Heavy flavour in high-energy nuclear collisions: overview of transport calculations

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Heavy Flavour in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why);
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NB At high-p_T the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)

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NB for realistic temperatures $g \sim 2$, so that one can wonder *whether a charm is really "heavy"*, at least in the initial stage of the evolution.

Heavy quarks as probes of the QGP

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    - However, *a source of systematic uncertainty for studies of parton-medium interaction*;
- Hadronic rescattering (e.g. $D\pi \to D\pi$), from effective Lagrangians, but no experimental data the on relevant cross-sections

Transport theory: general setup

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$[1]:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

  Neglecting **x**-dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k} [\underbrace{w(\mathbf{p}+\mathbf{k}, \mathbf{k}) f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}}]$$

  $w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p} - \mathbf{k}$

[1] Approach adopted by Catania, Nantes, Frankfurt, LBL... groups

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[2] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

[2] B. Svetitsky, PRD 37, 2484 (1988)

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The Boltzmann equation reduces to the *Fokker-Planck equation*

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = (\delta^{ij} - \hat{p}^i \hat{p}^j) B_0(p) + \hat{p}^i \hat{p}^j B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*, directly derived from the scattering matrix

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Approach to equilibrium in the FP equation

The FP equation can be viewed as a continuity equation for the phase-space distribution of the kind $\partial_t\rho(t,\vec{p})+\vec{\nabla}_p\cdot\vec{J}(t,\vec{p})=0$

$$\frac{\partial}{\partial t}\underbrace{f_Q(t,\mathbf{p})}_{\equiv\rho(t,\vec{p})}=\frac{\partial}{\partial p^i}\underbrace{\left\{A^i(\mathbf{p})f_Q(t,\mathbf{p})+\frac{\partial}{\partial p^j}[B^{ij}(\mathbf{p})f_Q(t,\mathbf{p})]\right\}}_{\equiv -J^i(t,\vec{p})}$$

admitting a steady solution $f_{\rm eq}(p)\equiv e^{-E_p/T}$ when the current vanishes:

$$A^i(\vec{p})f_{\rm eq}(p)=-\frac{\partial B^{ij}(\vec{p})}{\partial p^j}f_{\rm eq}(p)-B^{ij}(\mathbf{p})\frac{\partial f_{\rm eq}(p)}{\partial p^j}.$$

One gets

$$A(p)p^i=\frac{B_1(p)}{TE_p}p^i-\frac{\partial}{\partial p^j}\left[\delta^{ij}B_0(p)+\hat{p}^i\hat{p}^j(B_1(p)-B_0(p))\right],$$

leading to the Einstein *fluctuation-dissipation* relation

$$A(p)=\frac{B_1(p)}{TE_p}-\left[\frac{1}{p}\frac{\partial B_1(p)}{\partial p}+\frac{d-1}{p^2}(B_1(p)-B_0(p))\right],$$

quite involved due to the *momentum dependence* of the transport coefficients (*measured* HQ's are relativistic particles!)

The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark arising from the pQCD Monte Carlo simulation of the initial $Q\overline{Q}$ production: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \rangle = 0 \quad \langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p})\frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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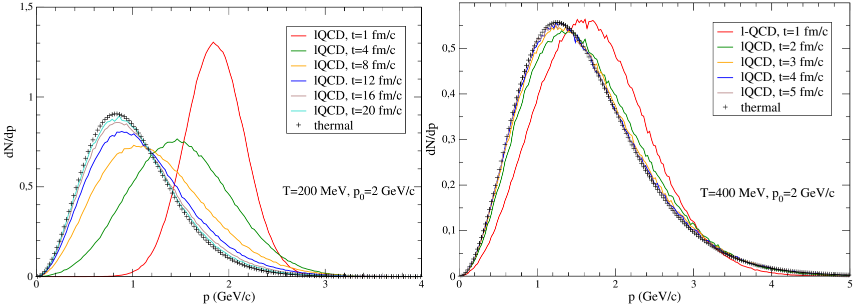
Transport coefficients related to the FP ones:

- *Momentum diffusion*: $\kappa_T(p) = 2B_0(p)$ and $\kappa_L(p) = 2B_1(p)$
- *Friction* term, in the *Ito pre-point discretization scheme*,

$$\eta_D^{\text{Ito}}(p) = A(p) = \frac{B_1(p)}{TE_p} - \left[\frac{1}{p}\frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2}(B_1(p) - B_0(p))\right]$$

A first check: thermalization in a static medium



(Test with a sample of c quarks with $p_0 = 2$ GeV/c).
For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution

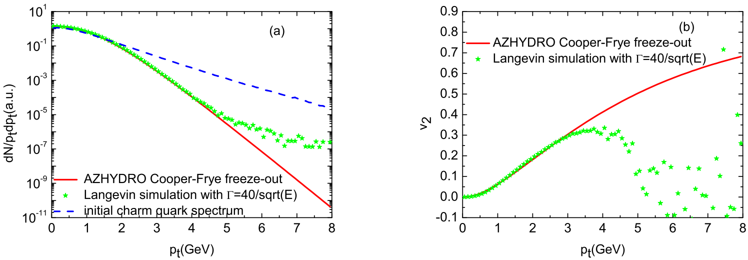
$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

The larger κ ($\kappa \sim T^3$), the faster the approach to thermalization.

Expanding fireball: testing the algorithm

In the limit of large transport coefficients heavy quarks should reach local thermal equilibrium and decouple from the medium as the other light particles, according to the Cooper-Frye formula:

$$E(dN/d^3p) = \int_{\Sigma_{\rm fo}} \frac{p^\mu \cdot d\Sigma_\mu}{(2\pi)^3} \exp[-p\cdot u/T_{\rm fo}]$$



This was verified to be actually the case (M. He, R.J. Fries and R. Rapp, PRC 86, 014903).

Transport coefficients

Transport coefficients: non-perturbative definition

One consider the non-relativistic limit of the Langevin equation for a HQ

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

in which the strength of the noise is given by a single number, the momentum-diffusion coefficient κ. Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\mathrm{HQ}} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t)F^i(0)\rangle_{\mathrm{HQ}}}_{\equiv D^>(t)},$$

For a static ($M = \infty$) HQ the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x}) t^a Q(t,\mathbf{x}) \mathbf{E}^a(t,\mathbf{x})$$

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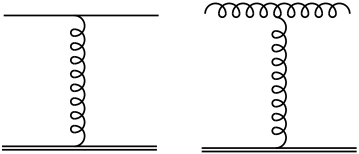
The above non-perturbative definition, referring to the $M \to \infty$ limit, is the starting point for a thermal-field-theory evaluation based on

- weak-coupling calculations (up to NLO);
- gauge-gravity duality ($\mathcal{N}=4$ SYM)
- lattice-QCD simulations

HQ momentum diffusion: weak-coupling calculation



In the $M \to \infty$ limit the HQ exchange momentum $q^\mu = (0, \vec{q})$, with $q \sim gT$, with the medium partons. The exchanged soft gluon is dressed by the Debye mass $m_D \sim gT$, which screens IR divergences

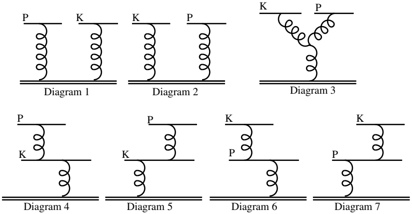
$$\kappa^{\mathrm{LO}} \equiv \frac{g^4 C_F}{12\pi^3} \int_0^\infty k^2 dk \int_0^{2k} \frac{q^3 dq}{(q^2 + m_D^2)^2}$$
$$\times \left[N_c n_B(k)(1+n_B(k)) \left(2 - \frac{q^2}{k^2} + \frac{q^4}{4k^2}\right) + N_f n_F(k)(1-n_F(k)) \left(2 - \frac{q^2}{2k^2}\right) \right]$$

Under the assumption that $q \ll k \sim T$ one can "expand" the results in a weak-coupling series

$$\kappa = \frac{C_F g^4 T^3}{18\pi} \left(\left[N_c + \frac{N_f}{2} \right] \left[\ln \frac{2T}{m_D} + \xi \right] + \frac{N_f \ln 2}{2} + \mathcal{O}(g) \right)$$

with the structure $\kappa \sim g^4 T^3 (\# \ln(1/g) + \# + \mathcal{O}(g))$, clearly meaningful only as long as $g \ll 1$.

HQ momentum diffusion: weak-coupling calculation



The weak-coupling expansion for κ receives $\mathcal{O}(g)$ corrections of various origin (S. Caron-Huot and G.D. Moore, JHEP 0802 (2008) 081):

- one part is contained in the unexpanded tree-level result, arising from the region $k \sim gT$ in which $n_B(k) \sim T/k \sim 1/g$ and the approximation $q \ll k$ no longer holds;
- another part arises from a NLO correction to the screened gluon propagator, which can be easily inserted in the tree-level result;
- a last part comes from overlapping statterings. Having a total scattering rate $\sim g^2 T$ and the duration of a single scattering $\sim 1/q \sim 1/gT$ entails that a fraction $\mathcal{O}(g)$ of scattering events overlap with each other (see diagrams).

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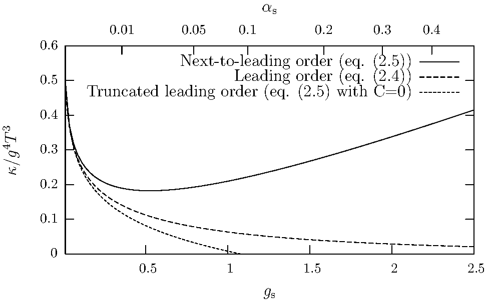

Collecting together the various terms one gets, for $N_f = N_c = 3$,

$$\kappa = \frac{16\pi}{3}\alpha_s^2 T^3 \left(\ln\frac{1}{g} + 0.07428 + 1.9026g + \mathcal{O}(g^2)\right)$$

which shows that, for realistic values of the coupling $\alpha_s \sim 0.3$, NLO corrections to κ are positive and large: what's the range of validity of a weak-coupling expansion if NLO corrections are so large?

HQ momentum diffusion from guage-gravity duality

Based on the AdS-CFT correspondence (Maldacena conjecture), the HQ momentum-diffusion coefficient was calculated in the strong-coupling regime in $\mathcal{N}=4$ SYM getting[3]

$$\kappa = \sqrt{\lambda}\pi T^3 \quad \text{with} \quad \lambda \equiv g^2_{SYM} N_c$$

Notice that, at variance with the $\eta/s = 1/4\pi$ ratio, for which the AdS-CFT conjecture provides a pure number as a result, here one gets a dependence on the coupling and the temperature, so that extending the prediction to QCD one gets an ambiguity from how to do exactly the mapping! Naively one can assume $T_{SYM} = T_{QCD}$, $g_{SYM} = g_s$ and $N_c = 3$, so that $g^2_{SYM} N_c = 12\pi\alpha_s$, but other choices are possible.

[3] J. Casalderrey-Solana and D. Teaney, PRD 74 (2006) 085012; C.P. Herzog *et al.*, JHEP 0607 (2006) 013; S.S. Gubser, NPB 790 (2008) 175-199

HQ momentum diffusion from lattice-QCD

The $(p \to 0)$ HQ momentum-diffusion coefficient

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is given by the $\omega \to 0$ limit of the FT of the electric-field correlator $D^>$. In a thermal ensemble, from the periodicity of the bosonic fields, one has $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega})D^>(\omega)$, so that

$$\kappa \equiv \lim_{\omega\to 0} \frac{D^>(\omega)}{3} = \lim_{\omega\to 0} \frac{1}{3}\frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

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On the lattice one evaluates then the *euclidean electric-field correlator* $(t = -i\tau)$

$$D_E(\tau) = -\frac{\langle {\rm Re\,Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle {\rm Re\,Tr}[U(\beta,0)]\rangle}$$

and from the latter one extract the spectral density according to

$$D_E(\tau) = \int_0^{+\infty}\frac{d\omega}{2\pi}\frac{\cosh(\tau-\beta/2)}{\sinh(\beta\omega/2)}\sigma(\omega)$$

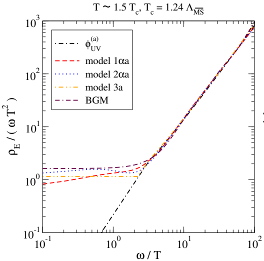
HQ momentum diffusion from lattice-QCD

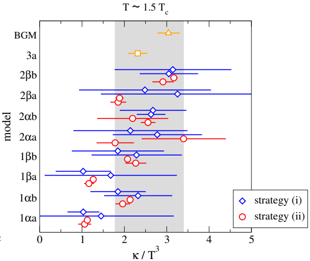
The direct extraction of the spectral density from the euclidean correlator

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

is a ill-posed problem, since the latter is known for a limited set (~ 20) of points $D_E(\tau_i)$, and one wish to obtain a fine scan of the the spectral function $\sigma(\omega_j)$. A direct χ^2-fit is not applicable. Possible strategies:

- Bayesian techniques (Maximum Entropy Method)
- Theory-guided ansatz for the behaviour of $\sigma(\omega)$ to constrain its functional form (A. Francis *et al.*, PRD 92 (2015), 116003)





From the different ansatz on the functional form of $\sigma(\omega)$ one gets a systematic uncertainty band:

$$\kappa/T^3 \approx 1.8 - 3.4$$

Collisional broadening in the non-static case

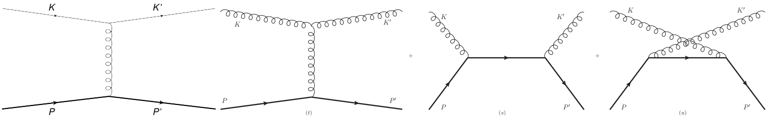
In the case of experimental interest HQ's have a large but finite mass and most of the p_T-bins for which data are available refer to quite fast, or even relativistic, HF hadrons: extending the estimates for the HQ transport coefficients to finite momentum is mandatory to provide theoretical predictions relevant for the experiment.

The effect of $2 \to 2$ collisions can be included in an "improved" tree-level calculation (W.M. Alberico *et al.*, EPJC 73 (2013) 2481) with an *Intermediate cutoff* $|t|^* \sim m_D^2$[4] separating the contributions of

- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation
- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation (*resummation of medium effects*)

[4]Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008)

Transport coefficients $\kappa_{T/L}(p)$: hard contribution



$$\kappa_T^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$

$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

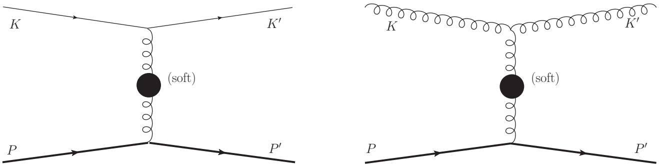
$$\kappa_L^{g/q(\mathrm{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$

$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

where: $(|t| \equiv q^2 - \omega^2)$.

NB At high momentum also Compton-like diagrams give a non-negligible contribution ($\neq$ static calculation)

Transport coefficients $\kappa_{T/L}(p)$: soft contribution



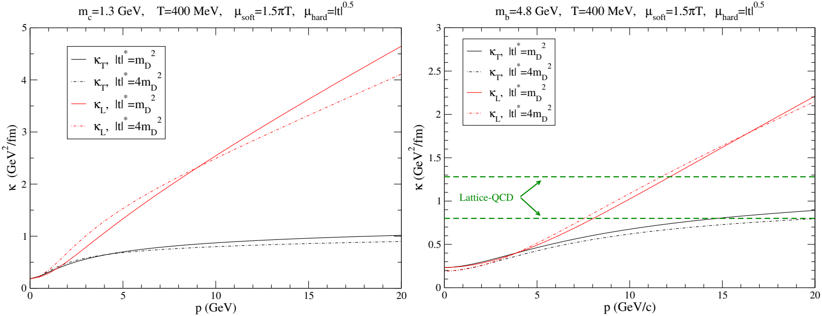
When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.
NB In the corresponding static calculation only longitudinal gluon exchange, dressed simply by a Debye mass, without any energy and momentum dependence

Transport coefficients: numerical results

Combining together the hard and soft contributions...



...the dependence on the intermediate cutoff $|t|^*$ is very mild!

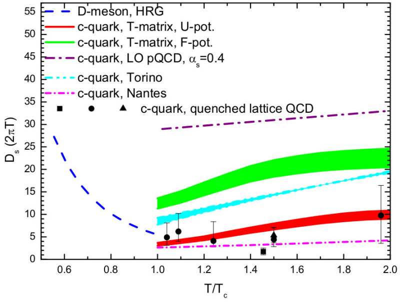
NB Notice, in the case of charm, the strong momentum-dependence of κ_L, much milder in the case of beauty, for which $\kappa_L \approx \kappa_T$ up to 5 GeV

Spatial diffusion coefficient D_s

In the *non-relativistic* limit an excess of HQ's initially placed at the origin will diffuse according to

$$\langle \vec{x}^2(t)\rangle \underset{t\to\infty}{\sim} 6D_s t \quad \text{with} \quad D_s = \frac{2T^2}{\kappa}.$$

For a strongly interacting system spatial diffusion is very small! Theory calculations for D_s have been collected (F. Prino and R. Rapp, JPG 43 (2016) 093002) and are often used by the experimentalists to summarize the difference among the various models (BUT momentum dependence, not captured by D_s, is important!)



- lattice-QCD

$$(2\pi T)D_s^{lQCD} \approx 3.7 - 7$$

- $\mathcal{N}=4$ SYM:

$$(2\pi T)D_s^{SYM} = \frac{4}{\sqrt{g^2_{SYM}N_c}} \approx 1.2$$

for $N_c = 3$ and $\alpha_{SYM} = \alpha_s = 0.3$.

In-medium hadronization

From quarks to hadrons

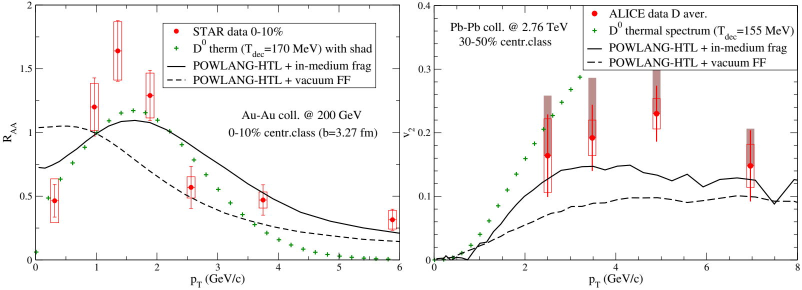
In the presence of a medium, rather then fragmenting like in the vacuum (e.g. $c \to cg \to c\overline{q}q$), HQ's can hadronize by recombining with light thermal quarks (or even *diquarks*) from the medium. This has been implemented in several ways in the literature:

- $2 \to 1$ (or $3 \to 1$ for baryon production) coalescence of partons close in phase-space: $Q + \overline{q} \to M$
- String formation: $Q + \overline{q} \to \mathrm{string} \to \mathrm{hadrons}$
- Resonance formation/decay $Q + \overline{q} \to M^\star \to Q + \overline{q}$

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective (radial and elliptic) flow* of light quarks. Furthermore, it can change the HF hadrochemistry, leading for instance to and enhanced productions of strange particles (D_s) and baryons (Λ_c): no need to excite heavy $s\overline{s}$ or diquark-antidiquark pairs from the vacuum as in elementary collisions, a lot of thermal partons available nearby! Selected results will be shown in the following.

From quarks to hadrons: *kinematic* effect on R_{AA} and v_2

Experimental D-meson data show a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: kinetic equilibrium, decoupling from FO hypersurface)



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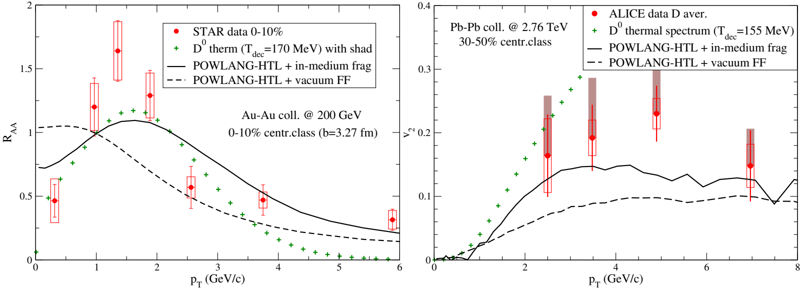
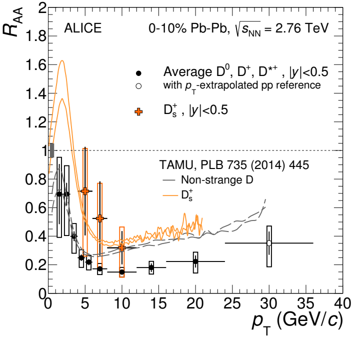


However, *comparing transport results with/without the boost* due to $u^{\mu}_{\rm fluid}$, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization (POWLANG results A.B. et al., in EPJC 75 (2015) 3, 121).

From quarks to hadrons: HF hadrochemistry (I)

The abundance of strange quarks in the plasma can lead e.g. to an enhanced production of D_s mesons wrt p-p collisions via $c + \bar{s} \rightarrow D_s$



ALICE data for D and D_s mesons (JHEP 1603 (2016) 082) compared with TAMU-model predictions (M- He et al., PLB 735 (2014) 445)

Langevin transport simulation in the QGP + hadronization modeled via

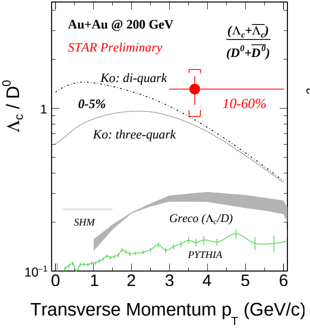
$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = -\underbrace{(\Gamma/\gamma_p) F_M(t, \vec{x}, \vec{p})}_{M \rightarrow Q + \bar{q}} + \underbrace{\beta(t, \vec{x}, \vec{p})}_{Q + \bar{q} \rightarrow M}$$

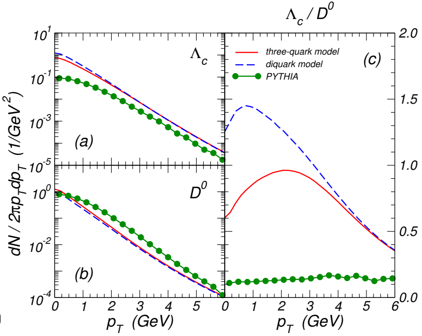
$$\text{with} \quad \sigma(s) = \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$$

From quarks to hadrons: HF hadrochemistry (II)

A possible enhanced production of Λ_c baryons in AA collisions (also from the feed-down of its excited states), may occur via coalescence with

- quarks: $Q + q + q \to \Lambda_Q$
- possible diquarks: $Q + (qq) \to \Lambda_Q$





Preliminary STAR data (arXiv:1704.04364) are compared with coalescence model predictions (C.M. Ko *et al.*, PRC 79 (2009) 044905).

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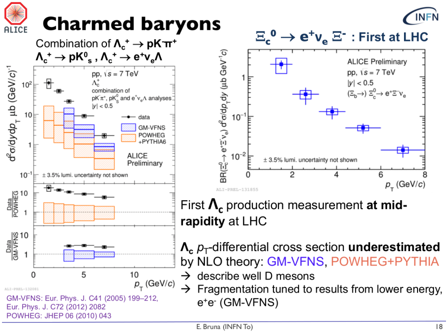

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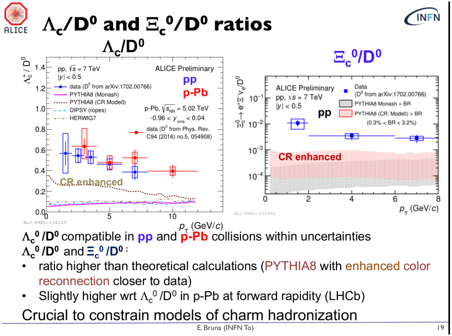

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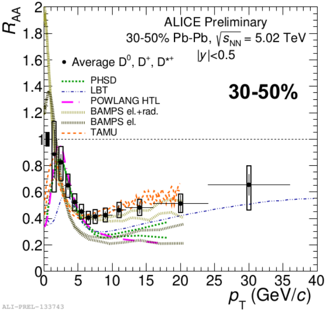
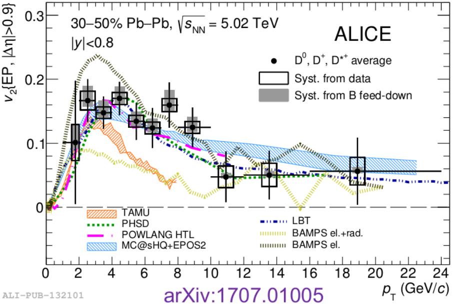
The challenge: extracting the transport coefficients

R_{AA} and v_2 : constraints to models





arXiv:1707.01005

Models where charm quarks pick up collective flow via recombination and/or subsequent elastic collisions in expanding hydrodynamic medium do better at describing both R_{AA} and v_2 at low p_T (BAMPS elastic, LBT, MC@sHQ+EPOS, TAMU, POWLANG, PHSD)

Models provide:

→ Diffusion coefficient $2\pi T D_s(T) \approx 1.5$–$7$ at critical temperature T_c

→ Charm thermalization time τ_{charm}~3-14 fm/c

Some new predictions

In the following, some new predictions by the POWLANG setup[5] will be shown, mostly focused on

- HF observables in small systems
- Higher flow harmonic (v_2, v_3)
- Time-development of flow

and compared to experimental data and independent theoretical studies

[5] A.B. *et al.*, EPJC 75 (2015) no.3, 121 and JHEP 1603 (2016) 123 + work in progress

The importance of event-by-event fluctuations

The study of small systems and of higher flow-harmonics in AA collisions requires a modeling of initial-state event-by-event fluctuations. We perform a Glauber-MC sampling of the initial conditions, each one characterized by a *complex eccentricity*

$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\rm coll}} \exp\left[-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}\right] \quad \longrightarrow \quad \epsilon_m e^{im\Psi_m} \equiv -\frac{\left\{r^2 e^{im\phi}\right\}}{\left\{r^2\right\}}$$

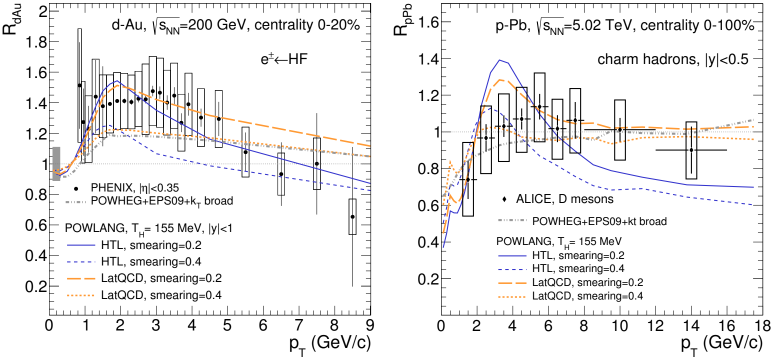
with orientation and modulus given by

$$\Psi_m = \frac{1}{m}\,\mathrm{atan2}\left(-\{r^2\sin(m\phi)\}, -\{r^2\cos(m\phi)\}\right)$$

$$\epsilon_m = \frac{\sqrt{\{r_\perp^2\cos(m\phi)\}^2 + \{r_\perp^2\sin(m\phi)\}^2}}{\{r_\perp^2\}} = -\frac{\{r^2\cos[m(\phi-\Psi_m)]\}}{\{r^2\}}$$

Exploiting the fact that, on an event-by-event basis, for $m = 2, 3$ $v_m \sim \epsilon_m$ one can again consider an *average background* obtained summing all the events of a given centrality class, each one rotated by its *event-plane* angle ψ_m, depending on the harmonics one is considering.

Heavy-flavor in small systems: model predictions

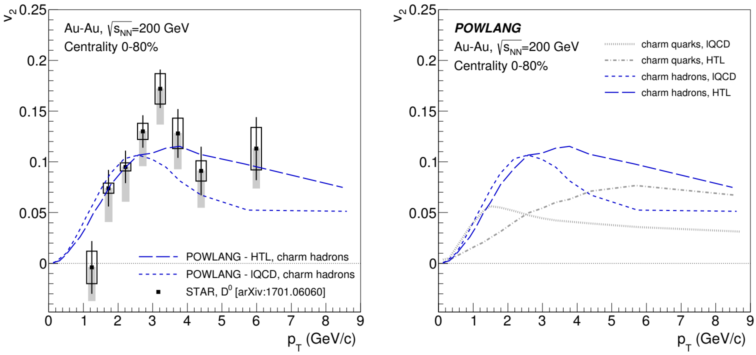


We display our predictions[6], with different initializations (source smearing) and transport coefficients (HTL vs lQCD), compared to

- HF-electron R_{dAu} by PHENIX at RHIC (left panel)
- D-mesons R_{pPb} by ALICE at the LHC (right panel)

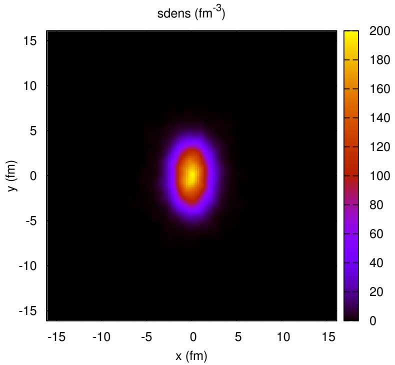
[6] A.B. et al., JHEP 1603 (2016) 123

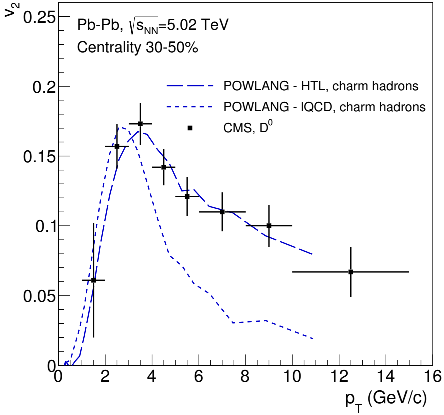
New results at 200 GeV: D-meson v_2 in Au-Au



Comparison with STAR data has been extendend to the *D*-meson elliptic flow in the 0-80% centrality class. Notice the differences *at the partonic level* between different transport coefficients (weak-coupling vs non-perturbative). In-medium hadronization necessary in order to reproduce the data.

New results at 5.02 TeV: D-meson v_2 and v_3 in Pb-Pb





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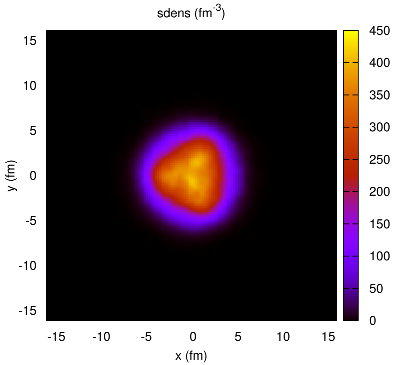

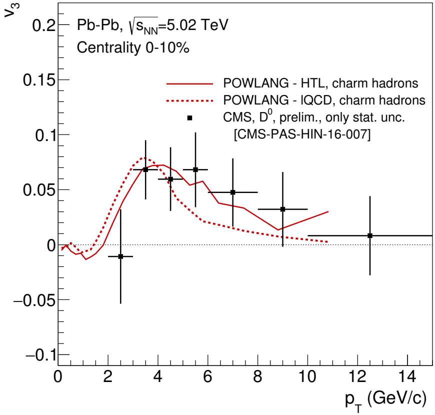

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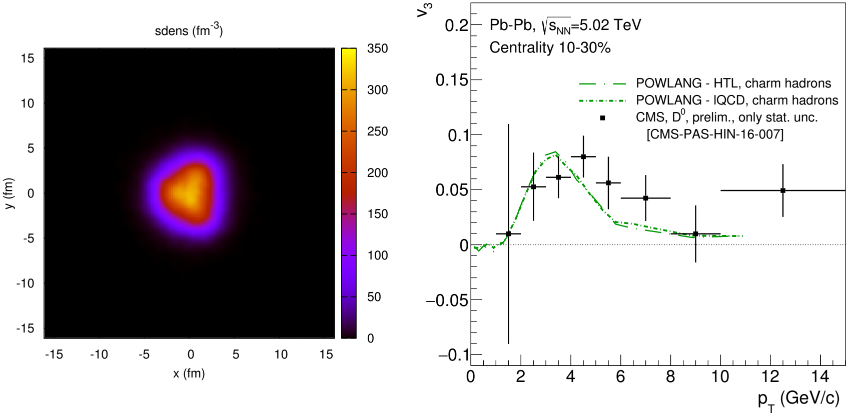

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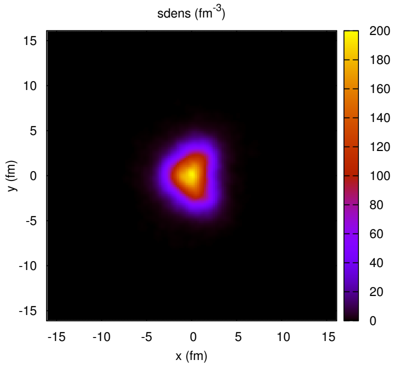

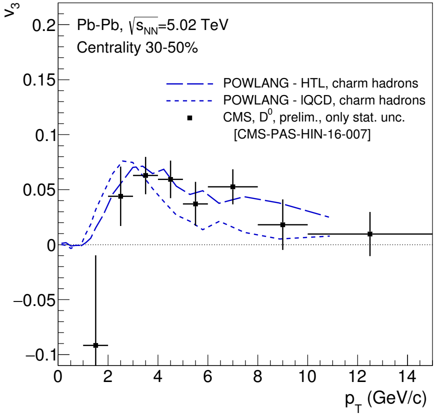

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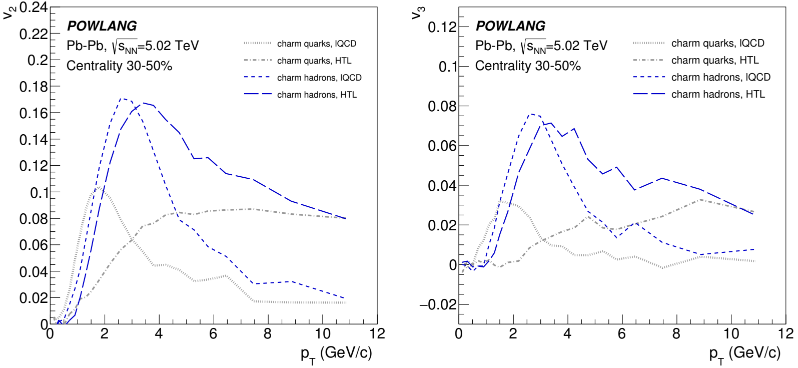
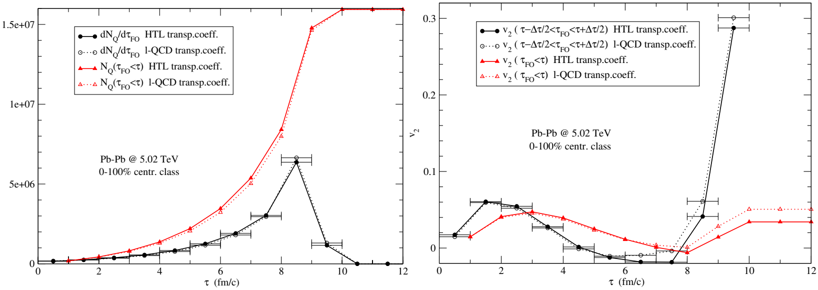


- CMS data for D-meson $v_{2,3}$ satisfactory described;
- Recombination with light quarks provides a relevant contribution;

Time-development of HF flow



- Most of the HQ's decouple quite late ($\sim$ 50% after 8 fm/c);
- Final elliptic flow from a complex interplay of contributions from the whole medium history;

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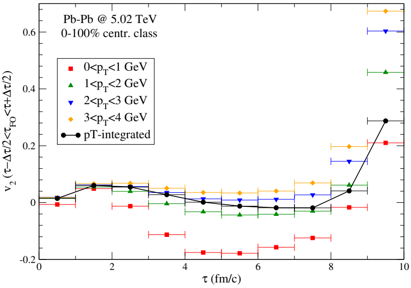

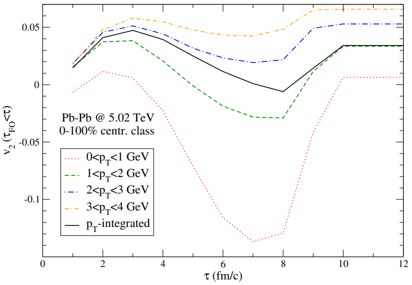

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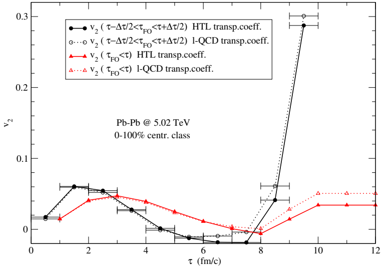

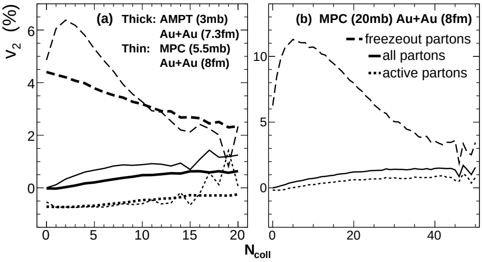

- Most of the HQ's decouple quite late ($\sim$ 50% after 8 fm/c);
- Final elliptic flow from a complex interplay of contributions from the whole medium history;
- supplementary information from p_T-differential analysis;
- interesting to compare with asymmetric escape-probability scenario suggested to interpret light-hadron flow (L. He *et al.*, PLB 753 (2016) 506-510)

Conclusions and future perspectives

Theory-to-experiment comparison allows one to draw some robust qualitative conclusions: c-quarks interact significantly with the medium formed in heavy-ion collision, which affects both their propagation in the plasma and their hadronization. As a result, HF-hadron spectra are quenched at high-p_T, while at low-p_T they display signatures of radial, elliptic and triangular flow.

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- Charm measurements down to $p_T \to 0$: flow/thermalization and total cross-section (of relevance for charmonium supression!)
- D_s and Λ_c measurements: change in hadrochemistry and total cross-section
- Beauty measurements in AA via exclusive hadronic decays: better probe, due to $M \gg \Lambda_{\rm QCD}, T$ (initial production, evaluation of transport coefficients and Langevin dynamics under better control)
- Charm in p-A collisions: which relevance for high-energy atmospheric muons/neutrinos (Auger and IceCube experiments)? Possible initial/final-state nuclear effects?

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The challenge is to become more quantitative, with the extraction of HF transport coefficients from the data (like η/s in hydrodynamics), goal for which beauty is the golden channel