Magnetic oscillations in holographic liquids (and monopoles)

Valentina Giangreco Puletti
University of Iceland

Based on arXiv: 1501.06459 with S. Nowling, L. Thorlacius, and T. Zingg

and on work in progress with T. Alho, and R. Pourhasan

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Outline

- Part I
  - Overview and Motivations from CMT-side (I) and AdS-side (I)
  - Prelude: Magnetic oscillations in cond-matt
  - Holographic liquids: our model
  - Results
  - Discussions and Open problems
- Part II
  - Motivations from CMT-side (II) and AdS-side (II)
  - Monopole operators: our model
  - Preliminary results
  - Discussion and outlook

A broad perspective from CMT

- Motivation: study of compressible quantum states of matter

  - Quantum: $T = 0$ or very low T ($T << \mu$)
  - Compressible [Huijse-Sachdev'11][Sachdev'12]:
    continuum, translationally invariant, with a global charge Q;
    Ground state of $H - \mu Q$ is characterised by $\langle Q \rangle$ a smooth non-constant function of μ
  - Known examples:

    1. Solids (translational symmetry is broken);
    2. Superfluids ($\mathrm{U}(1)$ is broken);
    3. Fermi liquids
    4. Non-Fermi liquids

- Zoom in: Non-Fermi liquids

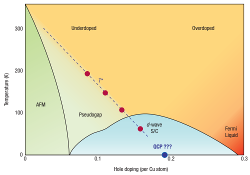
Fermi liquids vs non-Fermi liquids: intro

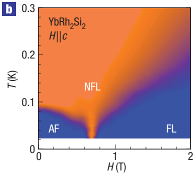
Fermi liquids (FL)

- resistivity $\rho \sim T^2$
- Fermi surfaces
- quasi-particles
- perturbative field theory

Non-Fermi liquids (NFL)

- resistivity $\rho \sim T$
- Fermi surfaces
- NO quasi-particles
- strongly interacting





- Left: Doping-temperature phase diagram of the hole-doped cuprate superconductors [Broun '08]
- Right: Magnetic field-Temperature in heavy-fermions metals [Gegenwart et al '08]

  - Quantum critical point behind NFL

A more specific perspective from CMT

Magnetic oscillations in finite density systems

- quantum oscillations in the magnetization as a function of $1/B$, present in metals at low temperature T and strong magnetic field B
- standard tool to diagnostic and analyse Fermi surfaces
- in ordinary metals (Fermi liquids): Landau-Kosevich-Lifshitz formula
- in "exotic phases": some surprises:

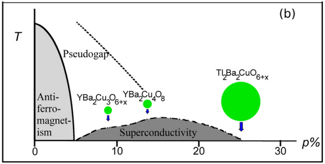


Figure: phase diagram from under-doped to over-doped high-T_c SC [Sebastian-Harrison-Lozarich'12]

In NFL: Magnetic oscillations and ARPES suggest gapless excitations and no quasi-particle descriptions [Sebastian-Harrison-Lozarich'12, Sebastian-Harrison-Lozarich'11]

Prelude: Magnetic oscillations in Fermi liquids

- Apply a magnetic field to a gas of electrons in $(3+1)$-dimensions:
  - Landau levels + Zeeman splitting:

$$\epsilon_\ell^2 = k^2 + m^2 + (2\ell+1)\gamma B \pm \gamma B\,, \qquad \ell = 0, 1\,, \ldots$$

  - B is parallel along z-axis and $k \equiv k_z$
  - γ is proportional to the gyromagnetic ratio

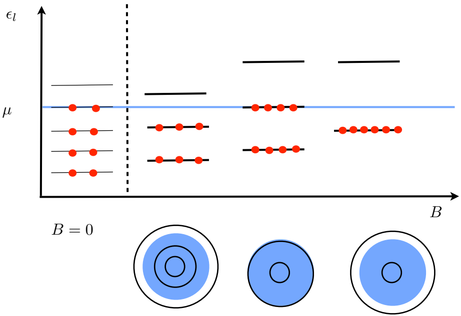
- What happens?
  - closed quantized orbits in the plane perpendicular to k
  - separation between two orbits: $\Delta_{\ell,\ell+1} \sim B$
  - degeneracy: $\delta \sim B$

- Recall: Free energy

$$\Omega \sim TB \sum_\ell \int dk \sum_{\sigma=\pm} \log\left(1 + \exp\frac{\mu - \epsilon_{\ell,\sigma}(k)}{T}\right)$$

Prelude: Magnetic oscillations in $2+1$-d Fermi liquids

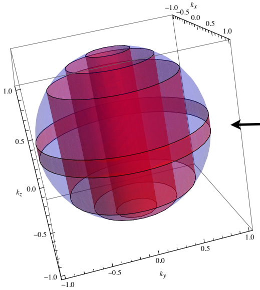
- Increasing *B*: what happens? $(2+1)$-d example



- Increasing *B*: then $\Delta_{\ell,\ell+1}$ increases but also the degeneracy (area enclosed) δ increases

Prelude: Magnetic oscillations in $3+1$-d Fermi liquids

- Increasing *B*: what happens? $(3+1)$-*d* example



$$\frac{\delta S}{\delta k} = 0$$

- extra degeneracy in k: the crossing of Fermi surface is maximized when this degeneracy is minimal: extremal cross-sectional Fermi surface!
- period is controlled by the (extremal) Fermi surface: A_F

Prelude: Magnetic oscillations in Fermi liquids: Summary

- Increasing *B*: what happens?
  - continuous jump of free energy (Ω) $\Longleftrightarrow$ oscillations of magnetization $M = -\frac{1}{V}\frac{\partial \Omega}{\partial \mu}$ (crossing of FS by highest occupied Landau level)
  - conditions
    1. Low *T* (thermal effects suppressed by oscillations) and strong *B*: $T \lesssim B << \mu$
    2. (pure metal)
- For FL: Landau-Kosevich-Lifshitz formula in $3+1$-*d*

$$\mathcal{M}_{\rm osc} \sim \sqrt{B} \sum_{\ell} \frac{m^\star T/B}{\sqrt{\ell}\,\sinh\left(\#\,\ell m^\star T/B\right)} \cos\left(A_F \ell/B\right)$$

- For FL: Landau-Kosevich-Lifshitz formula in $2+1$-*d*

$$\mathcal{M}_{\rm osc} \sim \frac{1}{B} \sum_{\ell} \frac{m^\star T/B}{\ell\,\sinh\left(\#\,m^\star T/B\right)} \sin\left(A_F \ell/B\right)$$

with $m^\star$ the effective mass, $m^\star \sim \frac{\partial A_F}{\partial \epsilon}$.

Questions, goals, and methods

- Questions
  - Can gauge/gravity duality give an alternative prediction and description for magnetic oscillations in strongly correlated systems?
  - Can we attack the problem from a different point of view?
- Our goals
  - Magnetic oscillations in systems at finite density and strongly correlated via gauge/gravity duality
  - bottom-up approach without introducing probe fermions
- Our approach
  - Extension of electron star/cloud model [Hartnoll-Silverstein-Polchinski-Tong '10][Hartnoll-Tavanfar '10][VGMP-Nowling-Thorlacius-Zingg'10][Hartnoll-Petrov'10] See also previous related works: [deBoer-Papadodimas-Verlinde'10][Arsiwalla-deBoer-Papadodimas-Verlinde'10]
  - Previous work on magnetic effects in holographic metals within probe approx [Denef-Hartnoll-Sachdev'09][Hartnoll-Hofman'09][Hartnoll-Hofman-Tavanfar'10][Gubankova-Brill-Cubrovic-Schalm-Schijven-Zaanen'11] [Blake-Bolognesi-Tong-Wong'12] [Albash-Johnson-MacDonald'12][Gubankova-Brill-Cubrovic-Schalm-Schijven-Zaanen'13] [Hartnoll-Hofman-Vegh'11]

The bulk model: Electron cloud at $B = 0$

- Action

$$
\begin{aligned}
S &= S_{EH} + S_M + S_{fl} \\
&= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,(R - 2\Lambda) - \frac{1}{4e^2}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu} - \int d^4x\sqrt{-g}\,\mathcal{L}_{fl}
\end{aligned}
$$

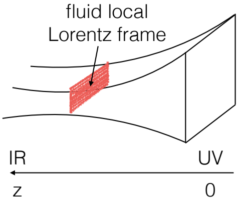
where on-shell $\mathcal{L}_{fl} = p$ [Schutz' 70][Brown '93][Bombelli-Torrence '90][de Ritis-Lavorgna-Platania-Stornaiolo '85]

- Ingredients
  - degenerate charged perfect fluid of non-interacting fermions of mass m in 4-dimensions
    coupled to
  - Maxwell-Einstein theory with $\Lambda = -\frac{3}{L^2}$ (asymptotically AdS_4, with L the AdS radius)
  - we search finite T configurations

- $\kappa^2 = 8\pi G_N$ is the gravitational coupling, e Maxwell coupling constant, here: $\frac{\kappa}{L} \ll 1$ (classical gravity regime)

- Dual: strongly correlated fermions at finite density $\mu = \lim_{r\to 0} A_t$ and at finite temperature $T = T_H$

The electron cloud at $B = 0$



- fermions are treated in a Thomas-Fermi approximation (or Tolman-Oppenheimer-Volkoff) [Hartnoll-Tavanfar'10]:

$$mL >> 1\,, \quad \text{and} \quad e^2 \sim \frac{\kappa}{L} \ll 1 \tag{1}$$

- actually it works beyond (1) [Allais-McGreevy'13][Gubankova-Brill-Cubrovic-Schalm-Schijven-Zaanen'13]
- fermions are characterised by a local chemical potential

$$\mu_{\rm loc}(r) = A_\mu(r) u^\mu(r) = A_t(r) e_0^t(r)$$

(static fluid: 4-velocity $u^\mu = (e_0^t, 0, 0, 0)$, with e_a^μ the tetrad)

The electron cloud at $B = 0$

- Eq. of state for the fermions in the rest frame: $\varepsilon^2 = k^2 + m^2$
- density of states

$$n(\varepsilon) = \beta\varepsilon\sqrt{\varepsilon^2 - m^2} \qquad \text{electrons:} \quad \beta = \frac{1}{\pi^2}$$

- fluid is described by pressure $p(r)$, energy density $\rho(r)$, and charge density $\sigma(r)$

$$p = p\left(\mu_{\rm loc}(r)\right)\ , \quad \rho = \rho\left(\mu_{\rm loc}(r)\right)\ , \quad \sigma = \sigma\left(\mu_{\rm loc}(r)\right)$$

for example

$$\sigma = \beta \int_m^{\mu_{\rm loc}(r)} d\varepsilon\,\varepsilon\sqrt{\varepsilon^2 - m^2} = \frac{\beta}{3}\left(\mu_{\rm loc}^2(r) - m^2\right)^{3/2}$$

- fluid is supported if

$$\mu_{\rm loc}(r) \geq m$$

The electron cloud at $B = 0$: Geometry

- Ansatz

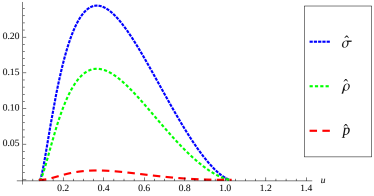
$$ds^2 = \frac{L^2}{r^2}\left(-\frac{\hat{c}(r)^2}{\hat{g}(r)^2}dt^2 + dx^2 + dy^2 + \hat{g}(r)^2 dr^2\right), \qquad A_t = \frac{eL}{\kappa}\frac{\hat{c}(r)\hat{a}(r)}{r\hat{g}(r)}$$

- Then

$$\mu_{\rm loc}(r) = \frac{e}{\kappa}\hat{a}(r)$$

where $\hat{}$ denotes dimensionless quantities

- Solutions at finite temperature



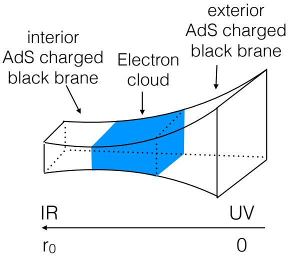


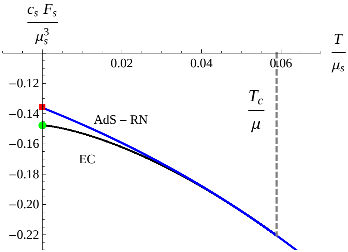
Figure: Left: fluid profile at finite T [VGMP-Nowling-Thorlacius-Zingg'10]. Right: Cartoon of the geometry

The electron cloud at $B = 0$: Thermodynamics

- Free energy

$$\mathcal{F} = \mathcal{E} - \mu\, \mathcal{Q} - s\, T$$

- The electron cloud geometry is the preferred solution for $T \leq T_c$ compared to AdS charged black brane [VGMP-Nowling-Thorlacius-Zingg'10].



- 3rd order phase transition [VGMP-Nowling-Thorlacius-Zingg'10][Hartnoll-Petrov'10]

The anisotropic electron cloud: $B \neq 0$

- Now: we add a magnetic field B supported by the black brane, B is constant and pointing along the radial direction
- Action

$$
\begin{aligned}
S &= S_{EH} + S_M + S_{fl} \\
&= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,(R - 2\Lambda) - \frac{1}{4e^2}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu} - \int d^4x\sqrt{-g}\,\mathcal{L}_{fl}
\end{aligned}
$$

  where on-shell $\mathcal{L}_{fl} = p$

- we search for finite T and B configurations
- charged perfect fluid of non-interacting fermions of mass m in 4-dimensions coupled to Maxwell-Einstein theory with $\Lambda = -\frac{3}{L^2}$
- Spin fluid models known since 70's
  [Schutz'70][Ray'72][Bailey-Israel'75][deOliveira-Salim'91][Brown'93][deOliveira-Salim'95]
- Dual: strongly correlated fermions at finite density $\mu = \lim_{r\to 0} A_t$, at finite temperature $T = T_H$ and at finite magnetic field $\mathcal{B} = \lim_{r\to 0} F_{xy}$.

The anisotropic electron cloud: $B \neq 0$

- degenerate gas of electrons experience

$$\mu_{\rm loc}(r) = A_\mu u^\mu \,, \qquad H_{\rm loc}(r) = e_1^{[\mu} e_2^{\nu]} F_{\mu\nu}$$

- Fluid equation of state: Landau levels and Zeeman splitting

$$\varepsilon_\ell^2 = m^2 + k^2 + (2\ell+1)\gamma H_{\rm loc} \pm \gamma H_{\rm loc}$$

- Density of states

$$n(\varepsilon) = \beta\gamma H_{\rm loc} {\sum_{\ell\geq 0}}' \theta(\varepsilon^2 - m^2 - 2\ell\gamma H_{\rm loc}) \frac{\varepsilon}{\sqrt{\varepsilon^2 - m^2 - 2\ell\gamma H_{\rm loc}}} \,,$$

- fluid is described by thermodynamic variables

$$p = p\left(\mu_{\rm loc}(r)\,, H_{\rm loc}(r)\right)\,,$$

same for $\rho(r)$, $\sigma(r)$, and the magnetization $\eta(r)$. For example

$$\begin{aligned} \sigma(r) &= \int_{\sqrt{m^2+2\ell\gamma H_{\rm loc}}}^{\mu_{\rm loc}} n(\varepsilon) d\varepsilon \\ &= \beta\gamma H_{\rm loc} {\sum_{\ell\geq 0}}' \theta(\mu_{\rm loc}^2 - m^2 - 2\ell\gamma H_{\rm loc}) \sqrt{\mu_{\rm loc}^2 - m^2 - 2\ell\gamma H_{\rm loc}} \,, \end{aligned}$$

The anisotropic electron cloud: some details

- Ansatz

$$
\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(-\frac{\hat{c}(r)^2}{\hat{g}(r)^2}dt^2 + dx^2 + dy^2 + \hat{g}(r)^2 dr^2\right) \\
A_t &= \frac{eL}{\kappa}\frac{\hat{c}(r)\hat{a}(r)}{r\hat{g}(r)}\,, \qquad A_y = \frac{eL}{\kappa}\hat{B}\,x
\end{aligned}
$$

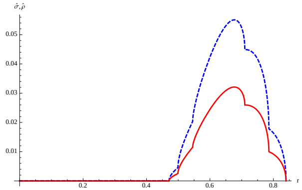
- Then

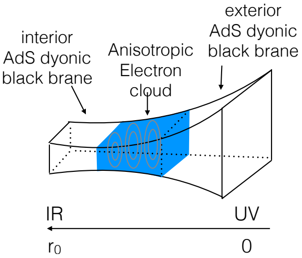
$$
\mu_{\rm loc}(r) = \frac{e}{\kappa}\hat{a}(r)\,, \qquad H_{\rm loc}(r) = \frac{e}{\kappa L}\hat{B}\,r^2
$$

- Solve for $\hat{c}(r)\,, \hat{g}(r)\,, \hat{a}(r)$.

The anisotropic electron cloud: Geometry

- Solution





- fluid is supported when

$$\ell_{\rm filled} = \left\lfloor \frac{\mu_{\rm loc}^2 - m^2}{2\gamma H_{\rm loc}} \right\rfloor \geq 0$$

Results: boundary magnetization $\mathcal{M}$

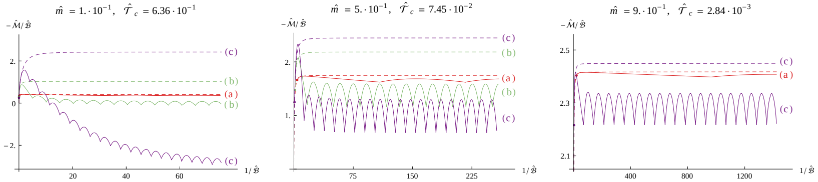
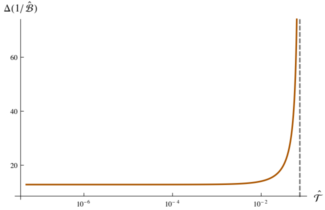


Figure: The labels denote temperatures $\hat{\mathcal{T}}/\hat{\mathcal{T}}_c = 0.9$ (*a*), 0.3 (*b*), $3\cdot 10^{-3}$ (*c*)

- The magnetization of the electron cloud (solid lines) is lower than that of a dyonic black brane (dashed lines) with the same parameters
- only one Fermi surface: the extremal (with respect to the radial direction) [Hartnoll-Hofman-Tavanfar'10][Hartnoll-Hofman-Vegh'11] $\neq$ observations
- overall amplitude of the magnetization $\mathcal{M}$ is linear in $\mathcal{B}$: $\neq$ from Landau-Fermi theory and $\neq$[Hartnoll-Hofman-Tavanfar'10]][Blake-Bolognesi-Tong-Wong'12]: we have back-reaction now!
- $\neq$ Friedel oscillations, which were not observed in the electron cloud [VGMP-Nowling-Thorlacius-Zingg'11]: continuum of bulk Fermi surfaces

Period of oscillations

- period of oscillations versus T:



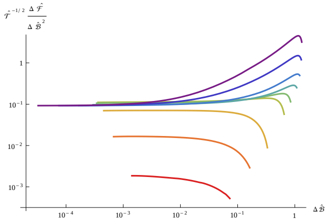
independent of the temperature T as expected at low T

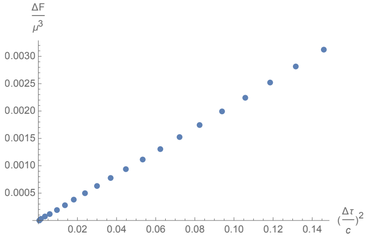
The anisotropic electron cloud: Thermodynamics

- Free energy and thermodynamics relation

$$\mathcal{F} = \mathcal{E} - s\mathcal{T} - \mu\mathcal{Q}\,, \qquad \frac{3}{2}\mathcal{E} = s\mathcal{T} + \mu\mathcal{Q} - \mathcal{M}\,\mathcal{B}$$

- free energy: 2nd order phase transition between anisotropic electron cloud and dyonic black hole





- Compare with $B = 0$: third order phase transition [VGMP-Nowling-Thorlacius-Zingg'10][Hartnoll-Petrov'10]

  Expected to be first order taking into account quantum corrections as in $B = 0$
  [Medvedyeva-Gubankova-Cubrovic-Schalm-Zaanen'13]

The anisotropic electron cloud: Phase transition

- Phase transitions can be studied numerically and analytically close to the critical point [Hartnoll-Petrov'10]

$$\Delta\mathcal{F} \sim \int \delta \mathsf{p} \sim \delta\mu^{3/2}\,\Delta \mathsf{r} \sim (\Delta \mathsf{r})^4 \sim \Delta\mathcal{T}^2$$

- Analytically: They match with numerics up to the third digit

  - vs $\mathcal{T}$ (at fixed $\mathcal{B}$)

$$\frac{\Delta\mathcal{F}}{\mu^3} = -\hat{f}\left(m, \frac{B}{\mu^2}, \star\right)\Delta\mathcal{T}^2$$

  - vs $\mathcal{B}$ (fixed $\mathcal{T}$)

$$\frac{\Delta\mathcal{F}}{\mu^3} = -\hat{g}\left(m, \frac{T}{\mu}, \star\right)\Delta\mathcal{B}^2$$

Summary and Outlook

- Summary
  - a holographic model for a 2+1 dimensional system of strongly correlated electrons in a magnetic field
  - The system shows magnetic oscillations dominated by a single sharp Fermi surface
  - The oscillation amplitude has a non-Fermi liquid character and it is different from earlier probe fermion computations
  - The model: 3+1 dimensional bulk fermions treated in a Thomas - Fermi approx in an asymptotically AdS dyonic black brane background with gravitational and electromagnetic back-reaction
  - our results confirmed later also by [Carney-Edalati'15]
- Oulook
  - beyond Thomas-Fermi approx: WKB along the lines of [Medvedyeva-Gubankova-Cubrovic-Schalm-Zaanen'13][Carney-Edalati'15] or approach of [Allais-McGreevy'13]
  - Other systems
  - Thermalisation effects