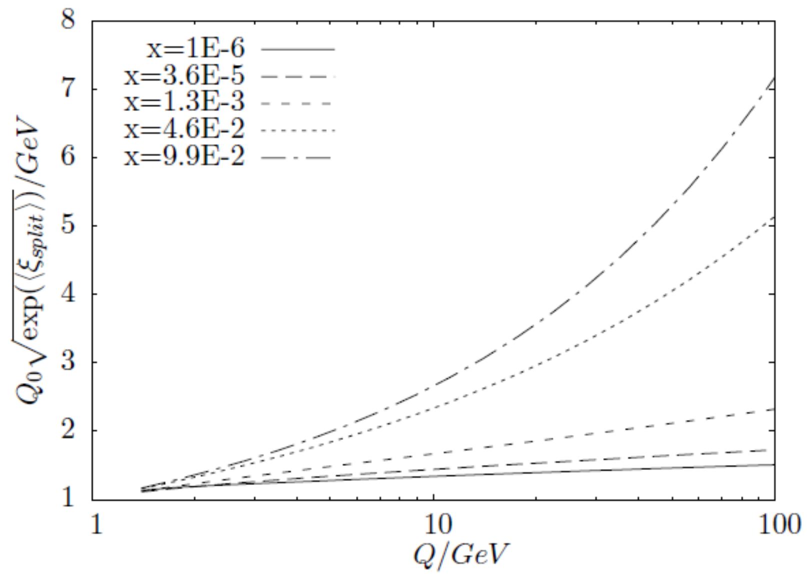


Physical explanation: Movement of colour by large transverse distance **b** in hadron.

Soft gluons with wavelengths larger than 1/**b** can't resolve colour transfer – cut off in Sudakov factor should really be 1/**b2**.

Typical x and k values of 1→2 splitting





$$Y = \ln(1/x),\ \xi = \ln(Q^2/Q_0^2),\ \xi_\Lambda = \ln(\Lambda_{QCD}^2/Q_0^2)$$

$$Q_0 = 1\ \text{GeV},\ \Lambda_{QCD} = 0.359\ \text{GeV},\ N_f = 3$$

ATLAS analysis improvement

Improvements to the analysis since ATLAS-CONF-2011-160

- Anti-$k_\perp$ jets with size parameter $R = 0.4$ instead of 0.6 are used.
- SHERPA no longer used to derive the central value (it is used for systematic studies) $\rightarrow$ AHJ is used as the nominal MC.
- CKKW matching threshold in SHERPA is lowered from 30 GeV (increasing the jet correlation of the no-DPI SHERPA sample.)
- DPI-off sample of AHJ is defined by rejecting events which have partons with $p_{\mathrm{T}}^{\mathrm{max}} > 15$ GeV (instead of 3.5 GeV). Partons are not matched to reconstructed jets.
- Additional background samples are subtracted from data; single top, diboson and $Z \rightarrow \tau\tau$.
- Pile-up calibration to jets improved. In addition, a pileup correction factor, $r_{\mathrm{pile-up}} = 1.17 \pm 0.15\,(\mathrm{stat.})$, is incorporated into calculation of $f_{\mathrm{DP}}^{(\mathrm{D})}$.

Differential Cross Section for DPS for q_T << Q

To calculate differential DPS cross sections for small $\mathbf{q}_A$, $\mathbf{q}_B$ where DPS is comparable with SPS, would actually require a different formula containing 'two parton transverse momentum dependent PDFs' or 2pGTMDs:

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

2pGTMD

$$\frac{d\sigma_D^{(A,B)}}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_h^{ik}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{b})\Gamma_h^{jl}(x_1', x_2', \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, \mathbf{b})$$
$$\times \hat{\sigma}_{ij}^A(x_1, x_1')\hat{\sigma}_{kl}^B(x_2, x_2')\mathrm{d}x_1\,\mathrm{d}x_1'\mathrm{d}x_2\,\mathrm{d}x_2'\mathrm{d}^2\mathbf{b}$$
$$\times \prod_{i=1,2}\int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta(\mathbf{k}_i + \bar{\mathbf{k}}_i - \mathbf{q}_i)$$

(Neglecting a possible soft factor + dependence of the 2pGTMDs on rapidity regulator)

Differential cross section can also be expressed in terms of **r** space 2pGTMDs – as in total cross section, one makes the replacement:

$$\int \Gamma_h^{ik}(\mathbf{b})\Gamma_h^{jl}(\mathbf{b})d^2\mathbf{b} \to \int \Gamma_h^{ik}(\mathbf{r})\Gamma_h^{jl}(-\mathbf{r})\frac{d^2\mathbf{r}}{(2\pi)^2}$$

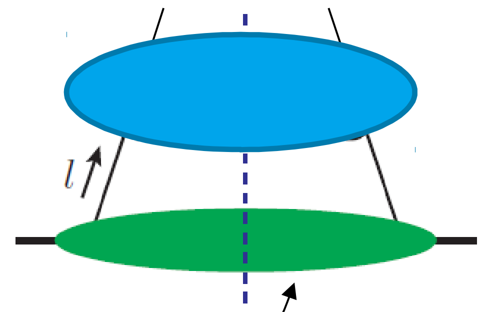
Relation between 2pGPDs and 2pGTMDs for qT >> Λ

SPS:

If |**q**| >> Λ (but still << Q), then TMD can be written in terms of collinear PDFs and a perturbative factor.

Collins, Soper, Sterman , Nucl.Phys. B250 (1985) 199
Collins, pQCD book, Ch. 13

$$F_h(x,\mathbf{k}) = T(x,\mathbf{k}) \otimes D_h(x,\mu^2=\mathbf{k}^2)$$



Collinear (single) PDF

Indeed, at double leading logarithmic order, we obtain the DDT formula for the differential SPS cross section for |**q**| >> Λ :

$$\frac{d\sigma}{dq^2\,dq_\perp^2} = \frac{d\sigma_{\rm tot}}{dq^2} \times \frac{\partial}{\partial q_\perp^2}\left\{ D_a^q\left(x_1,q_\perp^2\right) D_b^{\bar{q}}\left(x_2,q_\perp^2\right) S_q^2\left(q^2,q_\perp^2\right) \right\}.$$

Sudakov factor

We expect there to be a similar relation between 2pGPDs and 2pGTMDs. At the double leading log level, it has been shown that the Sudakov factor for DPS is the product of Sudakov factors for SPS:

$$\pi^2\frac{d\sigma^{(4\to4)}}{d^2\delta_{13}\,d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\,d\hat{t}_2}\cdot\frac{\partial}{\partial\delta_{13}^2}\frac{\partial}{\partial\delta_{24}^2}\left\{ {}_{[2]}D_a^{1,2}(x_1,x_2;\delta_{13}^2,\delta_{24}^2)\times{}_{[2]}D_b^{3,4}(x_3,x_4;\delta_{13}^2,\delta_{24}^2) \times S_1\left(Q^2,\delta_{13}^2\right)S_3\left(Q^2,\delta_{13}^2\right)\times S_2\left(Q^2,\delta_{24}^2\right)S_4\left(Q^2,\delta_{24}^2\right)\right\}.$$

➔ for |**q**| >> Λ there is a portion of the DPS differential σ that resembles the total σ

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))
BDFS, Eur.Phys.J. C72 (2012) 1963

Operator definition of 2pGPDs

$$D^{q_1 q_2}_{p(0)}(\xi_1, \xi_2) = \langle P|\, 2p^+ \int \sum_{i=1}^{2} \frac{dz_i^-}{2\pi} e^{i\xi_i z_i^- p^+} dy^- d^2\boldsymbol{y}$$
$$\times \bar{\psi}_{q_2,a(0)}(y - \tfrac{1}{2}z_2) \mathcal{G}_{ab}(y - \tfrac{1}{2}z_2, y + \tfrac{1}{2}z_2) \tfrac{1}{2}\gamma^+ \psi_{q_2,b(0)}(y + \tfrac{1}{2}z_2)$$
$$\times \bar{\psi}_{q_1,c(0)}(-\tfrac{1}{2}z_1) \mathcal{G}_{cd}(-\tfrac{1}{2}z_1, \tfrac{1}{2}z_2) \tfrac{1}{2}\gamma^+ \psi_{q_1,d(0)}(\tfrac{1}{2}z_1)\, |P\rangle_c \Big|_{z_i^+ = y^+ = 0, \boldsymbol{z}_i = 0}$$

$$\begin{aligned} M_{q,q}(\boldsymbol{y}^2) &= \int_0^1 dx_1 \int_0^1 dx_2 \Big[{}^1F_{q,q}(x_1, x_2, \boldsymbol{y}) \\ &\quad - {}^1F_{\bar{q},q}(x_1, x_2, \boldsymbol{y}) - {}^1F_{q,\bar{q}}(x_1, x_2, \boldsymbol{y}) \\ &\quad + {}^1F_{\bar{q},\bar{q}}(x_1, x_2, \boldsymbol{y}) \Big] \\ &= \frac{2}{p^+} \int dy^- \langle p|\, \mathcal{O}_q(0,0)\, \mathcal{O}_q(y,0) |p\rangle \,. \qquad (37) \end{aligned}$$

$$M_{q,q}(\boldsymbol{y}^2) = \int d(py)\; \langle \mathcal{O}\mathcal{O}\rangle(py, y^2) \Big|_{y^2 = -\boldsymbol{y}^2} .$$

$$(py)^2/(-y^2) = (\vec{p}\,\vec{y})^2/\vec{y}^2 \;\leq\; \vec{p}^{\,2} ,$$