Holographic Quantum Hall Ferromagnetism

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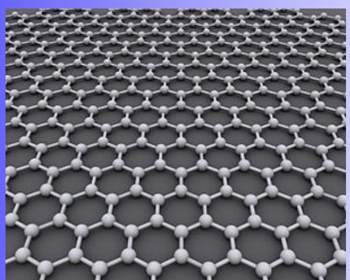
Based on:

- C. K. & G. Semenoff, arXiv:1212.5609 [hep-th], JHEP 1306 (2013) 048
- C. K., R. Pourhasan & G. Semenoff, arXiv:1311.6999 [hep-th], JHEP 1402 (2014) 097
- J. Hutchinson, C. K. & G. Semenoff, arXiv: 1408:3320, Phys.Lett.B (2014) 733

GGI Florence, Apr. 15, 2015

Original Motivation: Attempt to embed graphene in AdS/CFT

Graphene



Particle theory
(Gordon Semenoff '84)

String-theory

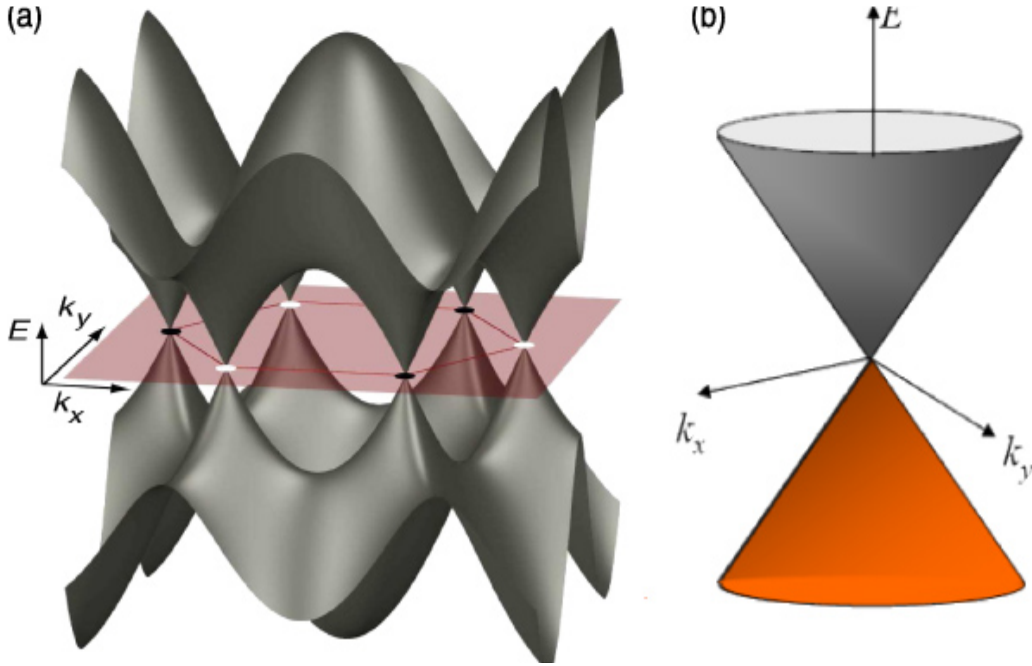
?

Strength of coupling, gauge theory →

Strength of coupling, string theory ←

Why would graphene fit into AdS/CFT

1. Relativistic dispersion relation



$$E = \pm v_F |\hbar k|$$

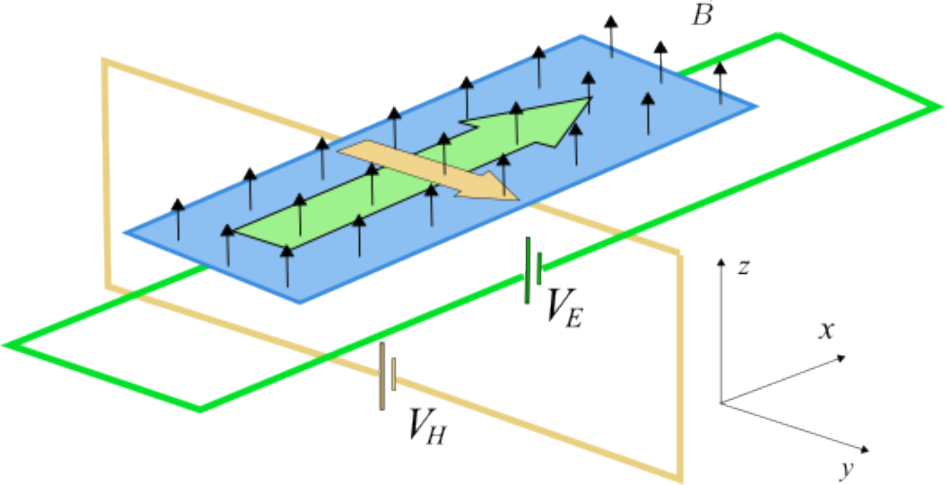
$$v_F = \frac{c}{300}$$

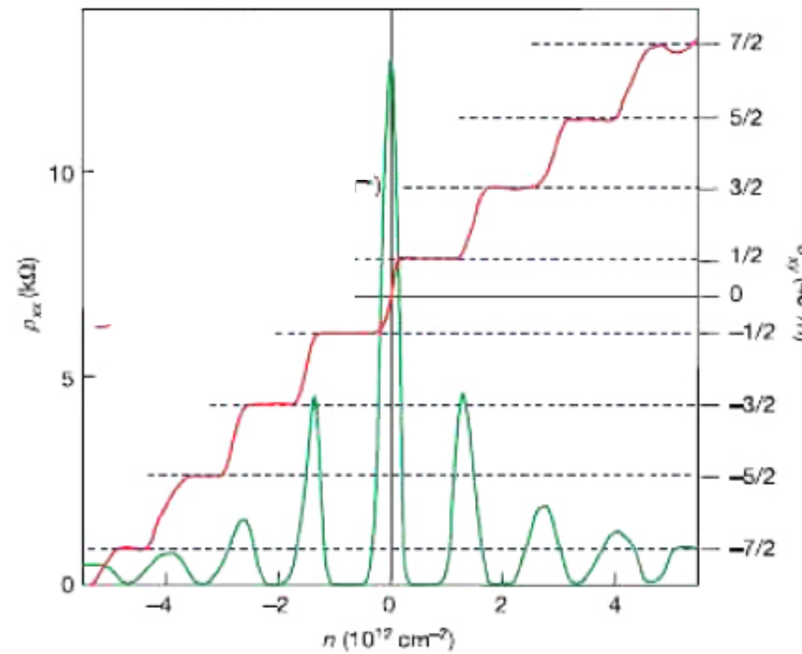
2. Strong coupling

In an electromagnetic field: $\frac{e^2}{4\pi\hbar c} \rightarrow \frac{e^2}{4\pi\hbar v_F} \approx \frac{300}{137}$

2 valleys and 2 spin states $\longrightarrow$ SU(4) symmetry

Quantum Hall effect in graphene



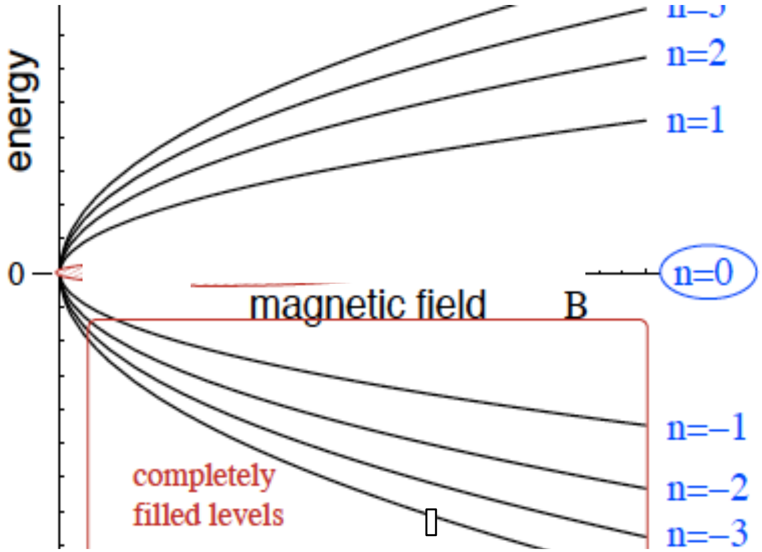


$$J_y = \sigma_{xy} E_x, \quad \sigma_{xy}$$ Hall conductivity

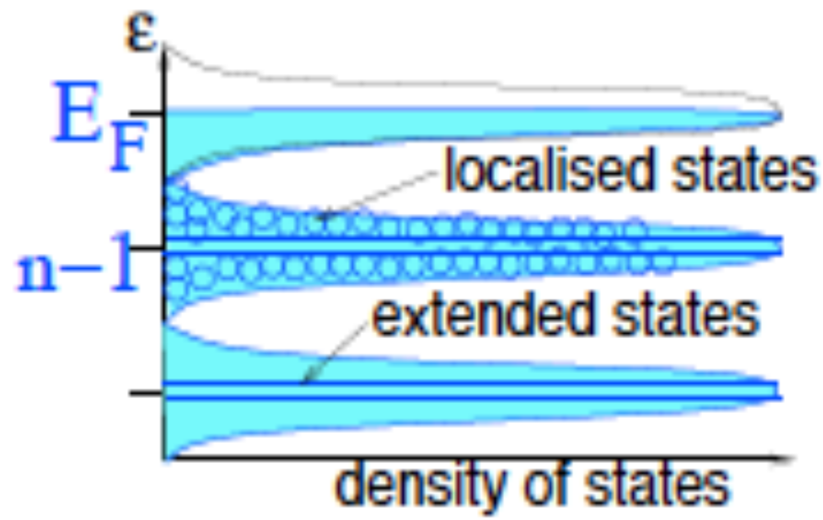
$$J_x = \sigma_{xx} E_x, \quad \sigma_{xx}$$ Longitudinal conductivity

Standard explanation of QHE

Landau levels = Gapped states



Localization



$$E_n = \sqrt{2nB}, \quad n = 0, \pm 1, \pm 2, \ldots$$

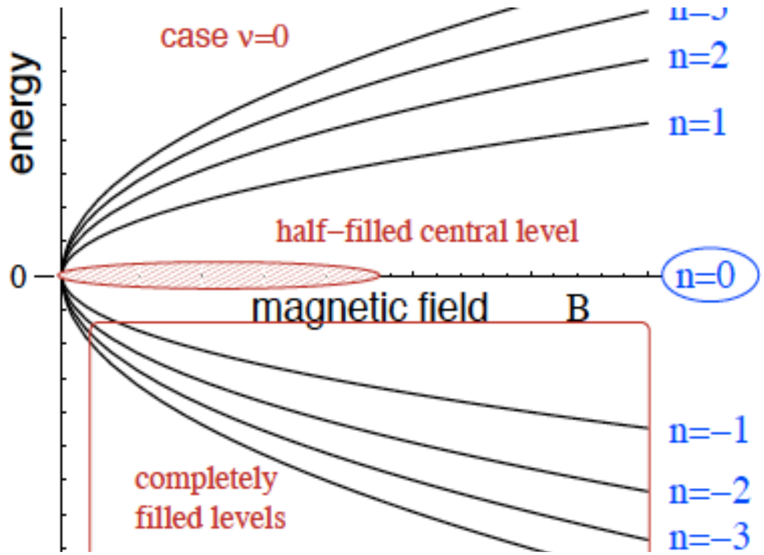
Filling fraction: $n = \dfrac{n_e}{n_L} = \dfrac{e^2}{h}\dfrac{\rho}{B}$

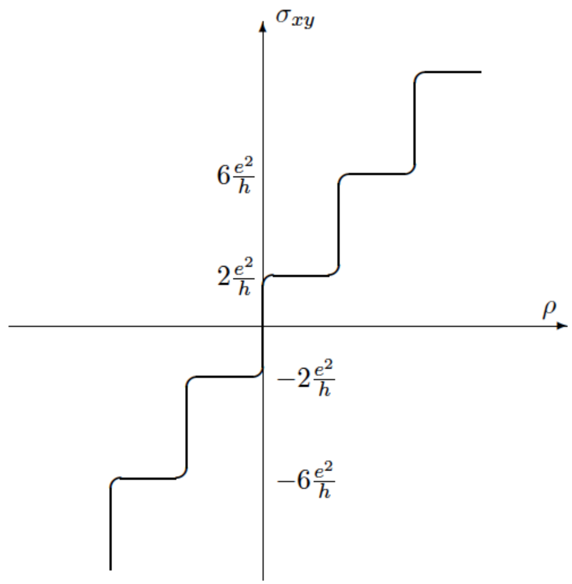
Half filling of level $\longleftrightarrow$ mid-point of step in QHE

Level filled $\longleftrightarrow$ mid-point of plateau in QHE

$$\sigma_{xy} = [n] \cdot \frac{e^2}{h}$$

QHE in graphene





Charge neutral state has level of zero-modes half filled $\longleftrightarrow$ step

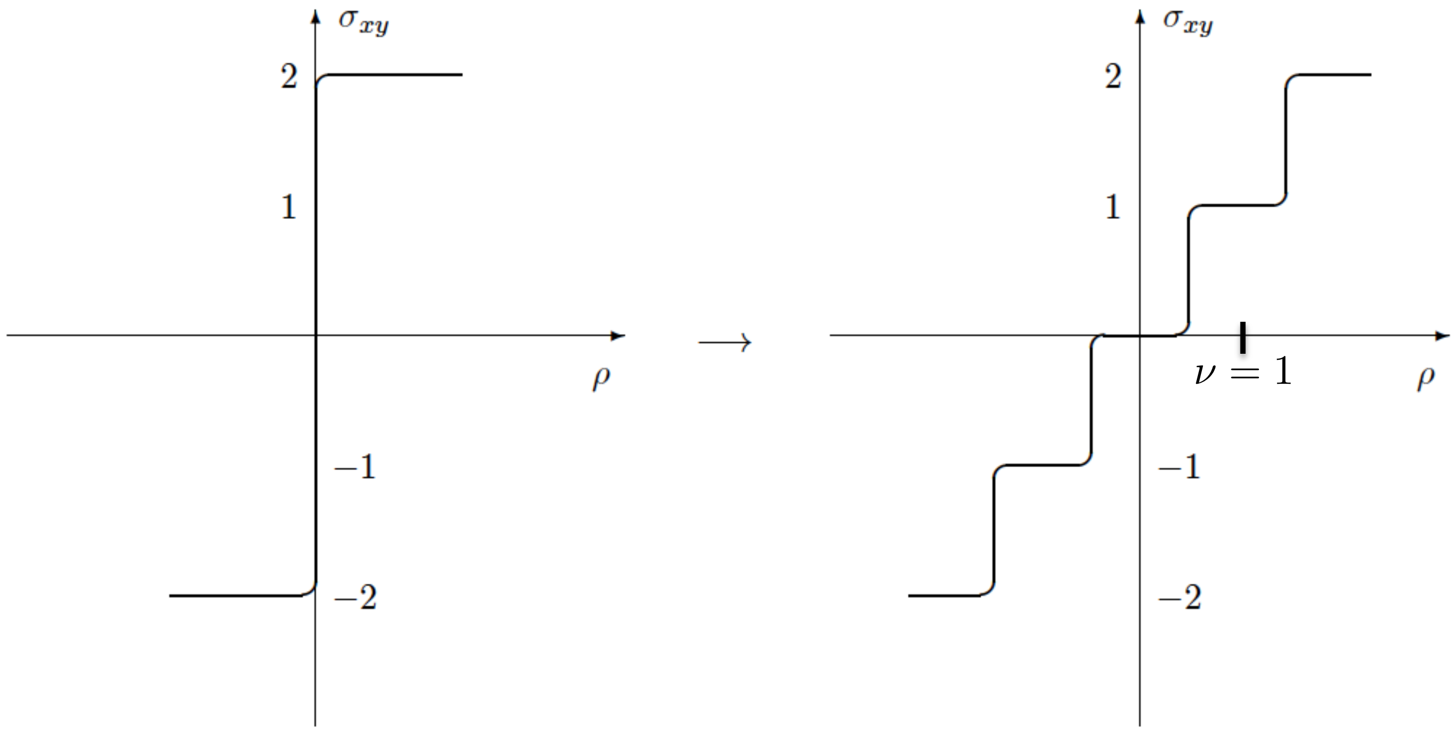
Electrons have SU(4) symmetry

Plateaux in Hall conductivity at

$$\sigma_{xy} = 4\left(m + \frac{1}{2}\right)\frac{e^2}{h}, \quad m = 0, \pm 1, \pm 2 \ldots$$

Magnetic catalysis of SU(4) symmetry breaking

For strong magnetic fields:

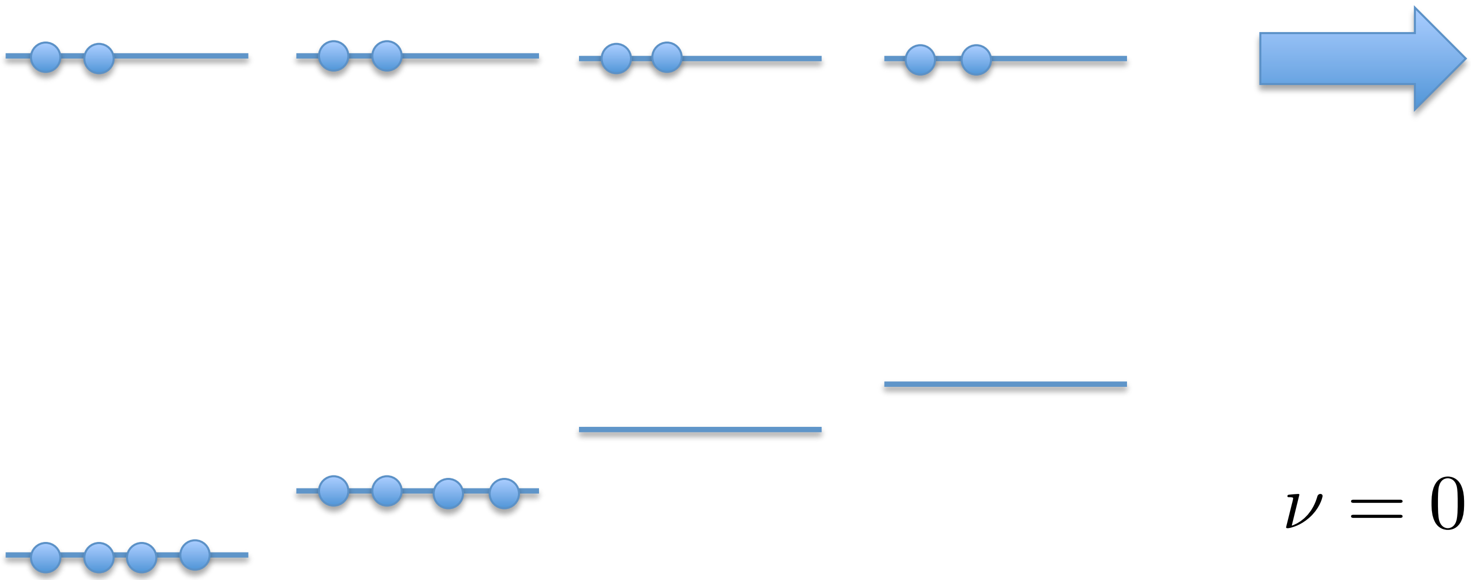


$$\sigma_{xy} = [\nu] \cdot \frac{e^2}{h},$$

$$\nu = \frac{e^2}{h}\frac{\rho}{B}, \quad \text{filling fraction}$$

Quantum Hall Ferromagnetism

Magnetically induced chiral symmetry breaking leads to novel gapped states



$$\nu = 0$$

Filling fraction: $\nu = \frac{e^2}{h}\frac{\rho}{B}$, from now on corresponds to these levels.

The string theoretical set-up

10 D space with geometry $AdS_5 \times S^5$

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$		
$D7$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$

N D3-branes (N rank of gauge group of dual field theory.)
N_5 D5 branes, N_7 D7-branes

Probe approximation: $N_7 = 1, 2, 3, \ldots \ll N_5 \ll N$

D3-D7: Only fermionic excitations in 3D

D3-D5: Fermions as well as bosons in 3D (supersymmetry)

Our model: D3-D5 brane type but some D5 branes can blow up to D7-branes

Landau levels in the string theory picture

Need: Charge density ρ
Magnetic field B $\Big\}$ Introduce background gauge field

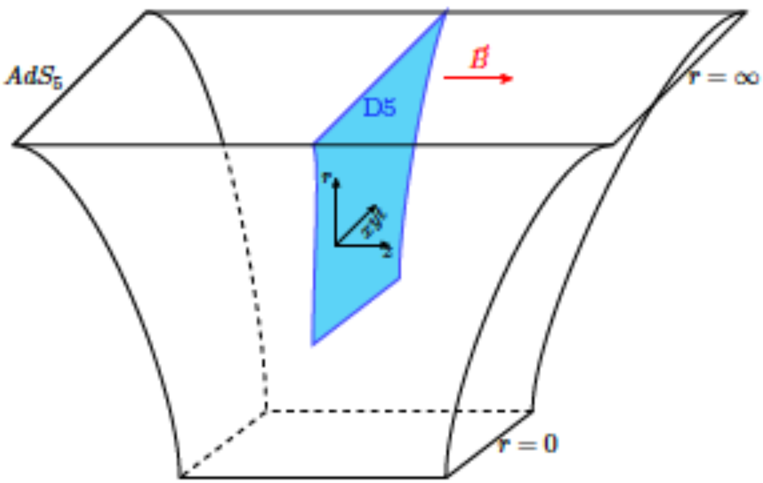
Filling fraction $\nu = \frac{2\pi}{N}\frac{\rho}{B}$

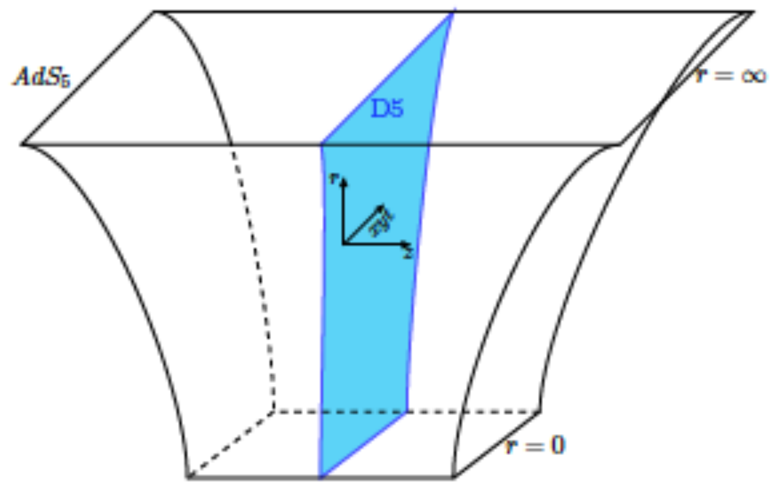
Outcome of studies (classical limit of string theory):

D5 branes: Gapped state at $\nu = 0$
Geometry: $AdS_4 \times S^2$ Filev et al. '09

D7 brane (Blown up D5 branes): Gapped state at $\nu = 1$
+ generalizations
Geometry: $AdS_4 \times S^2 \times S^2$

Gapped vs. un-gapped branes





Gapped branes do not reach the Poincaré horizon of AdS_5

Gapped branes have a gap in the energy spectrum of open strings ending on them

The phase structure for $\nu \in [0,1]$

Input symmetry: $U(N_5)$ x $SO(3)$ x $SO(3)$ via boundary conditions

$\nu = 0:$ N_5 gapped D5 branes

$0 < \nu \lesssim \frac{1}{2}:$ N_5 ungapped D5 branes

$U(N_5)$ x $SO(3)$ x $SO(2)$

Phase-transition

$\frac{1}{2} \lesssim \nu < 1:$ One ungapped D7 brane

$\nu = 1:$ One gapped D7 brane

$U(1)$ x $SO(3)$ x $SO(3)$

Higher Landau levels in string theory

$\nu = 0 :$ N_5 gapped D5-branes

$\nu = \pm 1 :$ All N_5 D5-branes blown up to a single gapped D7-brane

$\nu = \pm 2 :$ All N_5 D5-branes blown up to two gapped D7-branes

a.s.o

$\nu = \pm N_5$

OBS Interesting phase diagram in (ν, N_5) plane (composite solutions)

(Concentrate on $\nu \in [0, 1]$)

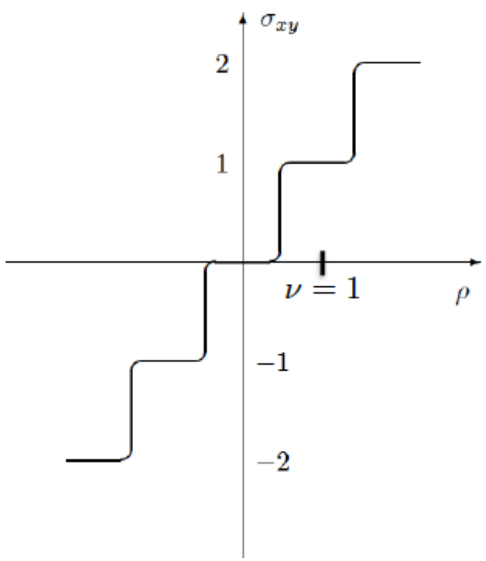
The Hall conductivity from string theory

Need to introduce E-field and temperature, T

$T \neq 0$ necessary for QHE, $\sigma_{xy} = \dfrac{N\nu}{2\pi}$ for T=0

Determine currents and extract σ_{xx}, σ_{xy}

Dream scenario:



Concentrate on $\nu \in [0, 1]$

The geometric set-up in $AdS_5 \times S^5$

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2 + dw^2) + \frac{dr^2}{r^2} + \right.$$
$$\left. + d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2) + \cos^2\psi(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2) \right]$$

	t	x	y	w	r	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$		
$D7$	$\times$	$\times$	$\times$		$\times$		$\times$	$\times$	$\times$	$\times$

D5: $w = const, \;\; \tilde{\theta} = const, \;\; \tilde{\phi} = const, \;\; \psi = \psi(r)$

D7: $w = const, \;\; \psi = \psi(r)$

Boundary condition: $\psi(r) \to \frac{\pi}{2} \quad \text{for } r \to \infty$

Consequence: D7-brane can be viewed as blown-up (giant) D5-brane

Gapped branes: $\psi(r) \to 0 \quad \text{for } r > 0$

Actions and Approximations

- 't Hooft limit $N \to \infty, \lambda$ fixed
- Strong coupling $\lambda \to \infty$

Classical limit of string theory

- Probe approximation $N_7 = 1, (2, 3, \ldots) \ll N_5 \ll N$

$$S = S_{DBI} + S_{WZ}$$ (WZ term does not play any role for the D5-brane)

World volume gauge fields and flux:

$$2\pi\alpha' \mathcal{F}_7 = \sqrt{\lambda}\alpha' \left[\frac{d}{dr} a(r) dr \wedge dt + b dx \wedge dy + \frac{f}{2} d\tilde{\Omega}_2 \right].$$

Parameters:

Filling fraction $\nu = \frac{2\pi}{N} \frac{\rho}{B}$, Number of D5-branes/Flux $f = \frac{2\pi N_5}{\sqrt{\lambda}}$

Temperature $T = \frac{r_h}{\pi}$ (of $AdS_5 \times S^5$ black hole background)

Mapping out the phase diagram

For given parameters $(\nu \in [0,1])$ various solutions compete:

- The trivial (chirally symmetric) solution $\psi(r) = \frac{\pi}{2}$
- The non-trivial D5 brane
- The D7 brane

Energies must be compared

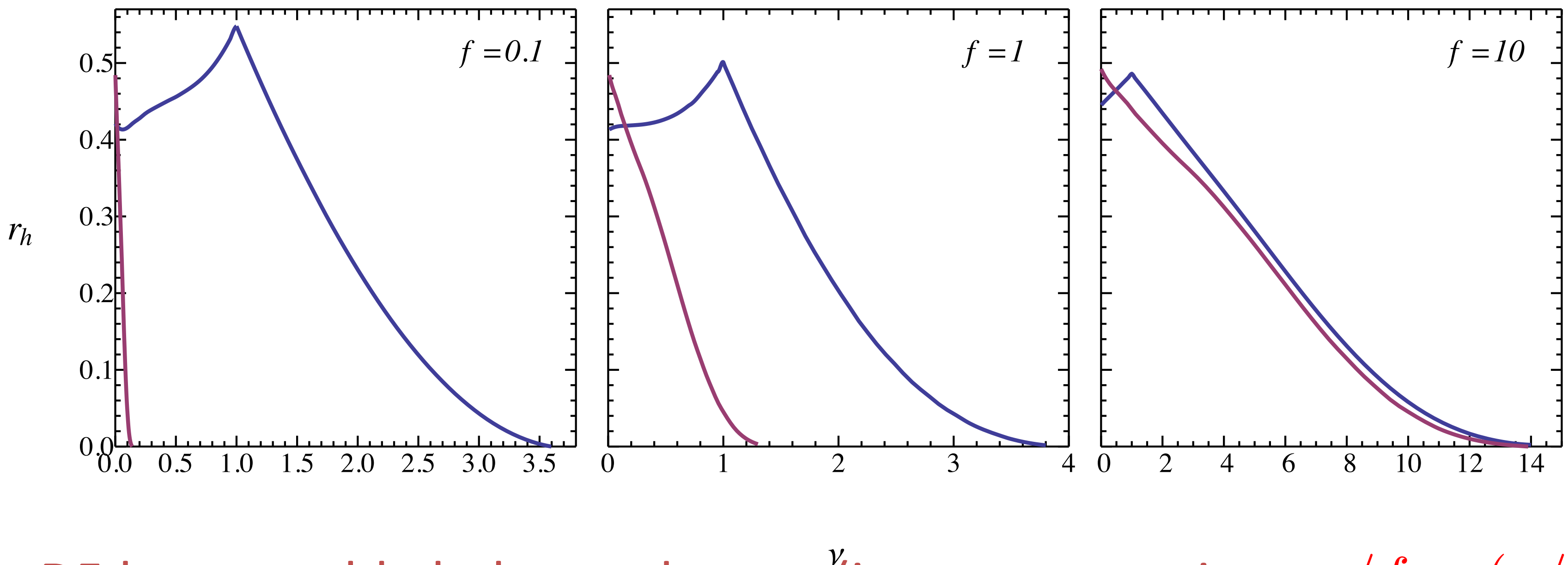
Expectation:

The constant solution $\psi = \frac{\pi}{2}$ (chirally symmetric phase) is favoured for large values of ν and T

Analytical result for T=0:
Chiral symmetry broken for: $\nu/f < (\nu/f)_c = \frac{2\sqrt{7}}{\pi} \approx 1.68$

Jensen,Karch, Son, Thompson '11

Stability regions



D5 brane stable below red curve (intersects x-axis at $\nu/f = (\nu/f)_c$)

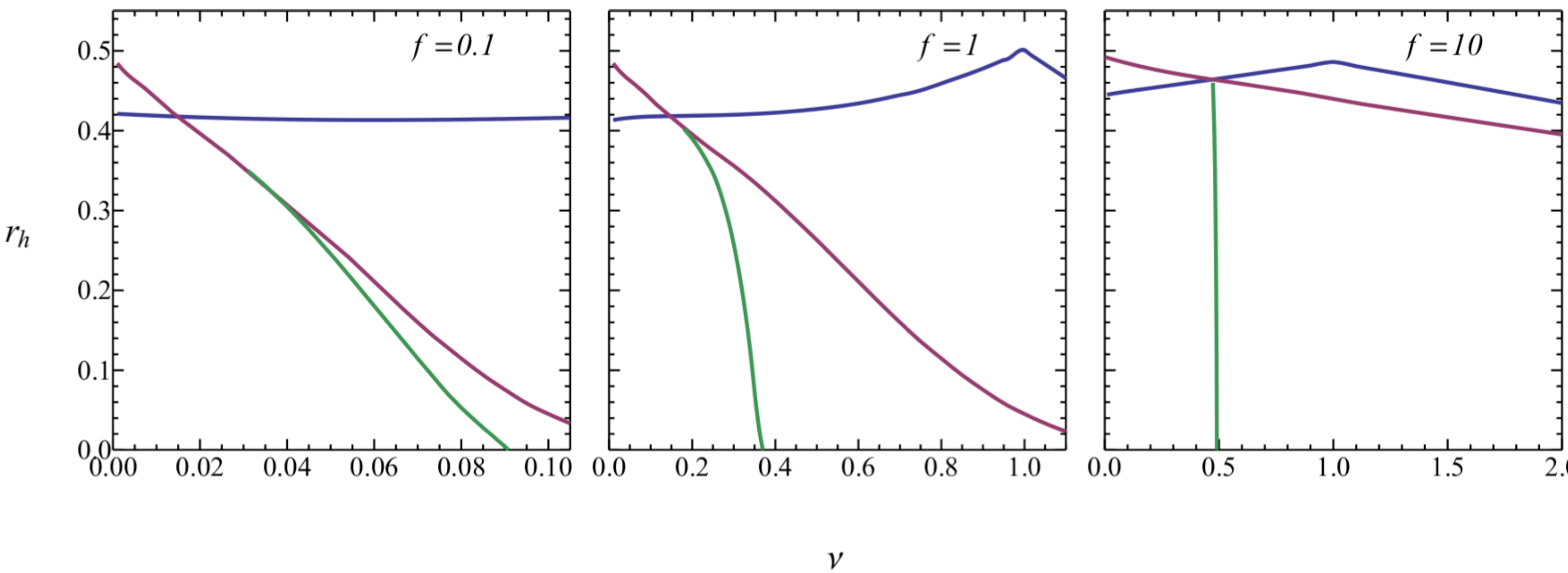
Evans, Gebauer, Kim & Magou, '10

D7 brane stable below blue curve, co-exist with constant solution in some region.

Notice that there is a dynamical upper bound on T

Which solution is preferred when both are stable?

Competition between D5 and D7



D7 brane preferred to the right of the green curve

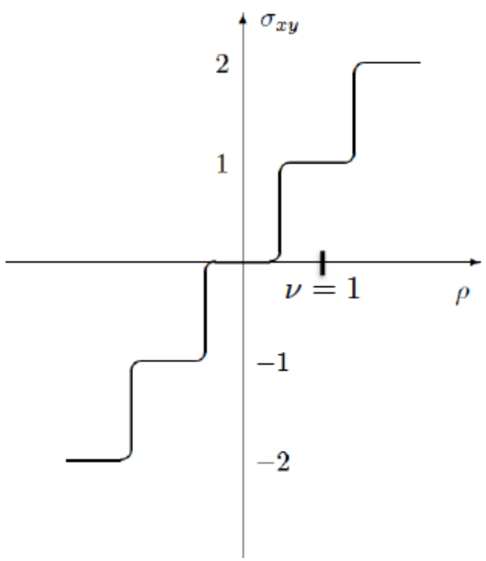
OBS: Composite solutions for $\nu > 1$

The Hall conductivity from string theory

Need to introduce E-field

Determine currents and extract σ_{xx}, σ_{xy}

Dream scenario:



Concentrate on $\nu \in [0, 1]$

The Hall conductivity from string theory

Introducing the world volume electric and magnetic fields

$$2\pi\alpha'\mathcal{F}_5 = \sqrt{\lambda}\alpha'\left[\frac{d}{dz}a(z)dz\wedge dt + b\,dx\wedge dy + \frac{d}{dz}f_y(z)dz\wedge dy + \frac{d}{dz}f_x(z)dz\wedge dx - e\,dt\wedge dx\right]$$

Three conserved charges

$$q \equiv \frac{\delta S_5}{\delta a'(z)}, \qquad j_x \equiv -\frac{\delta S_5}{\delta f_x'(z)}, \qquad j_y \equiv -\frac{\delta S_5}{\delta f_y'(z)}$$

Strategy

Fix q, b and e.

Determine j_x and j_y using Karch-O'Bannon technique

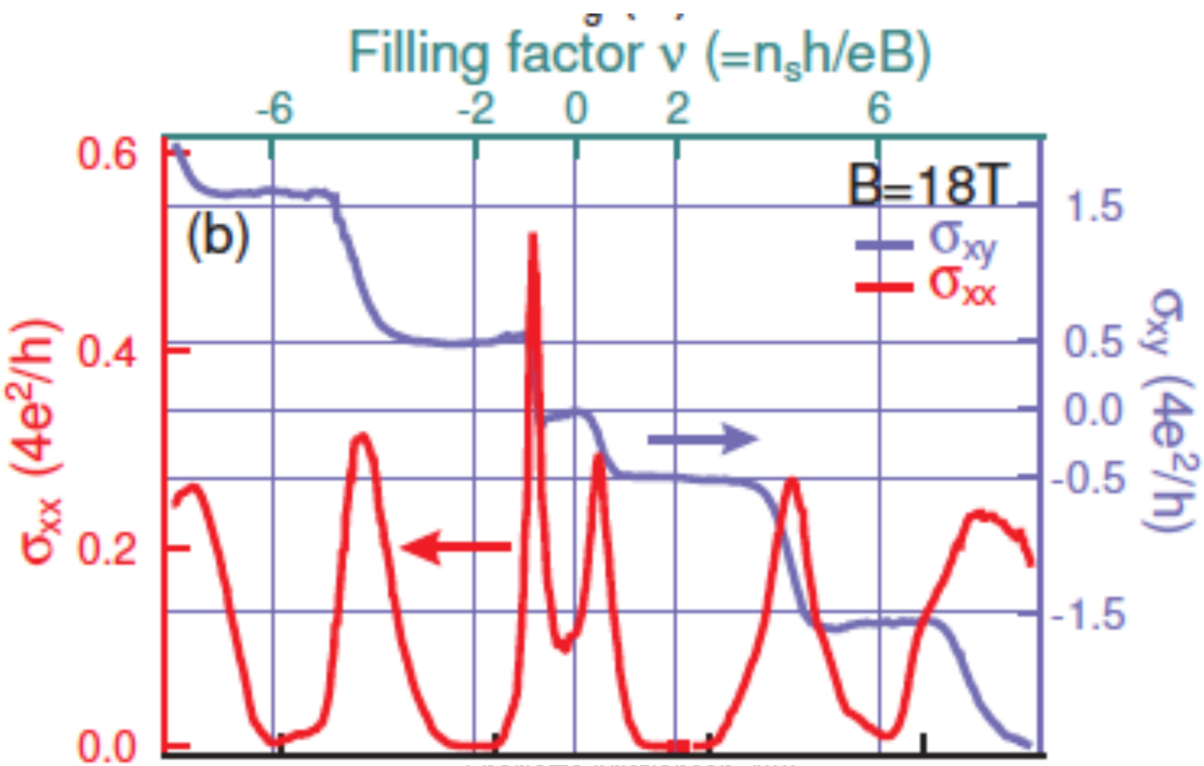
$$\sigma_{xy} = \left.\frac{j_y}{e}\right|_{e=0}, \qquad \sigma_{xx} = \left.\frac{j_x}{e}\right|_{e=0}$$

Hall conductivity — Longitudinal conductivity

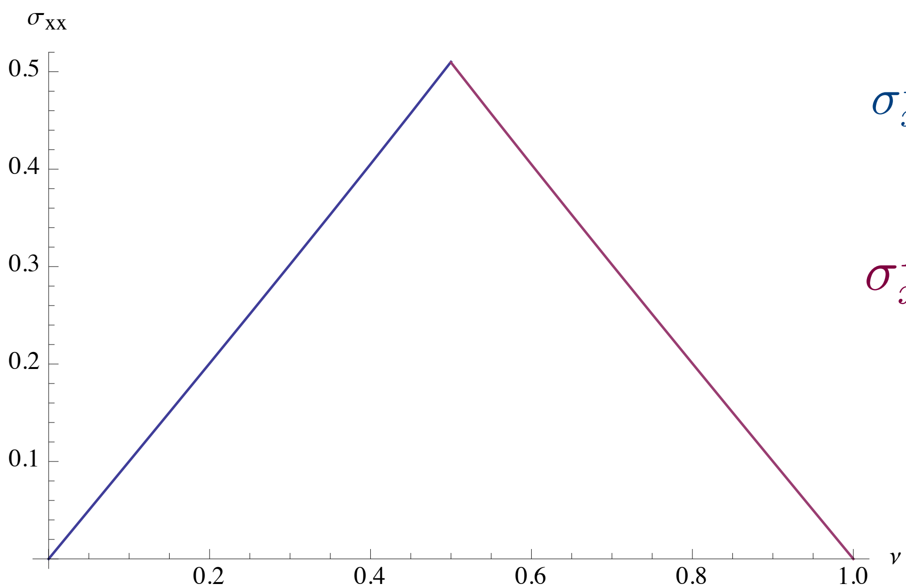
Expectation for the conductivities

σ_{xy} : Step at half-integer filling fraction
Plateau at integer filling fraction.

σ_{xx} : Peak at half-integer filling fraction
Vanish at integer filling fraction.



The longitudinal conductivity σ_{xx} for $f = \infty$ and $\nu \in [0,1]$

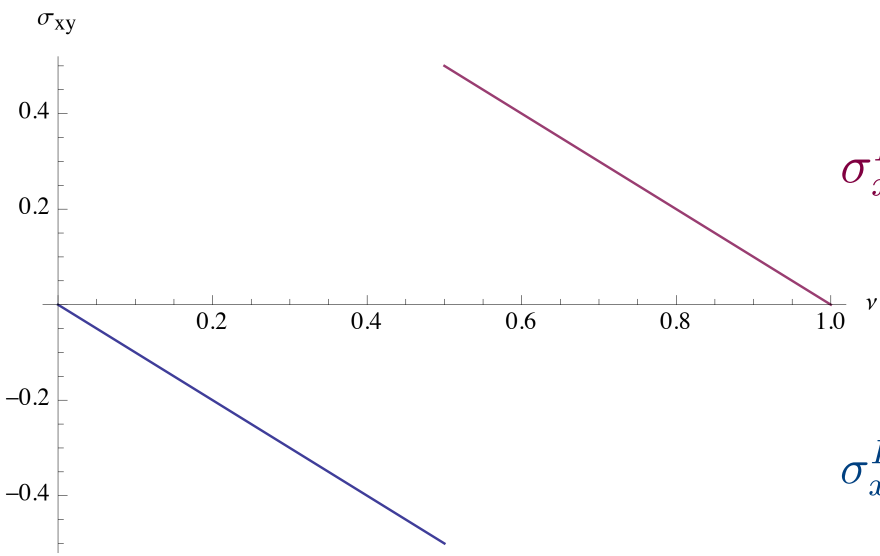


$$\sigma_{xx}^{D5} = \frac{N}{2\pi}\,\frac{\hat{r}_h^2}{1+\hat{r}_h^4}\,\nu\ , \qquad \hat{r}_h = \sqrt{\frac{\pi}{2B}}\,\lambda^{1/4}\,T$$

$$\sigma_{xx}^{D7} = \frac{N}{2\pi}\,\frac{\hat{r}_h^2}{1+\hat{r}_h^4}\,(1-\nu)$$

- The longitudinal conductivity vanishes for T=0 (as expected)
- σ_{xx} vanishes for the gapped states $\nu = 0$ and $\nu = 1$
- The longitudinal conductivity has a peak at $\nu = 1/2$ where the D7 brane takes over

The deviation of σ_{xy} from the linear form for $f = \infty$ and $\nu \in [0, 1]$

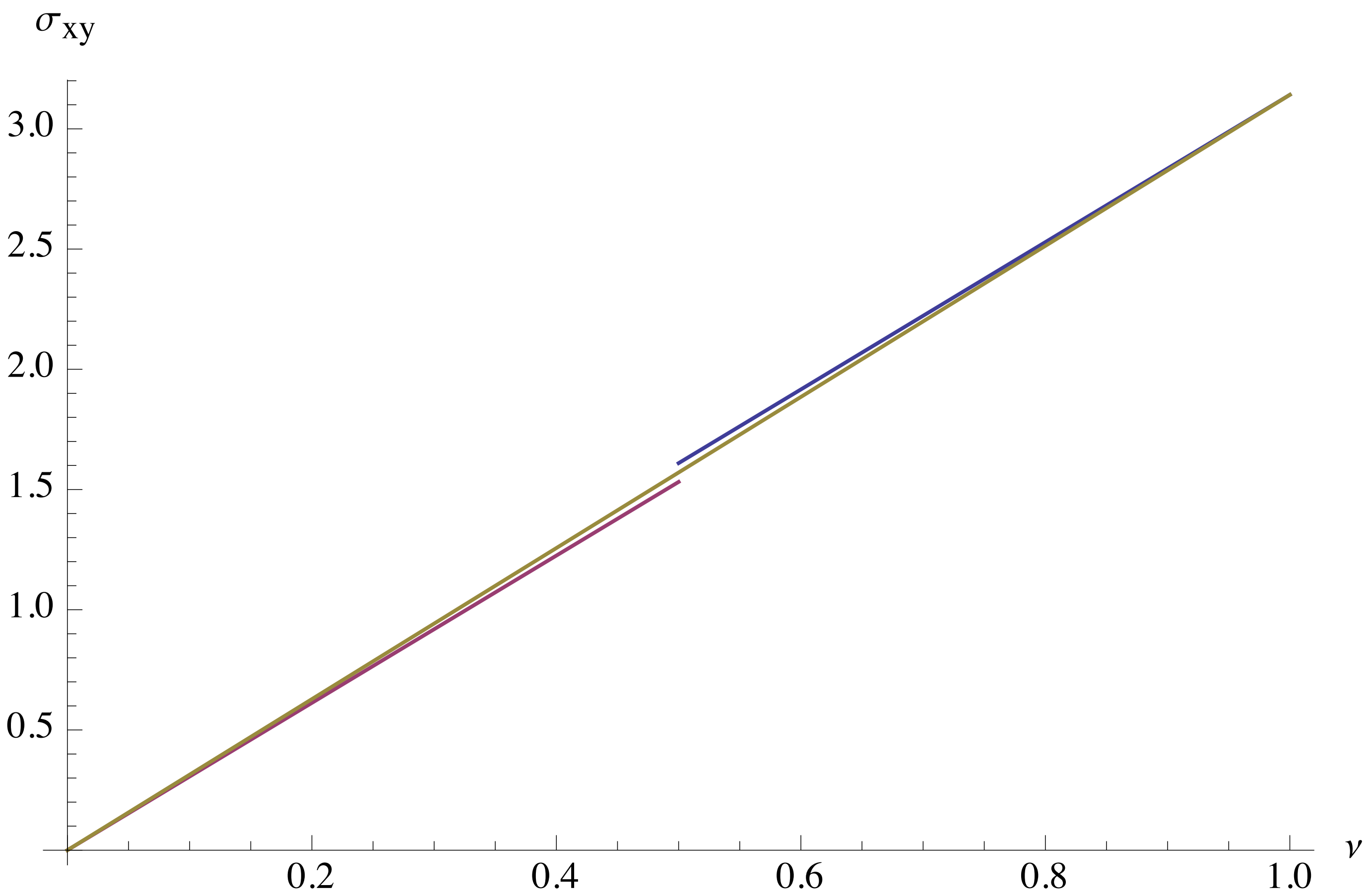


$$\sigma_{xy}^{D7} = \frac{N\nu}{2\pi} + \frac{N(1-\nu)}{2\pi}\frac{\hat{r}_h^4}{1+\hat{r}_h^4}.$$

$$\sigma_{xy}^{D5} = \frac{N\nu}{2\pi} - \frac{N\nu}{2\pi}\frac{\hat{r}_h^4}{1+\hat{r}_h^4}, \qquad \hat{r}_h = \sqrt{\frac{\pi}{2B}}\,\lambda^{1/4}\,T$$

- The correction vanishes for T=0 (as expected)
- The correction vanishes for the gapped states $\nu = 0 \text{ and } \nu = 1$
- There is a jump upwards in Hall conductivity at $\nu = 1/2$ where the D7 brane takes over
- For $T \to \infty$ one gets a perfect Hall step (but $\hat{r}_h \lesssim 0.4$)

Results for the Hall conductivity



$f = 10, \hat{r}_h = 0.4$ (for maximal effect)

Conclusion/Outlook

We have identified a $\nu = 1$ Hall state at strong coupling (+generalizations)

We have observed the equivalent of quantum Hall ferromagnetism at strong coupling

We have observed a strong coupling mechanism that gives some tendency towards plateau formation.

The observed strong coupling effect requires a finite temperature and its magnitude is small (for a clean sample)

Introduce impurities or look for other ways of breaking Lorentz invariance

Other solutions? Instabilities towards striped phases?

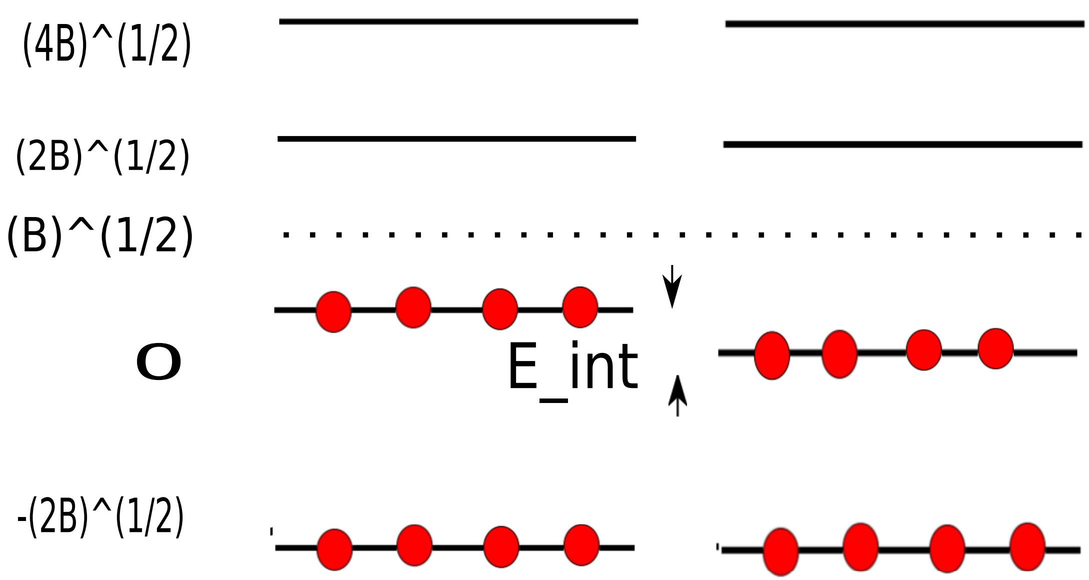
Bergman, Jokela, Lifschytz & Lippert '11

Issues with supersymmetry

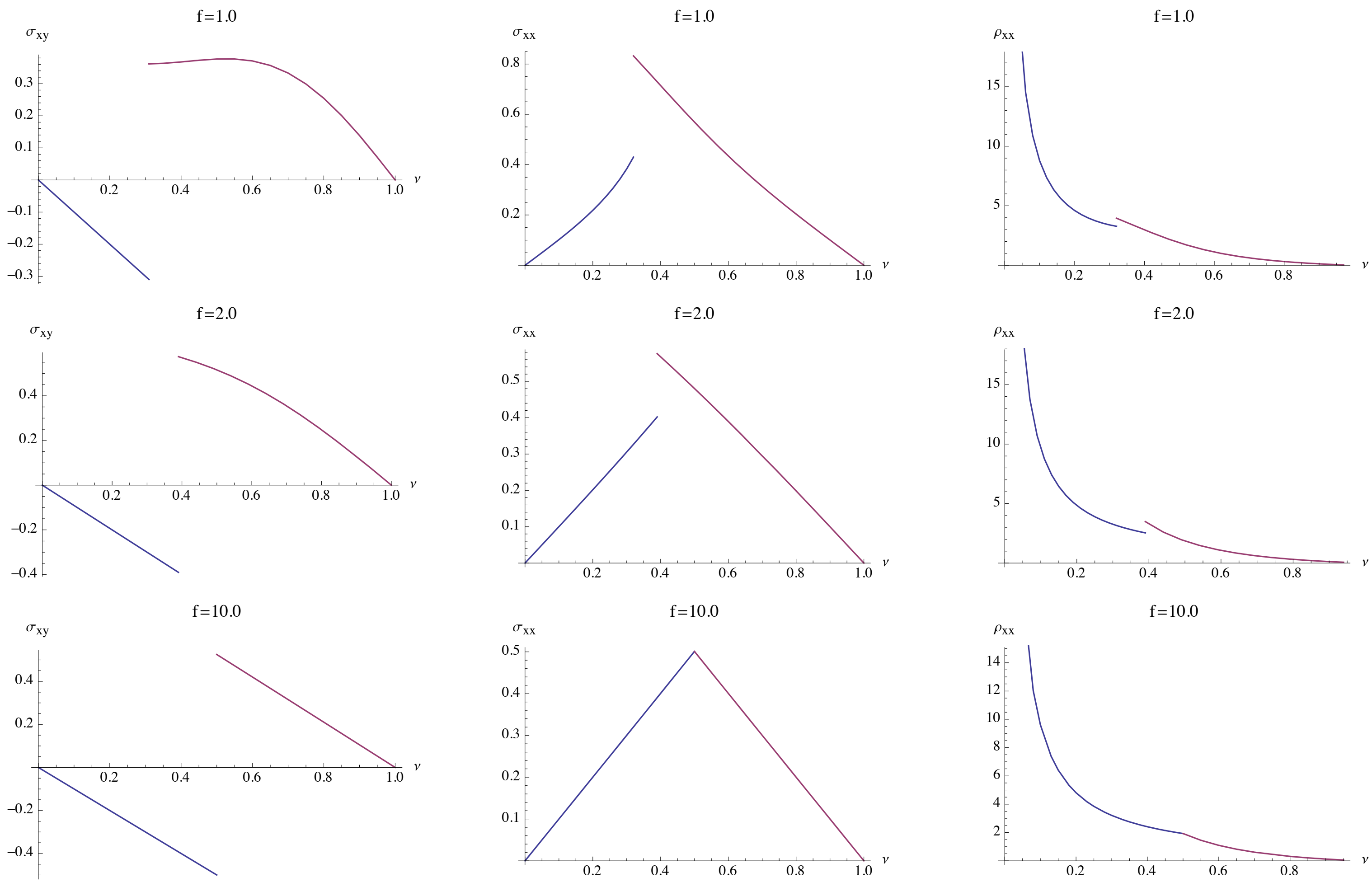
For fermions: $E_n = \pm\sqrt{2Bn}, \quad n = 0, 1, 2, \ldots$

For scalars: $\omega_n = \sqrt{(2n+1)B}, \quad n = 0, 1, 2, \ldots$

Quantum Hall states must fit under the treshold for creating a boson (here shown for N_5=1).



Numerical results for the conductivity tensor for finite f



The deviation of the Hall conductivity from the linear form, the longitudinal conductivity and the longitudinal resistivity for $\nu \in [0,1]$