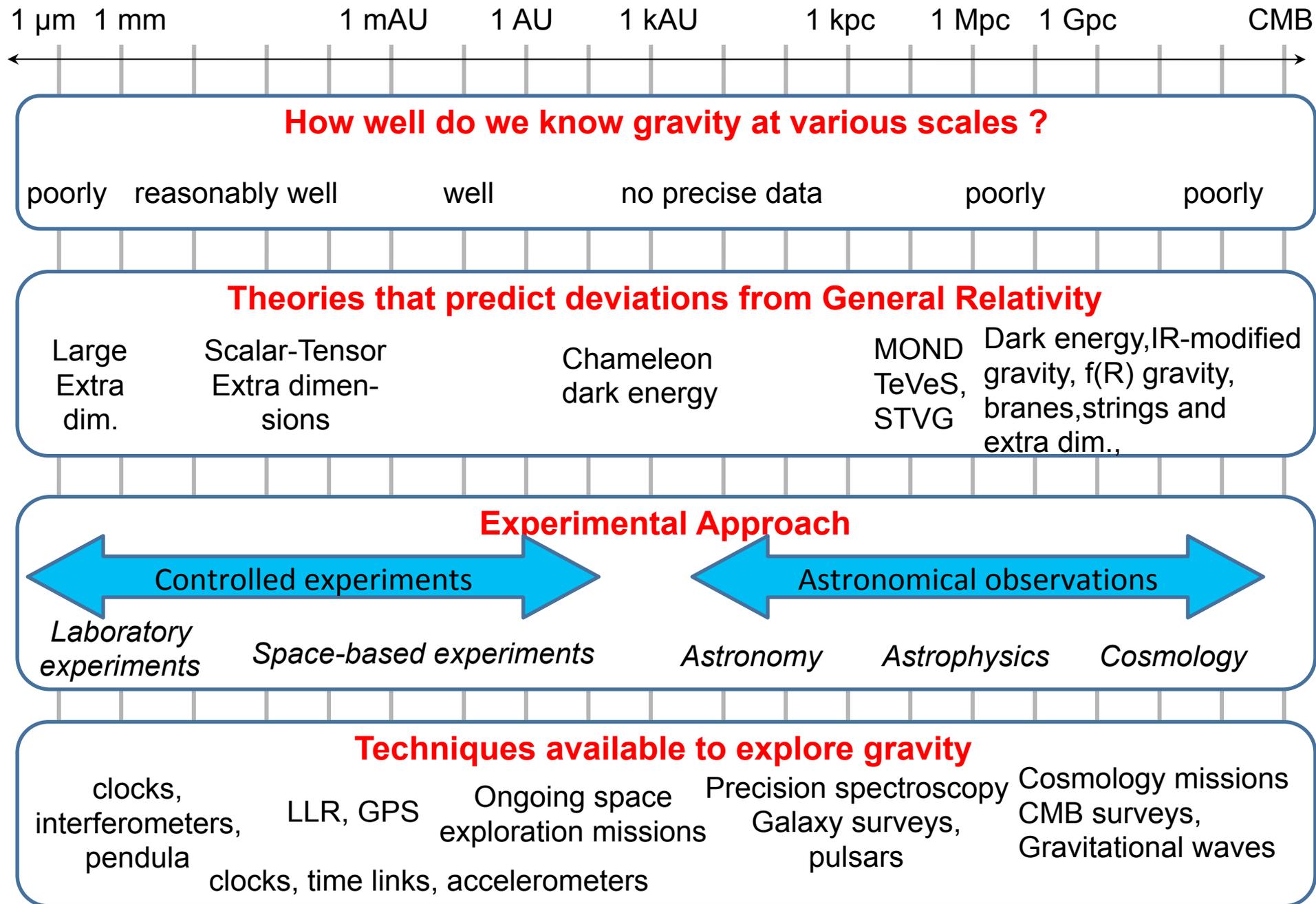


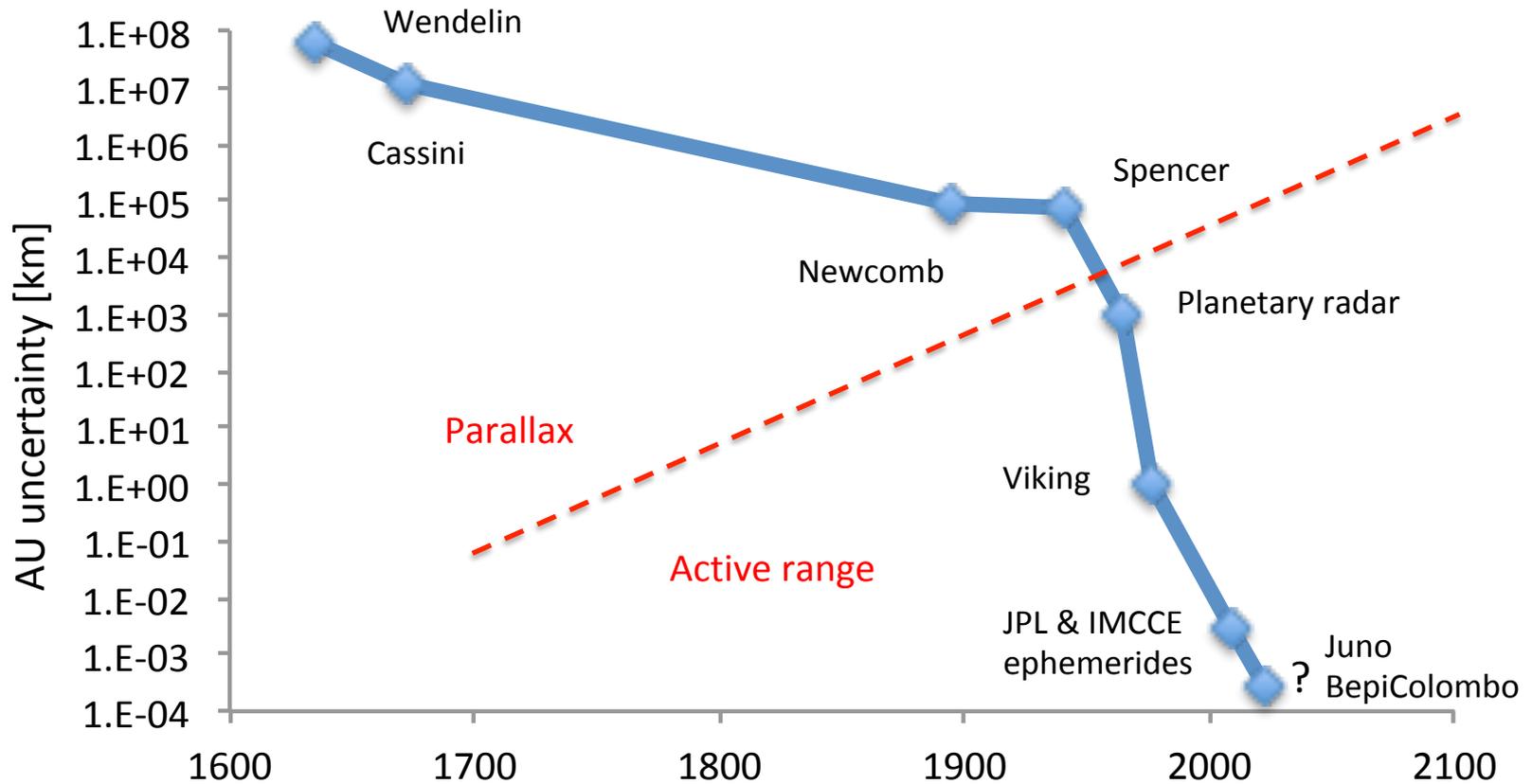
## Probing Laws of Gravity in the Solar System: from Cassini to BepiColombo

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# The yardstick of the solar system



# Which tools are available?

- Geodesic motion of test masses (deep space probes, solar system bodies)
- Propagation of photons in a gravity field
- Measurements of angles, distances and velocities

# Observables used in deep space navigation and solar system tests of GR

## Range (light travel time)

Phase comparison of modulation tones or codes in coherent radio links

Current accuracies (2-way):  
0.5-2 m (incl. station bias)  
0.02 m (BepiColombo Ka-band / multilink radio systems with wideband code modulation and delay calibration)

## VLBI (angles)

Time delay at two widely separated ground antennas

Current accuracies:  
 $\approx 0.5\text{-}1$  nrad ( $\Delta\text{DOR}$ )

(0.2-0.4 ns – further improvements limited by quasar position error)

## Range rate (velocity)

Phase comparison (carrier) in coherent radio links

Best accuracy attained (2-way, Cassini):  
 $1.5 \cdot 10^{-6}$  m/s @1000 s (Ka-band /multilink radio systems)

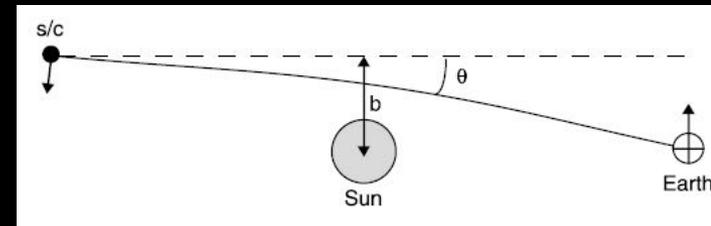
# Tests based on propagation of photons

## Deflection of light (GAIA)

$$\theta_{gr} = 2(1 + \gamma) \frac{M_{sun}}{b} = 4 \times 10^{-6} (1 + \gamma) \frac{R_{sun}}{b} \text{ rad}$$

**Main advantage:  
short time scale !  
[ 7-10 days]**

Solar Gravity



Time delay (BepiColombo)

$$\Delta t = (1 + \gamma) M_{sun} \ln \frac{l_0 + l_1 + l_{01}}{l_0 + l_1 - l_{01}}$$

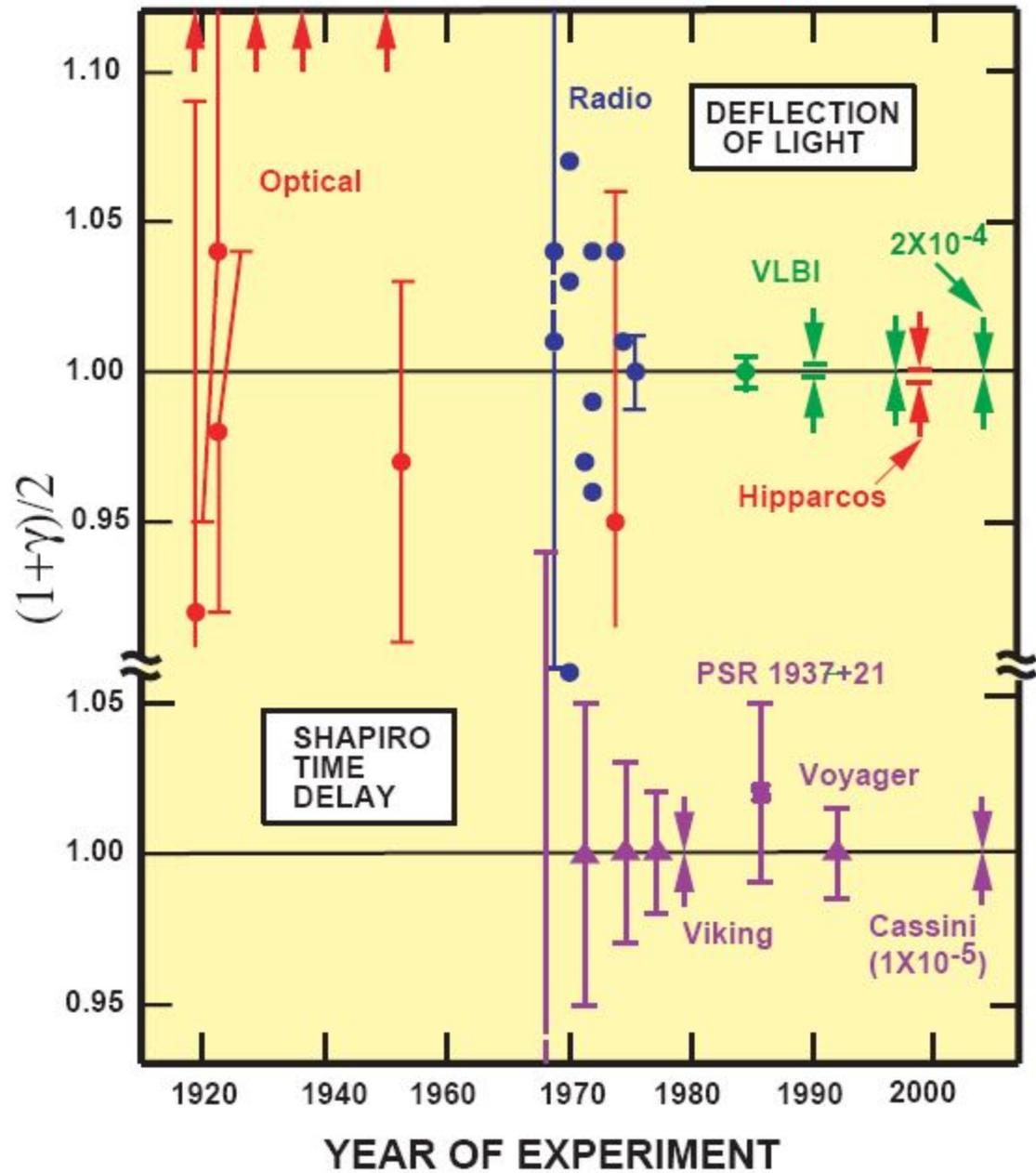
≈ 70 km for a grazing beam

Frequency shift (Cassini&BepiColombo)

$$\frac{\Delta \nu}{\nu} = \frac{d}{dt} \Delta t \cong 4(1 + \gamma) \frac{M_{sun}}{b} \frac{db}{dt}$$

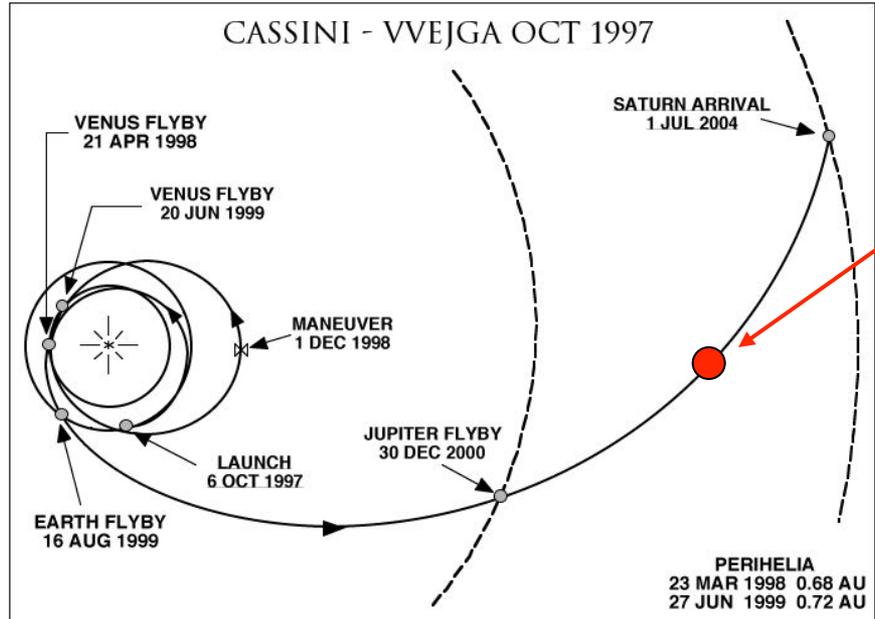
≈ 8 × 10<sup>-10</sup> for a grazing beam

# THE PARAMETER $(1+\gamma)/2$



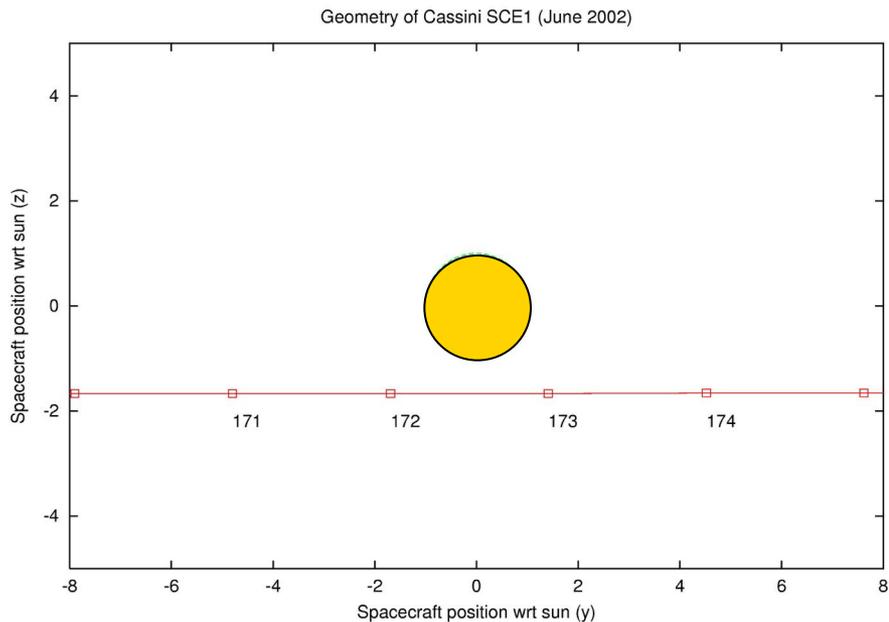
From:

Clifford M. Will,  
 "The Confrontation between General  
 Relativity and Experiment",  
 Living Rev. Relativity, 9, (2006), 3.  
<http://www.livingreviews.org/lrr-2006-3>



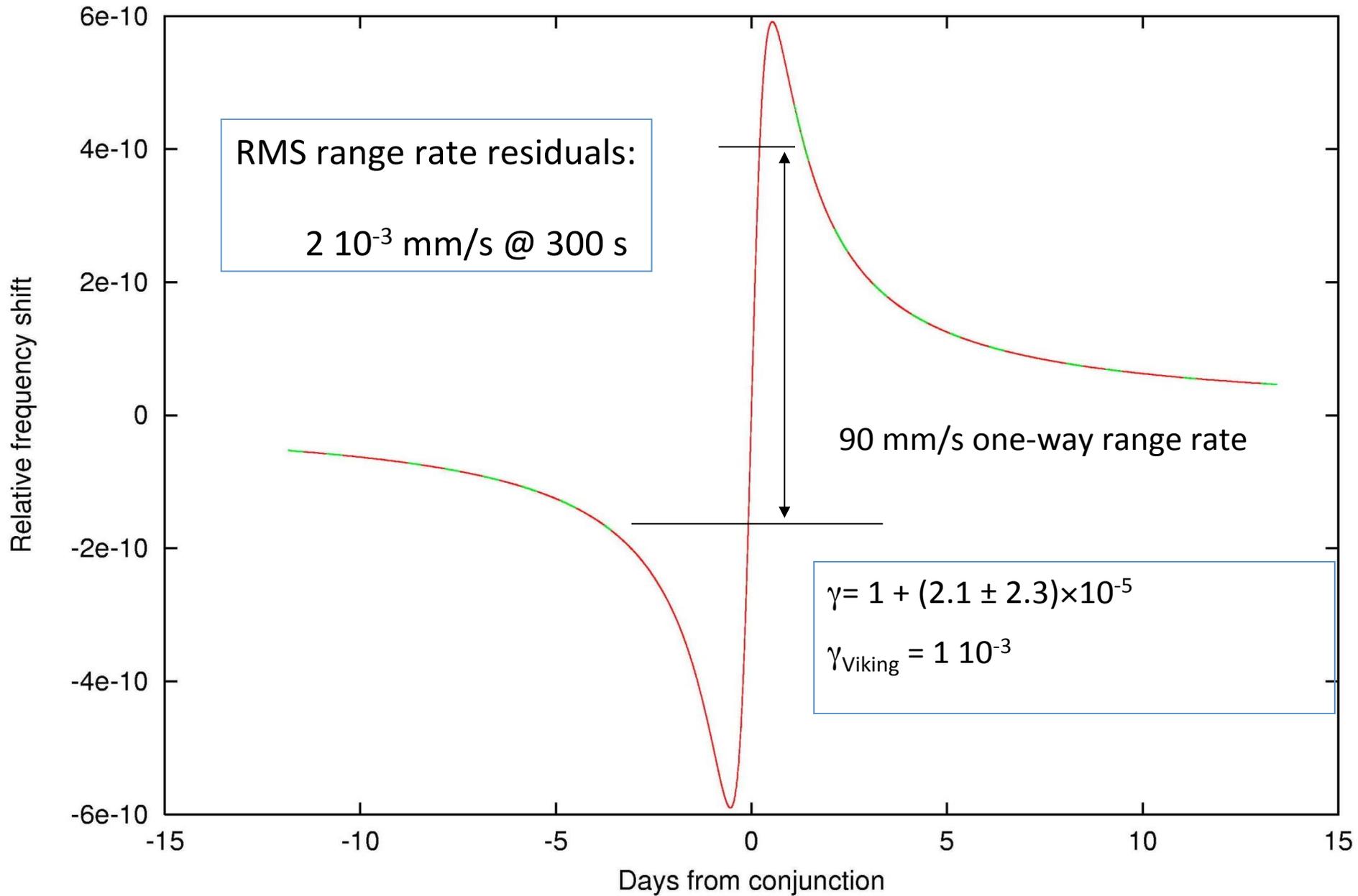
30 days coverage from DSN

June 6 to July 5, 2002



Heliocentric distance: 7.2 AU

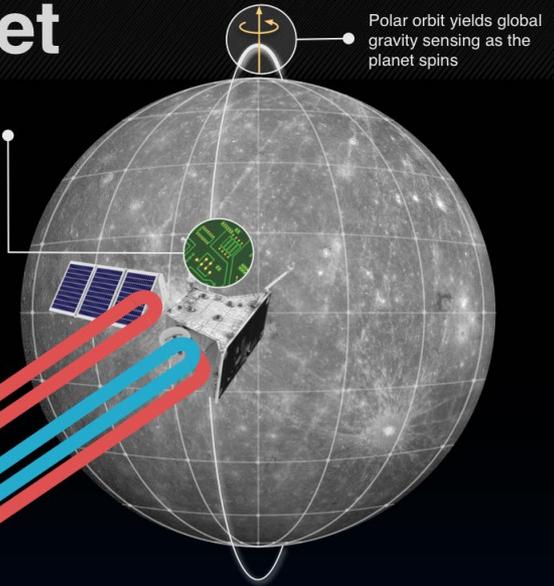
GR signal and GR signal + residuals (Cassini SCE1)



# BepiColombo - range rate error budget



- Allan deviation @ 60s
- 1.1e-15 □ TT&C system frequency stability
- 3.2e-14 □ RF Amp and distribution
- 1.3e-14 □ Antenna thermal/mech. stability
- 1.0e-13 □ Ka band Translator
- Deep Space Transponder



## Multi-Frequency link (X/X/Ka)

- Allows cancellation of dispersive noises (solar plasma, ionosphere) – up to very small impact parameter (~ 5÷8 solar radii)
- Is obtained by linear combination of Ka/Ka link from the onboard KaT with X/X and X/Ka links from the onboard DST
- Errors affecting the Ka/Ka link contributes with weight 1
- Errors affecting the X/X link contributes with weight ~1/13
- Errors affecting the X/Ka link contributes with weight ~1/35

Range accuracy:  
20 cm @ 5 s integration  
2 cm over a tracking pass

End-to-end Allan deviation  
 **$5.2 \times 10^{-14}$**

End-to-end range-rate error  
**0.016 mm/s**

- G/S frequency stability
- Frequency standard (Maser)
- RF receiver chain
- RF transmitter chain
- Antenna thermal and mechanical deformation

- Allan deviation @ 60s
- 3.6e-15 □
- 2.4e-15 □
- 1.2e-15 □
- 2.0e-14 □



- Dry troposphere (non dispersive)
- Minor error source for Doppler
- Scales with elevation (worst at horizon)
- Well reduced by ground meteo data



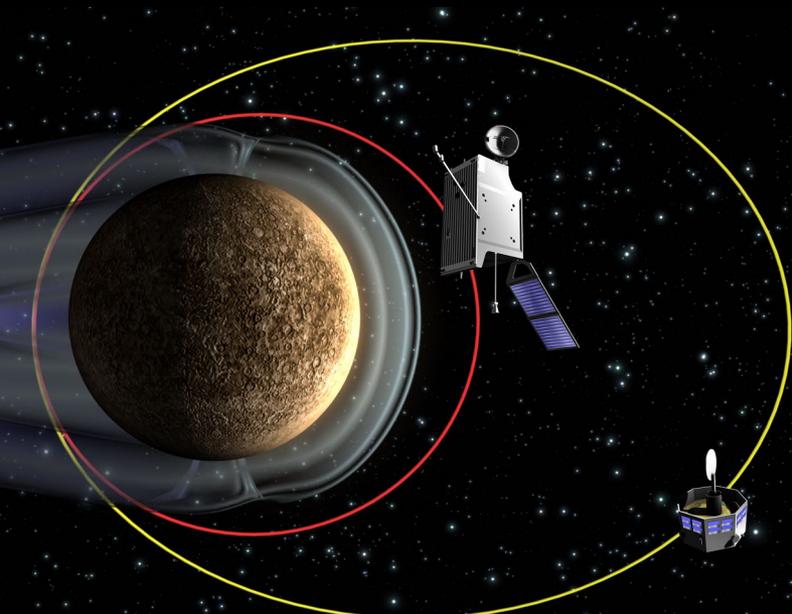
- Wet troposphere (non dispersive)
- Key error source for Doppler
- Scales with elevation (worst at horizon)
- Can be reduced by Water Vapour Radiometer (effective at long time scale)



- Allan deviation @ 60s
- 3.2e-14 □ (Wet + residual Dry)

multi-frequency link performance at 60s

# *BepiColombo: ESA's mission to Mercury*

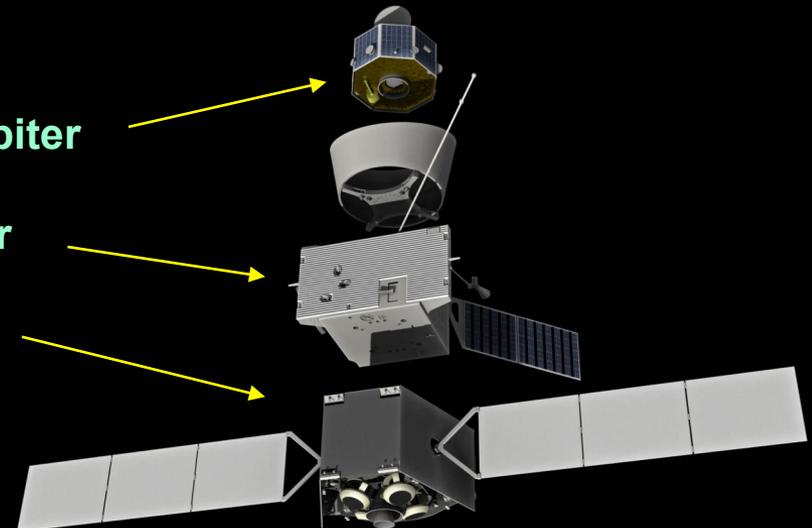


Launch: Ariane 5 (Oct. 2018)  
Solar Electric Propulsion  
Arrival at Mercury: 2025  
MPO orbit altitude: 400x1500 km

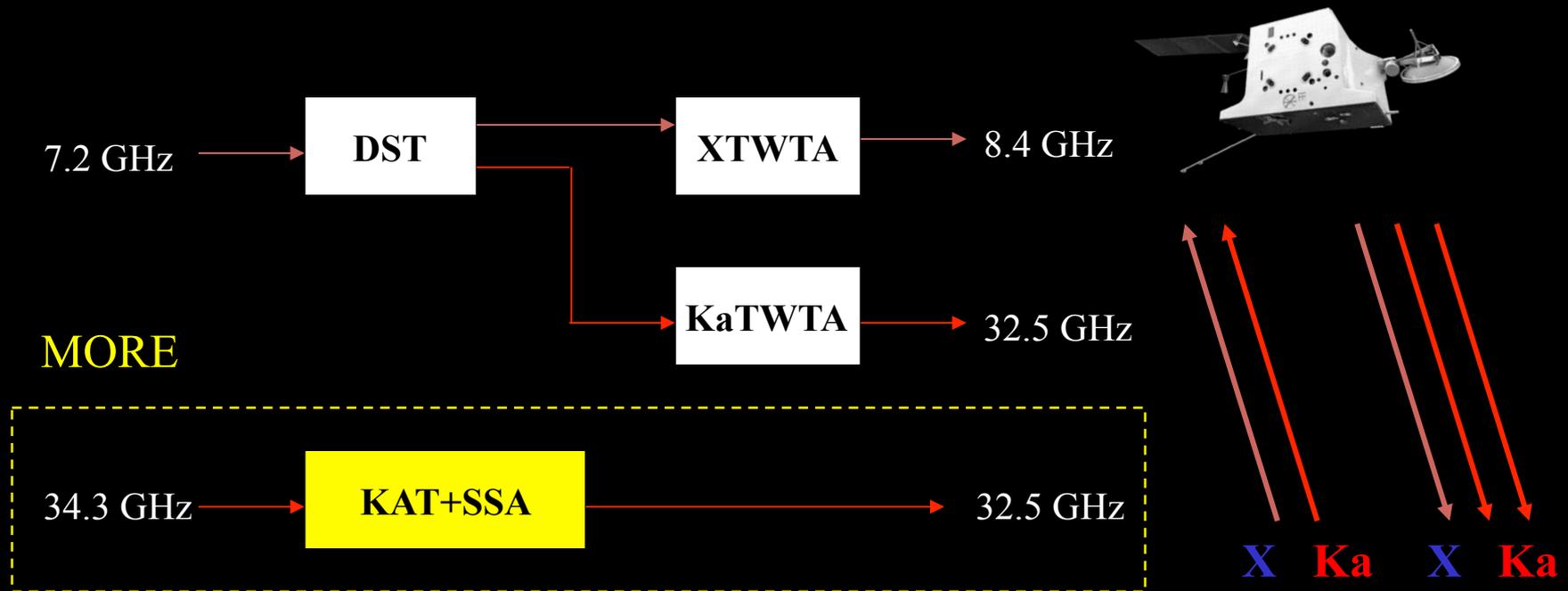
**Magnetospheric Orbiter**

**Planetary Orbiter**

**SEPM - CPM**



# Multifrequency link - BepiColombo



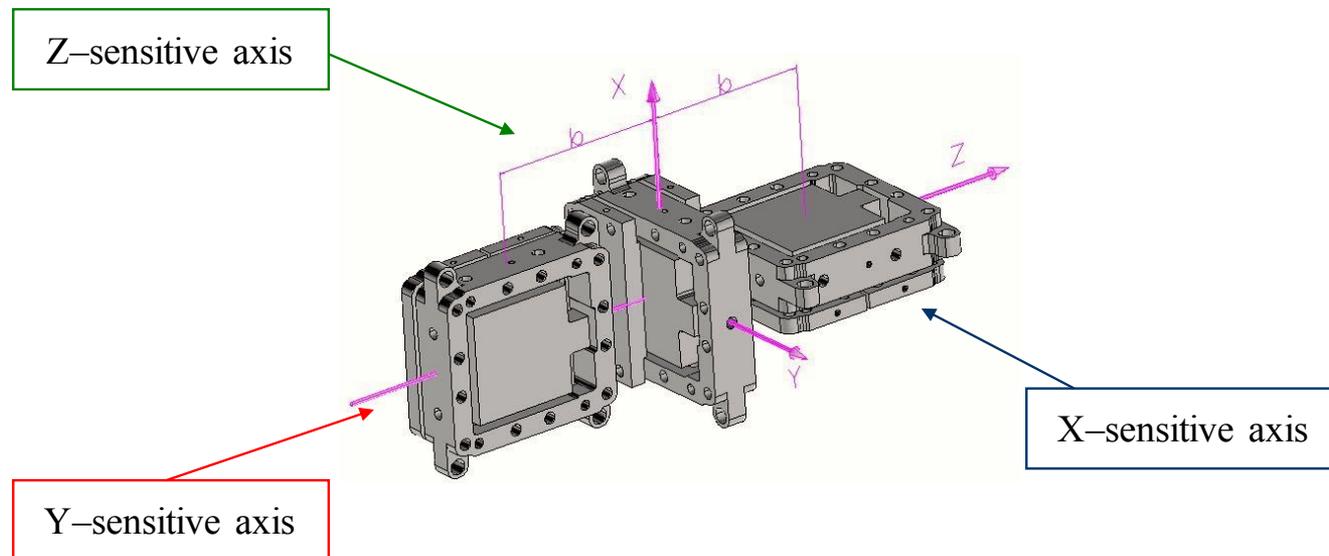
- Multilink needed for plasma noise cancellation
- 24 Mcps PN range modulation on Ka/Ka link
- Onboard range delay calibration
- End-to-end range accuracy (post-cal): 2-3 cm
- End-to-end 2-way range rate accuracy:  $3 \mu\text{m/s}$  @ 1000 s ( $\sigma_y=10^{-14}$ )



# The Italian Spring Accelerometer

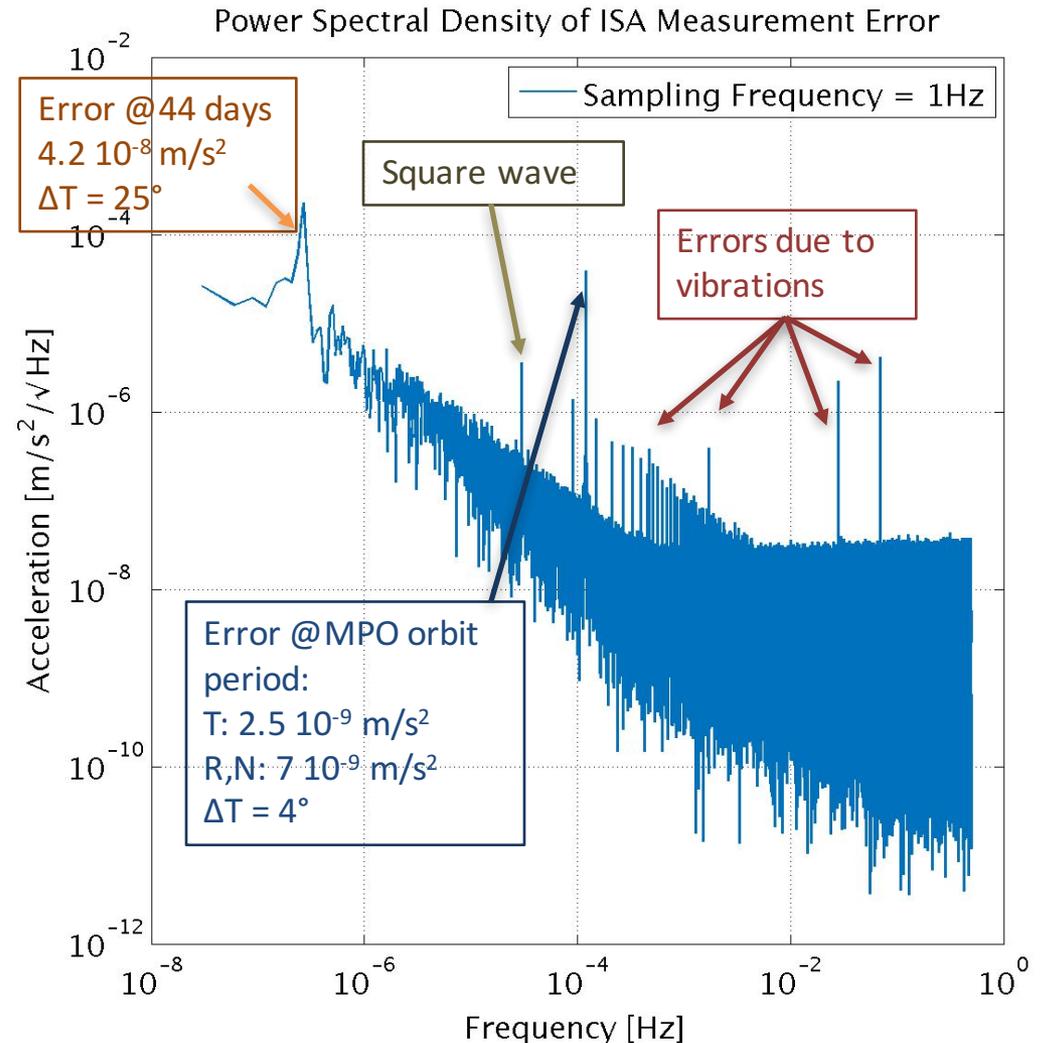
## Italian Spring Accelerometer (PI V. Iafolla, IAPS-INAF)

- Non-gravitational perturbations: direct solar radiation pressure, albedo radiation pressure, Mercury IR emission, thermal thrusts due to anisotropic IR spacecraft emission, gas leaks, etc.)
- NGA are large in the Hermean environment. Modelling errors lead to incorrect parameter estimation.
- Acceleration due to solar radiation pressure  $\approx 10^{-6} \text{ m/s}^2$ .



# The Italian Spring Accelerometer

- Scale (transduction) factor error  $\pm 10^{-2}$
- ISA random error:  $10^{-8} \text{ m/s}^2 / (\text{Hz})^{1/2}$
- ISA systematic error:
  - Square Wave due to thermal disturbance from adjacent units
  - 4 sinusoids due to S/C vibrations in the measurement bandwidth (wheels, appendages, etc)
  - Error at MPO period
  - Error at half of Mercury period (flip-over maneuver)
- FIR filtering to reduce the high frequency noise
- Measurement bandwidth:
  - $10^{-4} - 10^{-1} \text{ Hz}$



# Relativity experiment

MORE can determine the orbit of Mercury and Earth, using a fully relativistic dynamical model.

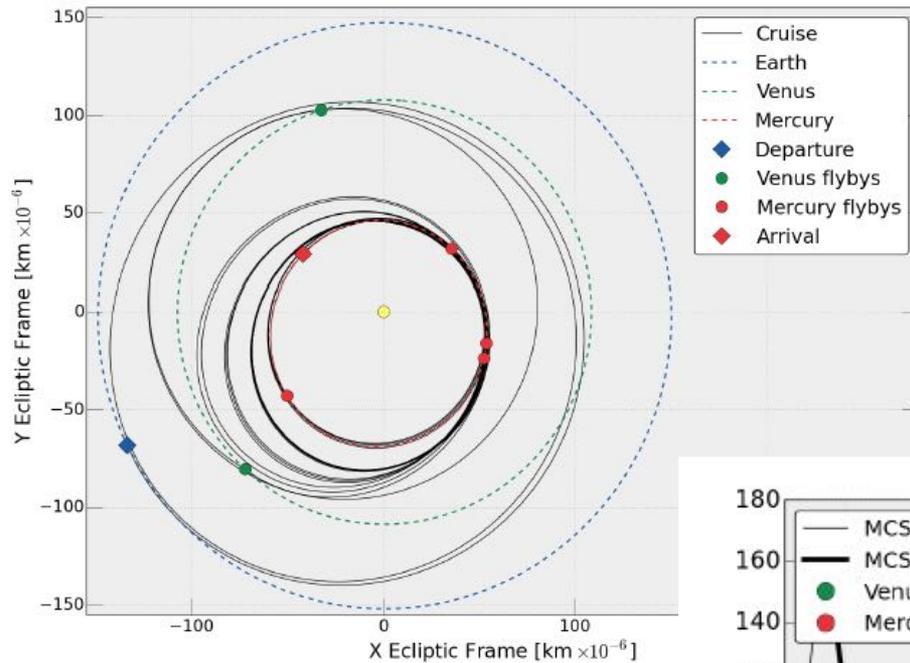
The PPN parameters of interest are:

- $\beta$ : related to non-linear 3-body general relativistic interaction ( $\beta=1$  in GR).
- $\gamma$ : parameterizes the velocity-dependent modification of the 2-body interaction and measures the space curvature produced by a unit mass ( $\gamma=1$  in GR).
- $\eta$ : measures the contribution of the gravitational self-energy to the violation of SEP ( $\eta=0$  in GR).
- $\alpha_1, \alpha_2$ : describe the preferred frame effects ( $\alpha_1=\alpha_2=0$  in GR).

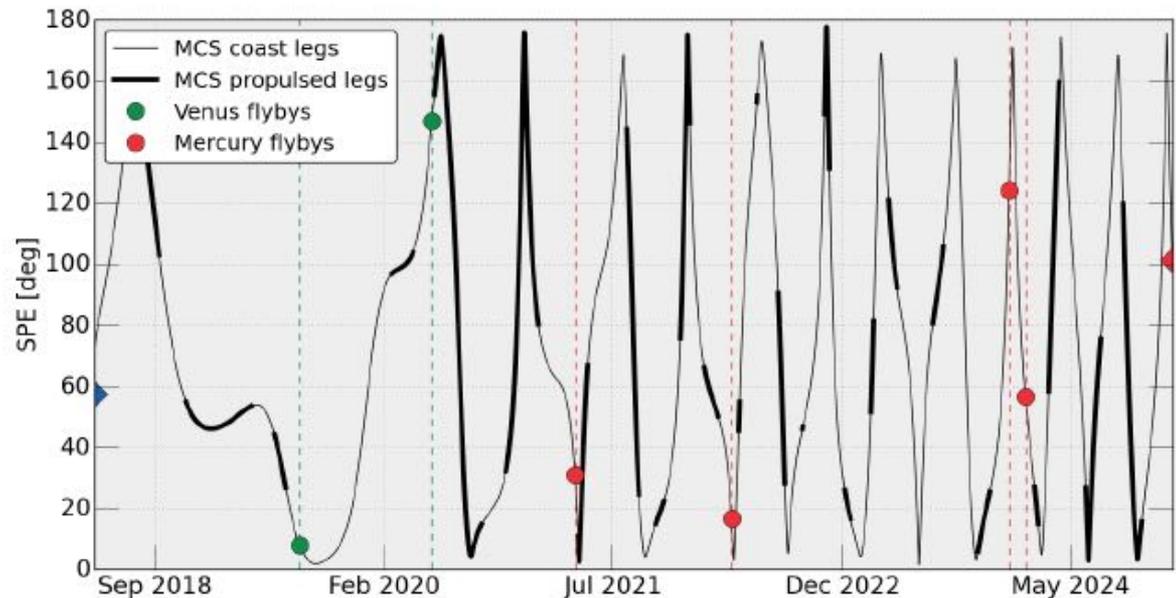
The addition parameters affecting the planetary orbits are:

- $\mu_{\text{Sun}}$ : gravitational parameter of the Sun.
- $\zeta$ : time derivative of  $\mu_{\text{Sun}}$ , from variations in  $M_{\text{Sun}}$  and  $G$ .
- $\mathbf{J}_{2\text{Sun}}$ : solar quadrupole coefficient.

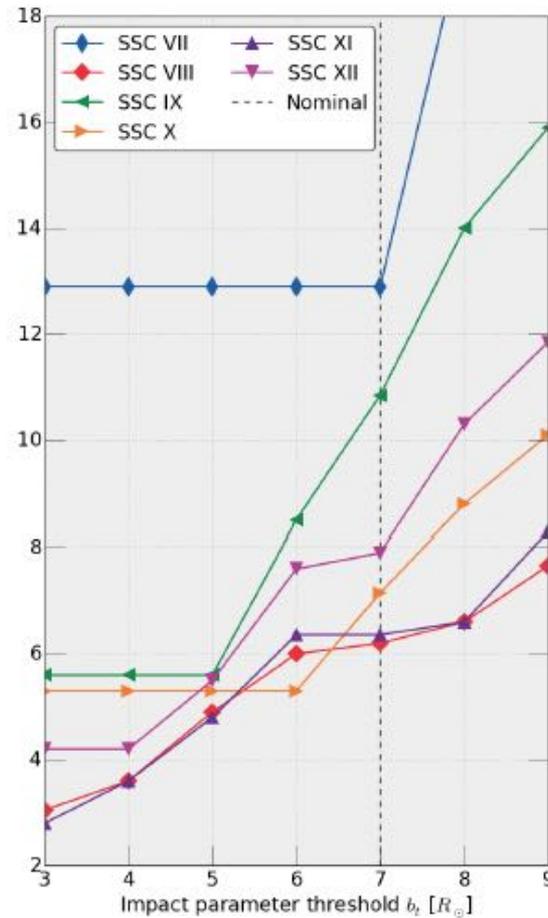
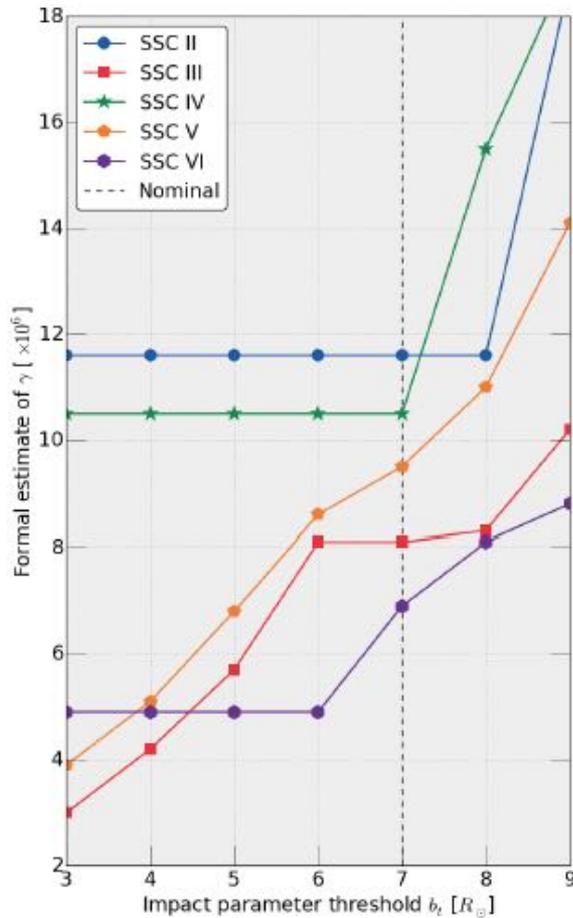
# Measuring $\gamma$ in cruise



SSC	Date	$b_{min}$ [ $R_{\odot}$ ]	$db/dt_{max}$ [ $R_{\odot}/day$ ]
#1	31 Aug 2019	5.72	0.01
#2	15 Aug 2020	8.31	1.70
#3	17 Apr 2021	3.04	3.84
#4	14 Sep 2021	7.71	1.36
#5	02 Apr 2022	3.78	3.45
#6	01 Aug 2022	6.65	3.32
#7	13 Jan 2023	7.91	1.09
#8	25 May 2023	1.71	3.99
#9	01 Oct 2023	5.52	1.55
#10	26 Feb 2024	6.75	2.37
#11	11 Jun 2024	3.38	3.57
#12	30 Sep 2024	4.99	1.97



(Imperi & less, 2017)



Accuracy as a function of minimum impact parameter

	#8 + #11	Thrust-free	All
Nominal tracking + $b_t = 7R_\odot$	4.4	3.3	2.5
h24 tracking + $b_t = 7R_\odot$	3.2	2.4	1.8
h24 tracking + $b_t = 5R_\odot$	2.4	1.8	1.3

Expected accuracies for different combinations of conjunctions and observation scenarios.

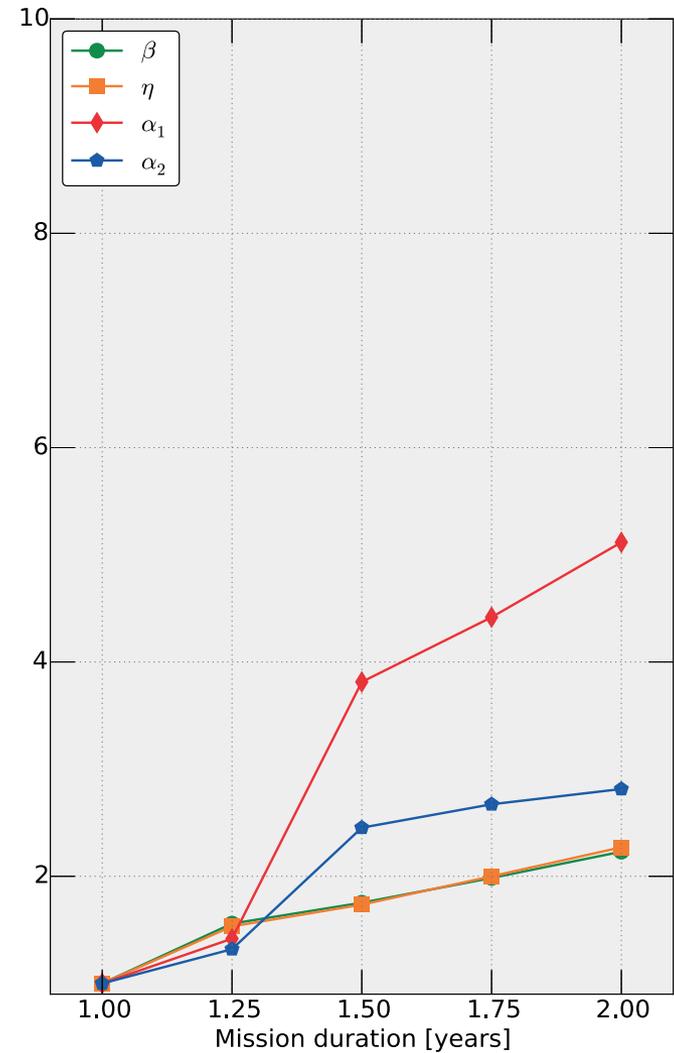
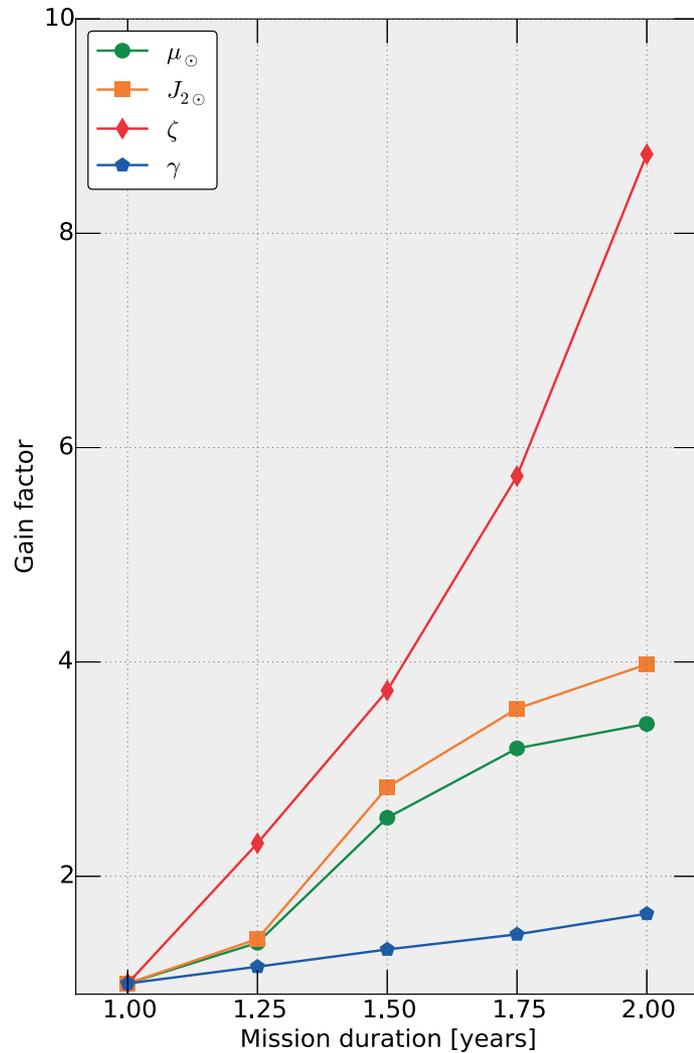
# PPN parameters

Parameter	Current limit	Method	MORE
$\beta-1$	$7 \times 10^{-5}$ $3.9 \times 10^{-5}$	IMPOP global planetary fit MESSENGER range data	$10^{-6}$
$\gamma-1$	$2.3 \times 10^{-5}$	Cassini SCE	$1.1 \times 10^{-6}$
$\eta$	$4.5 \times 10^{-4}$	LLR	$3.0 \times 10^{-6}$
$\alpha_1$	$6 \times 10^{-6}$ $4 \times 10^{-5}$	Solar system precession Pulser-white dwarf	$6.1 \times 10^{-7}$
$\alpha_2$	$3.5 \times 10^{-5}$ $1.6 \times 10^{-9}$	Solar system precession Milliseconds pulsar	$1.3 \times 10^{-7}$
$\mu_{\text{Sun}}$	10	EPM global planetary fit	$5.3 \times 10^{-2} \text{ km}^3/\text{s}^2$
$J_{2\text{Sun}}$	$10^{-8}$ $1.2 \times 10^{-8}$ $9 \times 10^{-9}$	Helioseismology IMPOP global planetary fit MESSENGER range data	$5.5 \times 10^{-10}$
$\zeta$	$4.3 \times 10^{-14}$ $1.6 \times 10^{-13}$	EPM global planetary fit MRO range data	$2.8 \times 10^{-14} \text{ y}^{-1}$

- Combining MESSENGER and BepiColombo data looks very promising, especially for some PPN parameters.

(Imperi, Iess & Mariani, 2017)

# PPN parameters



# Conclusions and Outlook

- The quest for violations of GR continues ... but the theoretical framework is uncertain.
- In this context, space agencies are reluctant to fund expensive dedicated missions.
- BepiColombo may push current limits by a significant factor, testing several aspects of GR. Further improvement is expected for some parameters by combining the MESSENGER and BepiColombo data set.
- GAIA is expected to attain similar improvements on  $\gamma$ . The two missions will strengthen each other.
- An interplanetary network of orbiters and landers (Mars, Moon, ...) could immensely strengthen the geometry of the observations, improve the dynamical model of the solar system and put GR to much more challenging tests.