Make Nature Natural Again

Part 1: criticise all the rest

SUSY, you are fired;

Repeal standard naturalness;

Ban mass from theory.

Part 2: good crazy alt-phys

Dynamical generation of M_h, $M_{\rm Pl}$;

Infinite Energy;

Agravity, Ghosts; Inflation.

Alessandro Strumia

Pisa U. & INFN & CERN

Roma, 20/12/2017

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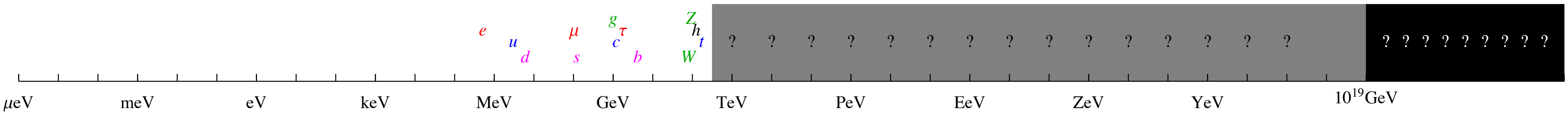
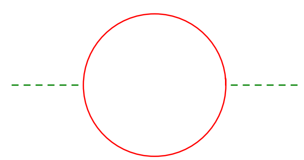
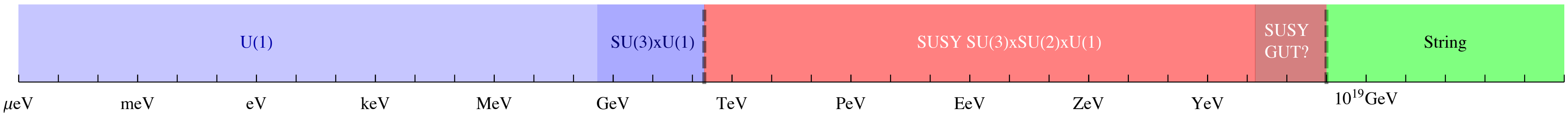
Mass scales in nature: traditional view



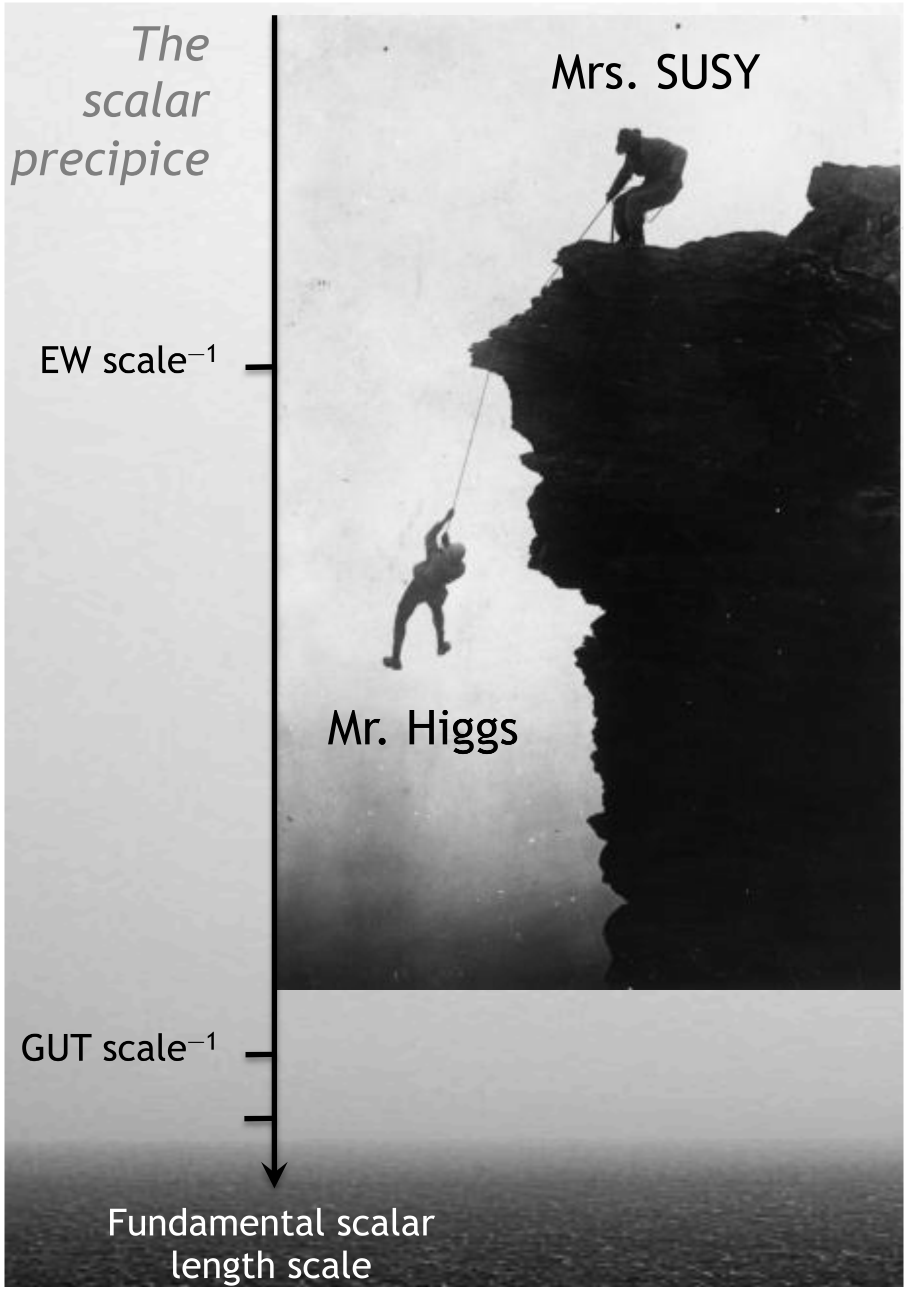
The Standard Model explains part of the mess, we still need to understand $M_h \ll M_{\mathsf{Pl}}$ apparently destabilized by quantum corrections:

$$\delta M_h^2 =$$  $$\sim \; g_{\mathsf{SM}}^2 \Lambda^2$$

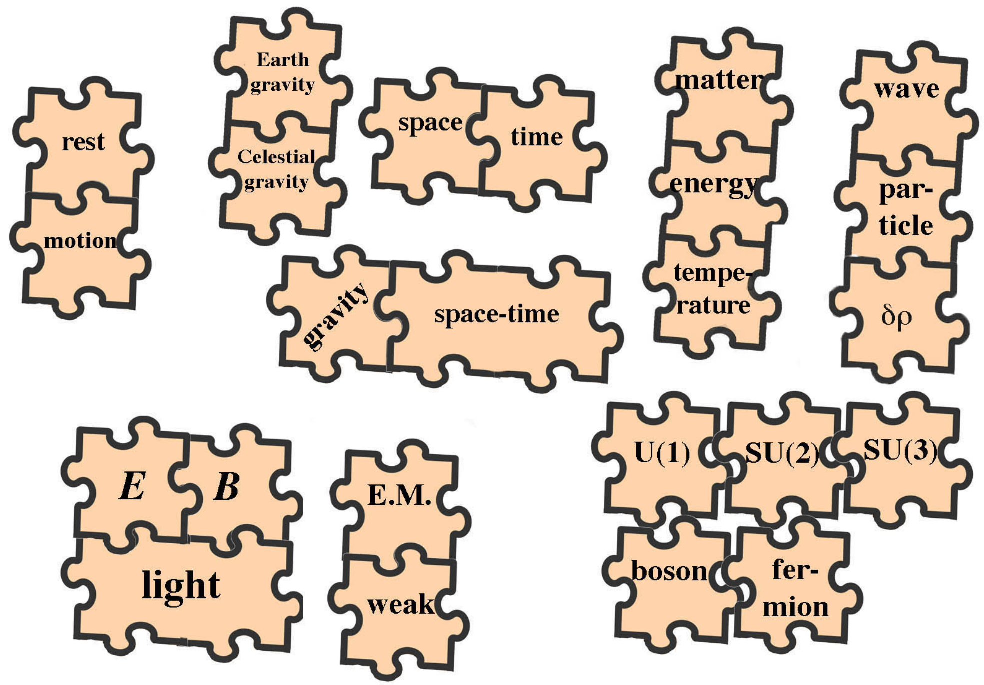
Dominant theory has two scales, the **string** scale and the EW/**SUSY** scale:



The establishment wants SUSY



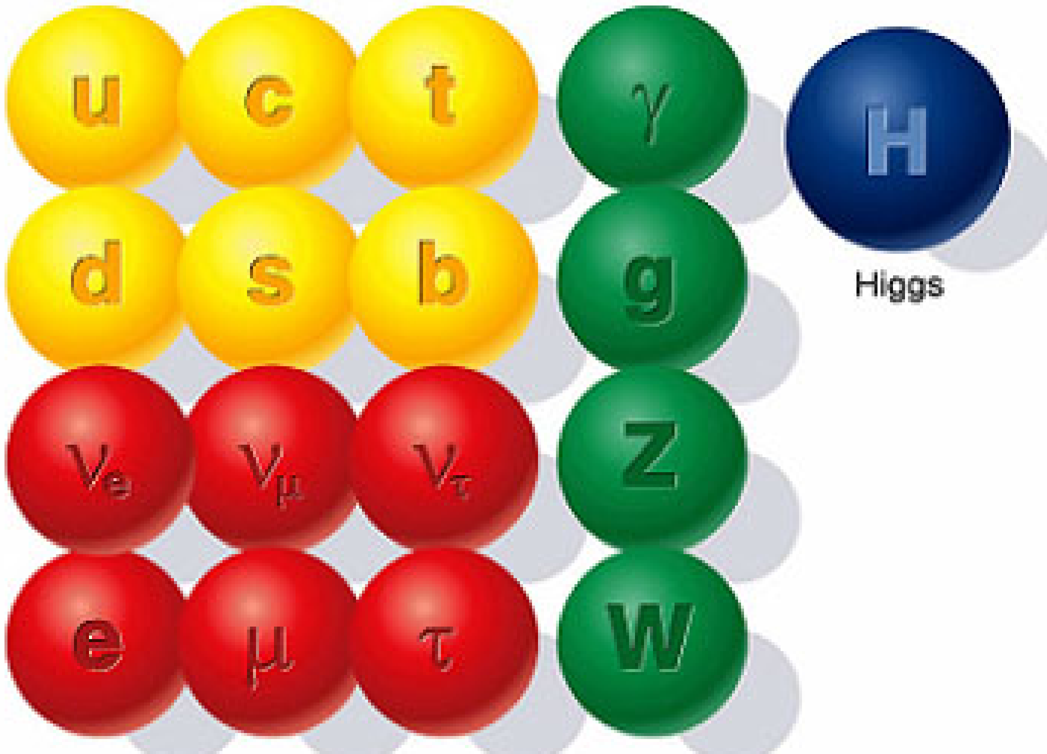
- ⋆ SUSY stabilizes Higgs.
- ⋆ SUSY extends Lorentz, allows spin 3/2.
- ⋆ SUSY unifies fermions with bosons.
- ⋆ SUSY unifies gauge couplings.
- ⋆ SUSY gives DM aka 'neutralino'.
- ⋆ SUSY is predicted by super-strings.



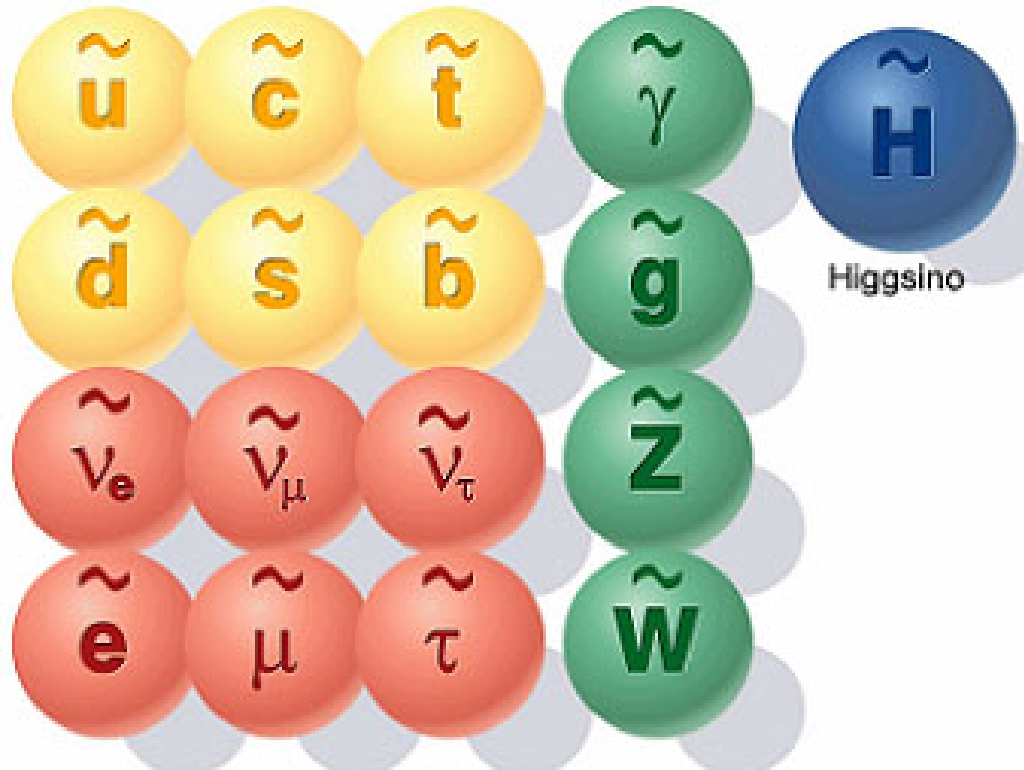
⋆ Worry: too many sparticles at LHC?

LHC inverse problem solved

SEEN

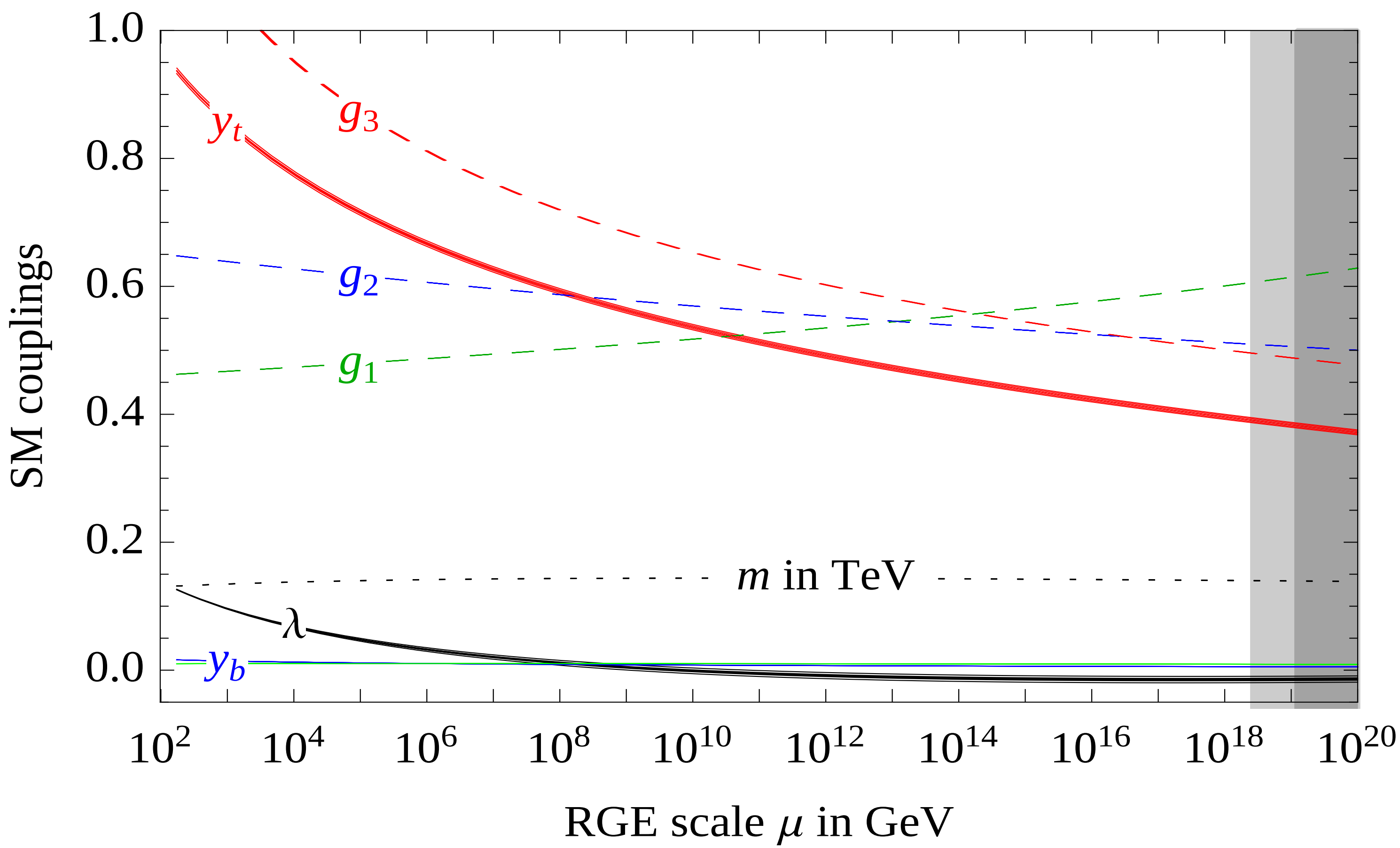


MISSING



News from the fronteer

No new physics at LHC. For the measured M_h, M_t the Standardissimo Model can be extrapolated up to $M_{\rm Pl}$ and above.



λ and its β-function nearly vanish around $M_{\rm Pl}$

Naturalness in trouble

SUSY was the best solution to a bigger issue: most theorists believe that

“light fundamental scalars must be accompanied by new physics that protects their lightness from quadratically divergent corrections”

But LHC observed the opposite: the Higgs and no new physics

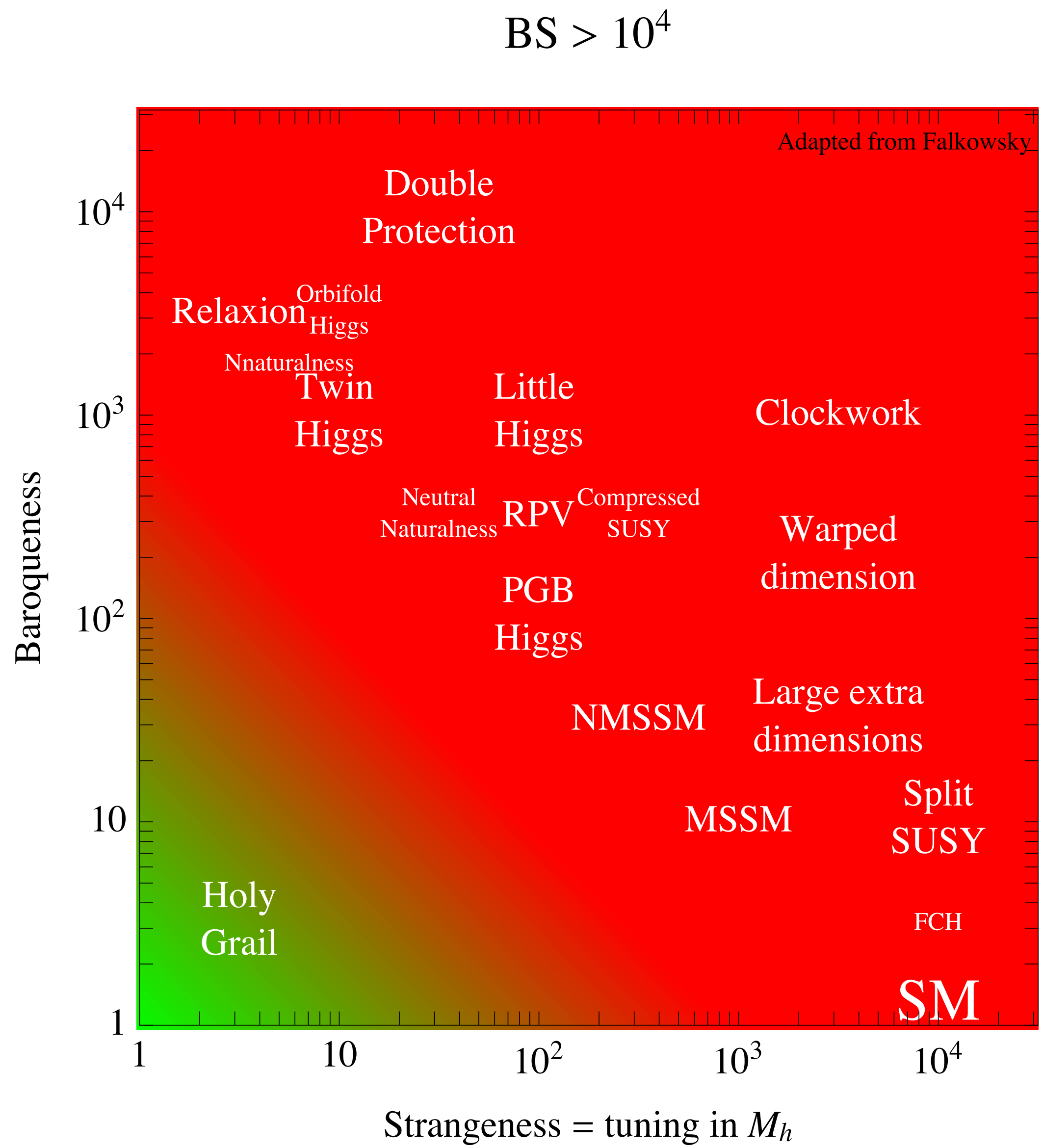
So many boring SM victories that the situation is interesting. All natural extensions of the SM in trouble: SUSY, X dims, technicolor, composite Higgs...

These models no longer can be natural: $\delta M_h^2 \gtrsim 100 M_h^2$

Reaction 1: weird theoretical ideas?

Add more smarter new physics to explain why we see nothing: compressed PGB flacky double orbifold RPV low-energy SUSY … twin hyperbolic composite relaxed little crooked Higgs …

Must be tried: no stone unturned.
Looks like therapeutic obstinacy:



2: anthropic selection in a multiverse?

The cosmological constant $V \sim 10^{-120} M_{\rm Pl}^4$ is one more unnaturally small mystery. No natural theory known. Weinberg: anthropic selection in a multiverse.

Anthropics explains $M_h \ll M_{\rm Pl}$ too?

- Needed to have systems made of many particles.
- Chemistry exists thanks to $y_d v \approx \alpha_{\rm em} \Lambda_{\rm QCD}$.

But natural solutions exist, difficult to argue that multiverse avoids them.
Even if we live in a multiverse, natural anthropic theories would be more likely:

- SM with a smaller y or $M_{\rm Pl}$;
- a QED+QCD alternative without a Higgs;
- weak scale SUSY.

Keep searching alternatives to anthropic nirvana

Subtle is the Lord

What is going on? We are confused but nature is surely following some logic

The goal of this talk is presenting an alternative: a renormalizable theory valid above M_{Pl} such that M_h is naturally smaller than M_{Pl} without new physics at the weak scale. It naturally gives inflation and a beautiful anti-graviton ghost.

Reconsidering naturalness

$\Rightarrow$

Make Nature Natural Again

If nature looks unnatural, maybe we misunderstood what naturalness means.

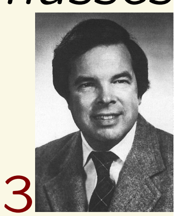
Power divergences and regulators are suggested by QFT equations. The æther was suggested by Maxwell equations. But power divergences are unphysical. Maybe we are again over-interpreting, adding realism to quantum mechanics.

Maybe there are no regulators: a SM-like theory holds up to infinite energy.

[Caution: this is when rotten tomatoes start to fly]

Wilson proposed usual naturalness attributing physical meaning to momentum shells of loop integrals, used in the 'averaged action'. Ipse undixit:

"The claim was that it would be unnatural for such particles to have masses small enough to be detectable soon. But this claim makes no sense".



Kenneth G. Wilson, hep-lat/0412043

Physical Naturalness

Demand that physical corrections only satisfy naturalness:

$$M_h \gtrsim \delta M_h \sim \begin{cases} g_{\rm SM} \Lambda_{\rm UV} & \text{Usual naturalness} \\ g_{\rm extra} M_{\rm extra} & \text{Physical naturalness} \end{cases}$$

The SM satisfies Physical Naturalness, for the measured $M_h \approx M_t$

This would be ruined by new heavy particles too coupled to the SM.

Unlike in the other scenarios, high-scale model building is very constrained.

Imagine there is no GUT. No flavour models too. Above us only sky.

Data demand some new physics: DM, neutrino masses, maybe axions...

Can this be added compatibly with Physical Naturalness?

Physical Naturalness and new physics

Neutrino mass models add extra particles with mass M

$$M \lesssim \begin{cases} 0.7\ 10^7\,\mathrm{GeV} \times \sqrt[3]{\Delta} & \text{type I see-saw model,} \\ 200\,\mathrm{GeV} \times \sqrt{\Delta} & \text{type II see-saw model,} \\ 940\,\mathrm{GeV} \times \sqrt{\Delta} & \text{type III see-saw model.} \end{cases}$$

Leptogenesis is compatible with PhysNat only in type I.

Axion and LHC usually are like fish and bicycle because $f_a \gtrsim 10^9\,\mathrm{GeV}$. Axion models can satisfy PN, e.g. KSVZ models employ heavy quarks with mass M

$$M \lesssim \sqrt{\Delta} \times \begin{cases} 0.74\,\mathrm{TeV} & \text{if } \Psi = Q \oplus \bar{Q} \\ 4.5\,\mathrm{TeV} & \text{if } \Psi = U \oplus \bar{U} \\ 9.1\,\mathrm{TeV} & \text{if } \Psi = D \oplus \bar{D} \end{cases}$$

Inflation: flatness implies small couplings.

Dark Matter: below about a TeV if weakly coupled.

DM with weak gauge interactions

Consider a Minimal Dark Matter n-plet. 2-loop quantum corrections to M_h^2:

$$\delta M_h^2 = \frac{cnM^2}{(4\pi)^4}\left(\frac{n^2-1}{4}g_2^4 + Y^2 g_Y^4\right) \times \begin{cases} 6\ln\frac{M^2}{\Lambda^2} - 1 & \text{for fermion DM} \\ \frac{3}{2}\ln^2\frac{M^2}{\Lambda\mu^2} + 2\ln\frac{M^2}{\Lambda^2} + \frac{7}{2} & \text{for scalar DM} \end{cases}$$

Quantum numbers $\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	Spin	DM could decay into	DM mass in TeV	$m_{\mathrm{DM}^\pm} - m_{\mathrm{DM}}$ in MeV	Physical naturalness bound in TeV, $\Lambda \sim M_{\mathrm{Pl}}$	σ_{SI} in $10^{-46}\,\mathrm{cm}^2$
2	1/2	0	EL	0.54	350	$0.4 \times \sqrt{\Delta}$	$(2.3 \pm 0.3)\,10^{-2}$
2	1/2	1/2	EH	1.1	341	$1.9 \times \sqrt{\Delta}$	$(2.5 \pm 0.8)\,10^{-2}$
3	0	0	HH^*	2.5	166	$0.22 \times \sqrt{\Delta}$	0.60 ± 0.04
3	0	1/2	LH	2.7	166	$1.0 \times \sqrt{\Delta}$	0.60 ± 0.04
3	1	0	HH, LL	1.6+	540	$0.22 \times \sqrt{\Delta}$	0.06 ± 0.02
3	1	1/2	LH	1.9+	526	$1.0 \times \sqrt{\Delta}$	0.06 ± 0.02
4	1/2	0	HHH^*	2.4+	353	$0.14 \times \sqrt{\Delta}$	1.7 ± 0.1
4	1/2	1/2	(LHH^*)	2.4+	347	$0.6 \times \sqrt{\Delta}$	1.7 ± 0.1
4	3/2	0	HHH	2.9+	729	$0.14 \times \sqrt{\Delta}$	0.08 ± 0.04
4	3/2	1/2	(LHH)	2.6+	712	$0.6 \times \sqrt{\Delta}$	0.08 ± 0.04
5	0	0	(HHH^*H^*)	9.4	166	$0.10 \times \sqrt{\Delta}$	5.4 ± 0.4
5	0	1/2	stable	11.5	166	$0.4 \times \sqrt{\Delta}$	5.4 ± 0.4

A new principle: nature has no scale

Physical Naturalness is phenomenologically viable, what about its theory?

A naive effective field theory suffers of the hierarchy problem:

$$\mathscr{L} \sim \Lambda^4 + \Lambda^2 H^2 + \mathscr{L}_4 + \frac{H^6}{\Lambda^2} + \cdots$$

Nature is singling out $\mathscr{L}_4$. Why?

Principle: "**Nature has no fundamental scales Λ**".

Then, the fundamental QFT is described by $\mathscr{L}_4$: only dimensionless couplings.

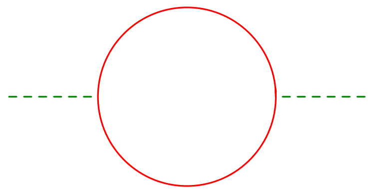
Power divergences have mass dimension. So they must vanish if there are no masses: $\int dE\, E = 0$. Anything different is dimensionally wrong.

The scale anomaly

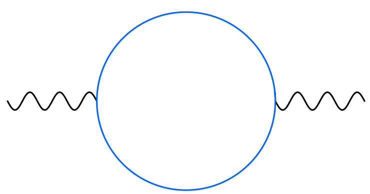
Is all this useless because quantum corrections break scale invariance?

- The chiral anomaly does not make fermions massive.
- The scale anomaly does not make scalars massive.

The one loop correction to a scalar mass^2 is quadratically divergent:

$\Pi(0) =$  $= -4y^2 \frac{1}{i} \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + m^2}{(k^2 - m^2)^2}$

The same happens for the photon:

$\Pi_{\mu\nu}(0) =$  $= -4e^2 \frac{1}{i} \int \frac{d^4k}{(2\pi)^4} \left(\frac{2k_\mu k_\nu}{(k^2 - m^2)^2} - \frac{\eta_{\mu\nu}}{k^2 - m^2} \right)$

And for the graviton $\Pi_{\mu\nu\alpha\beta}(0)$. A physical cut-off can respect gauge invariance and must break scale invariance (such as strings): keeps $M_\gamma = 0$, while $M_h \sim \Lambda$. In a theory with no cut-off M_γ and M_h have the same fate.

Can quantum corrections generate $M_h, M_{\rm Pl}$?

Yes, if dynamics generates vevs or condensates. 1) Models for M_h; 2) for $M_{\rm Pl}$.

1) What is the weak scale?

$M_h \sim g_{\text{extra}} M_{\text{extra}}$ where g_{extra} can be $\ll g_{\text{SM}}$, so M_{extra} can be $\gg M_h$

Physical naturalness does not imply new physics at the weak scale

- Could be generated from nothing by weak-scale dynamics.
  - Another gauge group might become strong around 1 TeV.
  - The quartic of another scalar might run negative around 1 TeV.

- Could be generated from nothing by heavier dynamics.
  - See-saw, axions, gravity...

Weakly coupled models for the weak scale

The Coleman-Weinberg mechanism can dynamically generate the weak scale

Model :

$G_{\mathsf{SM}} \otimes \mathsf{SU}(2)_X$ with one extra scalar S, doublet under $\mathsf{SU}(2)_X$ and potential

$$V = \lambda_H |H|^4 - \lambda_{HS} |HS|^2 + \lambda_S |S|^4.$$

1) **Dynamically generates** the weak scale and weak scale DM
2) **Preserves** the successful automatic features of the SM: B, L...
3) **Gets DM stability** as one extra automatic feature.

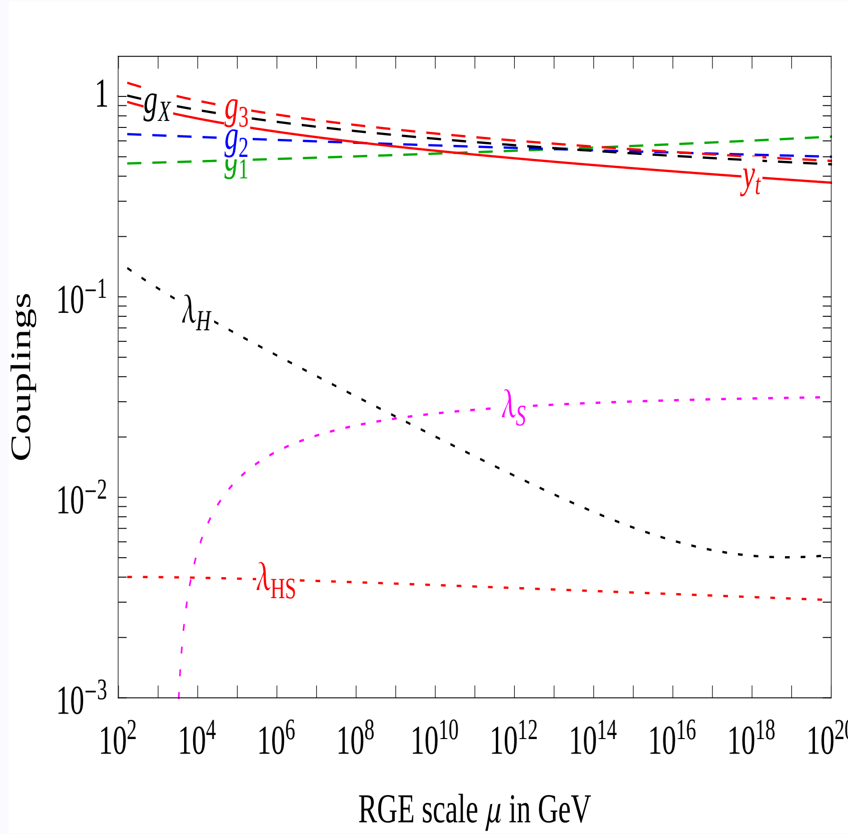
Weakly coupled SU(2) model

1) λ_S runs negative at low energy:

$$\lambda_S \simeq \beta_{\lambda_S} \ln \frac{s}{s_*} \qquad \text{with} \qquad \beta_{\lambda_S} \simeq \frac{9 g_X^4}{8(4\pi)^2}$$

$$S(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \text{[redacted]} \\ w + s(x) \end{pmatrix} \qquad w \simeq s_* e^{-1/4}$$

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \qquad v \simeq w \sqrt{\frac{\lambda_{HS}}{2\lambda_H}}$$



2) No new Yukawas.

3) SU(2)$_X$ vectors get mass $M_X = \frac{1}{2} g_X w$ and are automatically stable.

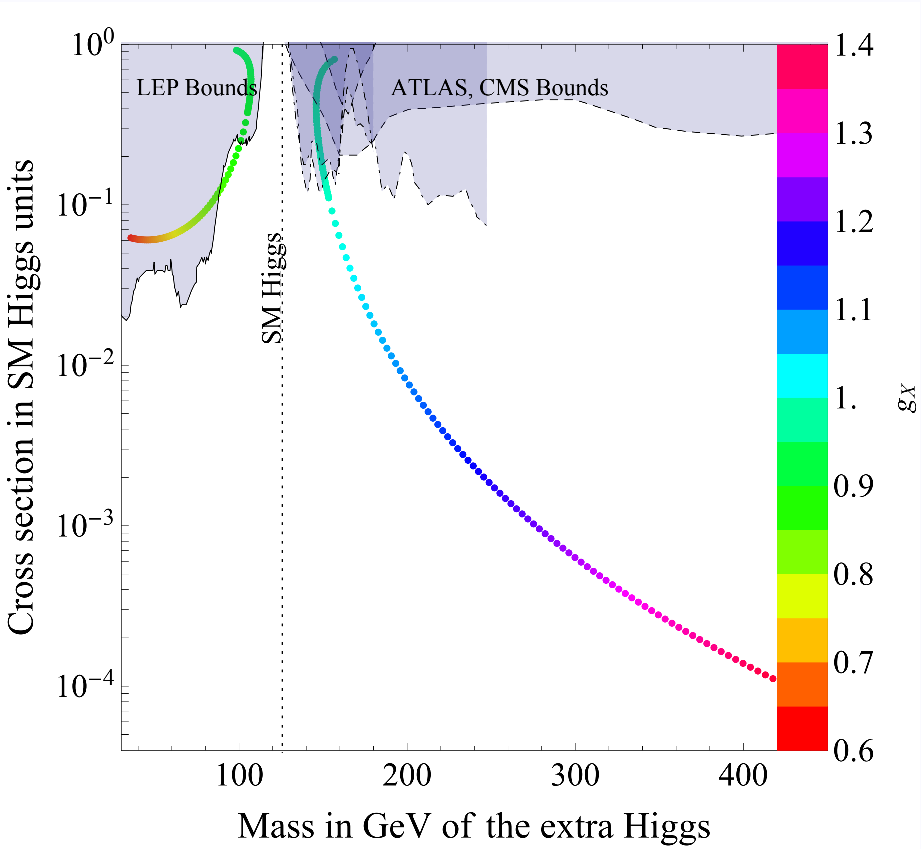
4) Bonus: threshold effect stabilises $\lambda_H = \lambda + \lambda_{HS}^2 / \beta_{\lambda_S}$.

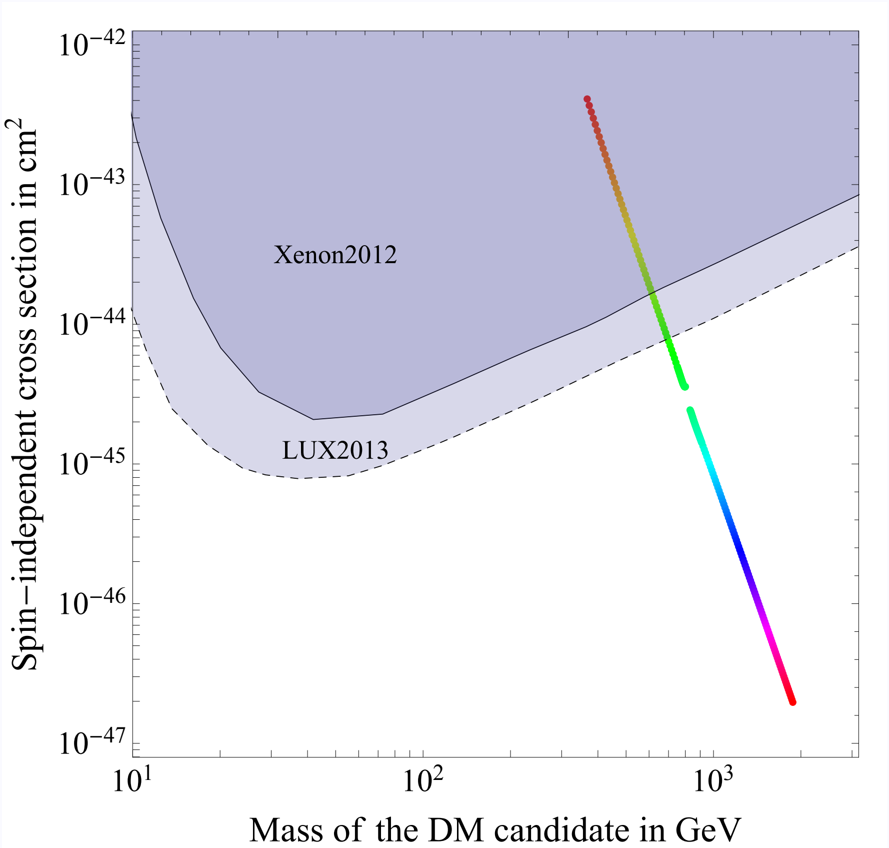
Experimental implications

1) New scalar s: like another h with suppressed couplings; $s \to hh$ if $M_s > 2M_h$.
2) Dark Matter coupled to s, h. Assuming that DM is a thermal relict

$$\sigma v_{\rm ann} + \frac{1}{2}\sigma v_{\rm semi-ann} = \frac{11 g_X^2}{1728\pi w^2} + \frac{g_X^2}{64\pi w^2} \approx 2.2\times 10^{26}\,\frac{\rm cm^3}{\rm s}$$

fixes $g_X = w/2\,\text{TeV}$, so all is predicted in terms of one parameter e.g. g_X:





Dark/EW phase transition is 1st order: gravitational waves, axiogenesis?

The weak scale from strong dynamics

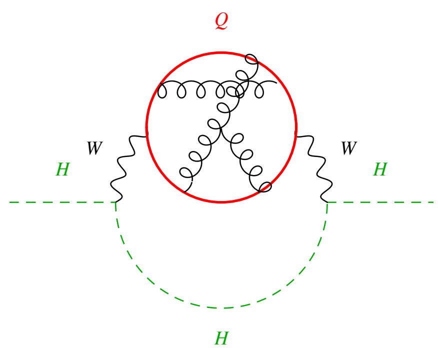
Model:

$G_{\mathsf{SM}} \otimes \mathsf{SU}(N)$ with one extra fermion in the $(0_Y, 3_L, 1_c, N \oplus \bar{N})$. $V = \lambda_H |H|^4$

No extra scalars, no masses: as many parameters as the SM!

The weak scale from strong dynamics

New QCD-like dynamics becomes strong at $\Lambda \sim$ few TeV inducing

$m_h^2 =$ 

$= \dfrac{9g_2^4}{4(4\pi)^2}\int dQ^2 \Pi_W(-Q^2)$

The W propagator contains strong dynamics. Dispersion relations proof $m_h^2 < 0$

$$\frac{\partial \Pi_W}{\partial \Lambda_{\rm TC}^2} = -\frac{q^2}{\Lambda_{\rm TC}^2}\frac{\partial \Pi_W}{\partial q^2}, \qquad \frac{\partial \Pi_W(q^2)}{\partial q^2} = \frac{1}{\pi}\int_0^\infty ds\, \frac{\overbrace{\text{Im}\,\Pi_W(s)}^{\sim -\sigma<0}}{(s-q^2)^2} < 0$$

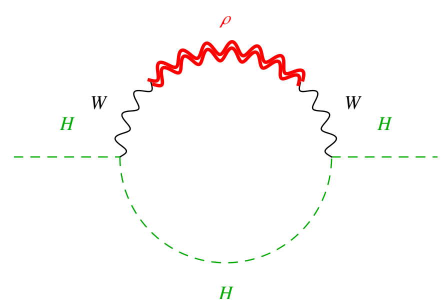
The weak scale from strong dynamics

Ignoring power divergences m_h^2 is UV-finite: use Operator Product Expansion

$$\Pi_W(q^2) \overset{q^2 \gg \Lambda^2}{\simeq} \underbrace{c_1(q^2)}_{\text{dimensionless}} + \underbrace{c_3(q^2)}_{-C/q^4} \underbrace{\langle 0|\frac{\alpha_{\rm TC}}{4\pi}\mathcal{G}^{A2}_{\mu\nu}|0\rangle}_{\text{positive}} + \cdots$$

Vector Meson Dominance estimates $\Pi_W(q^2) = m_\rho^2/g_\rho^2(q^2 - m_\rho^2 + i\epsilon)$

$$m_h^2 \sim$$



$$\sim -\frac{g_2^4 m_\rho^2}{(4\pi)^2 g_\rho^2}$$

All new physics univocally predicted: $m_\rho \sim 20\,\text{TeV}$, 'baryons' at $m_B \sim 50\,\text{TeV}$.

Lighter 'pions' in the $3 \otimes 3 - 1 = 3 \oplus 5$ of SU(2)$_L$ at $m_{\pi_n} \approx \frac{g_2 m_\rho}{4\pi}\sqrt{\frac{3}{4}(n^2-1)} \sim 2\,\text{TeV}$. π_5 decays via the anomaly $\pi_5 \to WW$.

Dark Matter from strong dynamics

The model has **two** accidentally stable composite DM candidates:

- **The lightest 'baryon'**, presumably subdominant:

$$\Omega_{\text{thermal}} \approx 0.1 \left(\frac{m_B}{200\,\text{TeV}}\right)^2$$

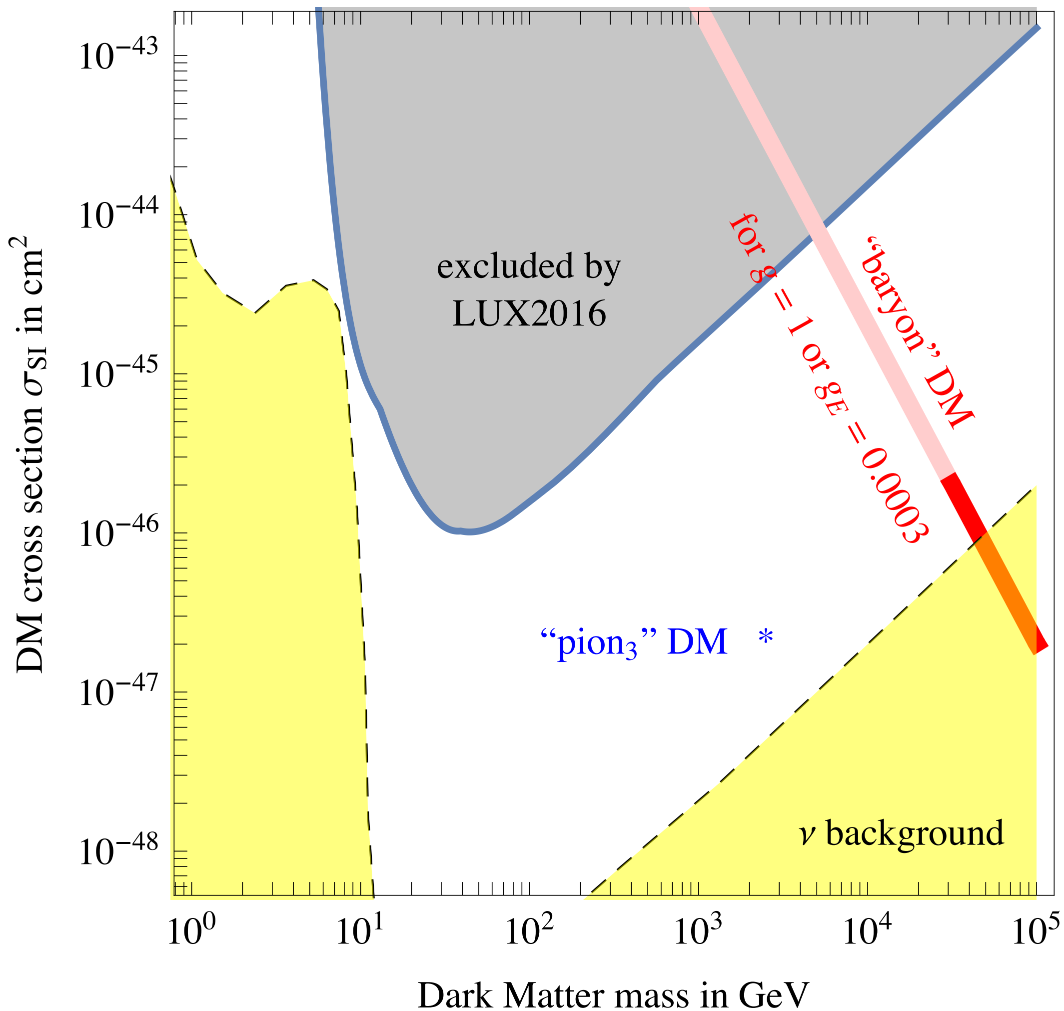
Characteristic magnetic dipole direct detection interaction.

- **The 'pion'** π_3. Thermal relic abundance predicted, ok for

$$m_{\pi_3} = 2.5\,\text{TeV}$$

Direct detection:

$$\sigma_{\text{SI}} \approx 0.2\ 10^{-46}\,\text{cm}^2.$$



Soft gravity

$$M_h \gtrsim \delta M_h \sim \begin{cases} g_{\rm SM}\Lambda_{\rm UV} & \text{Usual naturalness} \\ g_{\rm extra} M_{\rm extra} & \text{Physical naturalness} \end{cases}$$

The Einstein gravitational coupling grows with energy, blows up at $M_{\rm Pl}$

$$g_{\rm grav} \sim E/M_{\rm Pl}$$

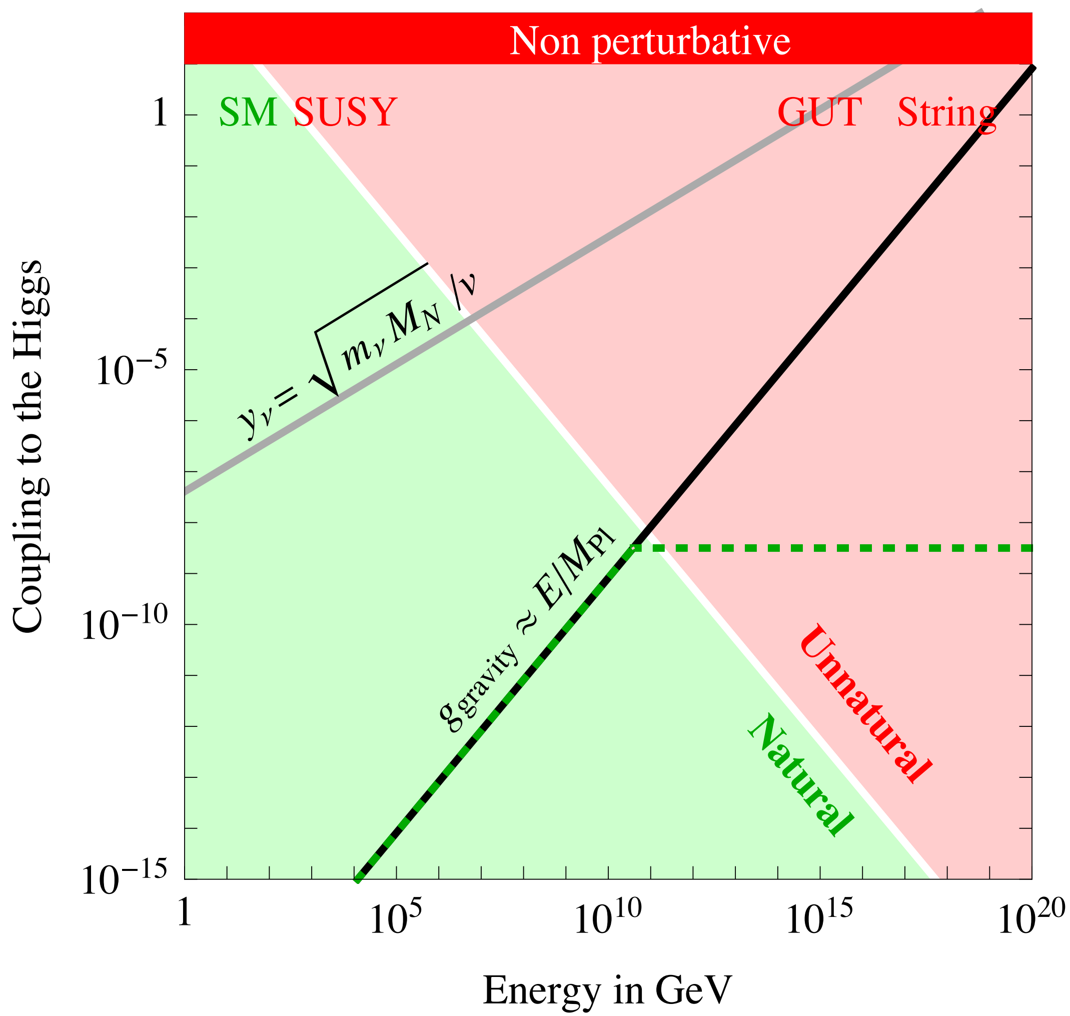
and couples everybody:

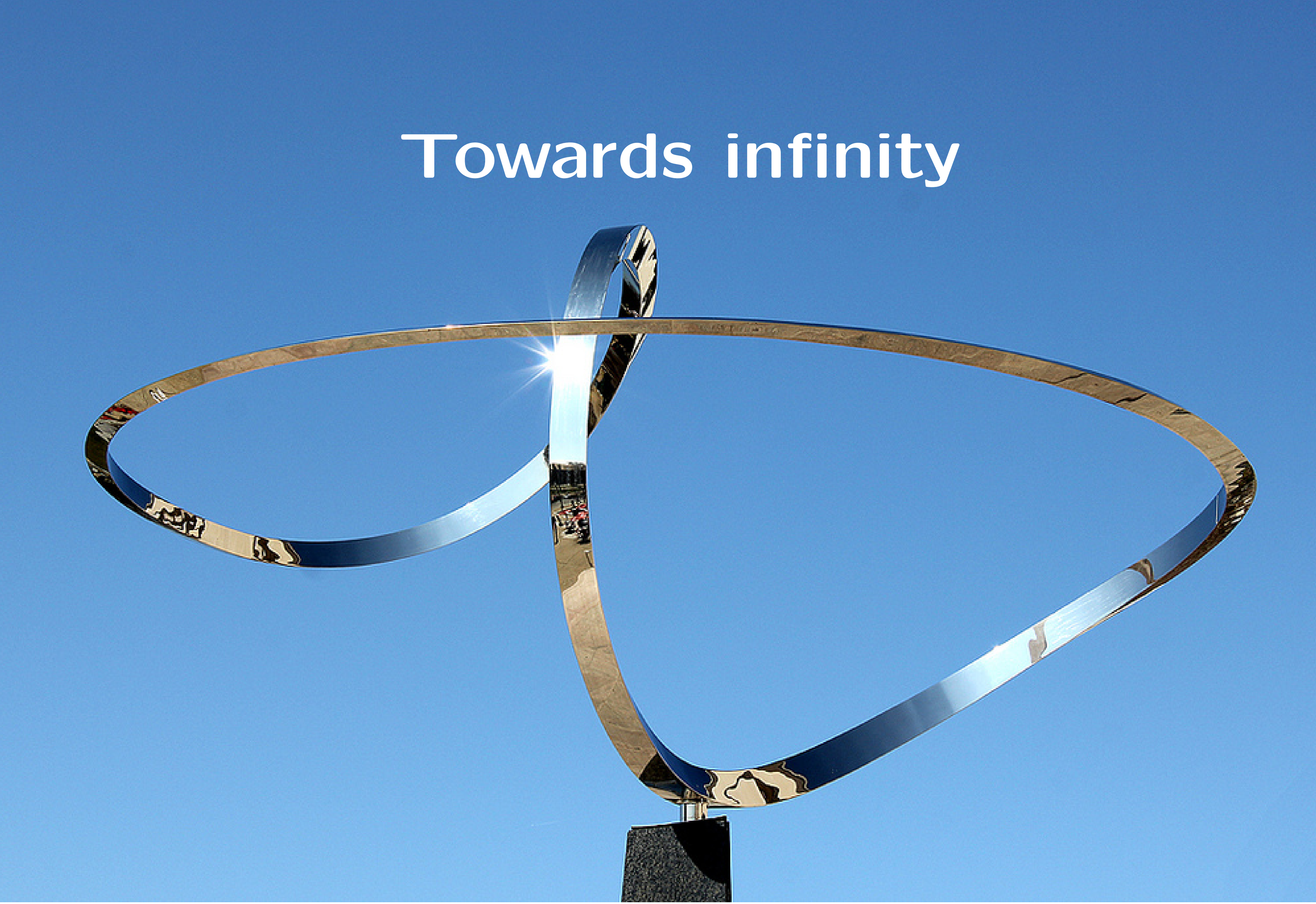
$$\delta M_h \sim g_{\rm grav} M_{\rm extra} \sim M_{\rm extra}^2/M_{\rm Pl}.$$

New physics must fix gravity when it is natural

$$g_{\rm grav} \lesssim 10^{-8}$$

$$M_{\rm extra} \lesssim 10^{12}\,\text{GeV}$$




Towards infinity

Motivation

If the theory has no cut-off Λ, it cannot give $\delta M_h^2 \sim \Lambda^2$

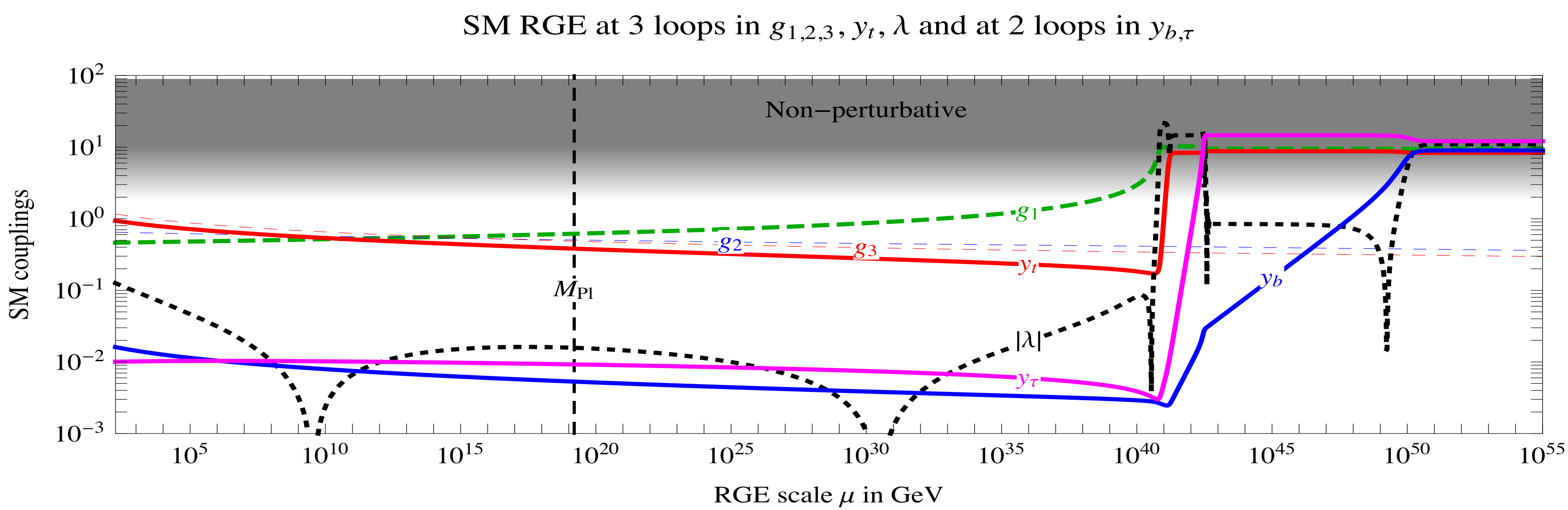
Models of soft gravity (agravity later) give RGE above $M_{\rm Pl}$.
We assume that the gravitational coupling is numerically small.
So RGE are dominated by the bigger QFT couplings: $g_{1,2,3}, y_t, \ldots$

Can the theory reach infinite energy?

Obstacle: Landau poles

Asymptotically safe Higgs?

In the SM, the abelian g_Y runs non-perturbative at $\Lambda \sim 10^{40}$ GeV.
SM dies there? Or $g_Y, y_t, \lambda, \ldots$ enter in Total Asymptotic Safety? Like this:



Would TAS imply an unnatural $\delta M_h^2 \sim \Lambda^2$?

Scalars remain naturally lighter than the transition scale Λ in a toy-SM where g, y, λ couplings enter perturbative TAS. Indeed, quantum dimensions of order $\epsilon \sim g^2/(4\pi)^2$ can make a mass^2 only at non-perturbative order $\epsilon^{2/\epsilon}$.

We don't know how to compute if the SM is TAS. So we explore TAF

Total Asymptotic Freedom?

Goal: compute if **all** couplings of a realistic QFT can run to 0 to $E = \infty$.

Naive attempt:

- solve the RGE for g, y, λ numerically
- up to infinite energy
- identify m-dimensional sub-spaces.

Result:

Analytic tools needed

TAF tools

Rewrite RGE in terms of $t = \ln \mu^2/(4\pi)^2$ and of $x_I = \{\tilde{g}_i, \tilde{y}_a, \tilde{\lambda}_m\}$ as

$$g_i^2(t) = \frac{\tilde{g}_i^2(t)}{t}, \qquad y_a^2(t) = \frac{\tilde{y}_a^2(t)}{t}, \qquad \lambda_m(t) = \frac{\tilde{\lambda}_m(t)}{t}.$$

Get

$$\frac{dx_I}{d\ln t} = V_I(x) = \begin{cases} \tilde{g}_i/2 + \beta_{g_i}(\tilde{g}), \\ \tilde{y}_a/2 + \beta_{y_a}(\tilde{g}, \tilde{y}), \\ \tilde{\lambda}_m + \beta_{\lambda_m}(\tilde{g}, \tilde{y}, \tilde{\lambda}). \end{cases}$$

Fixed-points $x_I(t) = x_\infty$ are determined by the <u>algebraic</u> equation $V_I(x_\infty) = 0$.

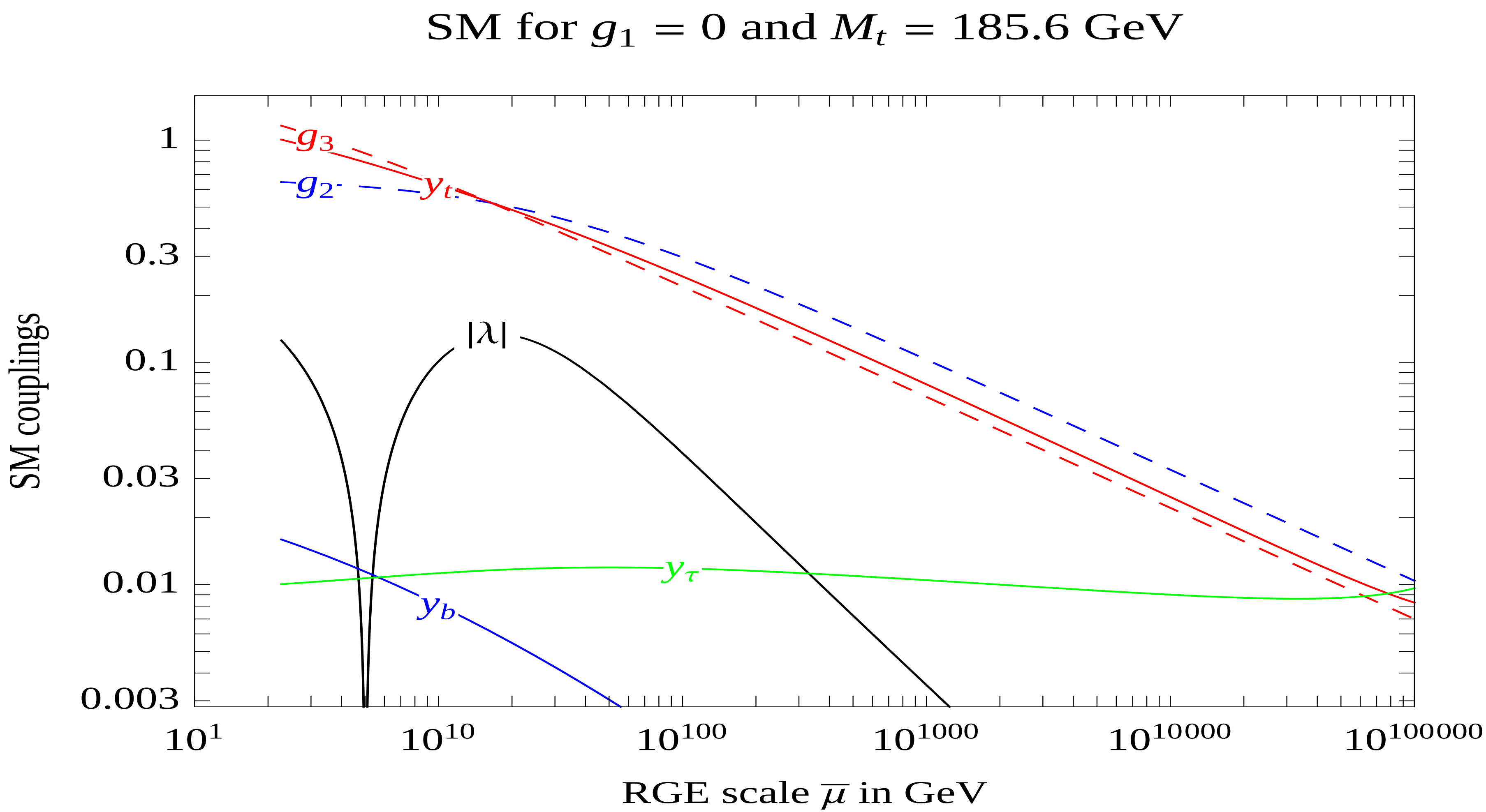
Linearize around each fixed-point:

$$V_I(x) \simeq \sum_J M_{IJ}(x_J - x_{J\infty}) \qquad \text{where} \qquad M_{IJ} = \left.\frac{\partial V_I}{\partial x_J}\right|_{x=x_\infty}$$

Negative eigenvalues of M are UV-attractive. Each positive eigenvalue implies a UV-repulsive direction: to reach the FP a coupling is univocally **predicted**.

SM up to infinite energy if $g_Y = 0$

Predictions: 1) $g_Y = 0$; in this limit 2) $y_t^2 \simeq 227/1197t$ i.e. $M_t = 186\,\mathrm{GeV}$; 3) $y_{\tau,\nu} = 0$; 4) $\lambda \simeq (-143 \pm \sqrt{119402})/4788t$ i.e. $M_h \leq 163\,\mathrm{GeV}$. Equality avoids $\lambda < 0$ at large energy, and too fast vacuum decay $\lambda < -1/12t$.



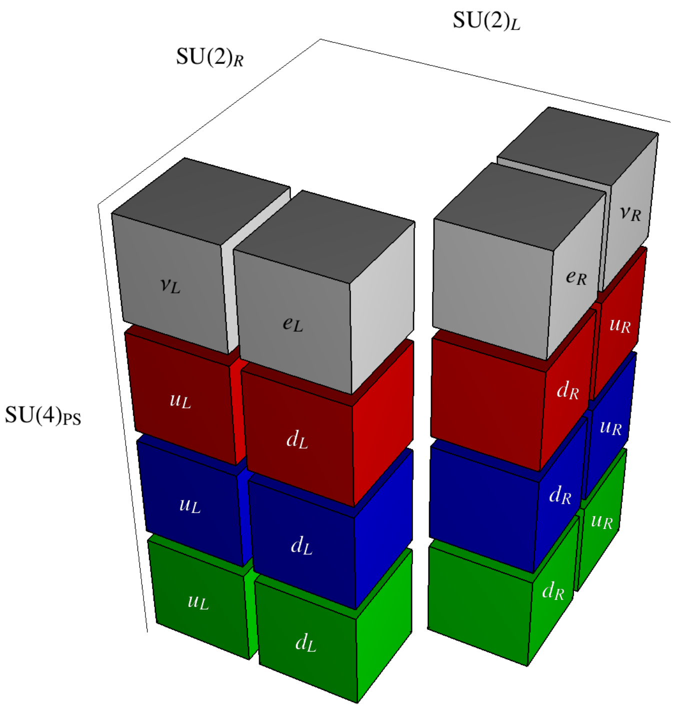
TAF extensions of the SM

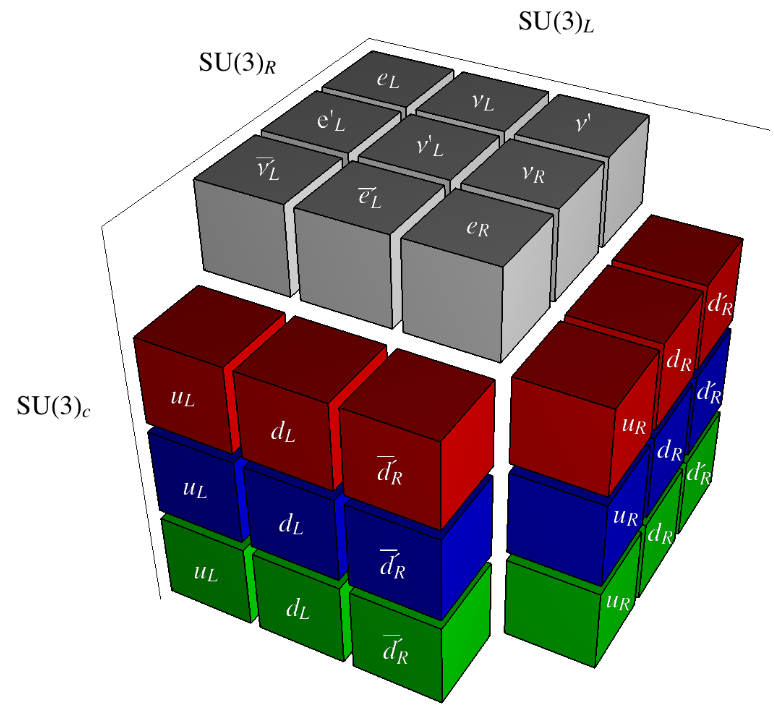
Can the SM be extended into a theory valid up to infinite energy?

Avoid Landau poles by making hypercharge non abelian.

We found realistic SU(5) TAF models. But GUTs are not compatible with finite naturalness, that demands a TAF extension at the weak scale. Making sense of $Y = T_{3R} + (B-L)/2$ needs $\mathrm{SU}(2)_R$. We see 2 possibilities:

$\mathrm{SU}(4)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ and $\mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$





Generic signals of natural TAF

- A W_R boson and a Z'_{B-L}: $M_{W_R} \gtrsim 2.2\,\text{TeV}$, $M_{Z'_{B-L}} > 2.6_{333}, 3.8_{224}\,\text{TeV}$

$$\delta M_h^2 = -\frac{9 g_R^2 M_{W_R}^2}{(4\pi)^2} \ln\left(\frac{M_{W_R}^2}{\bar{\mu}^2}\right) \approx M_h^2 \left(\frac{M_{W_R}}{2.5\,\text{TeV}}\right)^2$$

- The Higgs $(2_L, \bar{2}_R)$ contains 2 doublets coupled to u and d: new flavour violations controlled by a right-handed CKM matrix.

$$M_H > \begin{cases} 18\,\text{TeV} & \text{if } V_R = V_{\text{CKM}} \\ 3\,\text{TeV} & \text{if } V_R^{ij} = V_{\text{CKM}}^{ij} \times \min(m_i, m_j)/\max(m_i, m_j) \text{ (natural texture)} \end{cases}$$

- A lighter singlet that mixes with the higgs if $G_{\text{TAF}} \to G_{\text{SM}}$ dynamically.

- And TAF is tough: we still have to find models where y, λ obey TAF

Pati-Salam

Fields	spin	generations	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_R$	$\mathrm{SU}(4)_{\mathrm{PS}}$
$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
ϕ_R	0	1	1	2	$\bar{4}$
$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	2	2	$\bar{2}$	1
ψ	1/2	1, 2, 3	2	$\bar{2}$	1
Q_L	1/2	2	1	1	10
Q_R	1/2	2	1	1	$\overline{10}$
Σ	0	1	1	1	15

No extra chiral fermions. Two ways to get acceptable fermions masses:

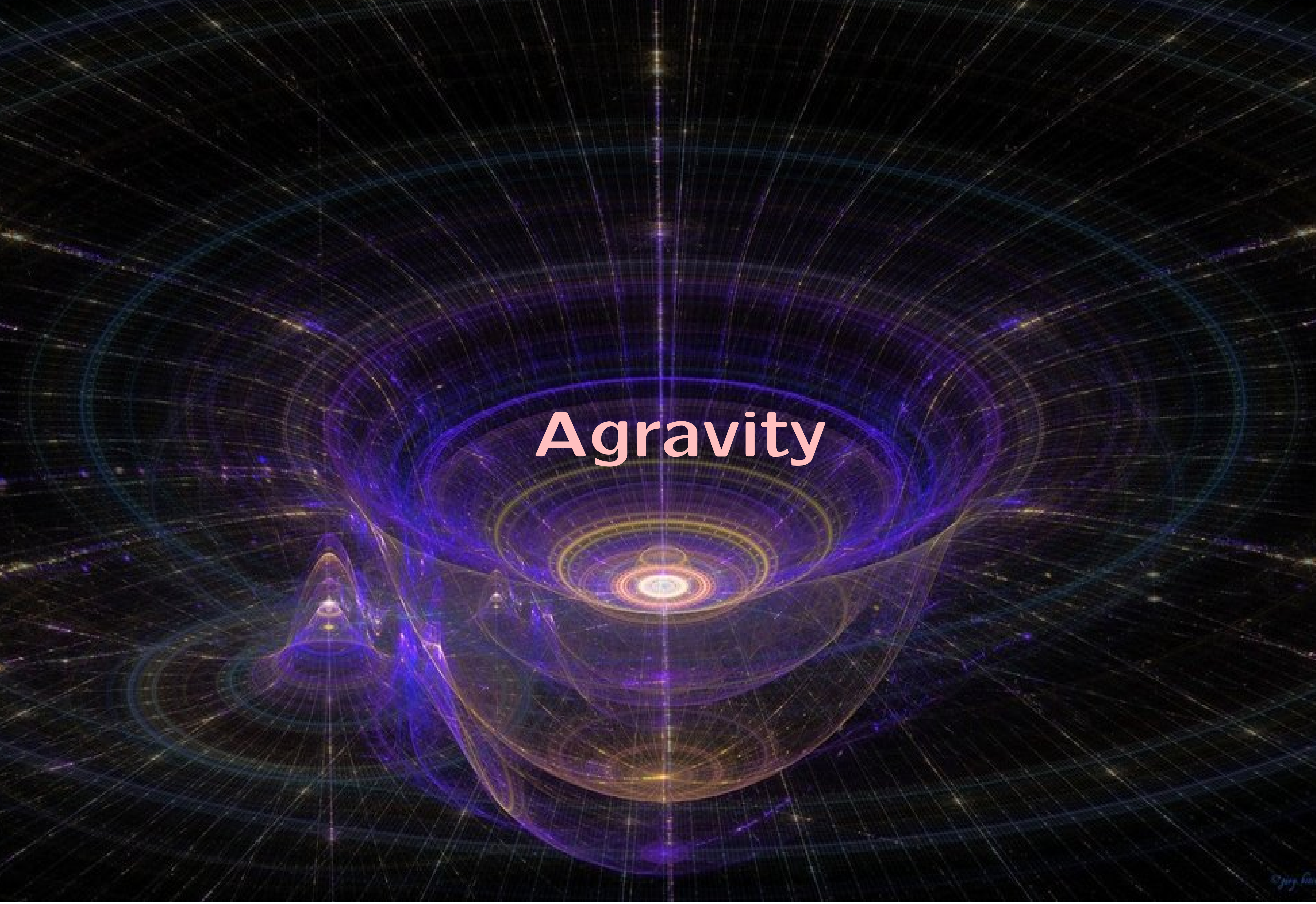
1) Foot: add ψ and ϕ_L: $-\mathscr{L}_Y = Y_N\,\psi_L\psi\phi_R + Y_L\,\psi\psi_R\phi_L + Y_U\psi_R\psi_L\phi + Y_D\,\psi_R\psi_L\phi^c$. Avoids ℓ_L/d_L unification so $M_{W'} > 8.8\,\mathrm{TeV}$. No TAF found for the 24 quartics.

2) Volkas: add $Q_{L,R}$ getting d_R mixing. Strong flavor bounds $M_{W'} > 100\,\mathrm{TeV}$ because of ℓ_L/d_L unification. **Unnatural**. TAF found adding Σ.

Trinification

Minimal weak-scale trinification model					
Matter fields	gen.s	spin	SU(3)$_L$	SU(3)$_R$	SU(3)$_c$
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	3	1/2	1	3	$\bar{3}$
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^1 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	3	1/2	$\bar{3}$	1	3
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	3	1/2	3	$\bar{3}$	1
$\langle H \rangle = \begin{pmatrix} v_u & 0 & 0 \\ 0 & v_d & v_L \\ 0 & V_R & V \end{pmatrix}$	3	0	3	$\bar{3}$	1

- Explains quantisation of Y. Needs $g_R = 2g_2 g_Y/\sqrt{3g_2^2 - g_Y^2} \approx 0.65 g_2$.
- No bad vectors: $V \approx$ few TeV allowed.
- Extra d', e', ν' fermions chiral under SU(3)^3 get mass $\sim yV$ from Yukawas $y_Q\, Q_L Q_R H + \frac{1}{2} y_L^n LLH^*$. $3H$ are needed to make d', e', ν' naturally heavy.
- TAF solutions found for H_1, H_2 (20 quartics) and for H_1, H_2, H_3 (90 λ).


Agravity

What about gravity?

Does quantum gravity give $\delta M_h^2 \sim M_{\mathsf{Pl}}^2$ ruining Physical Naturalness?

Yes in string models, where lots of new coupled particles exists around M_{Pl}.

Maybe M_{Pl}^{-1} is just a small coupling and there are no new particles around M_{Pl}.

Adimensional gravity

Applying the adimensional principle to the SM plus gravity and a scalar S gives:

$$\mathscr{S} = \int d^4x \ \sqrt{|\det g|}\, \mathscr{L}$$

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{R^2}{3f_0^2} + \frac{R^2 - 3R_{\mu\nu}^2}{3f_2^2} + |D_\mu S|^2 - \xi_S |S|^2 R - \lambda_S |S|^4 + \lambda_{HS} |HS|^2$$

where f_0, f_2 are the adimensional 'gauge couplings' of gravity and $R \sim \partial_\mu \partial_\nu g_{\mu\nu}$.

The theory is power-counting **renormalizable**, and the graviton propagator is:

$$\frac{-i}{k^4}\left[2f_2^2 P^{(\text{spin } 2)}_{\mu\nu\rho\sigma} - f_0^2 P^{(\text{spin } 0)}_{\mu\nu\rho\sigma} + \text{gauge-fixing}\right].$$

The Planck scale should be generated dynamically as $\xi_S \langle S\rangle^2 = \bar{M}_{\rm Pl}^2/2$.

Then, the spin-0 part of $g_{\mu\nu}$ gets a mass $M_0 \sim f_0 M_{\rm Pl}$ and the spin 2 part splits into the usual graviton and an anti-graviton with mass $M_2 = f_2 \bar{M}_{\rm Pl}/\sqrt{2}$ that acts as a Pauli-Villars in view its negative kinetic term [Stelle, 1977].

A ghost?

A ghost?

In presence of masses, ∂^4 can be decomposed as 2 fields with 2 derivatives:

$$\frac{1}{k^4} \to \frac{1}{k^4 - M_2^2 k^2} = \frac{1}{M_2^2}\left[\frac{1}{k^2} - \frac{1}{k^2 - M_2^2}\right]$$

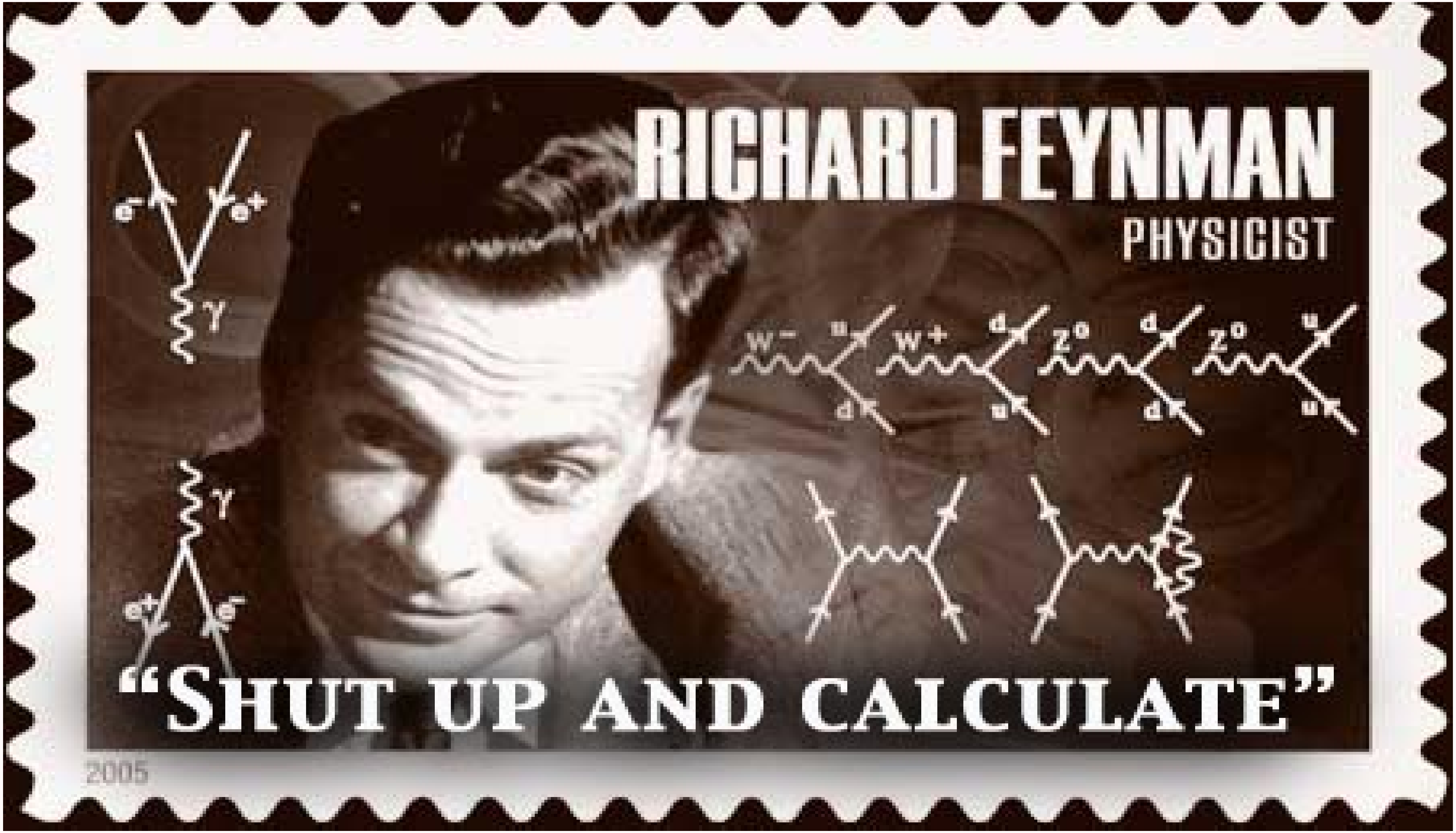
Ostrogradski showed in 1850 that higher derivatives are always bad:

$\partial^4 \Rightarrow$ unbounded **negative energy** $\Rightarrow$ the **classical** theory is dead.

Who cares, nature is quantum. ∂^4 can be quantized as:
i) negative energy, or as ii) **negative norm and positive energy**.
This is the $\pm i\epsilon$ choice that makes agravity renormalizable.

For the moment, let's ignore the issue and compute. Anti-particles teach us that sometimes we get the right equations before understanding their meaning.

A ghost?



One loop RGE in agravity...

The quantum behaviour of a renormalizable theory is encoded in its RGE.

- f_2 is asymptotically free:

$$(4\pi)^2\frac{df_2^2}{d\ln\mu} = -f_2^4\left[\frac{133}{10}+\frac{N_V}{5}+\frac{N_f}{20}+\frac{N_s}{60}\right]$$

- f_0 grows with energy

$$(4\pi)^2\frac{df_0^2}{d\ln\mu} = \frac{5}{3}f_2^4+5f_2^2f_0^2+\frac{5}{6}f_0^4+\frac{f_0^4}{12}\sum_s(1+6\xi_s)^2$$

- Gauge couplings are unaffected.

- Yukawa couplings get an extra multiplicative RGE correction:

$$(4\pi)^2\frac{dy_t}{d\ln\mu} = \frac{9}{2}y_t^3 - y_t(8g_3^2 - \frac{15}{8}f_2^2)$$

...One loop RGE in agravity

- Quartics get smaller at low energy:

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} = \xi_H^2[5f_2^4 + f_0^4(1+6\xi_H)^2] - 6y_t^4 + \frac{9}{8}g_2^4 + \cdots$$

- Mixed quartics are unavoidably created:

$$(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} = \frac{\xi_H\xi_S}{2}[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)] + \text{multiplicative}$$

- f_0 appears at the denominator if the RGE for ξ-couplings:

$$(4\pi)^2 \frac{d\xi_H}{d\ln\mu} = -\frac{5}{3}\frac{f_2^4}{f_0{}^2}\xi_H + f_0^2\xi_H(6\xi_H+1)(\xi_H+\frac{2}{3}) + (6\xi_H+1)\left[2y_t^2 - \frac{3}{4}g_2^2 + \cdots\right]$$

RGE simplify and couplings at the denominator disappear using

$$\tilde\lambda_\sigma \equiv f_0^2, \qquad \tilde\lambda_{H\sigma} \equiv f_0^2(\xi_H + \frac{1}{6}), \qquad \tilde\lambda_H \equiv \lambda_H + \frac{3}{8}f_0^2(\xi_H+\frac{1}{6})^2$$

that are the quartic couplings involving the conformal mode of the agraviton σ in a perturbatively equivalent formulation.

Relation with conformal gravity

Conformal gravity is agravity for $f_0 = \infty$, $\xi = -1/6$. New local scale symmetry

$$g_{\mu\nu} \to e^{2\sigma(x)} g_{\mu\nu}, \quad H \to e^{-\sigma(x)} H, \quad \psi \to e^{-3\sigma(x)/2}\psi, \qquad V_\mu \to V_\mu$$

Conformal gravity is not a complete theory: the scale (Weyl) invariance is broken by running of couplings, at multi-loop level anomalies give back agravity

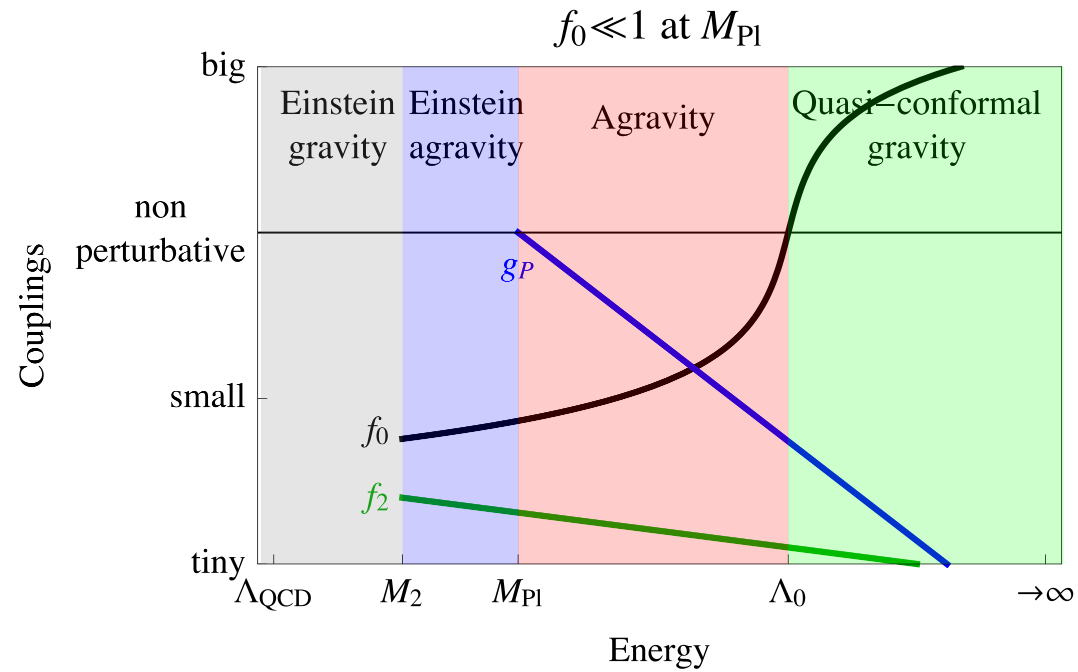
$$\begin{aligned}
\lim_{f_0\to\infty} \frac{d}{d\ln\bar{\mu}}\frac{1}{f_0^2} &= -\frac{665 g_2^6}{216(4\pi)^8} + \frac{728 g_3^6}{9(4\pi)^8} + \frac{416\lambda_H^5}{5(4\pi)^{12}} + \cdots \\
\lim_{f_0\to\infty} \frac{d}{d\ln\bar{\mu}}(\xi_H + \frac{1}{6}) &= 48\frac{\lambda_H^4}{(4\pi)^8} + \cdots
\end{aligned}$$

Up to infinite energy

Agravity can flow to **conformal gravity** at infinite energy.

Reasonably solid claim:

f_0 grows until the conformal mode σ of the agraviton, $g_{\mu\nu} = e^{2\sigma}\eta_{\mu\nu}$, gets strongly self-coupled, and decouples from other particles if $\xi \to -1/6$. The strongly coupled sector is



$$\int d^4x \sqrt{|\det g|}\frac{R^2}{6f_0^2} = \frac{6}{f_0^2}\int d^4x \left[\Box\sigma + (\partial\sigma)^2\right]^2 \overset{f_0\gg 1}{=} \frac{6}{f_0^2}\int d^4x\, (\partial\sigma)^4$$

because σ fluctuates wildly at $f_0 \gg 1$. Conformal and shift symmetries imply that $(\partial\sigma)^4$ is a 'free' theory, so $\beta(f_0) \overset{f_0\to\infty}{\simeq} 1/f_0^2$: **no Landau pole**, f_0 grows up to ∞ where σ becomes a Weyl gauge redundancy.

Generation of $M_{\rm Pl}$

Mechanisms that can generate dynamically the Planck scale:

Non-perturbative: Some coupling g runs non-perturbative at $M_{\rm Pl}$
Perturbative: Some quartic λ_S runs negative at $M_{\rm Pl}$

A non-perturbative model:
$G_{\rm SM} \otimes G$ with one extra fermion in the $(0_Y, 1_L, 1_c, \text{adj})$.

$\langle\lambda\lambda\rangle$ induces $\pm M_{\rm Pl}^2$: its sign depends on the (uncomputable?) strong dynamics.

No cosmological constant at order $V \sim -M_{\rm Pl}^4$ because of accidental SUSY of strong dynamics.

Generation of $M_{\rm Pl}$: perturbative

Add a scalar Planckion s with quantum potential $V(s) \approx \frac{1}{4}\lambda_S(\bar\mu \sim s)s^4$.
The gravitational coupling ξ_S makes the vacuum equation non-standard:

$$\frac{\partial V}{\partial s} - \frac{4V}{s} = 0 \qquad \text{i.e.} \qquad \frac{\partial V_E}{\partial s} = 0$$

Usual Coleman-Weinberg recovered in terms of the Einstein-frame potential:

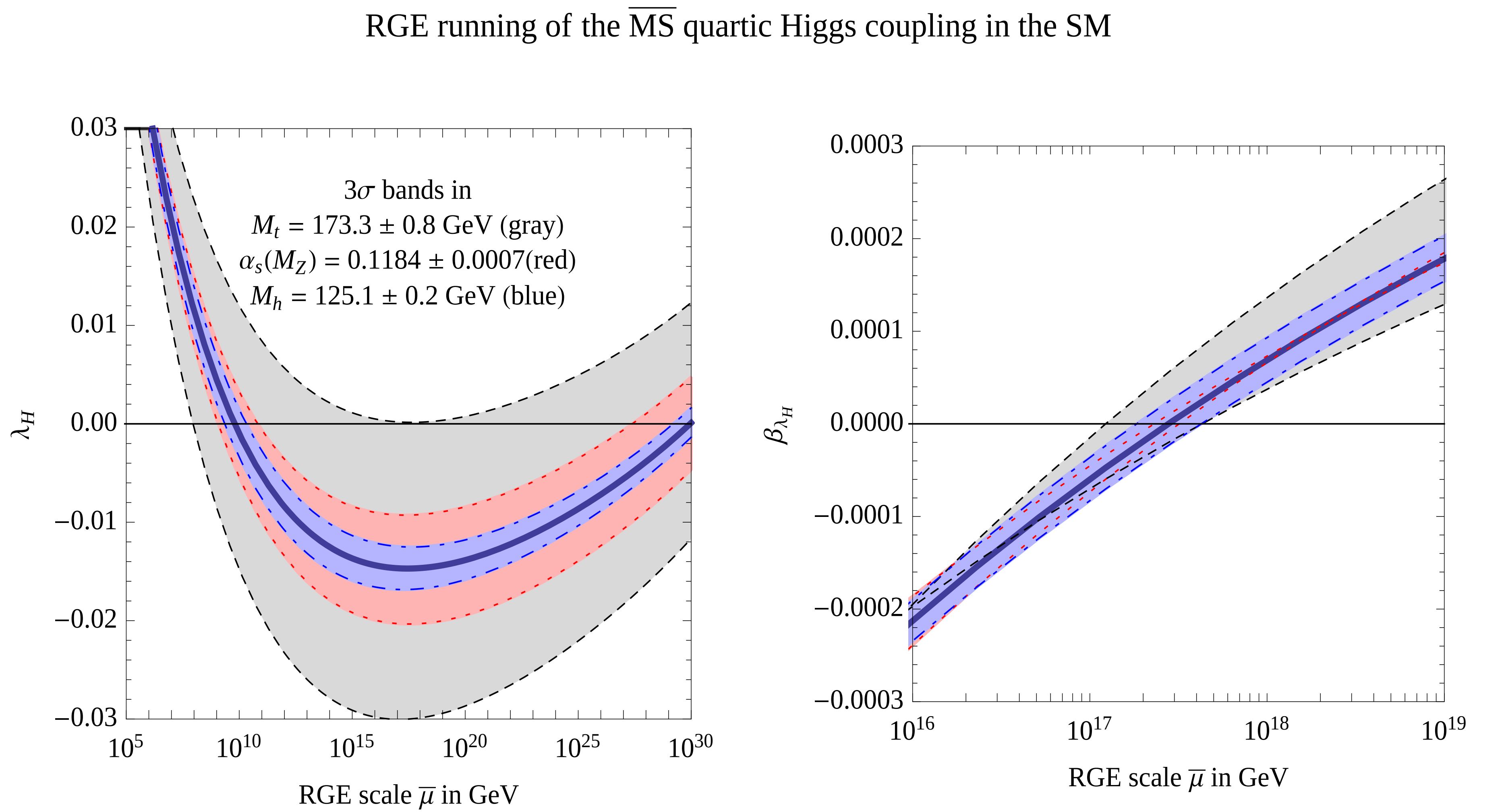
$$V_E = \frac{V}{(\xi_S s^2)^2} \sim \frac{\lambda_S(s)}{\xi_S^2(s)} \qquad \frac{\partial V_E}{\partial s} \propto \frac{\beta_{\lambda_S}(\bar\mu \sim \langle S\rangle)}{\lambda_S(\bar\mu \sim \langle S\rangle)} - 2\frac{\beta_{\xi_S}(\bar\mu \sim \langle S\rangle)}{\xi_S(\bar\mu \sim \langle S\rangle)} = 0$$

Needed running: $\lambda_S(\bar\mu)$ must kiss 0

$$\begin{cases} \lambda_S(\langle s\rangle) = 0 & \text{vanishing cosmological constant} \\ \beta_{\lambda_S}(\langle s\rangle) = 0 & \text{minimum at } \langle s\rangle = \bar M_{\rm Pl}/\sqrt{\xi_S} \end{cases}$$

Is this fine-tuned running possible?

This is how λ_H runs in the SM



We do not live in the $h \sim 10^{17.5}$ GeV minimum. Another scalar needed: a SM mirror, or something else with gauge and Yukawa interactions.

Generation of the Weak scale

RGE running from the ghost mass $M_{0,2}$ to $M_{\rm Pl}$:

$$(4\pi)^2 \frac{dM_h^2}{d\ln\bar\mu} = -\xi_H \left[5 f_2^4 + f_0^4(1+6\xi_H)\right] \bar M_{\rm Pl}^2 + \cdots$$

The weak scale arises if $f_{0,2} \sim \sqrt{M_h/M_{\rm Pl}} \sim 10^{-8}$ i.e. $M_{0,2} \sim 10^{11}\,\mathrm{GeV}$

All small parameters such as $f_{0,2}$ and $\lambda_{HS} \sim f_{0,2}^4$ are naturally small

Non-perturbative quantum gravity

Einstein gravity becomes strongly-coupled at $M_{\rm Pl}$. Black holes with mass $M_{\rm BH} \sim M_{\rm Pl}$ can give unnaturally large non-perturbative corrections to M_h:

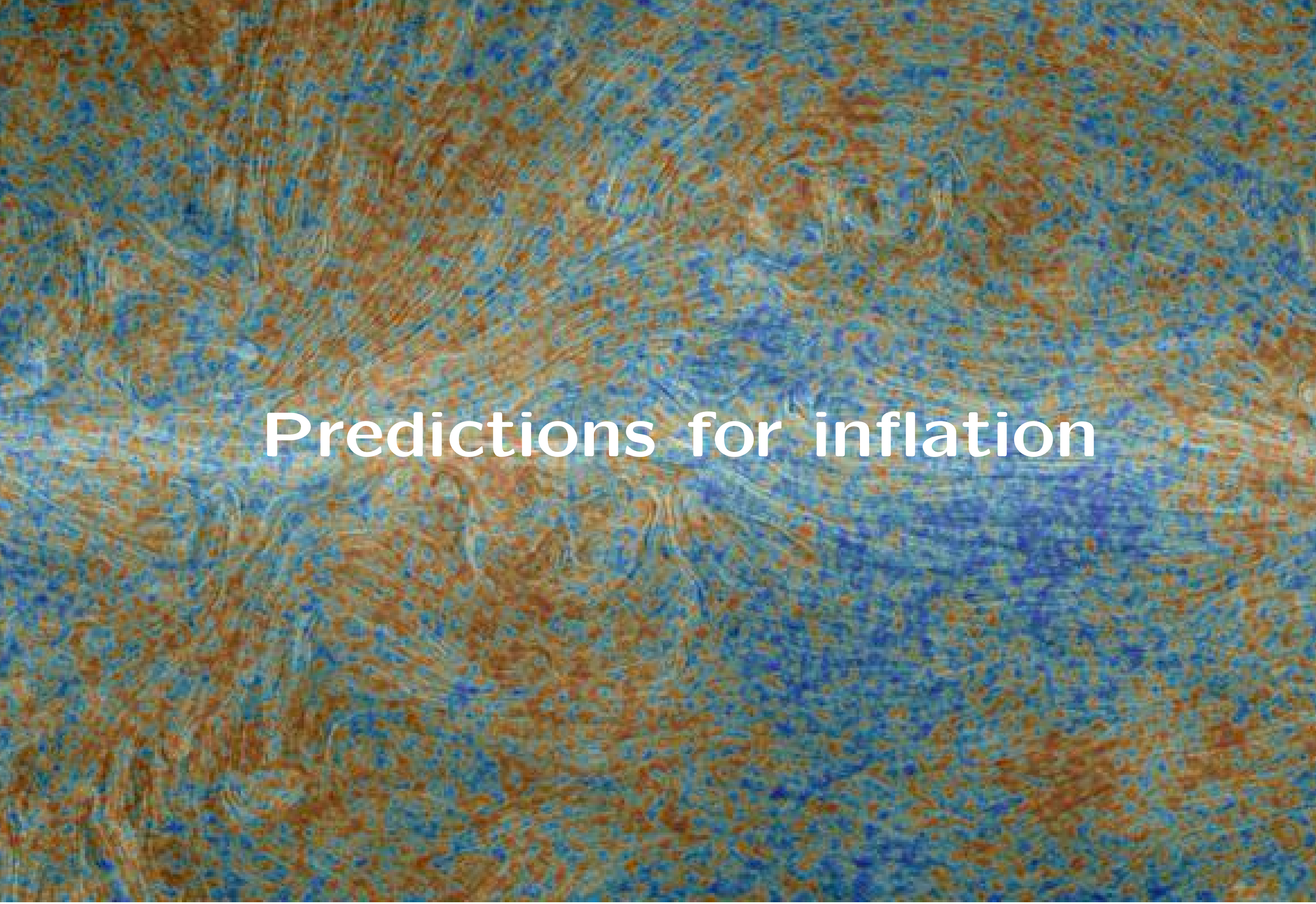
$$\delta M_h^2 \sim \int M_{\rm BH}^2 \, e^{-S}, \qquad S = 4\pi \frac{M_{\rm BH}^2}{M_{\rm Pl}^2} \sim \frac{4\pi}{g_{\rm grav}^2}.$$

In agravity $g_{\rm grav} \to f_{0,2} \lesssim 10^{-8}$, so non-perturbative effects should be negligible. Indeed states with $M_{\rm BH} \lesssim M_{\rm Pl}/f_{0,2}$ get modified by an healtier

$$V_{\rm Newton} = -\frac{GM}{r}\left[1 - \frac{4}{3}e^{-M_2 r} + \frac{1}{3}e^{-M_0 r}\right]$$

Light Schwarzschild BH are unstable.

Predictions for inflation



Inflation = perturbative agravity

Inflation is not a generic phenomenon: one needs to flatten potentials or justify hilltop initial conditions or consider super-Planckian field variations, which are forbidden in string theory where the scalar field space is compact with $M_{\rm Pl}$ size.

Inflation is a generic phenomenon in agravity: V is flat in Planck units if all M and $M_{\rm Pl}$ come from $\langle$scalars$\rangle$. The slow-roll parameters are given by the β-functions, which are small if the theory is perturbative. E.g.

$$\epsilon = \frac{1}{2}\frac{\xi_S}{1+6\xi_S}\left[\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right]^2,$$

More technically

Consider a generic inflaton s

$$\mathscr{L} = \sqrt{\det g}\left[-f(s)\frac{R}{2} + \frac{(\partial_\mu s)^2}{2} - V(s) + \cdots\right]$$

Make gravity canonical via a Weyl transformation $g_{\mu\nu} = g^E_{\mu\nu} \times \bar{M}^2_{\rm Pl}/f$:

$$\mathscr{L} = \sqrt{\det g_E}\left[-\frac{\bar{M}^2_{\rm Pl}}{2}R_E + \underbrace{\bar{M}^2_{\rm Pl}\left(\frac{1}{f} + \frac{3f'^2}{2f^2}\right)\frac{(\partial_\mu s)^2}{2}}_{\text{If desired make } s \text{ canonical}} - V_E + \cdots\right]$$

where $V_E = \bar{M}^4_{\rm Pl} V/f^2$ is the Einstein-frame potential. If V and f are generic functions, V_E is generic: ad hoc assumptions were invoked to make V_E flat.

In quantum agravity $f = \xi_S(\bar{\mu} \sim s)s^2$ and $V = \frac{1}{4}\lambda_S(\bar{\mu} \sim s)s^4$

So $V_E = \frac{1}{4}\bar{M}^4_{\rm Pl}\lambda_S(s)/\xi_S(s)^2$ is quasi-flat, even above $M_{\rm Pl}$.

Inflaton candidates in agravity

In agravity all scalars can be inflatons, and there are at least 3 scalars:

s The scalar 'Planckion' that breaks scale invariance generating $M_{\rm Pl}$.
It can be light, being the pseudo-Goldstone boson of scale invariance:

$$M_s \sim g_s^2 M_{\rm Pl}/(4\pi)^2$$

If it is the inflation one has $n_s \approx 0.967$ and $r \approx 0.13$.

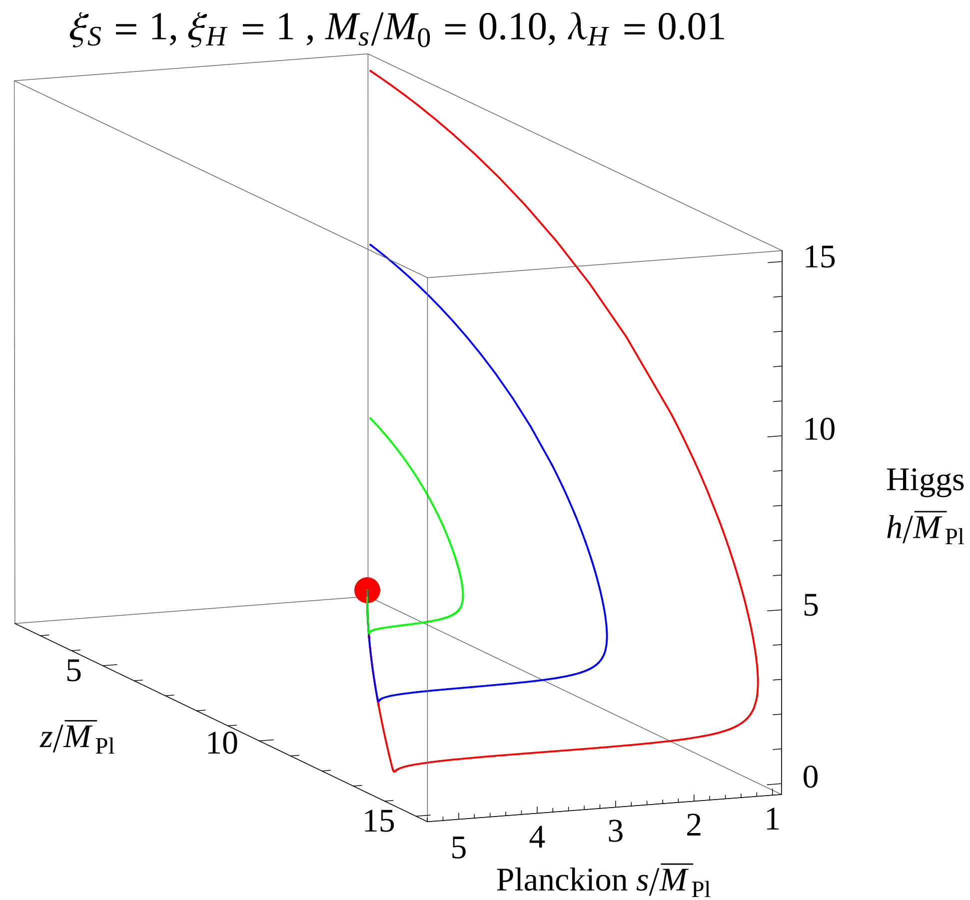
z The scalar component of the graviton, $M_0 \sim f_0 M_{\rm Pl}$.
If it is the inflaton one has Starobinski inflation: $n_s \approx 0.967$ and $r \approx 0.003$.

h The Higgs.
If it is the inflation one has Higgs inflation: $n_s \approx 0.967$ and $r \approx 0.003$?

For the moment we ignore the spin 2 ghost.

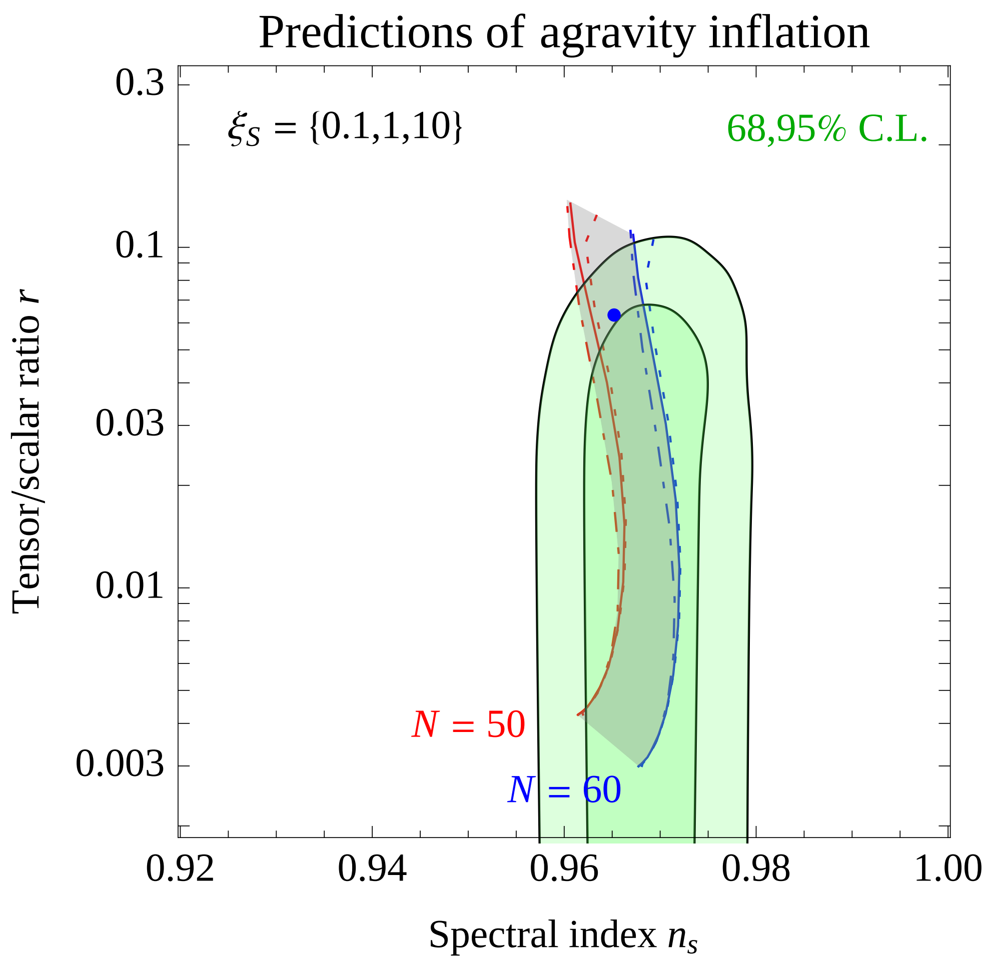
Who is the inflaton?

Predictions might depend on the initial condition. We find that, whatever is the starting point, slow-roll converges towards a **unique attractor solution**, probably because a dimensionless-potential has $V'' \sim \lambda\,$field^2.



The Higgs is never relevant because of its large λ_H.

Predictions for inflation



$$P_R \sim M_h/\bar{M}_{\rm Pl}$$

Any super-Planckian theory gives inflation, but don't explain $P_R \sim 10^{-9} \ll 1$.

Agravity relates the smallness of the amplitude of inflationary perturbations P_R to the smallness of $M_h/\bar{M}_{\rm Pl}$, up to couplings and loops and powers of $N \approx 60$.

Consider ainflation in the Starobinski limit:

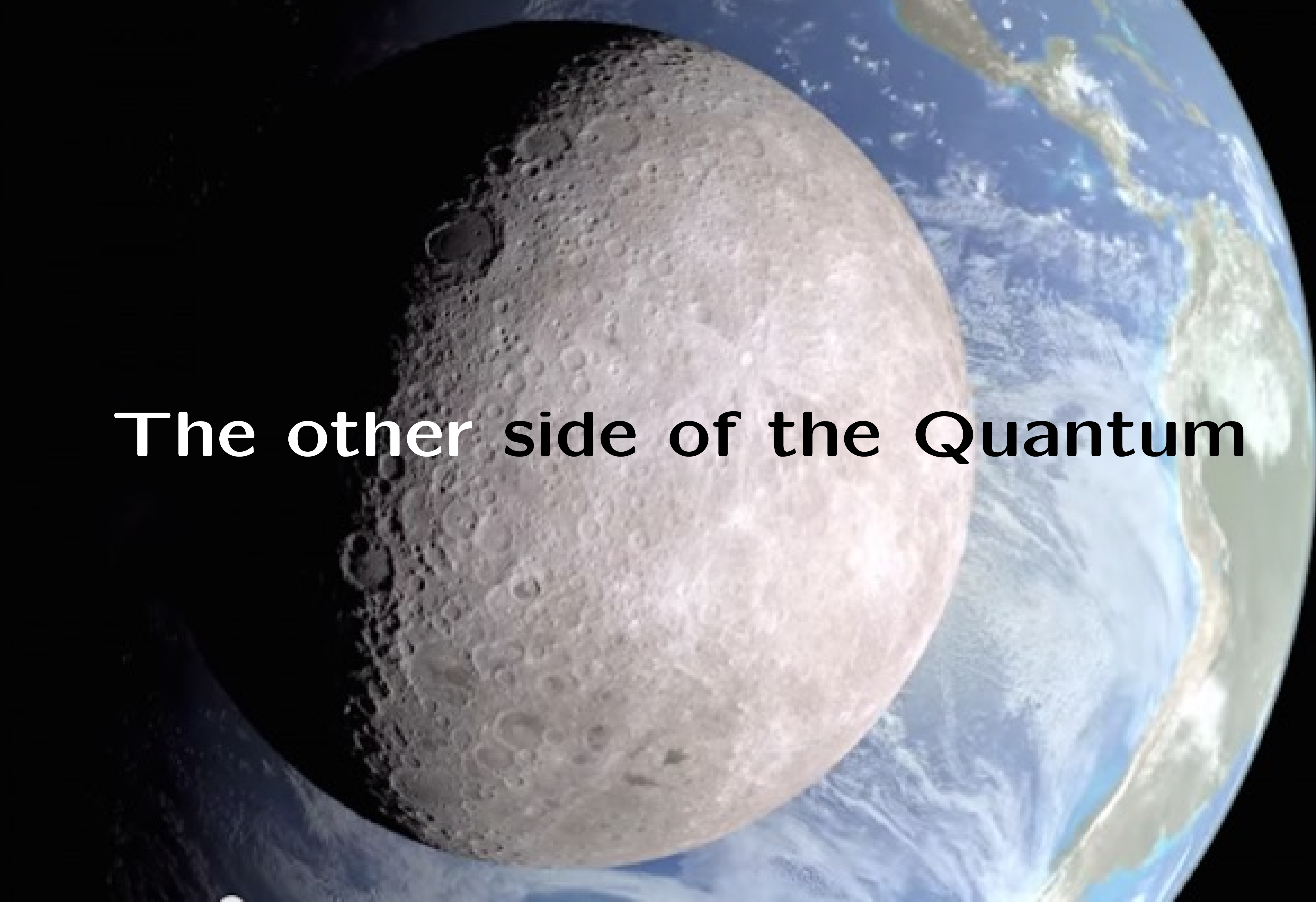
$$P_R = \frac{f_0^2 N^2}{48\pi^2} \qquad \text{i.e.} \qquad f_0 = 1.8\ 10^{-5}.$$

The quantum correction to the Higgs mass is dominated by the RGE:

$$\frac{dM_h^2}{d\ln\bar{\mu}} = -\xi_H(1+6\xi_H)f_0^4\bar{M}_{\rm Pl}^2 + \cdots$$

So finite naturalness demands $f_0 \lesssim 10^{-5-8}$ (at tree-loop level).

In minimal models, the two values of f_0 are compatible if ξ is close to 0 or $-\frac{1}{6}$.


The other side of the Quantum

Quantisation of 4-derivative systems

Quantisation was first understood for spin 1 and 0 particles with **2 derivatives**.

4 derivative systems have a problem: negative (indefinite) classical H.

Spin 1/2 fields have **1 derivative**. $\mathscr{L} = \bar{\Psi}[i\partial\!\!\!/ - m]\Psi$ classically leads to negative

$$H = \int \frac{d^3p}{(2\pi)^3} E_p[a^\dagger_{p,s} a_{p,s} - b_{p,s} b^\dagger_{p,s}].$$

Quantisation allows positive energy. The two-state solution to $\{b, b^\dagger\} = 1$ shows that one can redefine b into $\tilde{b}^\dagger$ by choosing $|1\rangle$ to have lower energy than $|0\rangle$:

$$b = \begin{array}{c} \\ \langle 0| \\ \langle 1| \end{array}\!\!\begin{array}{c} \begin{array}{cc} |0\rangle & |1\rangle \end{array} \\ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{array} = \begin{array}{c} \\ \langle 1| \\ \langle 0| \end{array}\!\!\begin{array}{c} \begin{array}{cc} |1\rangle & |0\rangle \end{array} \\ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{array} = \tilde{b}^\dagger.$$

'Ghosts' are avoided like a plague by serious theorists and explored only by crackpots such as Dirac, Pauli, Heisenberg, Pais and Uhlenbeck, Lee, Wick and Cutkosky, Coleman, Feynman, Boulware and Gross, Hawking and Hertog...

Ostrogradski classical no go

Gravity $g_{\mu\nu}(x,t) \approx$ QFT $\phi(x,t) \approx \int_p$ harmonic oscillators in QM... so

Focus on a single mode $q(t)$ with 4 time derivatives

$$\mathscr{L} = -\frac{1}{2} q (\frac{d^2}{dt^2} + \omega_1^2)(\frac{d^2}{dt^2} + \omega_2^2) q - V(q)$$

Describe as $2+2$ canonical form using the auxiliary coordinate $q_2 = \dot{q}$:

$$\begin{cases} q_1 = q, & p_1 = \dfrac{\delta S}{\delta \dot{q}_1} = (\omega_1^2 + \omega_2^2)\dot{q} + \dddot{q}, \\ q_2 = \dot{q}, & p_2 = \dfrac{\delta S}{\delta \dot{q}_2} = -\ddot{q} \end{cases}$$

The Hamiltonian is unbounded from below

$$H = \sum_{i=1}^{2} p_i \dot{q}_i - \mathscr{L} = p_1 q_2 - \frac{\lambda^2}{2} p_2^2 - \frac{\omega_1^2 + \omega_2^2}{2} q_2^2 + \frac{\omega_1^2 \omega_2^2}{2} q_1^2 + V(q_1).$$

Quantization

The classical free solution is ok:

$$q(t) = \frac{a_1 e^{-i\omega_1 t}}{\sqrt{2\omega_1(\omega_1^2-\omega_2^2)}} + \frac{a_2 e^{-i\omega_2 t}}{\sqrt{2\omega_2(\omega_1^2-\omega_2^2)}} + \text{h.c.}$$

Adding interactions *some* classical solutions run away. Presumably dead.

Usual quantisation $a_1^\dagger|\tilde{0}\rangle = 0$ and $a_2|\tilde{0}\rangle = 0$ gives negative energy. Dead?

Alternative quantization $a_{1,2}|0\rangle = 0$ give negative norms

$$[a, a^\dagger] = -1 \qquad |E_k\rangle = \frac{(a^\dagger)^k}{\sqrt{k!}}|0\rangle \qquad \langle E_{k'}|E_k\rangle = (-1)^k \delta_{kk'}$$

and positive H eigenvalues

$$H = -\frac{p^2+q^2}{2} = -\frac{aa^\dagger + a^\dagger a}{2} \qquad H|E_k\rangle = (k+\frac{1}{2})|E_k\rangle$$

so no run-away in transition amplitudes: $\int dt\, e^{-i(E_i-E_f)t} \to \delta(E_i - E_f)$.

But literature claims that wave-functions are non-normalizable

$$\psi_0(q_1,q_2) \propto \exp\left(\frac{-q_1^2\omega_1\omega_2 + q_2^2}{2}(\omega_1+\omega_2) - iq_1q_2\omega_1\omega_2\right).$$

Mistake: ψ computed using $q|x\rangle = x|x\rangle$ i.e. positive norm $\langle x'|x'\rangle = \delta(x-x')$.

Pauli-Dirac coordinate representation

Indefinite-norm coordinate representation

$$\hat{q}|x\rangle = ix|x\rangle, \qquad \hat{p}|x\rangle = +\frac{d}{dx}|x\rangle.$$

$\hat{q}$ and $\hat{p}$ are self-adjoint with respect to the indefinite norm $\langle x'|x\rangle = \delta(x' + x)$:

$$\langle x'|\hat{q}^\dagger|x\rangle \equiv \langle x|\hat{q}|x'\rangle^* = [ix'\delta(x+x')]^* = ix\delta(x+x') = \langle x'|q|x\rangle$$

For real interacting H, time evolution e^{-iHt} conserves the indefinite norm

Anti-symmetric $\psi(x)$ (odd levels of harmonic oscillator) have negative norm

$$\langle\psi'|\psi\rangle = \int dx\, \psi'^*(x)\psi(-x)$$

Ground state $\psi_0 \propto e^{-x^2/2}$, obtained solving $\langle x|a|0\rangle = 0$ with $a = (q+ip)/\sqrt{2}$:

Wave functions are normalizable

Pauli-Dirac is demanded by 4-derivative

A pair of canonical coordinates $[q, p] = i$ admits two coordinate representations:

proposed by	$\langle x\|\widehat{q}\|\psi\rangle$	T-parity	$\langle x\|\widehat{p}\|\psi\rangle$	T-parity	norm $\langle x'\|x\rangle$
Schroedinger	$x\psi(x)$	even	$-i\, d\psi/dx$	odd	$\delta(x - x')$, positive
Dirac-Pauli	$-ix\psi(x)$	odd	$d\psi/dx$	even	$\delta(x + x')$, indefinite

In both cases, q, p, H are self-adjoint and the eigenvalues of H are positive.

A 4-derivative $q(t)$ is canonically rewritten as two 2-derivative $q_1 = q$ and $q_2 = \dot{q}$.
T-even q implies T-odd $\dot{q}$, which *must* follow the indefinite norm quantisation.

$$\widehat{q}_1|q_1, q_2\rangle = q_1|q_1, q_2\rangle \qquad \widehat{p}_1|q_1, q_2\rangle = i\frac{\partial}{\partial q_1}|q_1, q_2\rangle$$

$$\widehat{q}_2|q_1, q_2\rangle = iq_2|q_1, q_2\rangle \qquad \widehat{p}_2|q_1, q_2\rangle = \frac{\partial}{\partial q_2}|q_1, q_2\rangle$$

Path integral

This gives a healthy quantum-like theory. Its transition amplitudes are naive continuation of the Euclidean path integral

$$\langle q_f, q'_f, t_{Ef} | q_i, q'_i, t_{Ei} \rangle \propto \int Dq \, \exp\left[- \int dt_E \; \mathscr{L}_E(q) \right].$$

For example ψ_0 is reproduced as

$$\langle q, q', t_E = 0 | 0, 0, t_E = -\infty \rangle \propto \exp\Big[- \frac{q^2 \omega_1 \omega_2 + q'^2}{2} (\omega_1 + \omega_2) + q q' \omega_1 \omega_2 \Big].$$

Probabilistic interpretation?

The deterministic part of QM is extended, what about the interpretation?

Born: "when an observable corresponding to a hermitian operator A is measured in a state $|\psi\rangle$, the result is an eigenvalue A_i with **probability** $p_i = |\langle A_i|\psi\rangle|^2$"

Probability enters if $|\psi\rangle$ is a mixed state. Not if $|\psi\rangle$ is an eigenstate:

"when an observable corresponding to a self-adjoint operator A is measured in an eigenstate $|i\rangle$ of A, the result is the eigenvalue A_i".

Important but unknown: the deterministic statement implies the Born probability.

Probability = certainty about repeated events. $|\boldsymbol{\psi}^{(\boldsymbol{n})}\rangle = |\boldsymbol{\psi}\rangle \otimes \cdots \otimes |\boldsymbol{\psi}\rangle$ **becomes an eigenstate of the repeated measurement** $A^{(n)}$ **in the limit** $n \to \infty$.

The power of powers

n-th power of $|\psi\rangle = \sum_i c_i |A_i\rangle$

$$|\psi^{(n)}\rangle = \sum_{k_1+\cdots+k_N=n} e^{ik_i\delta_i} \sqrt{|c_1^2|^{k_1}\cdots|c_N^2|^{k_N}\binom{n}{k_1\cdots k_N}}\,|A_1^{k_1}\cdots A_N^{k_N}\rangle.$$

Phases $c_i = e^{i\delta_i}|c_i|$ can be rotated away.
Unit norm kets:

$$|A_1^{k_1}\cdots A_N^{k_N}\rangle = \frac{\sum_{\text{perm}} |A_1\rangle^{k_1}\cdots|A_N\rangle^{k_N}}{\sqrt{\binom{n}{k_1\cdots k_N}}}$$

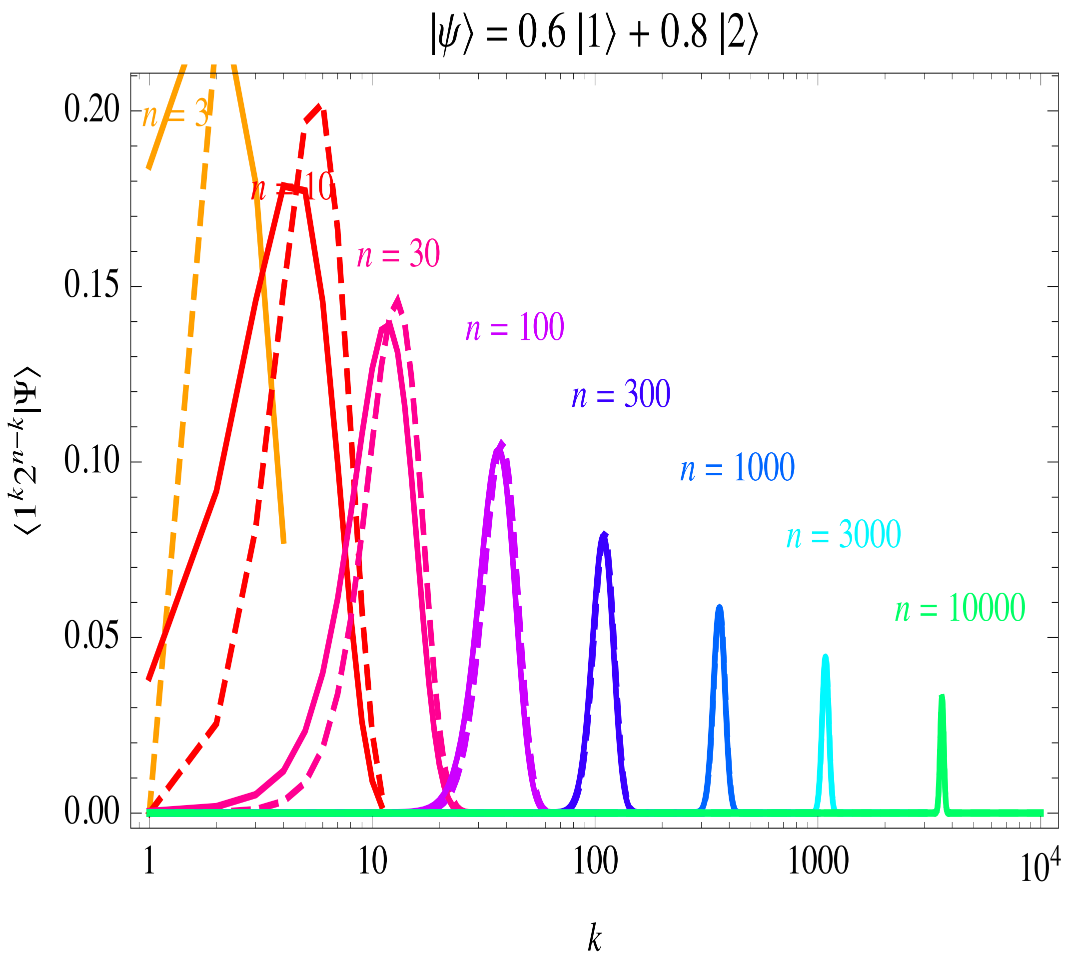
The $\sqrt{\ }$ contains a multinomial with

$$p_i = \frac{|c_i|^2}{\sum_j |c_j^2|}.$$

Large n: tends to a Gaussian with

$$\mu_i = np_i, \qquad \sigma_{ij}^2 = n(p_i\delta_{ij} - p_i p_j).$$

Infinite n: $\delta(k_i - \mu_i)$.



Interpretation of positive norm

Measure averages $A^{(n)}$ such as $A^{(3)} = \frac{1}{3}(A \otimes 1 \otimes 1 + 1 \otimes A \otimes 1 + 1 \otimes 1 \otimes A)$.

Basic observable: projector Π_i over $|A_i\rangle$. $\Pi_i^{(n)}$ counts the rate of $|A_i\rangle$:

$$\Pi_i^{(n)} |A_1^{k_1} \cdots A_N^{k_N}\rangle = \frac{k_i}{n} |A_1^{k_1} \cdots A_N^{k_N}\rangle.$$

$$\Pi_i^{(n)} |\psi^{(n)}\rangle \overset{n\to\infty}{=} p_i |\psi^{(n)}\rangle \qquad A^{(n)} |\psi^{(n)}\rangle \overset{n\to\infty}{=} \sum_i A_i p_i |\psi^{(n)}\rangle$$

- Coefficients of $\Pi_i^{(n)} |\psi^{(n)}\rangle$ projectively converge to those of $p_i |\psi^{(n)}\rangle$.
- Norm of $(\Pi_i^{(n)} - p_i) |\psi^{(n)}\rangle$ projectively converge to 0.

Interpretation of indefinite norm

Take non-degenerate self-adjoint A with eigenvectors $\langle A_i|A_j\rangle = N_i\delta_{ij}$, $N_i = \pm 1$:

- Norm of $(\Pi_i^{(n)} - w_i)|\psi^{(n)}\rangle$ projectively converge to 0, $w_i = N_i|c_i^2|/\sum_j N_j|c_j^2|$. But: 1) norm is indefinite; 2) w_i is indefinite; 3) $1 = (3-2)^{100}$ is immoral.

- Coefficients of $P_i|\psi^{(n)}\rangle$ converge to those of $p_i|\psi^{(n)}\rangle$ where $p_i = |c_i^2|/\sum_j |c_j^2|$. Physical intuition: coefficients of $(\sqrt{3}|+\rangle + \sqrt{2}|-\rangle)^{100}$ explode, renormalize.

Formal rephrasing

The above intuition can be formalized defining linear 'ghost operators'

$$G|\psi\rangle = \pm|\psi\rangle \qquad \pm = \text{sign}\,\langle\psi|\psi\rangle$$

There are many. Try to associate one G_A to each self-adjoint operator imposing $[A, G_A] = 0$. Self-adjoint operators split in 3 classes:

Self-adjoint A	$[A, \text{Ghost}] = 0$	$[A, \text{'boost'}]$
observable	one solution	$\neq 0$
unobservable	zero solution	$= 0$
ambiguous	multiple solutions	$= 0$

When $[A, G_A] = 0$ has **one** solution, it is $G_A|A_i\rangle = N_i|A_i\rangle$, which defines an associated positive A-norm $\langle\psi'|\psi\rangle_A \equiv \langle\psi'|G_A|\psi\rangle$ such that

$$\text{Coefficient convergence} \Leftrightarrow A\text{-norm convergence.}$$

PT-symmetric quantum mechanics postulated G_H norm. If $[A, H] = 0$ then $G_A = G_H$: A is conserved. Otherwise extra violation.

Bad observables

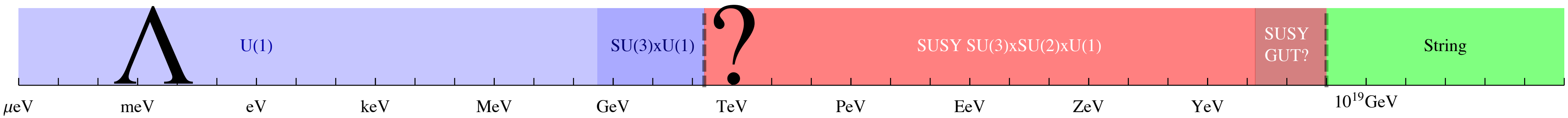
$[A, G] = 0$ has 0 solutions when pairs of eigenvectors lie along the 'null cone', where 'boosts' act diagonally. A is 'boost' invariant: does not fix a basis, like the 1 operator. Represents a bad apparatus that does not force $|\psi\rangle$ to choose.

Example: $\hat{q}$ is not observable, like a fermion field. $\hat{q}$ anti-commutes with ghost-parity: one can observe $\hat{q}^2$. Ambiguous, but fixed by $|x + \delta\rangle = e^{-i\hat{p}\delta}|x\rangle$.

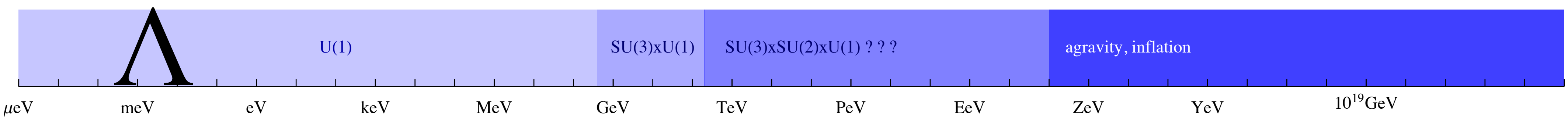
Back to agravity. Matter couples to the graviton g_+ (mass 0) and to its ghost g_- (mass M_2) as $(g_+ + g_-)E/M_{\rm Pl}$. How can they cancel at high energy $E \gg M_2$? Interactions give $H_{\rm eff} \propto (g_+ + g_-)^2$, so that coupling suppressed by M_2/E...

Conclusions

The standard view of mass scales in nature is in trouble with M_h and Λ:



New collider needed to fully clarify. Possible alternative for M_h:



Quantisation of $\partial^4 \Rightarrow$ physical naturalness + quantum gravity + inflation.

Remaining problems: give an interpretation to 'ghosts'.

References

Not clear? Skipped details can be found in:

1303.7244 and JHEP: physical naturalness.

1306.2329 and PRD: M_h^2 from weak dynamics.
1410.1817 and JHEP: M_h^2 from strong dynamics.

1412.2769 and JHEP: searching realistic asymptotically free theories.
1507.06848 and JHEP: trinification from the weak scale to infinite energy.
1701.01453 and FrontPhys: Higgs mass and asymptotic safety.

1403.4226 and JHEP: agravity.
1705.03896: agravity up to $E = \infty$.

1502.01334 and JHEP: inflation in agravity.

1512.01237 and EPJ: quantum mechanics of 4-derivative theories.
1709.04925: interpretation of indefinite quantum norm.

The Crackpot Index

"40 points for claiming that the 'scientific establishment' is engaged in a 'conspiracy' to prevent your work".

Incremental papers about ill traditional topics are (too) easily published, but referees tend to get scared by non-standard ideas. JHEP seemed the most negative, until a referee wrote that "PT quantum mechanics" already solved all problems about ghosts. This would be great.