(Electromagnetic) Form factor measurements (review)

Michael O. Distler for the A1 collaboration @ MAMI

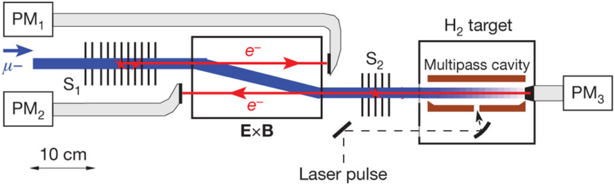
Institut für Kernphysik
Johannes Gutenberg-Universität Mainz

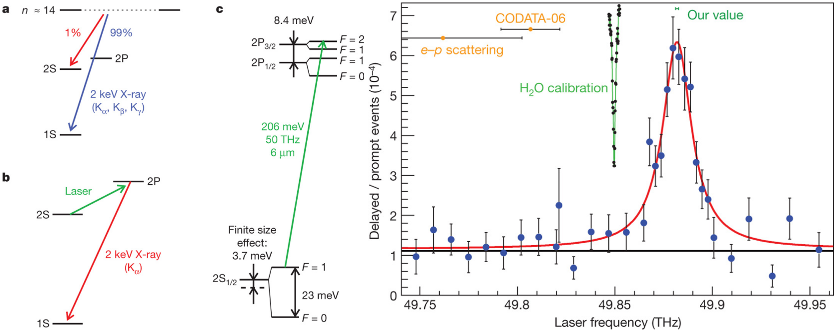
Outline

1. Introduction I: The size of the proton from the Lamb shift in muonic hydrogen and electron scattering.
2. Introduction II: Electric and magnetic form factors of the Proton.
3. The Mainz high-precision p(e,e')p measurement.
4. Impact on PV Asymmetries
5. Conclusion
6. (The muonic/electronic puzzle of the electric radius.)

Introduction I: The size of the proton

Nature **466**, 213-216 (8 July 2010)





Cross section and form factors for elastic e-p scattering

The cross section:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\,(1+\tau)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

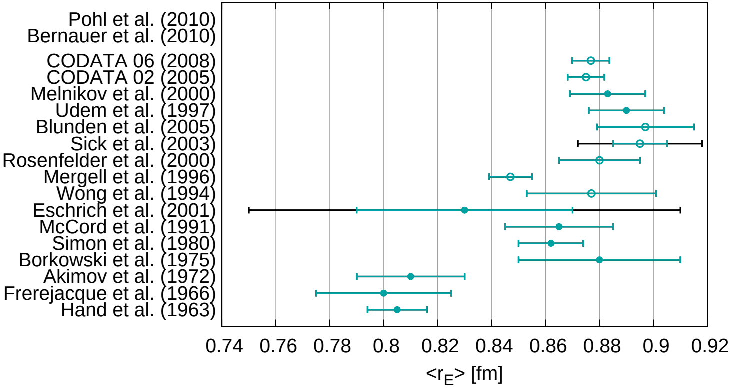
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\,(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Fourier-transform of $G_E,\ G_M \rightarrow$ spatial distribution (Breit frame)

$$\left\langle r_E^2\right\rangle = -6\hbar^2\,\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\bigg|_{Q^2=0} \qquad \left\langle r_M^2\right\rangle = -6\hbar^2\,\frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Overview of different proton charge-radius results



Filled dots: Results from new measurements.

Hollow dots: Reanalysis of existing data.

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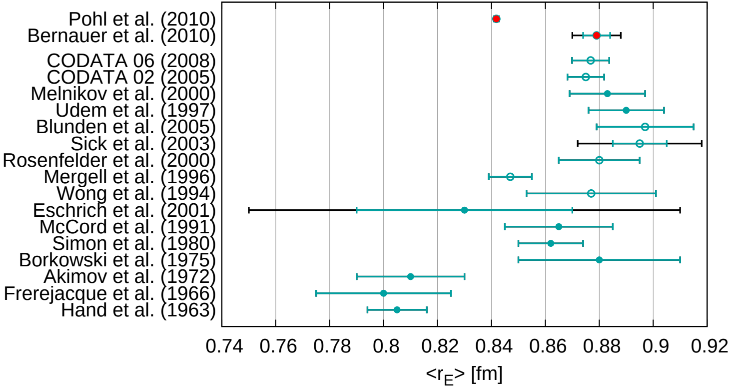


Filled dots: Results from new measurements.

Hollow dots: Reanalysis of existing data.

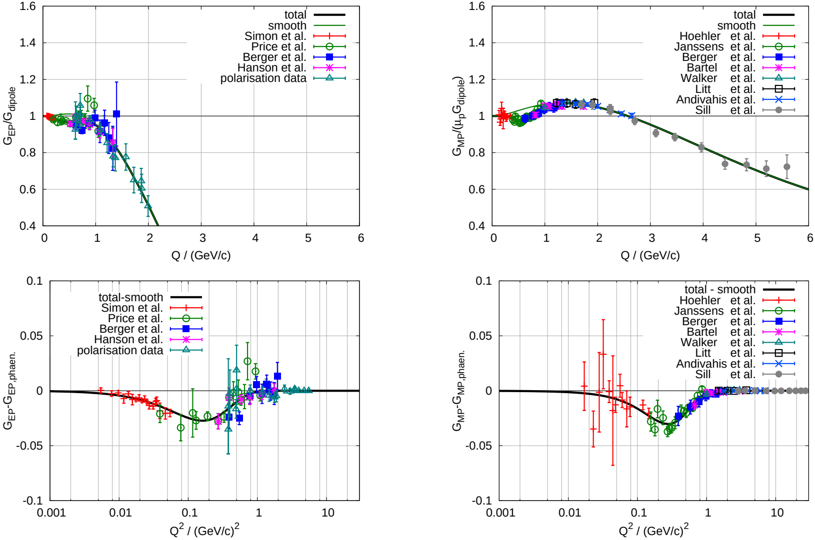
Introduction II: Original Motivation

Form factors from elastic e-p scattering
two methods:

- unpolarized elastic e-p scattering: “Rosenbluth separation” separated $G_E(Q^2)$ and $G_M(Q^2)$, but two photon exchange (TPE)?
- polarized scattering:
  - polarization transfer from electron to proton
  - polarized electrons scattered from polarized targets

  only $G_E(Q^2)/G_M(Q^2)$, but little contribution from two photon exchange (TPE)?
- as always in physics: What precision can be reached?

Introduction II: Original Motivation



(see J. Friedrich and Th. Walcher, Eur. Phys. J. A **17** (2003) 607)

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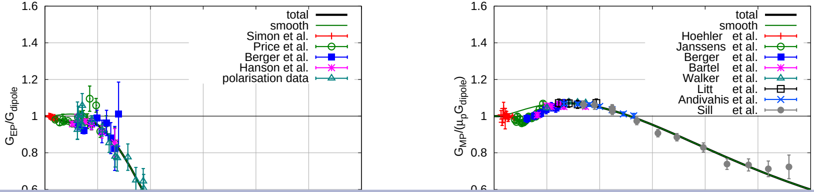
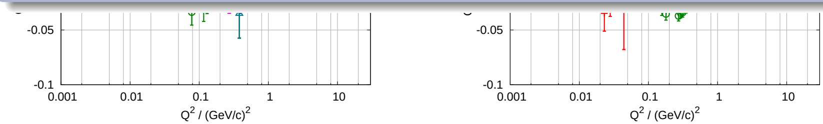


Discrepancy of existing values for proton electric radius:

- 0.809(11) fm: standard dipole at HEPL (Hand et al. 1963)
- 0.862(12) fm: low Q^2 at Mainz (Simon et al. 1979)
- 0.847(09) fm: dispersion relation (Mergell et al. 1996)
- 0.890(14) fm: Hydrogen Lamb shift (Udem et al. 1997)



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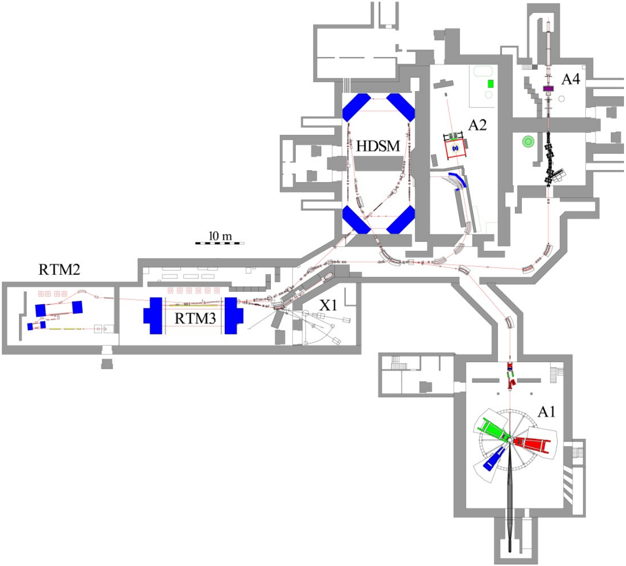
The Mainz Microtron MAMI



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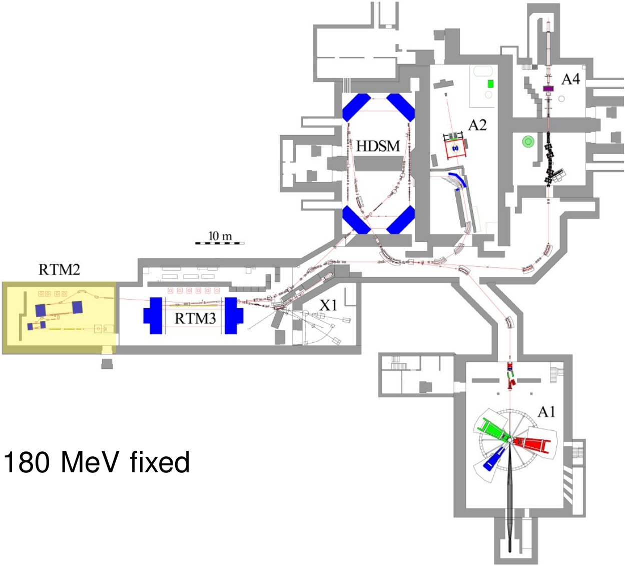

- MAMI-A: 180 MeV fixed

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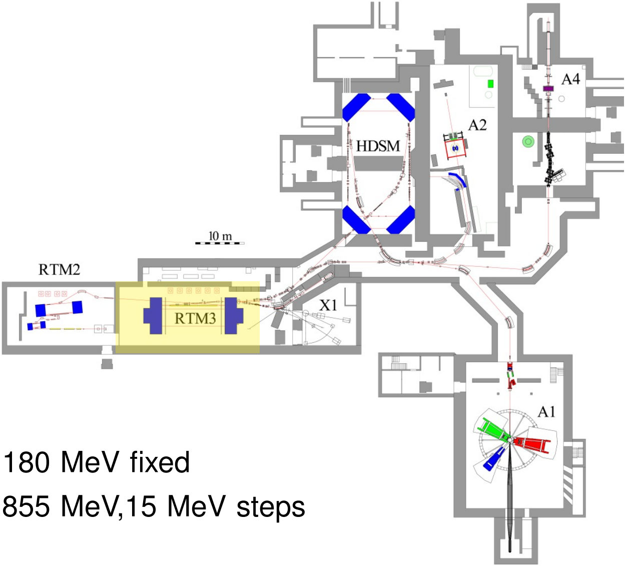

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- MAMI-B: 855 MeV,15 MeV steps

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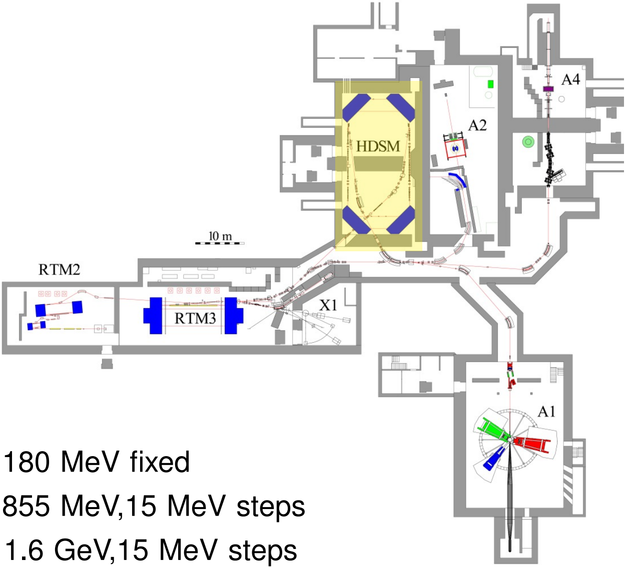

- MAMI-A: 180 MeV fixed
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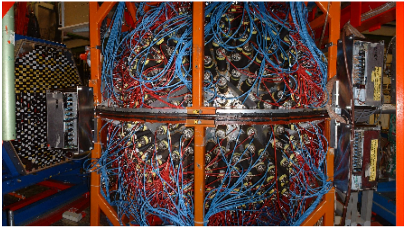

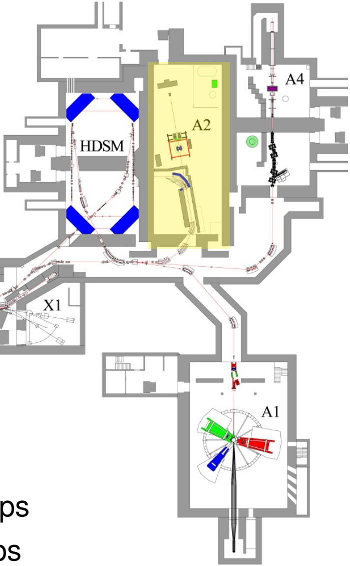
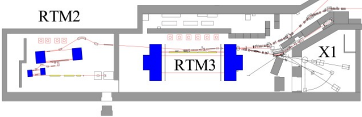

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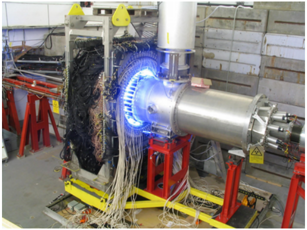

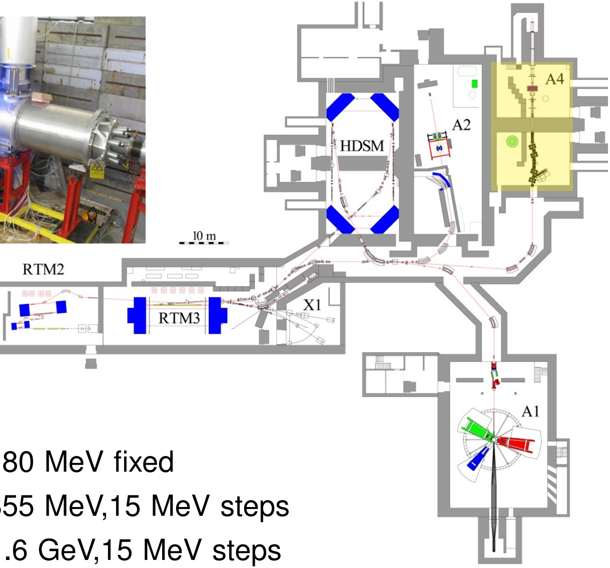

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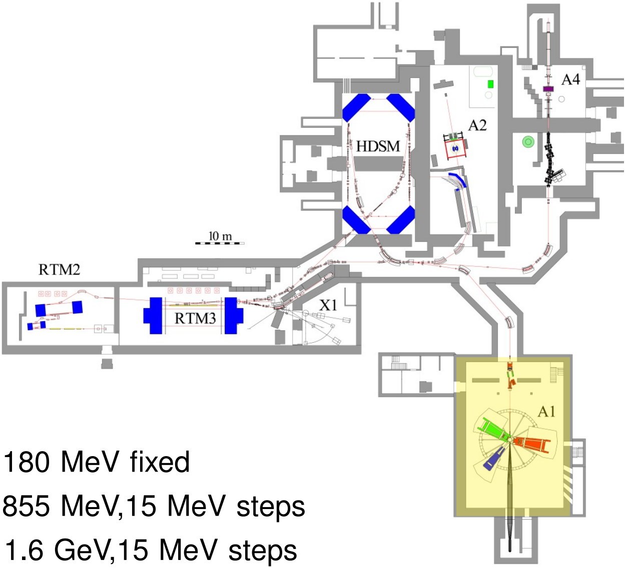

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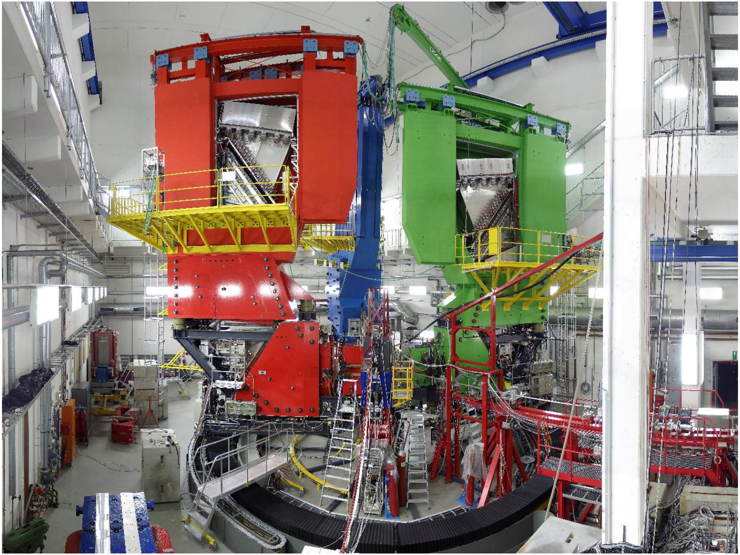
The Mainz high-precision p(e,e')p measurement: Three spectrometer facility of the A1 collaboration



Design goal: High precision

- Statistical precision of cross sections <0.1%,
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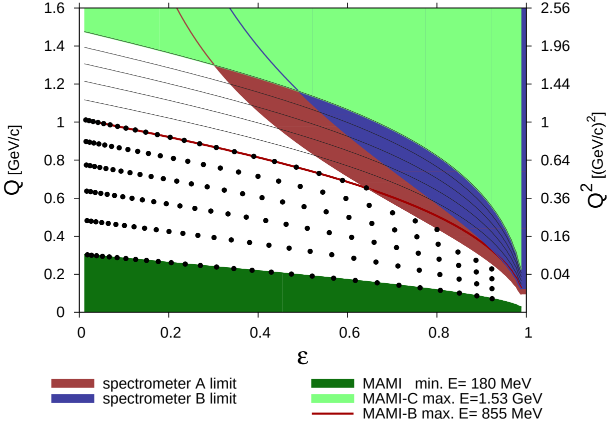
    $\Longleftrightarrow$ third magnetic spectrometer as monitor
  - Overlapping spectrometer acceptances
    most settings measured by different spectrometers more than once

Measured settings and future (high Q^2) expansion

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{\varepsilon\,(1+\tau)} \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$



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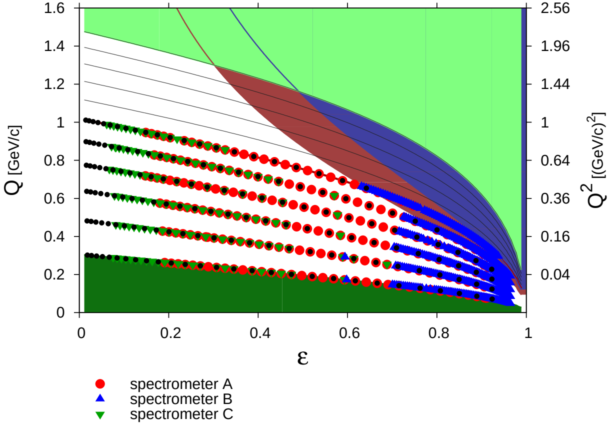
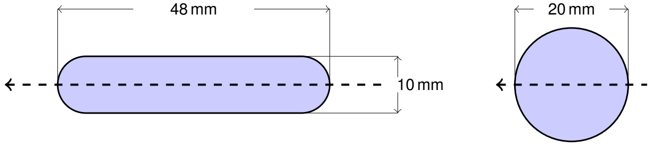


- ~90 GByte on disc
- ~1400 Settings
- $> 10^9$ events.

Background



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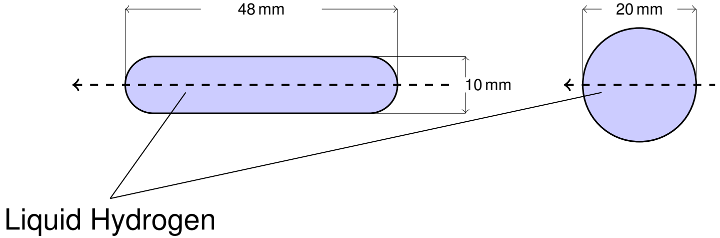

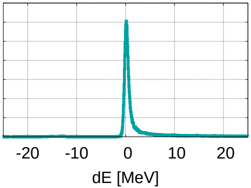

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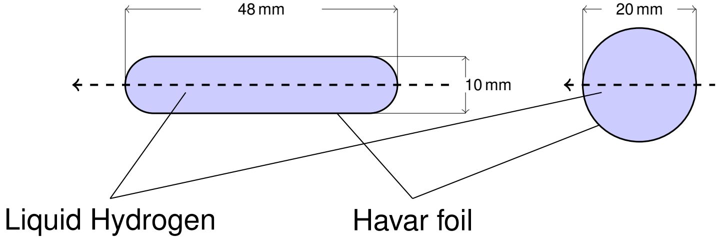
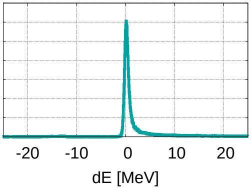
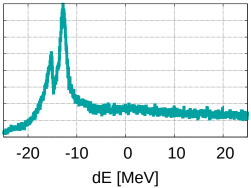


48 mm
20 mm
10 mm
Liquid Hydrogen
Havar foil



-20
-10
0
10
20
dE [MeV]

+



-20
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0
10
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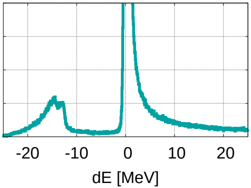
→



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-10
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20
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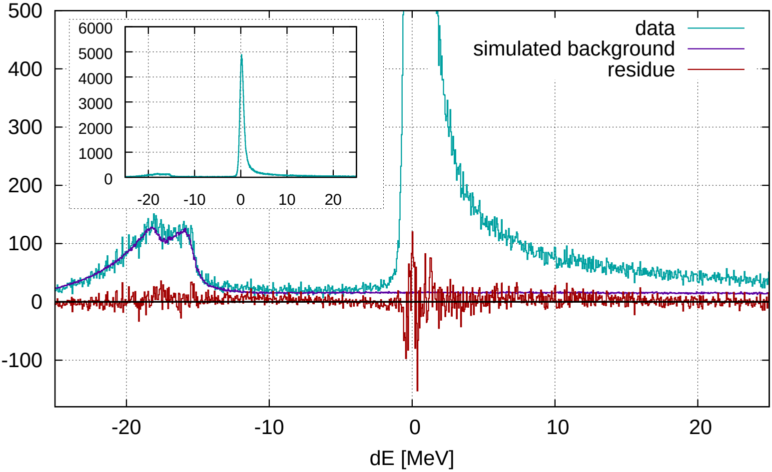
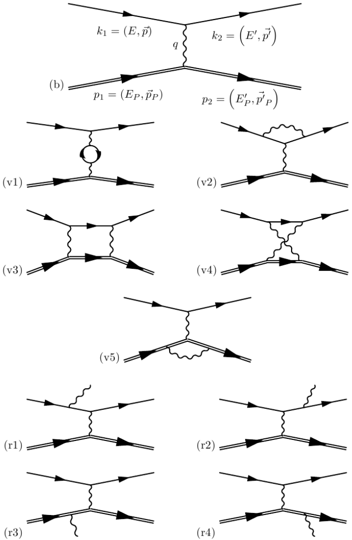
Simulation:

- Model for energy loss and small angle scattering
- Input: momentum-, angular-, vertex resolution



Feynman graphs of leading and next to leading order for elastic scattering



All graphs are taken into account:

- vacuum polarization (v1):
  e, (μ, τ)
  Maximon/Tjon (2000) and Vanderhaeghen et al. (2000)
- electron vertex correction
- Coulomb distortion
  (two photon exchange)
- real photon emission

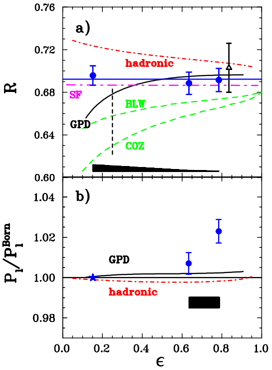
Comments on Coulomb distortion and TPE

- Coulomb distortion:
  Exchange of one hard and multiple soft photons
  Feshbach (1948), Mo and Tsai (1969).

- Two-photon exchange (TPE) with and w/o excited intermediate states:
  Exchange of two hard photons

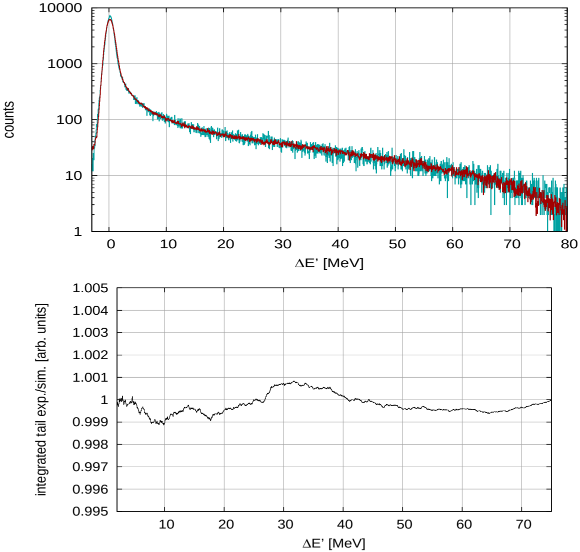
  Still not reliable and highly debated

  Figure shows a recent experimental result from JLab.

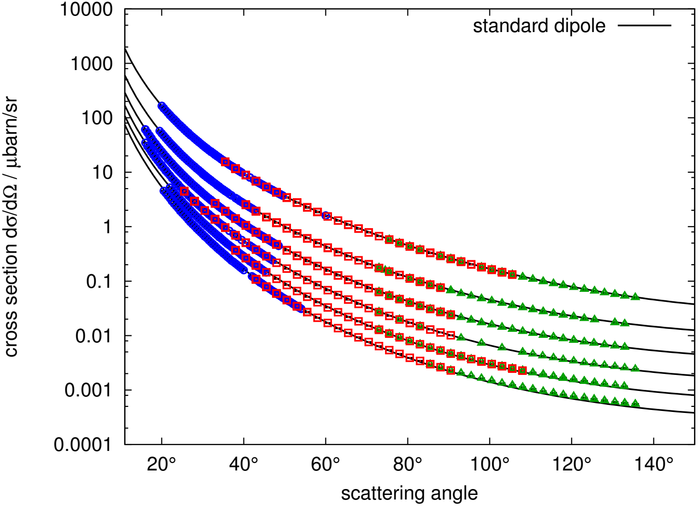


Meziane, M. et al.: *Search for effects beyond the Born approximation in polarization transfer observables in $\vec{e}p$ elastic scattering*, PRL 106, 132501 (2011), arXiv:1012.0339

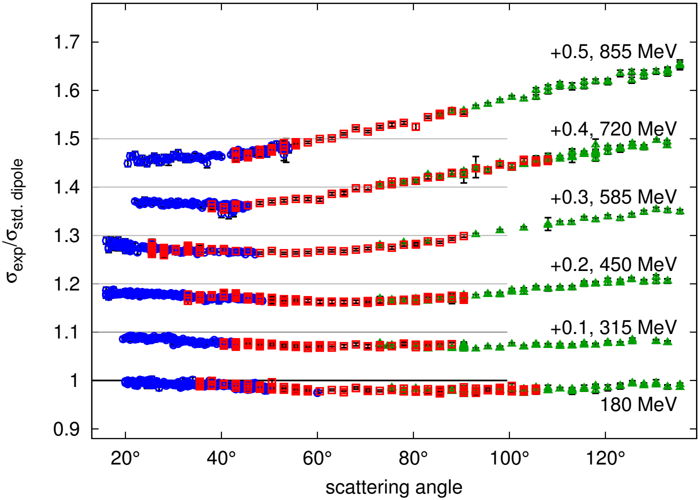
Description of the radiative tail



Cross sections



Cross sections / standard dipole



Extraction of form factors

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   fit of form factor models directly to the cross sections

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- Feasible due to fast computers.
- All data at all Q^2 and ε values contribute to the fit
  no projection to constant Q^2 $\curvearrowright$ no limit of kinematics
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For radii extraction: Needs a fit anyway!
Classical Rosenbluth: Extracted G_E and G_M highly correlated!
$\Longrightarrow$Error propagation very involved.

Extraction of form factors

Short excursion to statistical theory:

two views on χ^2 are possible:

Extraction of form factors

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$\curvearrowright$ "error band" of form factors

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$\curvearrowright$ "error band" of form factors

But, models are not arbitrary: "Priors", i.e. here physical constraints

- "standard dipole" is a first approximation

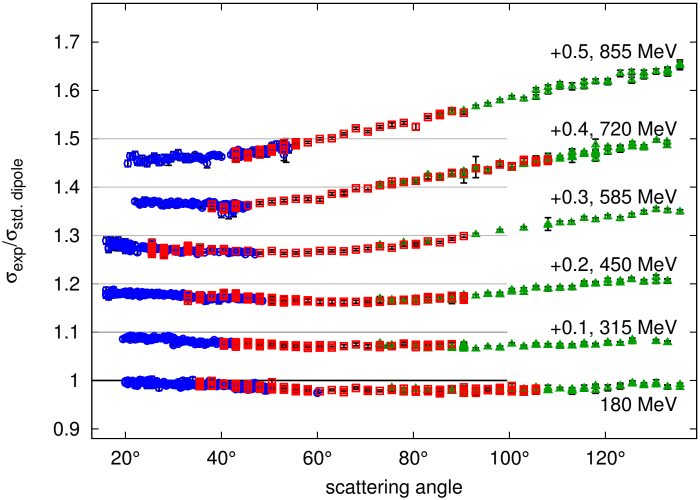
$$G\left(Q^2\right) = \frac{1}{\left(1 + Q^2/0.71\ (\mathrm{GeV/c})^2\right)^2}$$

further decisive consequence of Bayesian approach:

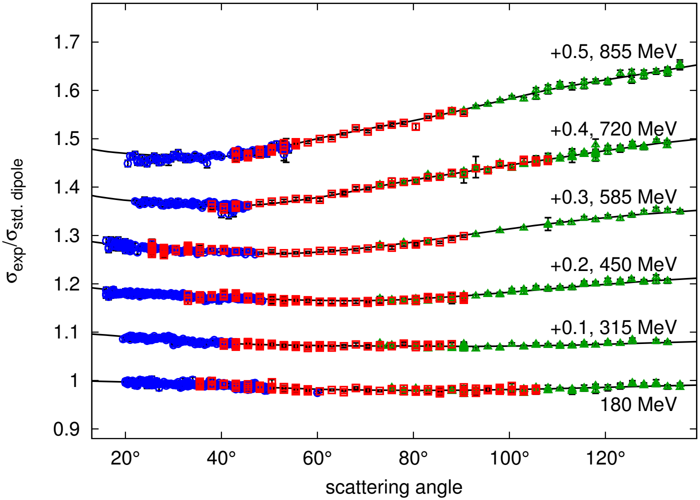
- globally to $G_E(0) = 1$ and $G_M(0)/\mu_p = 1$
- consistent normalization of data subsets, e.g. same incident energy

in MAMI experiment about 15 calibration constants of about 50 total parameters for fitting 1400 points.

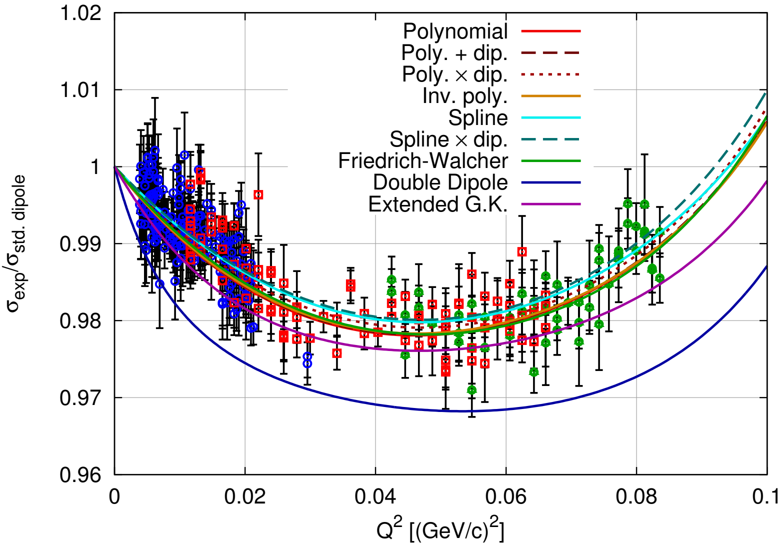
Cross sections / standard dipole



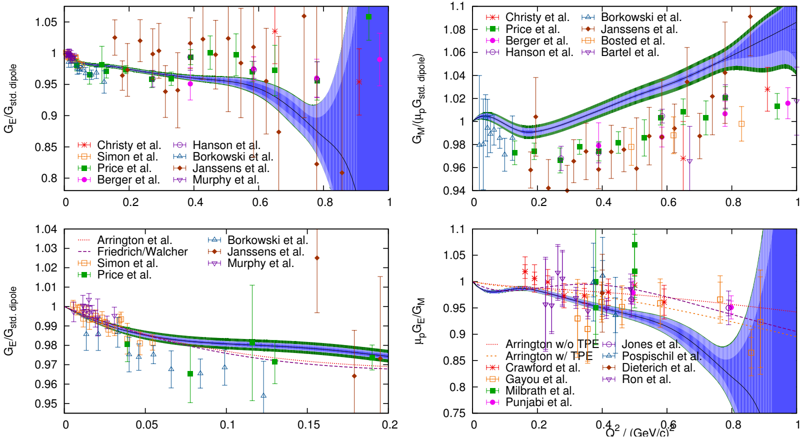
Cross sections + spline fit



Cross sections: 180 MeV



Form factor results

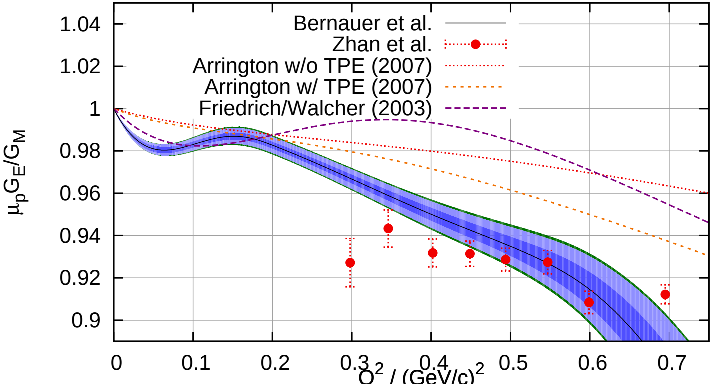


Jan C. Bernauer *et al.*, “High-precision determination of the electric and magnetic form factors of the proton”,
PRL 105, 242001 (2010), arXiv:1007.5076

Form factors from polarization transfer

$$
\begin{aligned}
P_x &= -h\,P_e\,\frac{2\sqrt{\tau(1+\tau)}\tan(\theta/2)G_E G_M}{G_E^2+\tau G_M^2\left(1+2(1+\tau)\tan^2(\theta/2)\right)}\\
P_y &= 0\\
P_z &= h\,P_e\,\frac{2\tau\sqrt{1+\tau+(1+\tau)^2\tan^2(\theta/2)}\tan(\theta/2)G_M^2}{G_E^2+\tau G_M^2\left(1+2(1+\tau)\tan^2(\theta/2)\right)}\\
R &= \frac{G_E}{G_M} = \frac{P_x}{P_z}\sqrt{\tau\left(1+(1+\tau)\tan^2(\theta/2)\right)}
\end{aligned}
$$

Form factor results: G_E/G_M ratio



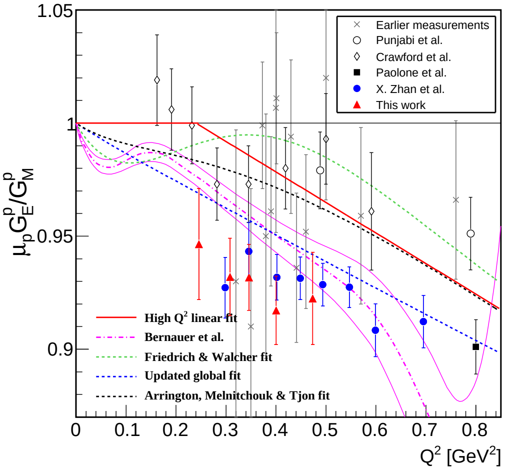
Jan C. Bernauer *et al.*, PRL 105, 242001 (2010), arXiv:1007.5076

X. Zhan *et al.*, arXiv:1102.0318

J. Arrington *et al.*, Phys. Rev. C76 (2007) 035205, arXiv:0707.1861

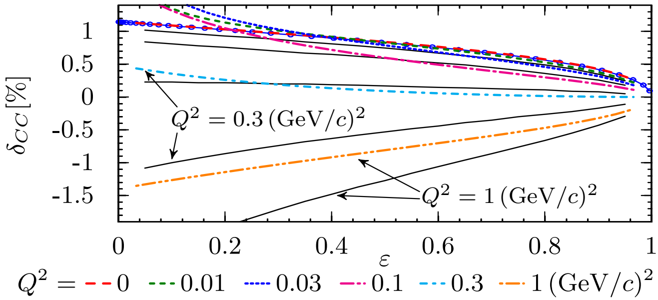
John Arrington's critic

But, the Coulomb correction is wrong, i.e. "not modern"



Ron, G. et al.: *Low Q^2 measurements of the proton form factor ratio $mu_p G_E/G_M$*, arXiv:1103.5784

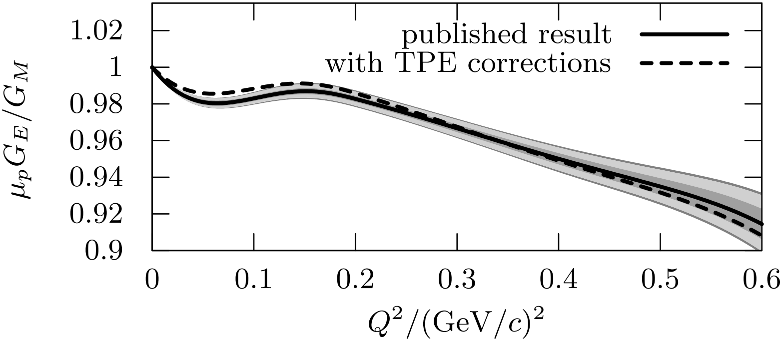
Coulomb corrections by John Arrington



Coulomb corrections $\longleftrightarrow$ two photon exchange TPE

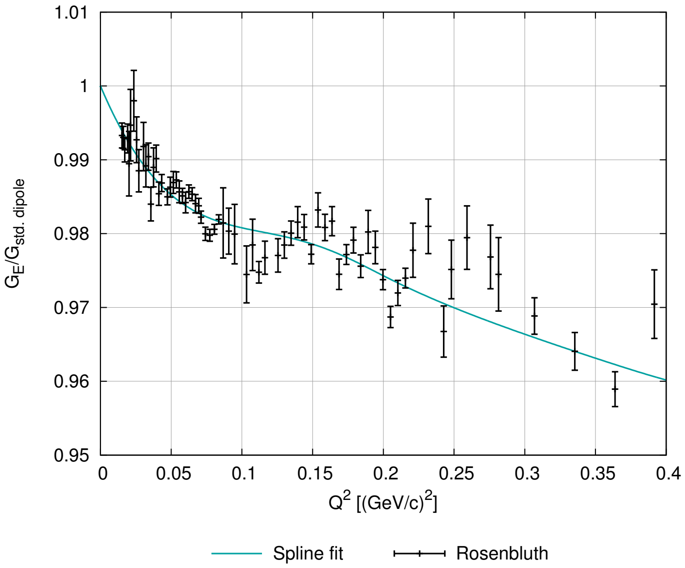
- J. Arrington and I. Sick, Phys. Rev. C **70** (2004) 028203: R. R. Lewis, "Potential Scattering of High-Energy Electrons in Second Born Approximation," Phys. Rev. **102** (1956) 537.
- W. A. McKinley and H. Feshbach, Phys. Rev. **74** (1948) 1759.
- D. Borisyuk and A. Kobushkin, "Two-photon exchange at low Q**2," Phys. Rev. C **75** (2007) 038202 [arXiv:nucl-th/0612104].

Form factor ratio G_E/G_M by J. Bernauer et al. TPE corrected

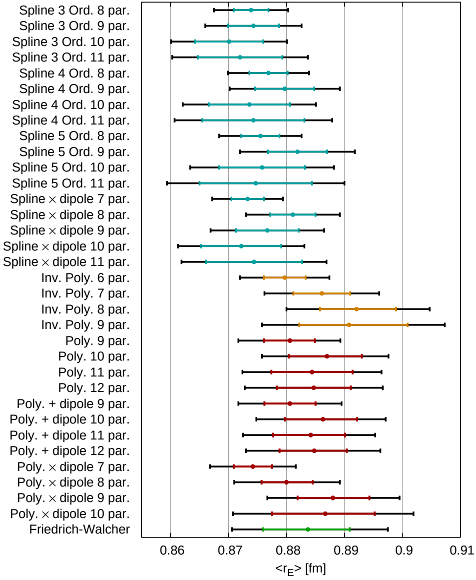


D. Borisyuk and A. Kobushkin, “Two-photon exchange at low Q**2,” Phys. Rev. C **75** (2007) 038202 [arXiv:nucl-th/0612104].

Comparison: Rosenbluth vs. Spline fit



Charge radius of the proton



- Two groups:
  - Splines
  - Polynomials
- No clear cause found
- Similar for r_M, less difference

Additional uncertainty: Half of difference

Radii of the proton from electron scattering

MAMI result with Coulomb correction (McKinley and Feshbach):

$$\begin{aligned}\sqrt{\langle r_e^2 \rangle} &= 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{mod.}} \pm 0.004_{\text{grp}}\ \text{fm},\\ \sqrt{\langle r_m^2 \rangle} &= 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{mod.}} \pm 0.002_{\text{grp}}\ \text{fm}.\end{aligned}$$

MAMI result with TPE correction (Borisyuk and Kobushkin):

$$\begin{aligned}\sqrt{\langle r_e^2 \rangle} &= 0.876 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{mod.}} \pm 0.004_{\text{grp}}\ \text{fm},\\ \sqrt{\langle r_m^2 \rangle} &= 0.803 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{mod.}} \pm 0.002_{\text{grp}}\ \text{fm}.\end{aligned}$$

best fit of world data without MAMI result with TPE by John Arrington:

$$\begin{aligned}\sqrt{\langle r_e^2 \rangle} &= 0.875 \pm 0.008_{\text{exp.}} \pm 0.006_{\text{fit}}\ \text{fm},\\ \sqrt{\langle r_m^2 \rangle} &= 0.867 \pm 0.009_{\text{exp.}} \pm 0.018_{\text{fit}}\ \text{fm}.\end{aligned}$$

Magnetic radius from electronic measurements

- electron scattering

$$\sqrt{\langle r_m^2 \rangle} = 0.777 \pm 0.013_{\mathrm{stat.}} \pm 0.009_{\mathrm{syst.}} \pm 0.005_{\mathrm{mod.}} \pm 0.002_{\mathrm{grp}}\ \mathrm{fm}$$
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- hyperfine splitting in electronic hydrogen

$$\sqrt{\langle r_m^2 \rangle} = 0.778(29)\,\mathrm{fm}$$

A. V. Volotka, V. M. Shabaev, G. Plunien and G. Soff:
“Zemach and magnetic radius of the proton from the hyperfine splitting in hydrogen,”
Eur. Phys. J. D **33** (2005) 23 [arXiv:physics/0405118].

Impact on PV Asymmetries

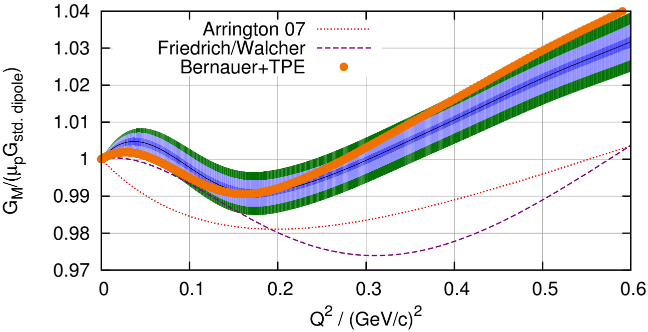
PV asymmetry

$$A_{LR}(\vec{e}p) = A_V + A_s + A_A$$

in elastic scattering cross section of right- and left-handed electrons.

$$\begin{aligned}
A_V &= -a\rho'_{eq}\{(1 - 4\hat{\kappa}'_{eq}\hat{s}_Z^2) - \frac{\epsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2}\}, \\
A_s &= a\rho'_{eq}\frac{\epsilon G_E^p G_E^s + \tau G_M^p G_M^s}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2}, \\
A_A &= a\frac{(1 - 4\hat{s}_Z^2)\sqrt{1-\epsilon^2}\sqrt{\tau(1+\tau)}G_M^p \tilde{G}_A^p}{\epsilon (G_E^p)^2 + \tau (G_M^p)^2}.
\end{aligned}$$

Impact on PV Asymmetries - magn. Form Factor



Conclusion

- High precision e-p scattering data from MAMI.
  PRL 105, 242001 (2010), arXiv:1007.5076.
- Q^2 range from 0.003 to 1 $(\mathrm{GeV/c})^2$.
- Consistent data set.
- "Super-Rosenbluth" fit to determine form factors and radii.
- The charge and magnetic rms radii are determined as

$$\begin{aligned}\langle r_e \rangle &= 0.879 \pm 0.005_{\mathrm{stat.}} \pm 0.004_{\mathrm{syst.}} \pm 0.002_{\mathrm{model}} \pm 0.004_{\mathrm{group}}\ \mathrm{fm},\\ \langle r_m \rangle &= 0.777 \pm 0.013_{\mathrm{stat.}} \pm 0.009_{\mathrm{syst.}} \pm 0.005_{\mathrm{model}} \pm 0.002_{\mathrm{group}}\ \mathrm{fm}.\end{aligned}$$

Supported by the "Deutsche Forschungsgemeinschaft (DFG)" with a "Sonderforschungsbereich (SFB443)".

Conclusion and Outlook

- High precision form factors from MAMI provide constraints for the charge distribution of the proton.
- Standard dipole approximation is not sufficient for correction of the muonic hydrogen Lamb shift.
- The proton size discrepancy is between the Lamb shift of muonic hydrogen and every ”electronic” determination.
- Explanation for the discrepancy?

- Outlook: Low- and high Q^2 measurements @ MAMI
- Lamb shift measurements on D, ^{3,4}He @ PSI
  Form factor and polarizability of D, ^{3,4}He @ MAMI

Backup

Discussion of the Lamb shift / electron scattering discrepancy

- **Muonic hydrogen (Lamb Shift)**
  $r_p = 0.84184(67)\,\mathrm{fm}$
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Discrepancy is between
muonic **and** *electronic* **measurements**

What could be wrong?

The muonic/electronic puzzle of the charge radius

What could be wrong?or Is it “new” physics?

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electron scattering:

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  Part of charge distribution?
- Models don’t extrapolate right to $Q^2 \to 0$?
  But, a plathora of models tried. All give same result.
- Coulomb corrections, resp. two photon exchange (TPE) is incomplete?
  But, effect on charge radius $< r_E >$ is negligible at $Q^2 \lesssim 1$ GeV for all TPE calculations.

Possible explanations of the discrepancy

- **Exotic particles**
  e.g. V. Barger *et al.*, arXiv:1011.3519 and references.
- **Contributions to the Lamb shift in μp**
  C.E. Carlson and M. Vanderhaeghen, arXiv:1101.5965
  U.D. Jentschura, Annals Phys. **326**, 500-515 (2011)
  E. Borie, arXiv:1103.1772
- **Higher moments of the charge distribution and Zemach radii**
  M.O.D., J.C. Bernauer, and Th. Walcher,
  Phys. Lett. **B696**, 343-347 (2011)

Speculation about the discrepancy

muonic hydrogen

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- Perturbative approximation for QED is not valid for bound system?
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  Relativistically covariant Bethe-Salpeter equation needed?
- Hadronic corrections incomplete?
  Only pion and higher masses are considered. What about quarks?

Speculation about the discrepancy

- Reminder: The muon g-2 experiment has a $2-3\sigma$ discrepancy. Hadronic corrections may provide an explanation.
- The main contribution to the **Lamb shift** in ...

Speculation about the discrepancy

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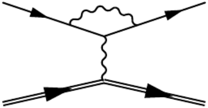
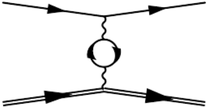
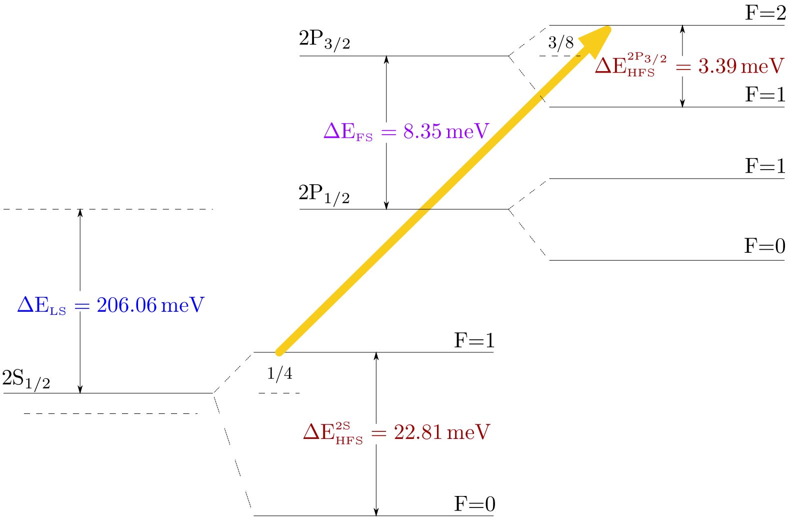
	'electronic' hydrogen	muonic hydrogen
		
vertex and self-energy	$1011.41\,\mathrm{MHz}$	
vacuum polarization	$-27.13\,\mathrm{MHz}$	$-205.028\,\mathrm{meV}$
anom. magn. moment	$67.82\,\mathrm{MHz}$	
+ higher order		
theoretical value	$1057.864(14)\,\mathrm{MHz}$	$-206.057\,\mathrm{meV}$
experimental value	$1057.862(20)\,\mathrm{MHz}$	$\Delta : 0.341\,\mathrm{meV}$

Discussion of the Lamb shift / electron scattering discrepancy

The following tables are taken from the 'QED supplement' published in *Nature* **466**, 213-216 (8 July 2010).
'**All known radius-independent contributions**' and '**all relevant radius-dependent contributions**' to the Lamb shift in μp from different authors are listed.

2S – 2P splitting in muonic hydrogen



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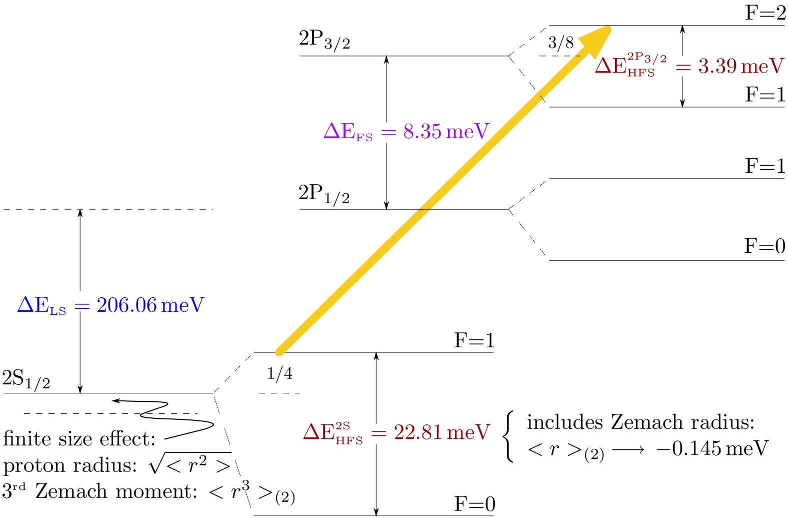

Discussion of the Lamb shift / electron scattering discrepancy

#	Contribution		Our selection		Pachucki[1–3]		Borie[5]	
		Ref.	Value	Unc.	Value	Unc.	Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Discussion of the Lamb shift / electron scattering discrepancy

$$\Delta E = 209.9779(49) - 5.2262\, r_p^2 + 0.0347\, r_p^3$$

Values are in meV and radii in fm.

Contribution	Ref.	our selection		Pachucki[2]	Borie[5]
Leading nuclear size contribution	[26]	-5.19745	$< r_{\mathrm{p}}^2 >$	-5.1974	-5.1971
Radiative corrections to nuclear finite size effect	[2,26]	-0.0275	$< r_{\mathrm{p}}^2 >$	-0.0282	-0.0273
Nuclear size correction of order $(Z\alpha)^6 < r_{\mathrm{p}}^2 >$	[1,27–29]	-0.001243	$< r_{\mathrm{p}}^2 >$		
Total $< r_{\mathrm{p}}^2 >$ contribution		-5.22619	$< r_{\mathrm{p}}^2 >$	-5.2256	-5.2244
Nuclear size correction of order $(Z\alpha)^5$	[1,2]	0.0347	$< r_{\mathrm{p}}^3 >$	0.0363	0.0347

Discussion of the Lamb shift / electron scattering discrepancy

Zemach-Moments:

- A. C. Zemach, *Proton Structure and the Hyperfine Shift in Hydrogen*, Phys. Rev. **104**, 1771 (1956).

$$< r^3 >_{(2)} = \int_0^\infty \frac{dq}{q^4} \left(G_E^2(q^2) - 1 + q^2 < r^2 >_p /3 \right)$$

$$< r^3 >_{(2)} = 2.27\,\mathrm{fm}^3 \qquad \longrightarrow \qquad r_p = 0.84184(67)\,\mathrm{fm}$$

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$$< r^3 >_{(2)} = 2.85(8)\,\mathrm{fm}^3 \quad \longrightarrow \quad r_p = 0.84245(67)\,\mathrm{fm}$$

- M.O.D., J.C. Bernauer, and Th. Walcher, *The RMS Charge Radius of the Proton and Zemach Moments*, in Press doi:10.1016/j.physletb.2010.12.067, arXiv:1011.1861.

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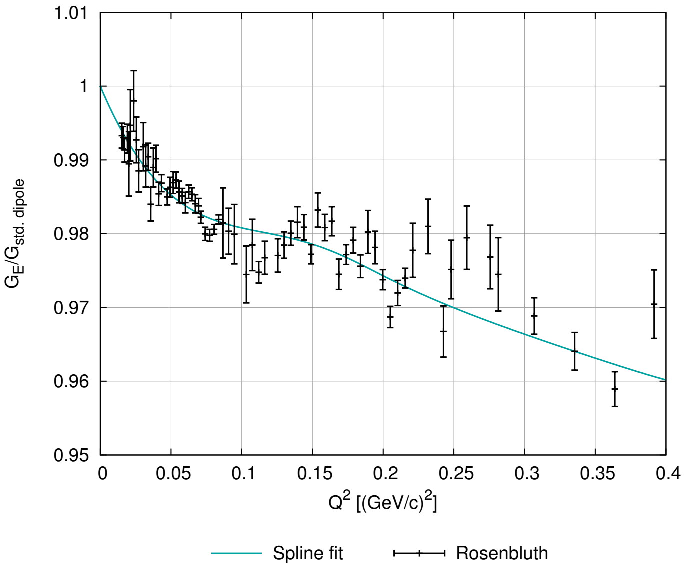

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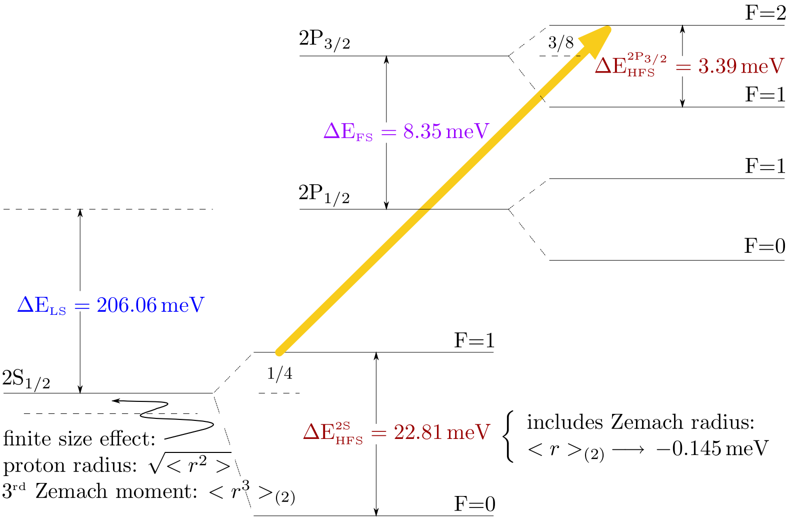

De Rújula's toy model

- A. De Rújula, "QED is not endangered by the proton's size", Phys. Lett. **B693**, 555 (2010).
- **Sum of "single pole" and "dipole"**

$$\rho_{\text{Proton}}(r) = \frac{1}{D}\left[\frac{M^4 e^{-Mr}\cos^2(\theta)}{4\pi r} + \frac{m^5 e^{-mr}\sin^2(\theta)}{8\pi}\right]$$

$$D \equiv M^2\cos^2(\theta) + m^2\sin^2(\theta)$$

using $M = 0.750\,\mathrm{GeV/c^2}$, $m = 0.020\,\mathrm{GeV/c^2}$, and $sin^2(\theta) = 0.3$ and

$$\rho_{(2)}(r) = \int d^3r_2\,\rho_{\mathrm{charge}}(|\vec{r} - \vec{r}_2|)\,\rho_{\mathrm{charge}}(r_2)$$

we get the **third Zemach moment**:

$$\langle r^3\rangle_{(2)} = \int d^3r\, r^3 \rho_{(2)}(r) = 36.2\,\mathrm{fm}^3$$

We put $\langle r^3 \rangle_{(2)} = 36.2\,\mathrm{fm}^3$ in the Lamb shift formular:

$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \left(209.9779(49) - 5.2262\,\frac{\langle r^2 \rangle}{\mathrm{fm}^2} + 0.00913\,\frac{\langle r^3 \rangle_{(2)}}{\mathrm{fm}^3} \right) \mathrm{meV}$$

and get $r_p = 0.878\,\mathrm{fm}$

De Rújula's toy model – ...

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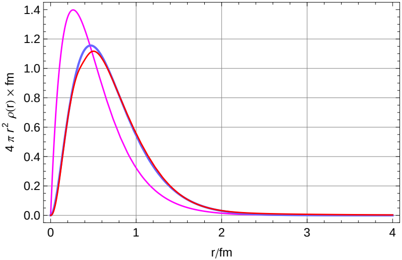
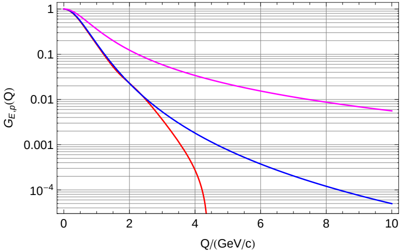
problem solved

De Rújula's toy model – is excluded by experiment

charge distribution

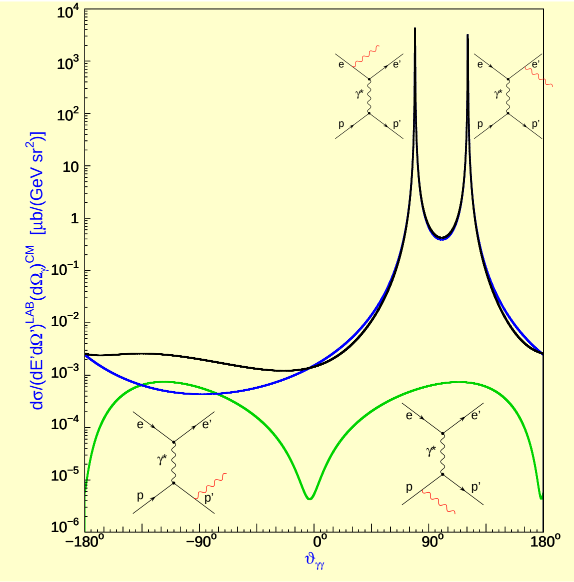


form factor



- De Rújula's toy model
- standard dipole
- Bernauer-Arrington fit assembly

Outlook: Initial state radiation



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