S-matrix approach to 2 and 3 hadron interactions

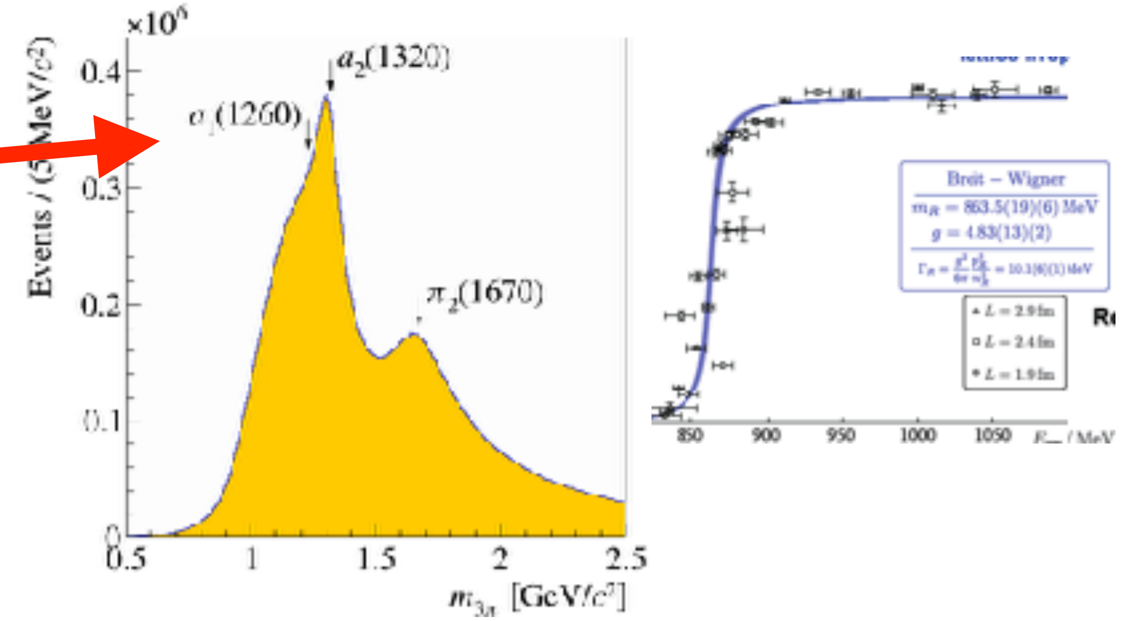
Adam Szczepaniak, Indiana University/Jefferson Lab

Outline

- Opportunities and challenges in hadron spectroscopy
- Amplitudes from S-matrix analysis: 2-to-2 scattering 1-to-3 decays : how virtual exchanges become real
- Three particle scattering : the framework

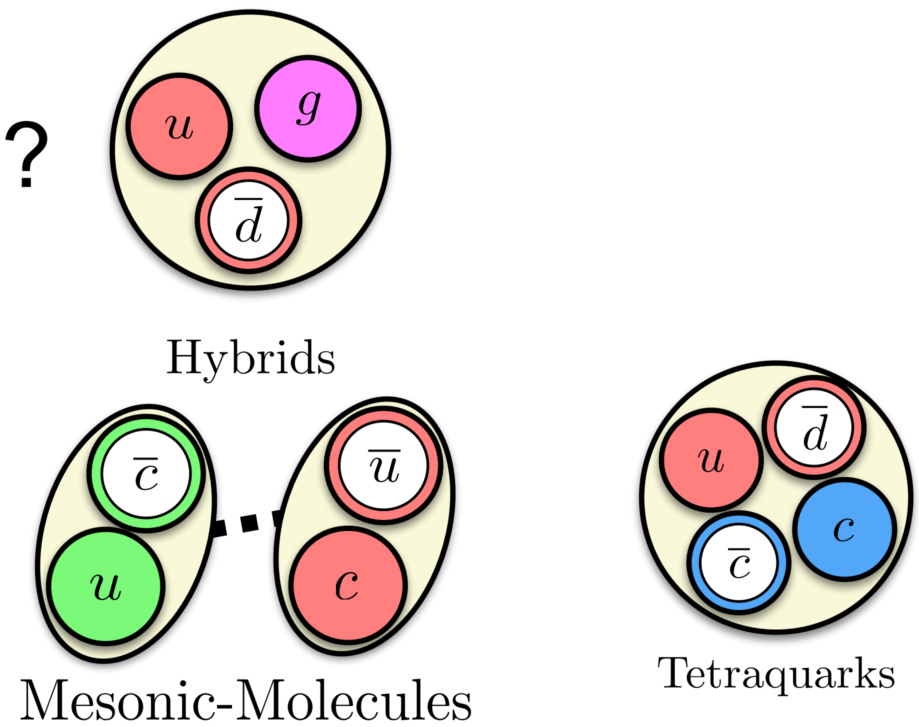
How Hadrons Emerge from QCD

- Experimental or lattice signatures (real axis data: cross section bumps and dips, energy levels)



- Theoretical signatures (complex plane singularities: poles, cusps)

- What is the interpretation (constituent quarks, molecules, …) ?



Reaction amplitudes

How Hadrons Emerge from QCD

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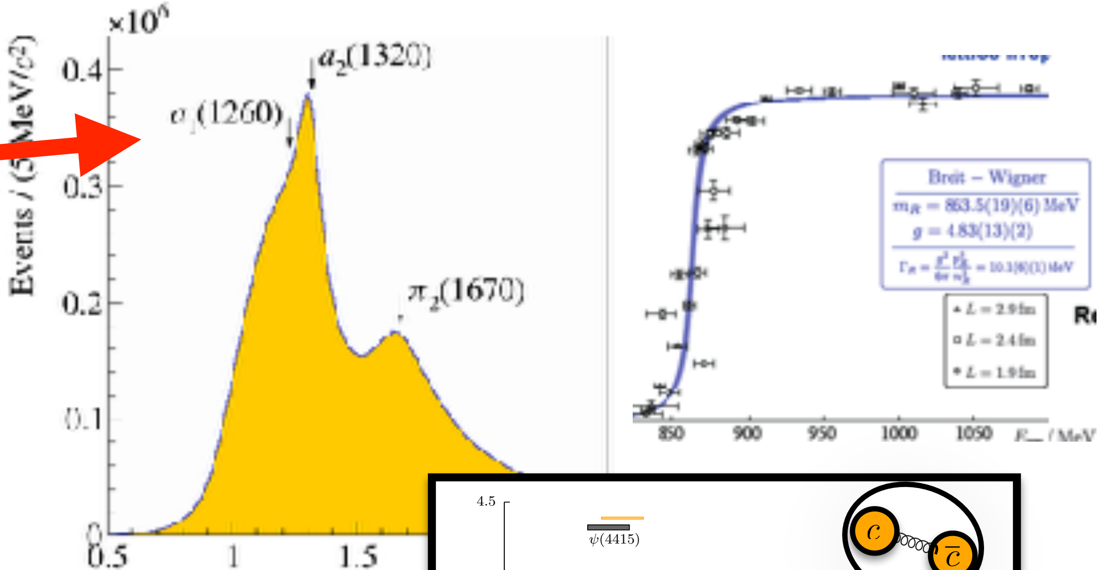

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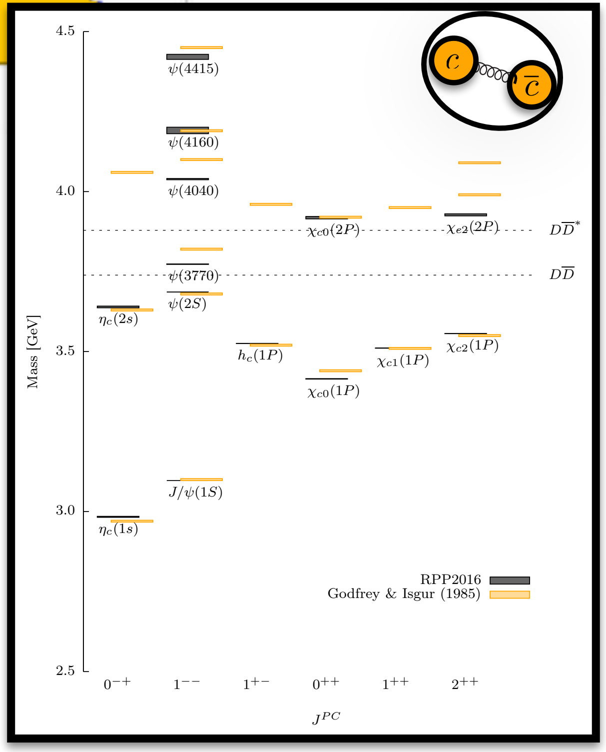

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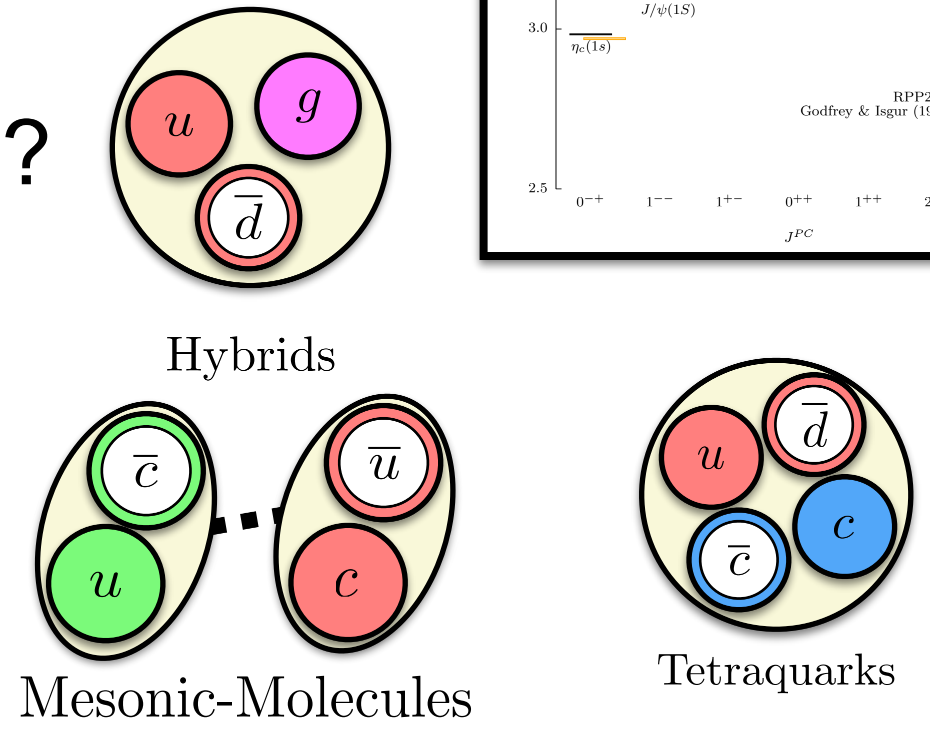
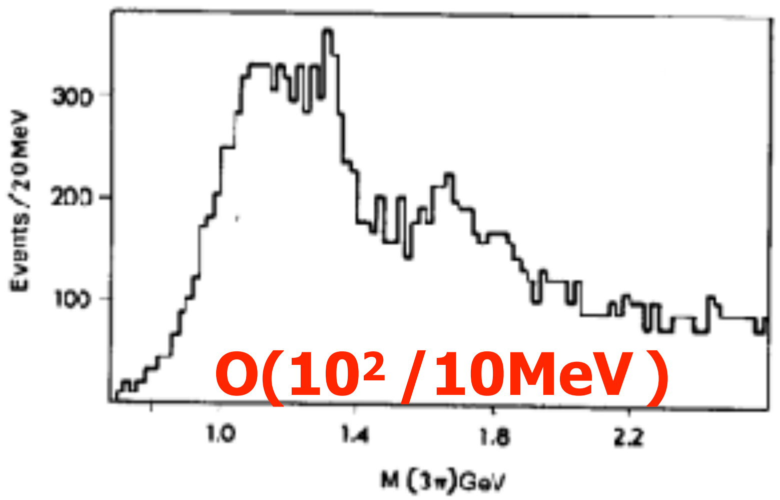


Reaction amplitudes

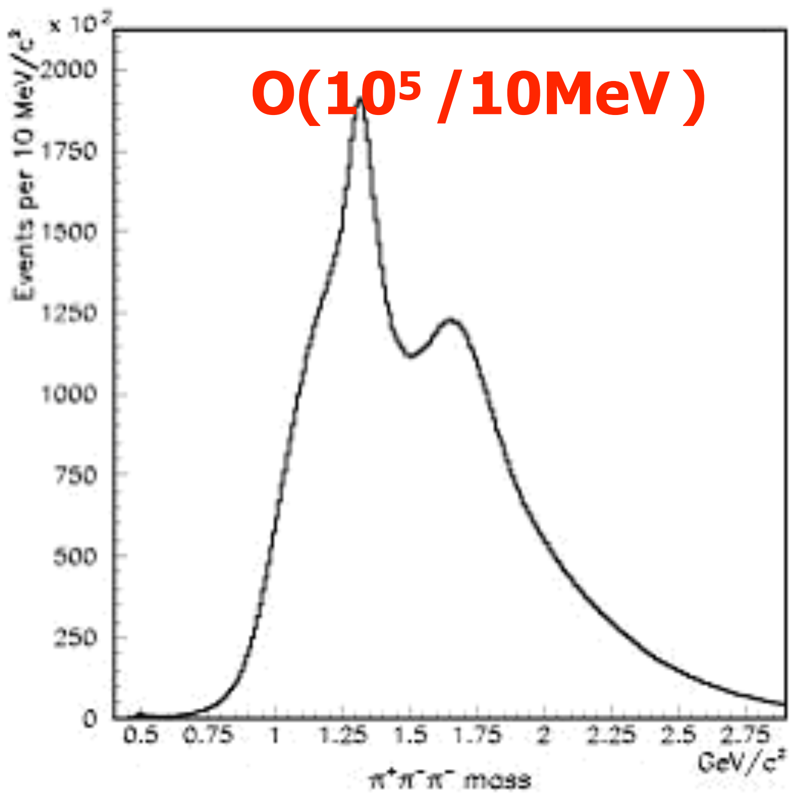
3pion challenge

evolution in statistics $\pi^- p \rightarrow \pi^-\pi^+\pi^- p$

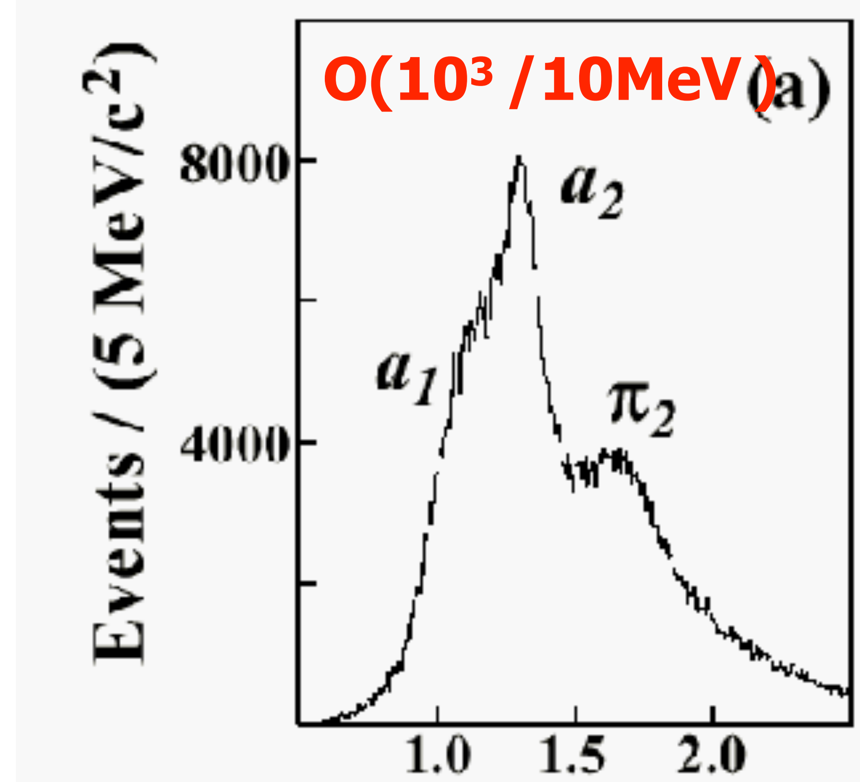
CERN ca. 1970



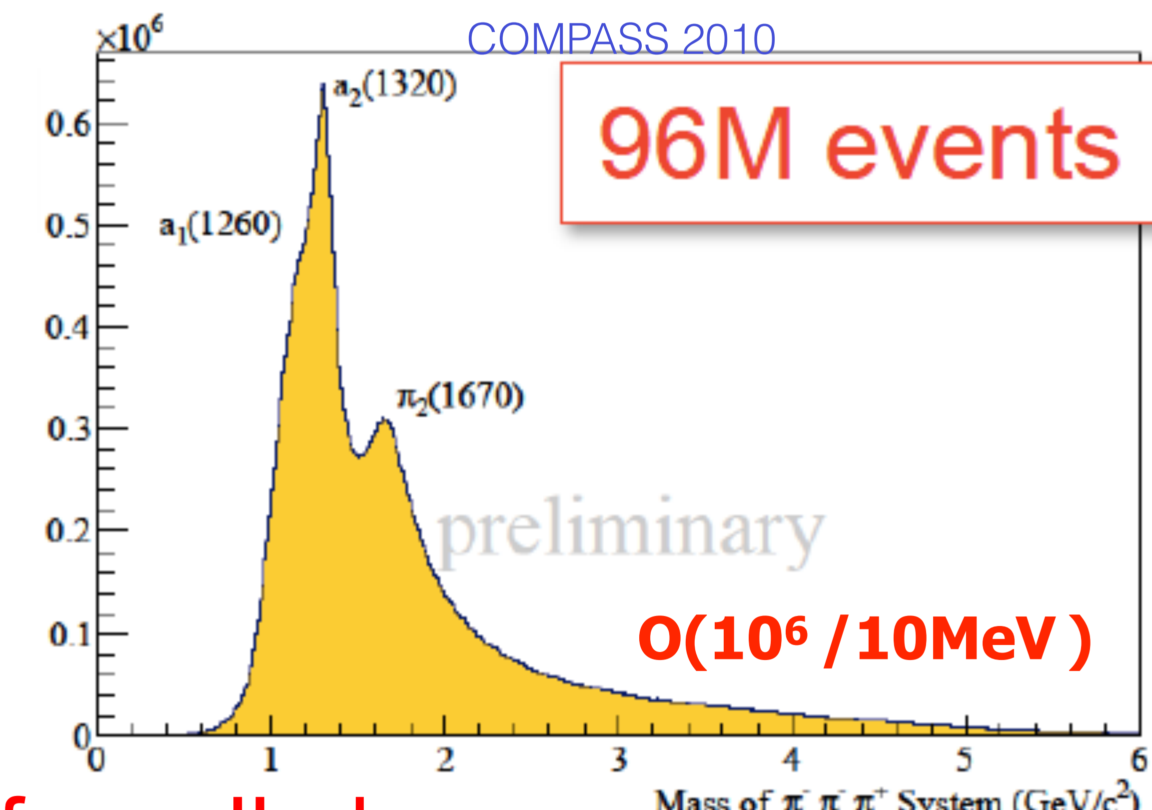
E852 (Full sample)



BNL (E852) ca 1995

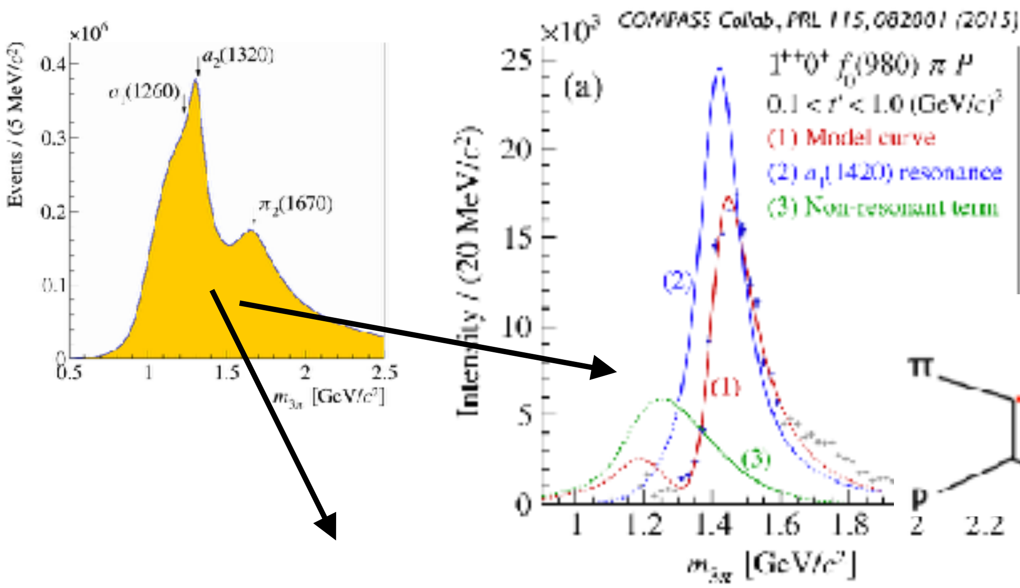


COMPASS 2010

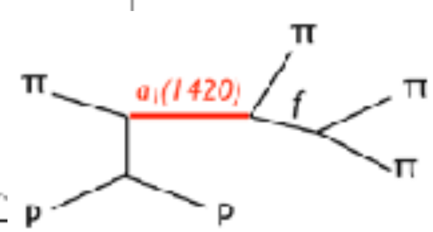


Comparable statistics expected from JLab

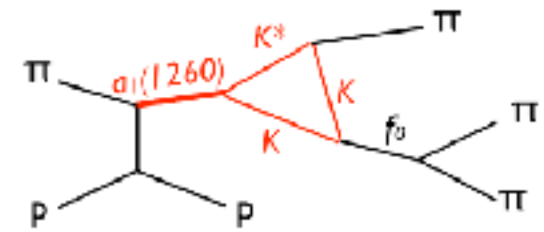
Signatures of new, unusual light resonances



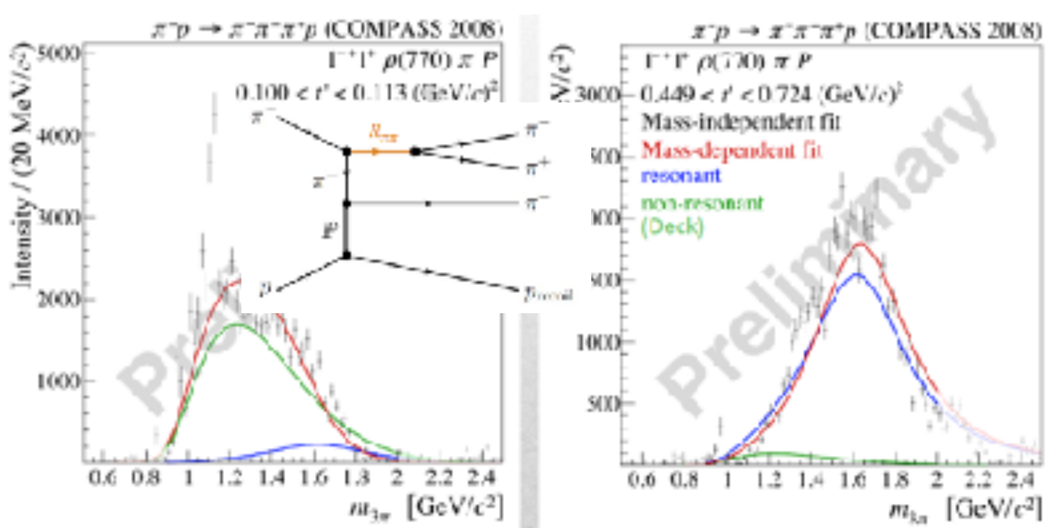
- High precision PWA of 3pi diffractive association yields a new $a_1(1460)$ incompatible with the quark model/Regge expectations.



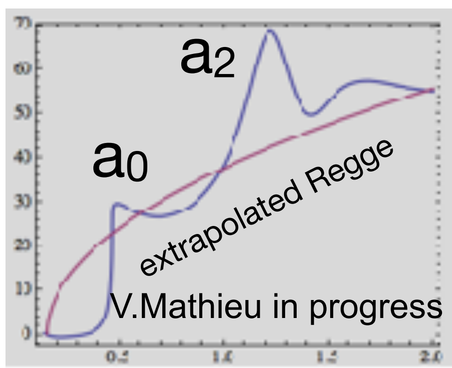
Or ?

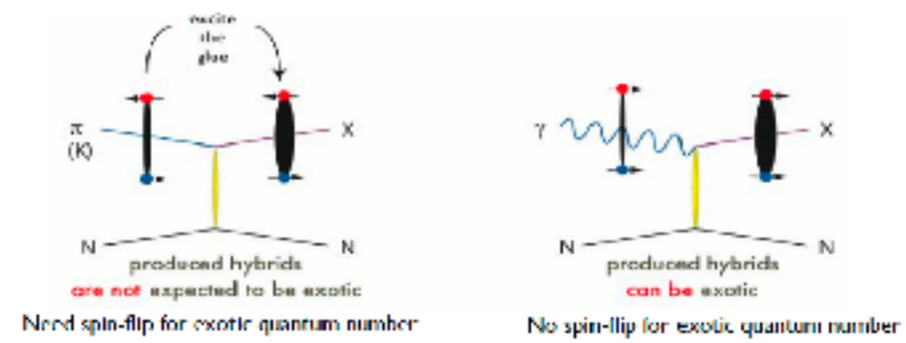


- At low-t exotic wave production compatible with one pion exchange

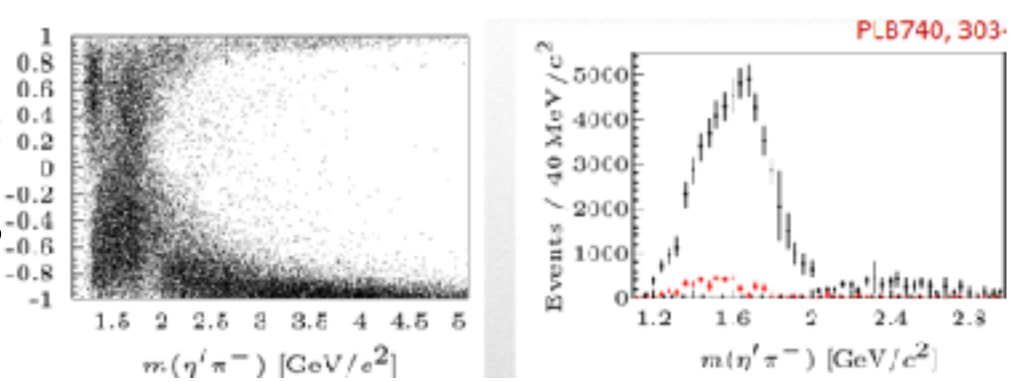


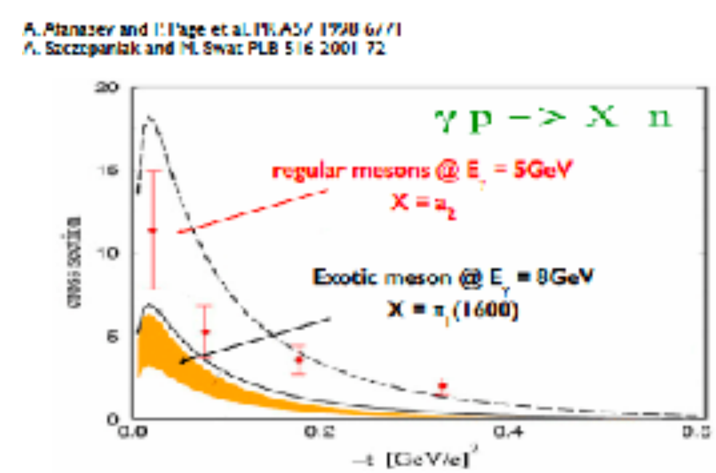
- In photoproduction exotic mesons be produced via pion exchange



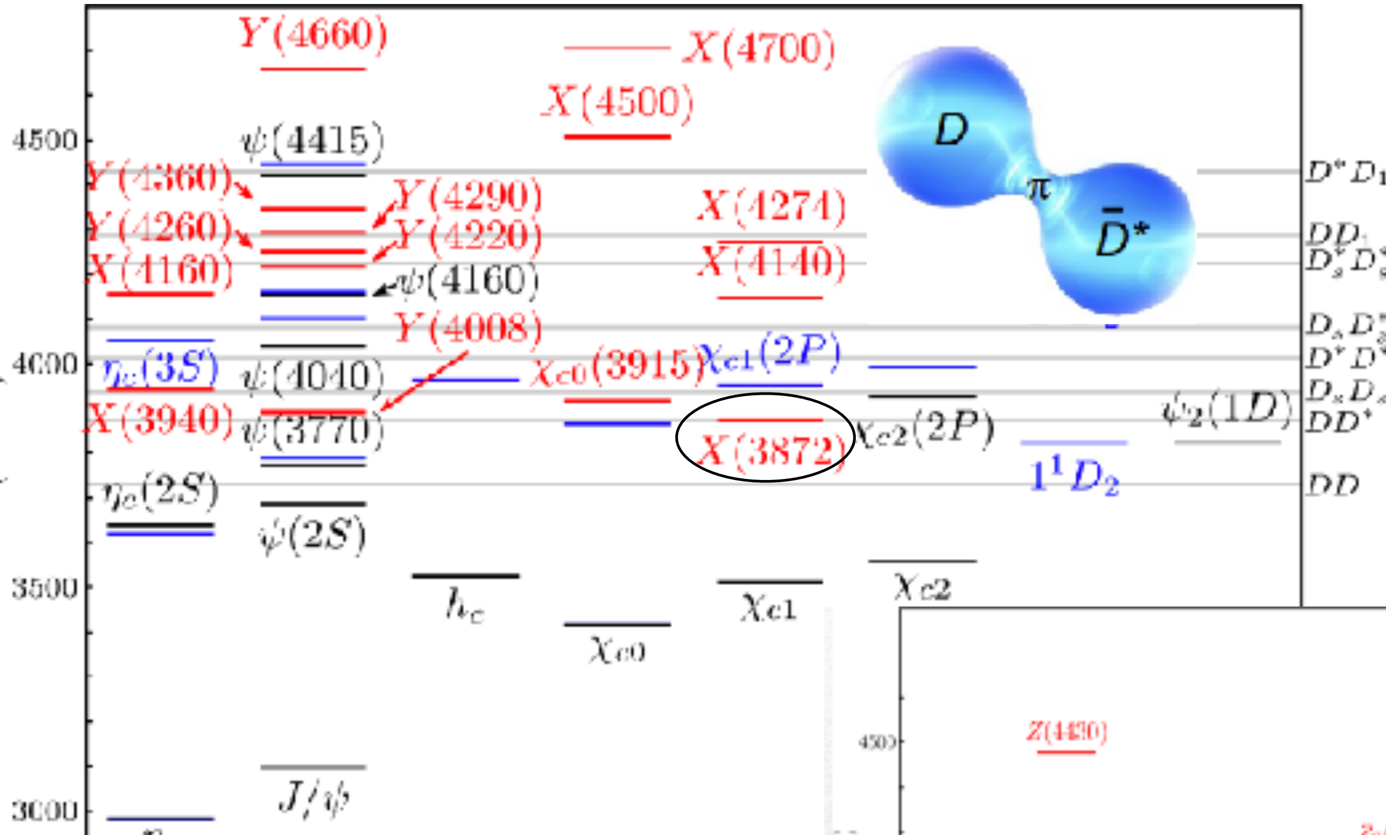


- Large exotic wave seen in $\eta^{(\prime)}\pi$ production : FESR's to constrain P-wave

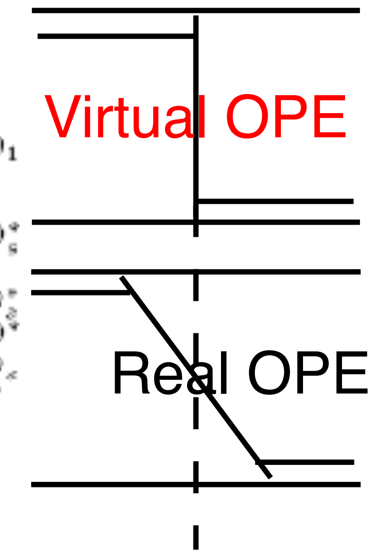


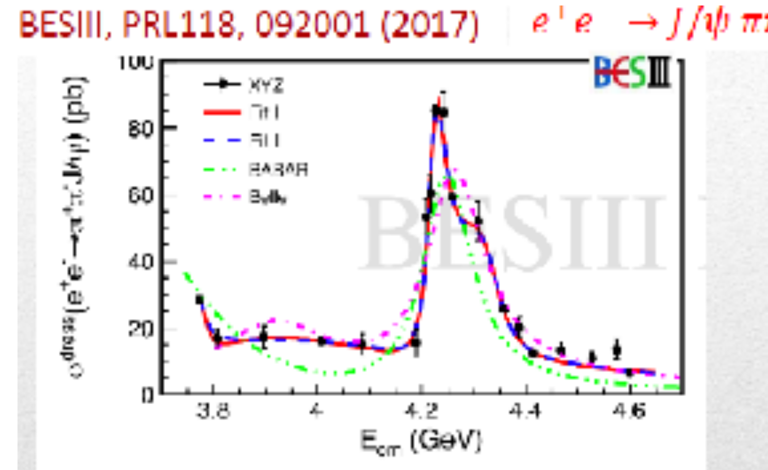


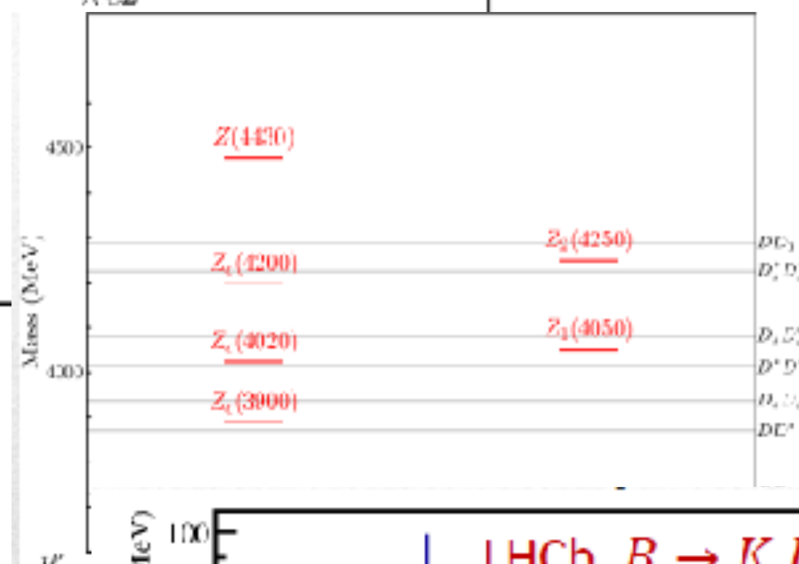
Signatures of unusual heavy quark resonances



Esposito, Pilloni, Polosa, Phys. Rep.





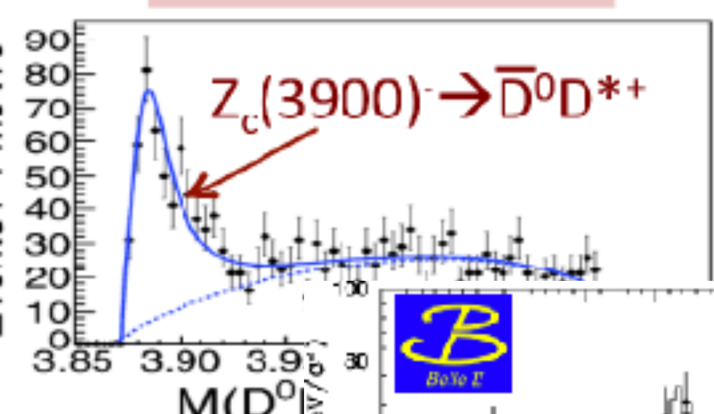


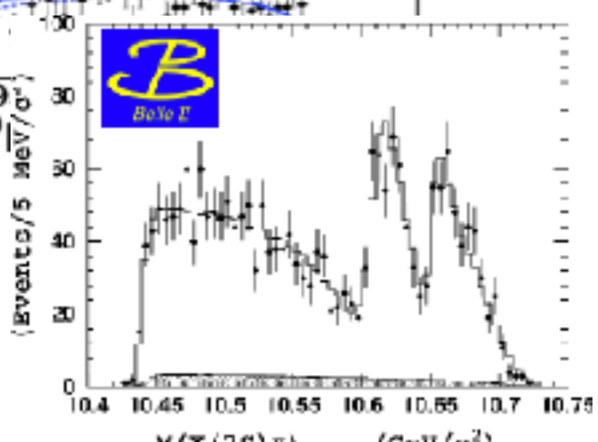
EMARK ON ENERGY PEAKS IN MESON SYSTEMS

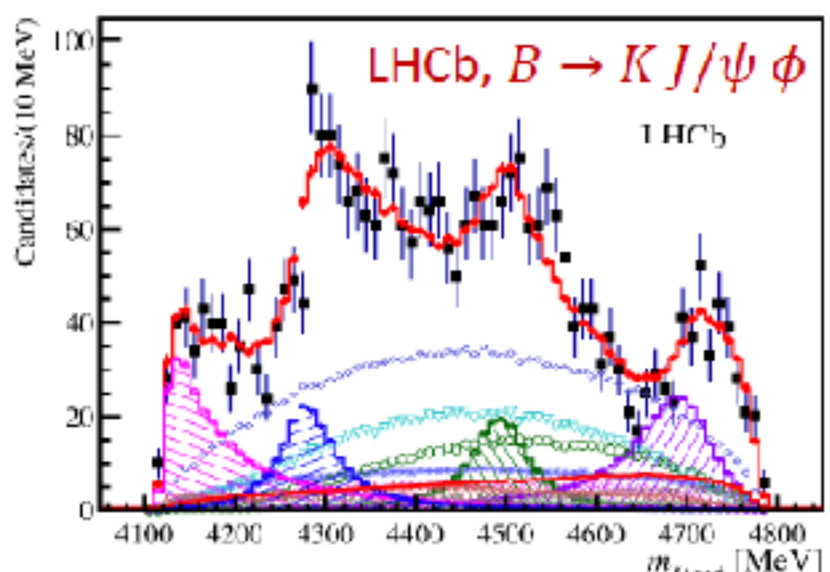
M. Nauenberg A. Pais

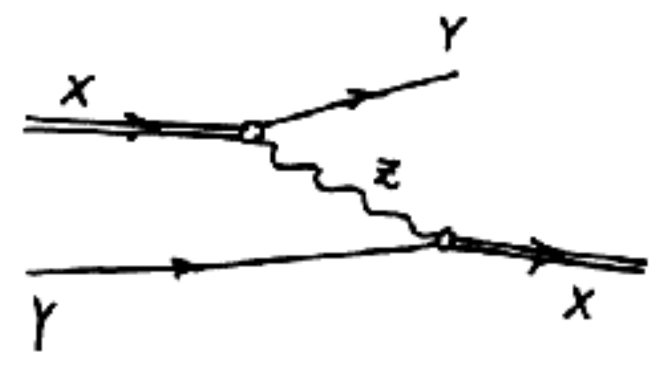
> If the width of particle X is not very large we will stay close to the physical region. This almost singular behavior of $A(s)$ for certain physical s causes the peaking effect to which we refer as an (X, Y, Z) peak.

BESIII PRL 112, 022001

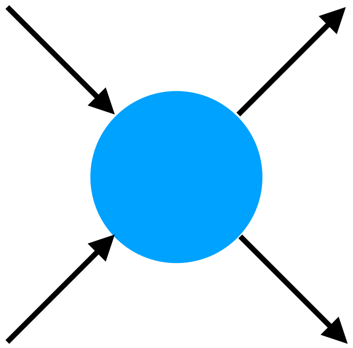






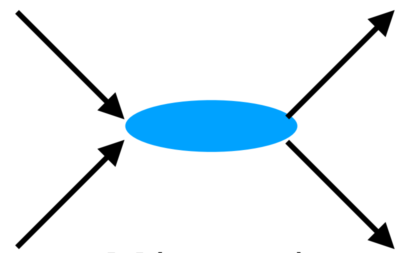


Amplitude signatures

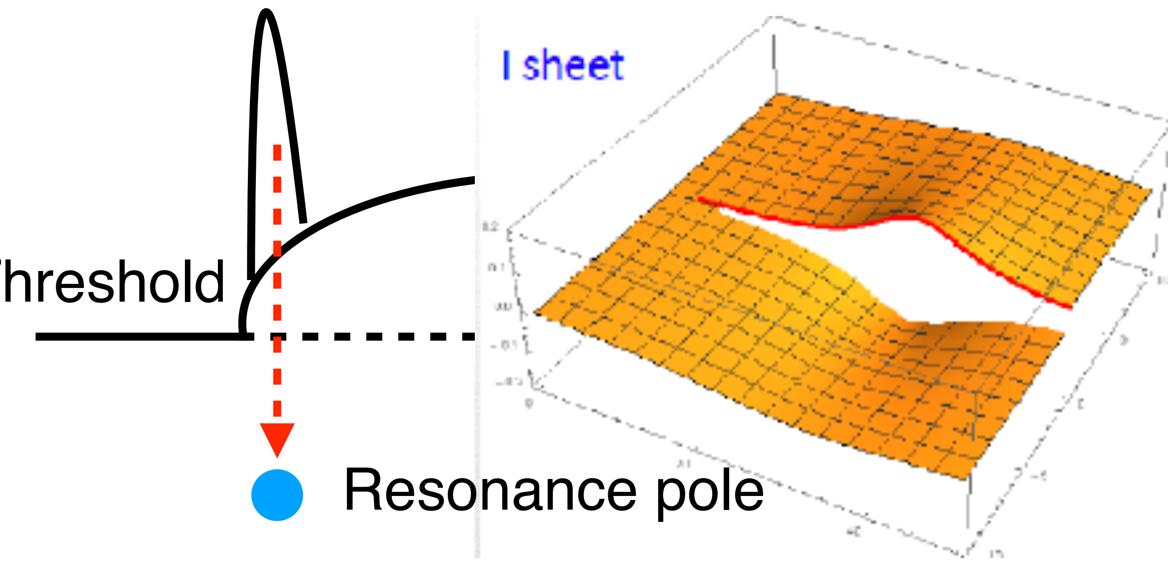


$\bar{D}\,D^* \rightarrow \bar{D}\,D^*$

New particles in the QCD spectrum

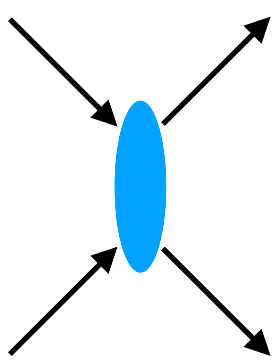


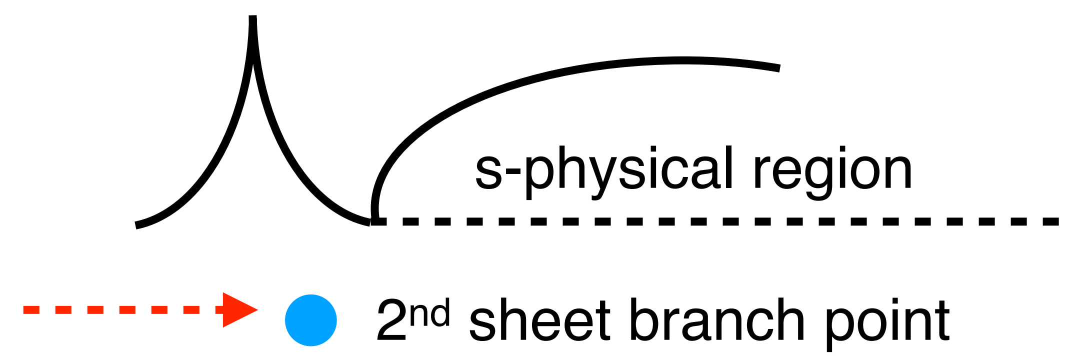
X(3872)



Other effects can be "generated" exchanges forces" (*)

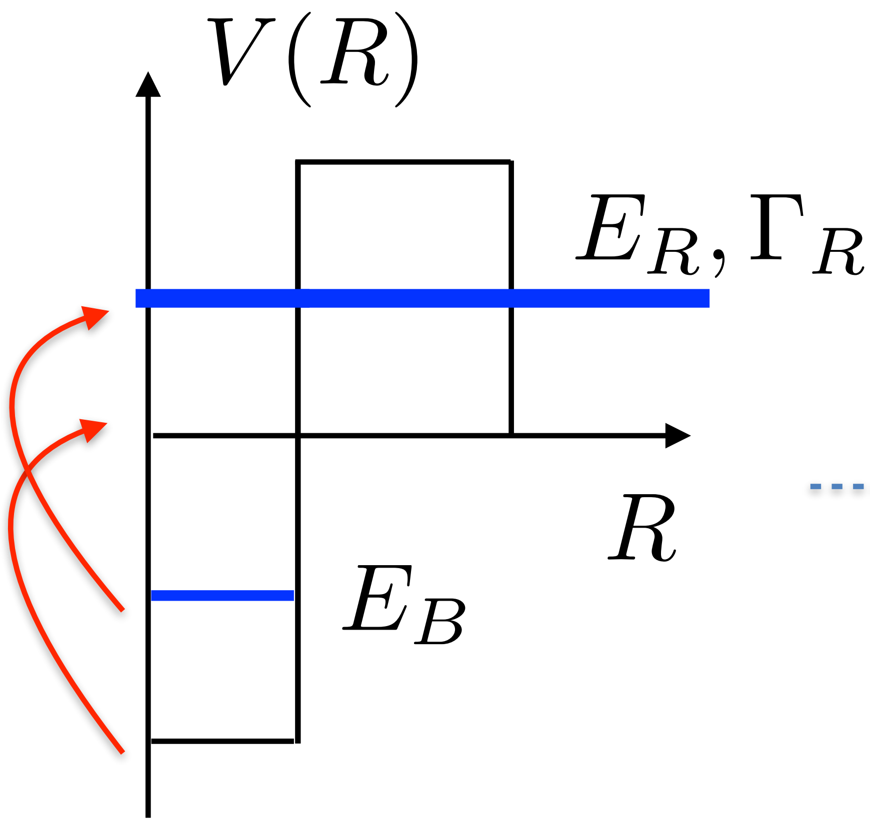
The "interesting stuff" happens on unphysical sheets. When singularities are close to the physical region rapid variations in amplitudes (cross sections) appear

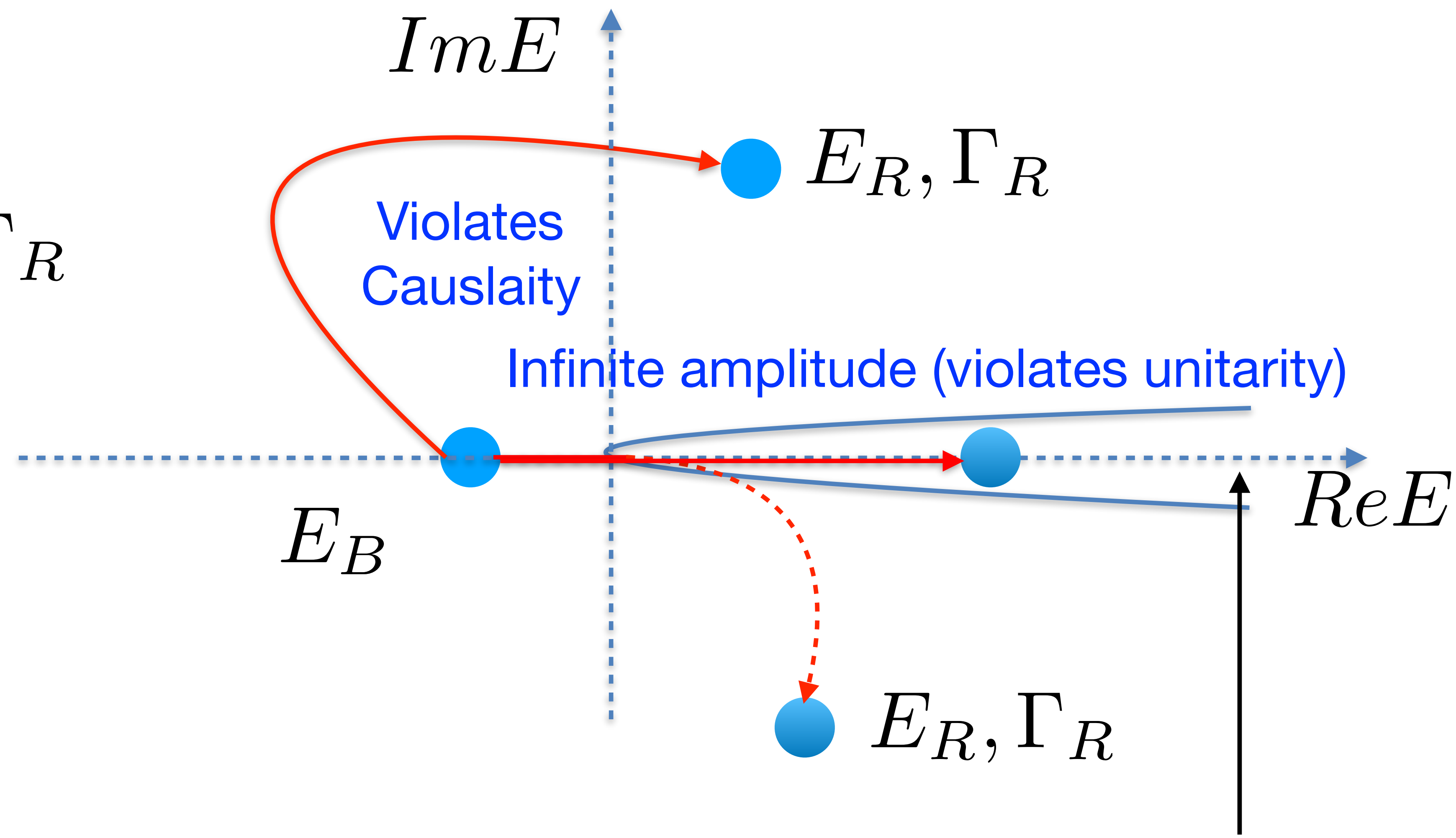




(*) (?) singularities because of confinement

Anatomy of resonances





When strength of interactions is reduces bound states become resonances

The only place for bound stets pole to migrate is onto an unphysical sheet connected to the open channel branch point

S-matrix formalism to 3 hadron interactions

Properties of reaction amplitudes are determined by

Causality: Reaction amplitudes are smooth (analytical) functions of kinematical variables with singularities reflecting existence of constraints (laws)

Unitarity: Determines singularities.

Crossing: Dynamical relations: reaction amplitudes in the exchange channel (forces) are analogous to amplitude in the direct channel (resonance)

Old vs New take on the S-matrix

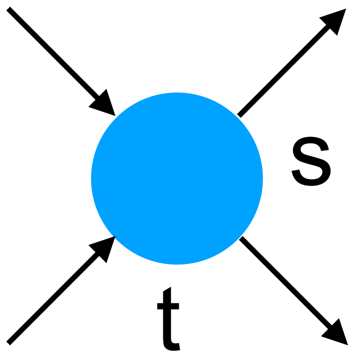
Old:

S matrix constraints -> “equations” (the Bootstrap program). Misses the specific “microscopics” e.g. Regge slope, CDD poles. Non unique solutions —> resonances appear not to be “self-generated” .

New:

Use the S-matrix principles to test QCD phenomenology.

2-to-2 scattering



$$A(s,t) = \sum_l (2l+1) a_l(s) P_l(z_s)$$

Partial waves test resonance hypothesis.

- Dispersion relation (Analiticity)

- Unitarity …

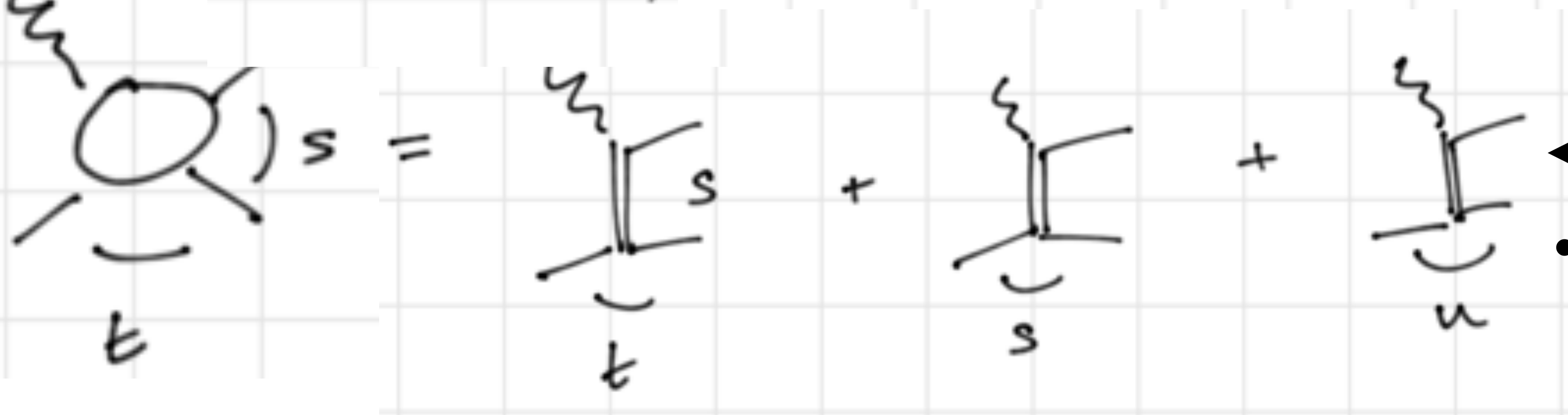
$$a_l(s) = B_l(s) + \frac{1}{\pi} \int_{tr} ds' \frac{Im a_l(s')}{s'-s} \longleftarrow |a(s'+i\epsilon)|^2 \rho(s')$$

…implies a relation between Re a_l and Im a_l

- B_l from cross channel interactions. Given B_l (e.g. for elastic $\pi\pi$ scattering it can be related to $a_l(s)$'s. Unitarity "converts" a dispersion relation to an integral equation (eg Roy eq.) for the partial waves). Solution is non unique — there are resonances/bound states not constrained by "exchange forces"

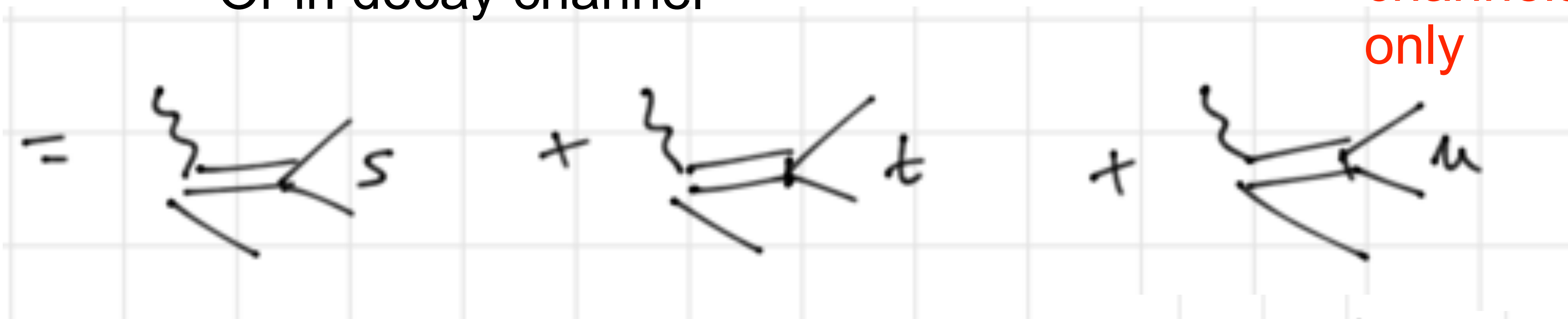
Beyond 2 particle production Khuri-Treiman (KT) model

$$A(s,t,M^2) = a(t,M^2) + a(s,M^2) + a(u,M^2)$$



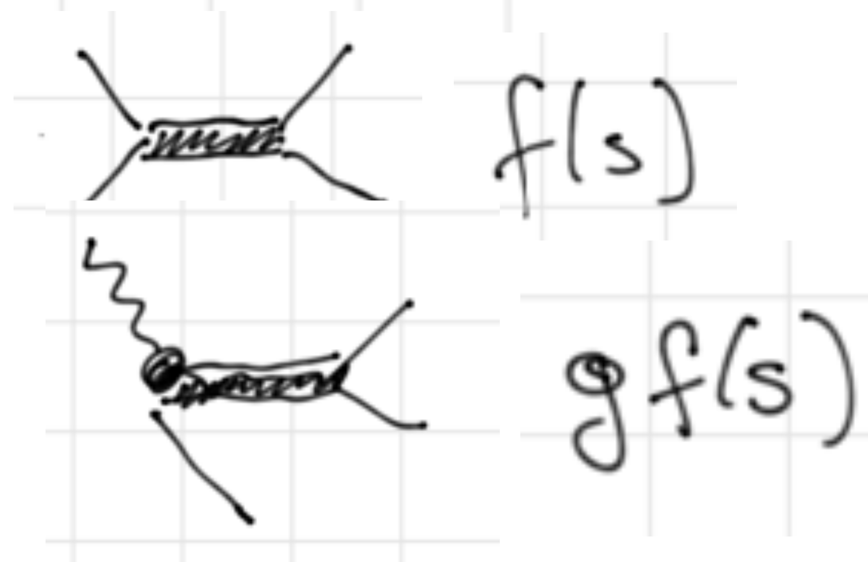
- Isobars are not partial waves. They are model amplitudes with direct channels branch points only

Or in decay channel

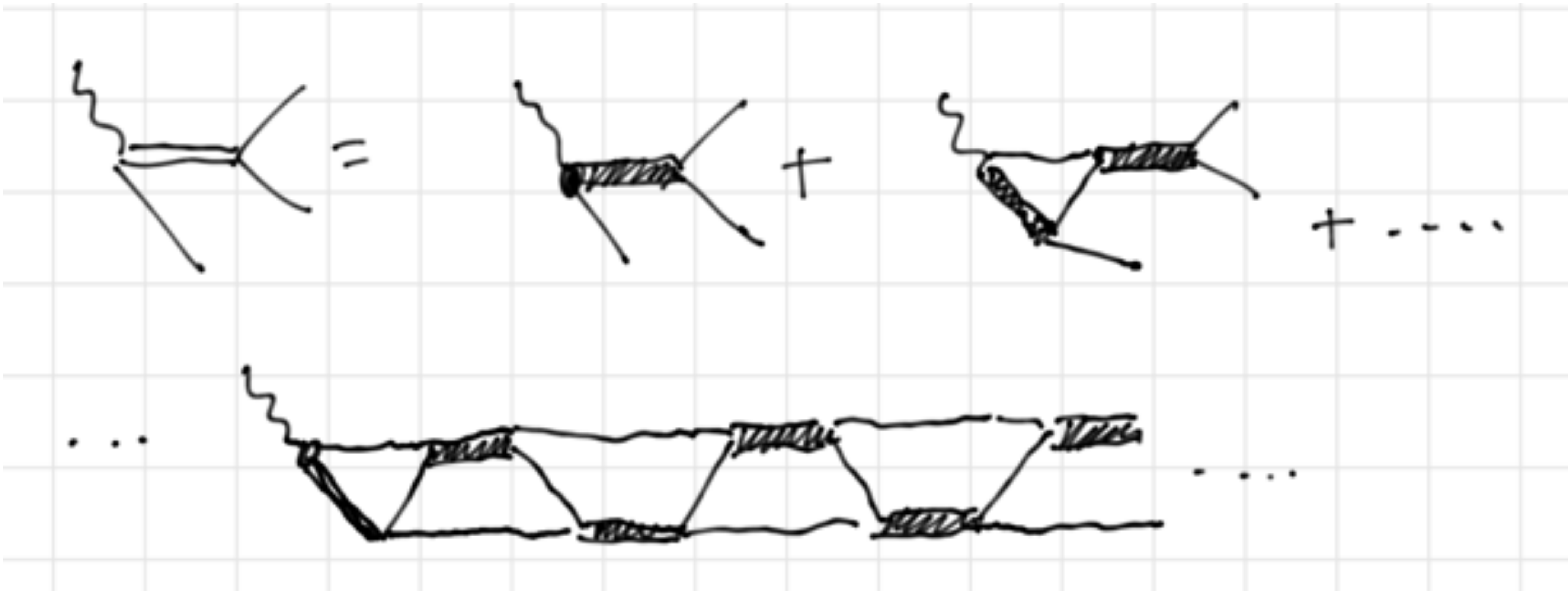


- Isobars to be determined in terms of 2-to-2 scattering amplitude and a coupling, g to the production channel (g=1)

- unitarity : t-channel process acts as a "driving" term for s-channel isobar



KT equations

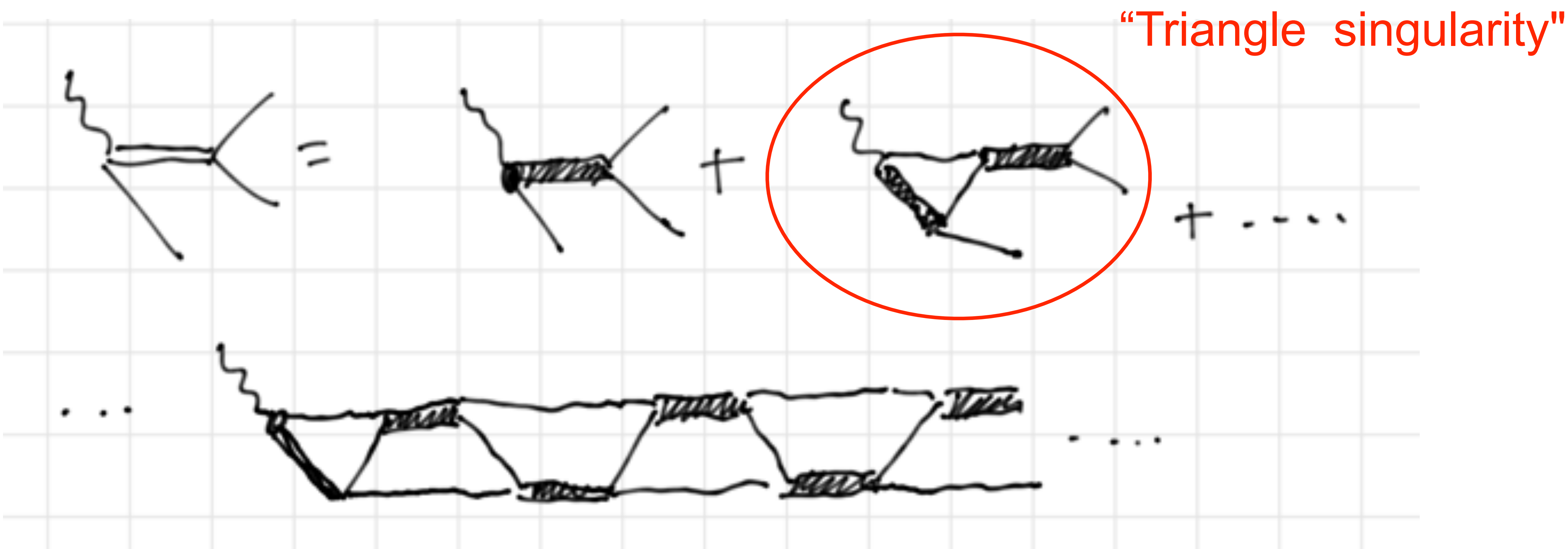


Isobar amplitudes have only the
normal threshold singularities

$$a(s, M^2) = f(s) + f(s) \int_{tr} \frac{ds'}{\pi} \frac{\rho(s')b(s', M^2)}{s' - s}$$

$$b(s, M^2) = \int_{-1}^{1} dz a(t(s, z, M^2), M^2)$$

KT equations



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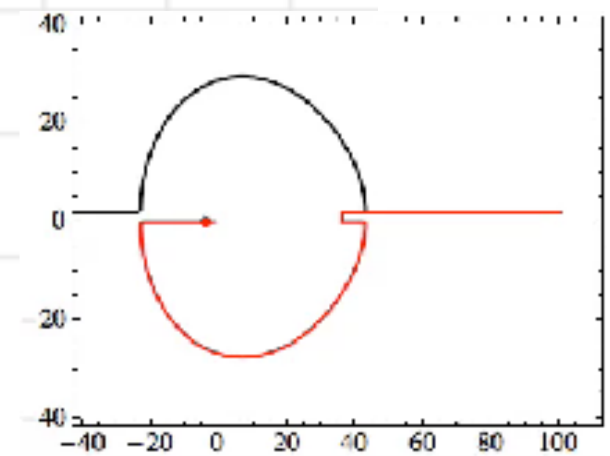
KT equations

"Triangle singularity"



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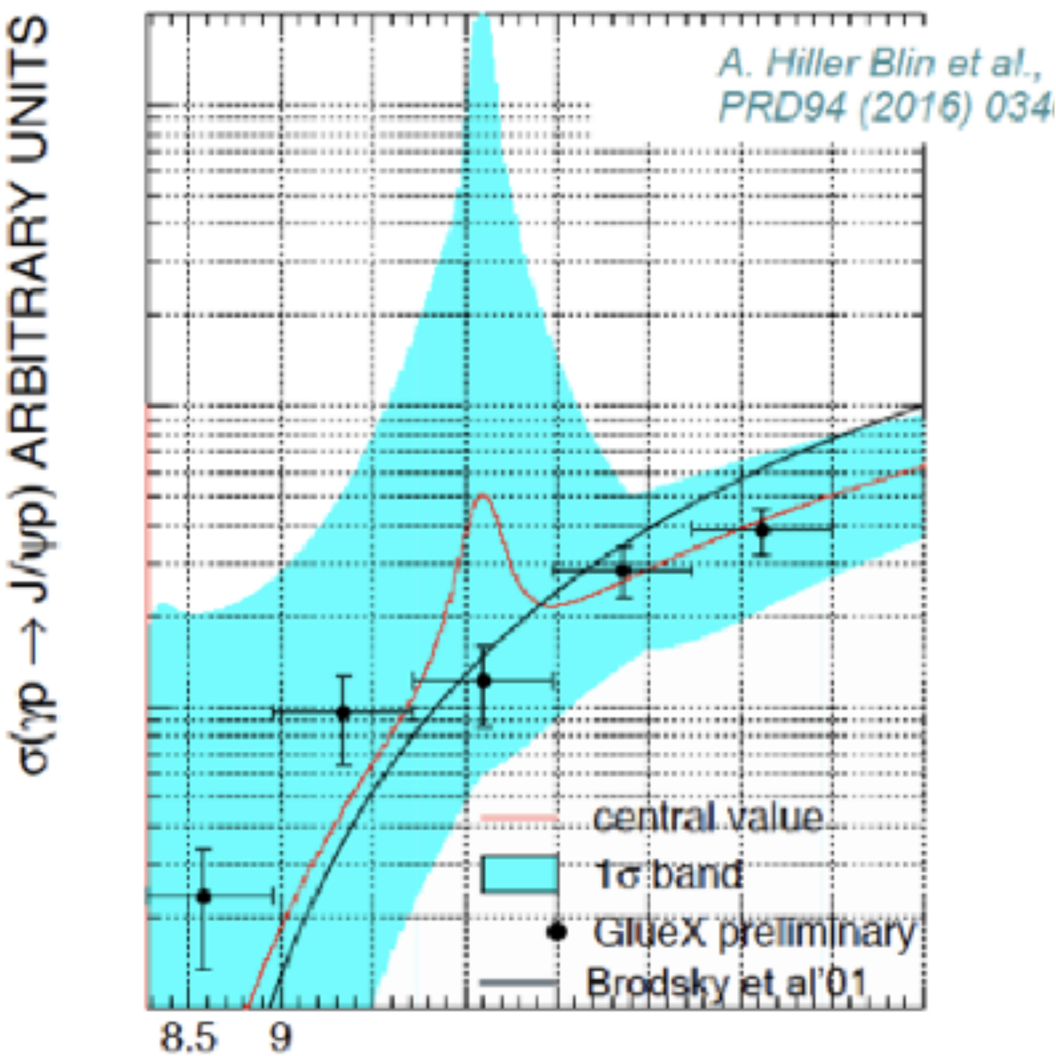
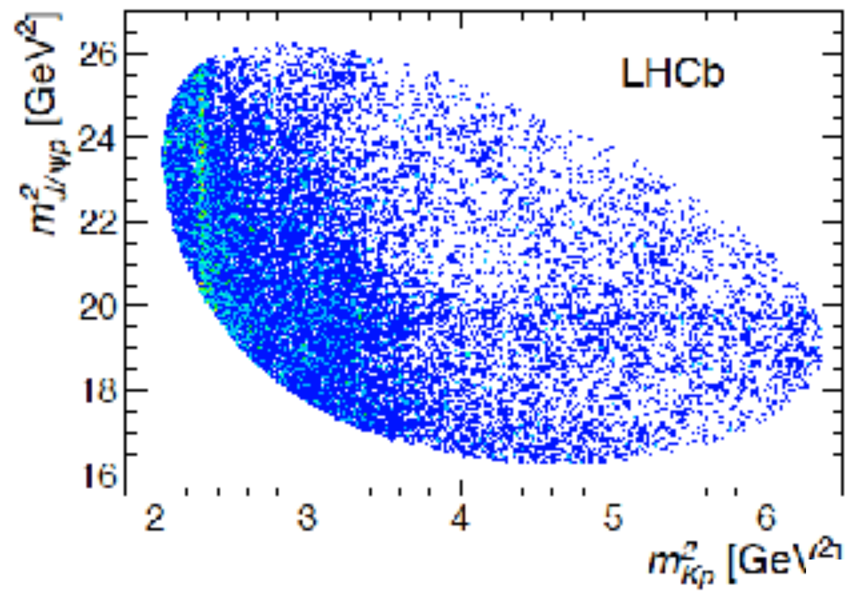
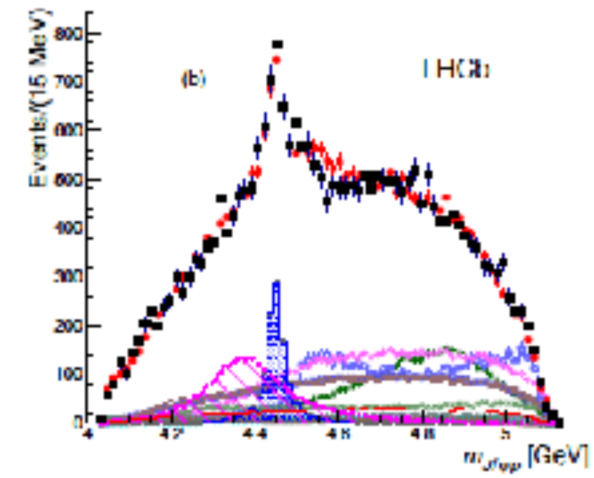
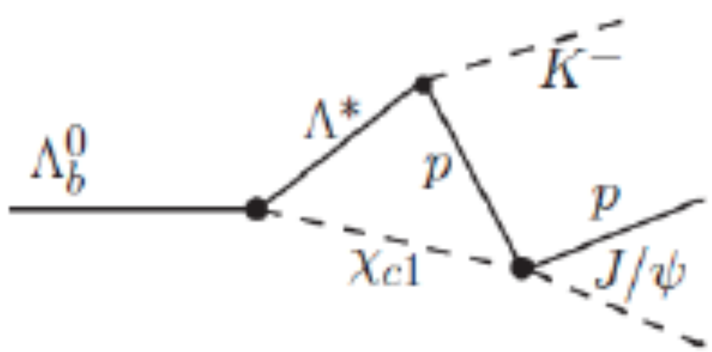
$$b(s, M^2) = \int_{-1}^{1} dz a(t(s, z, M^2), M^2)$$

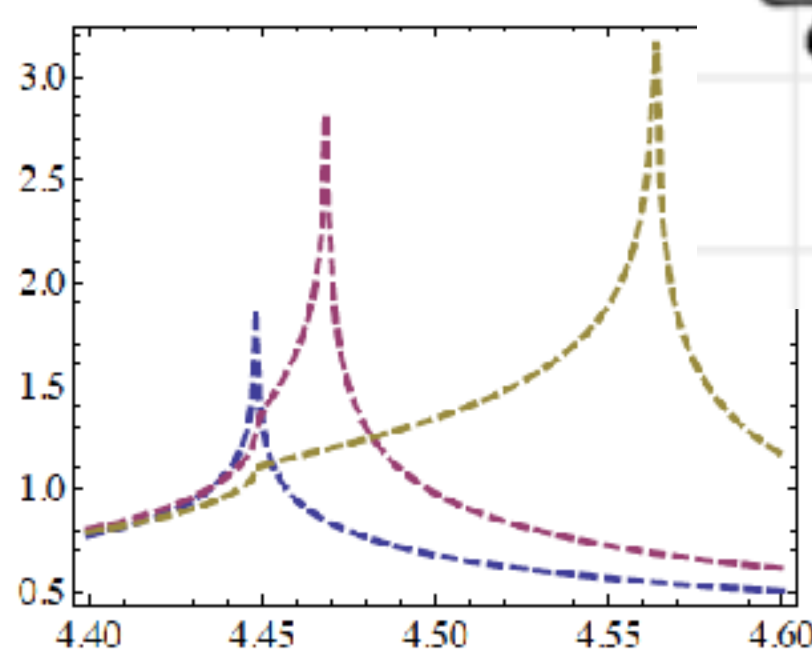
Pentaquark as a triangle singularity ?







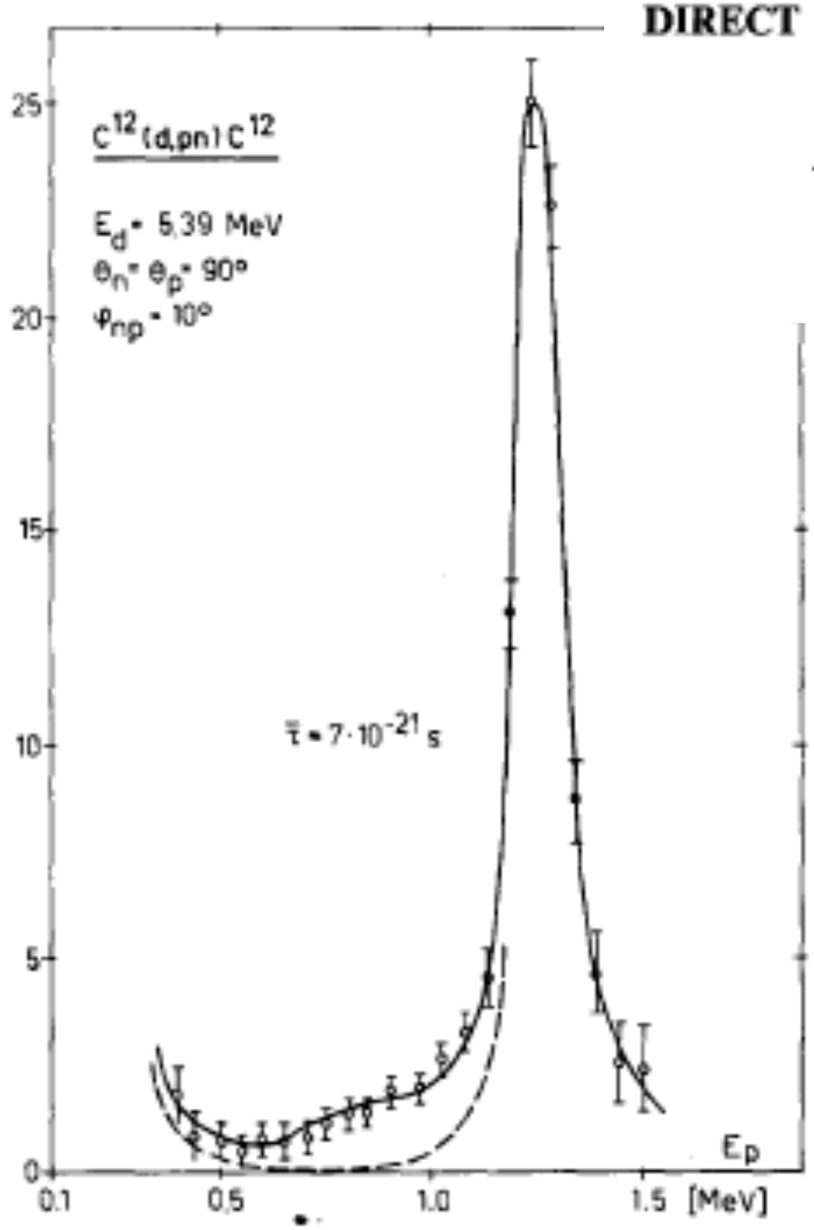




Axes Abs[$T(s)$], $\sqrt{s}$

Lines : blue (λ = 1.89 GeV), red (λ = 1.99 GeV), yellow (λ = 2.09 GeV)

Have cusps been seen



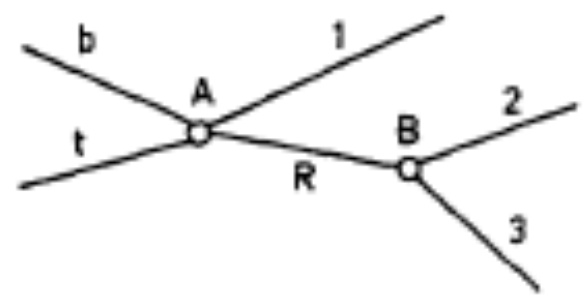
DIRECT DETERMINATION OF A SHORT NUCLEAR LIFETIME ($\approx 10^{-20}$ s) BY THE PROXIMITY SCATTERING METHOD

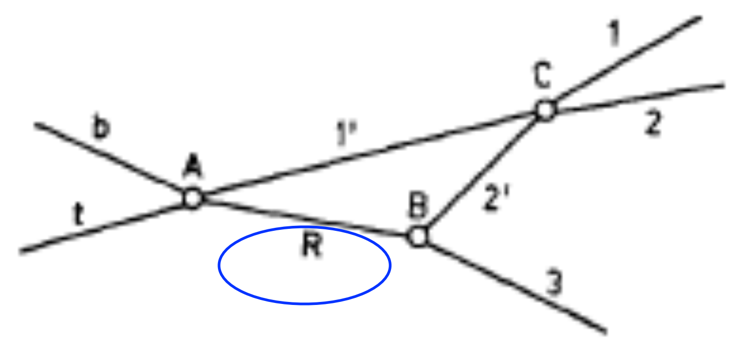
J. LANG, R. MÜLLER, W. WÖLFLI, R. BÖSCH and P. MARMIER

Laboratorium für Kernphysik, Eidg. Techn. Hochschule, Zürich †

Received 4 February 1966

$$b+t \rightarrow 1+2+3 \qquad [d+{}^{12}C \rightarrow n+p+{}^{12}C],$$





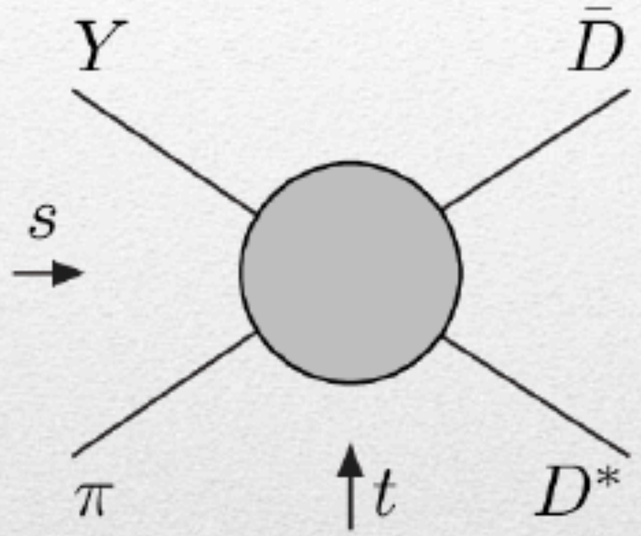
$$b+t \rightarrow 1+R, \qquad \text{with } Q\text{-value } Q_1 \qquad [d+{}^{12}C \rightarrow n+{}^{13}N^*, \qquad Q_1 = -3.82 \text{ MeV}],$$

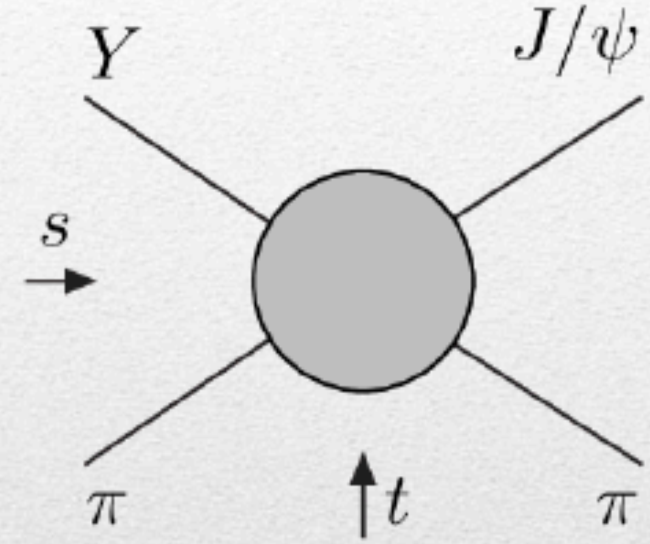
$$R \rightarrow 2+3 \qquad \text{with } Q\text{-value } Q_2 \qquad [{}^{13}N^* \rightarrow p+{}^{12}C, \qquad Q_2 = 1.59 \text{ MeV}],$$

Amplitude analysis for $Z_c(3900)$

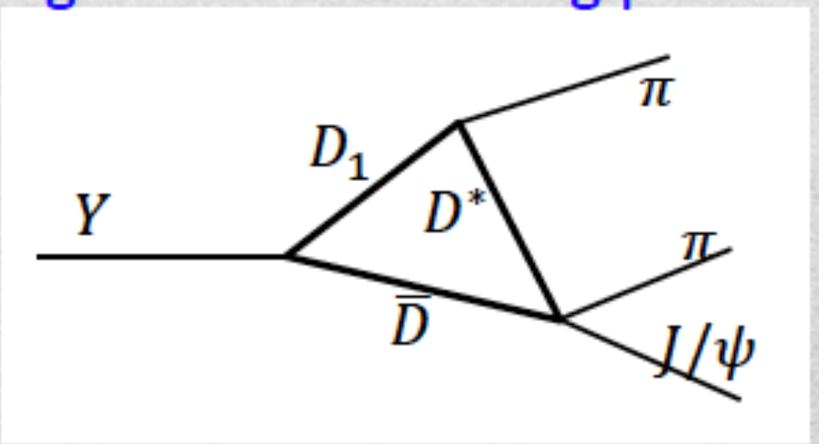
One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), arXiv:1612.06490





Triangle rescattering,
logarithmic branching point

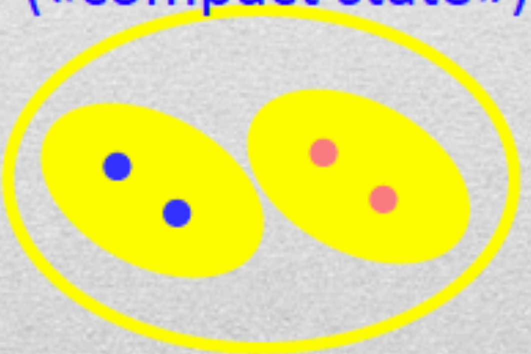


Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo *et al.* PRD92, 071502

(anti)bound state,
II/IV sheet pole
(«molecule»)

Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

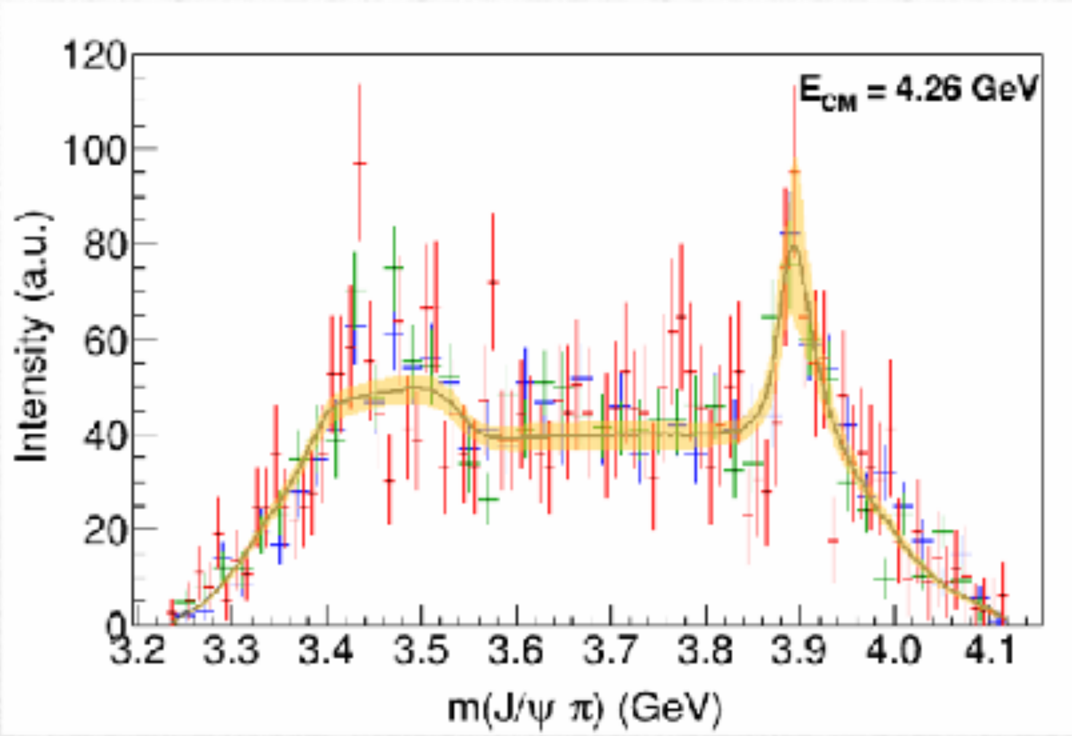
Resonance,
III sheet pole
(«compact state»)



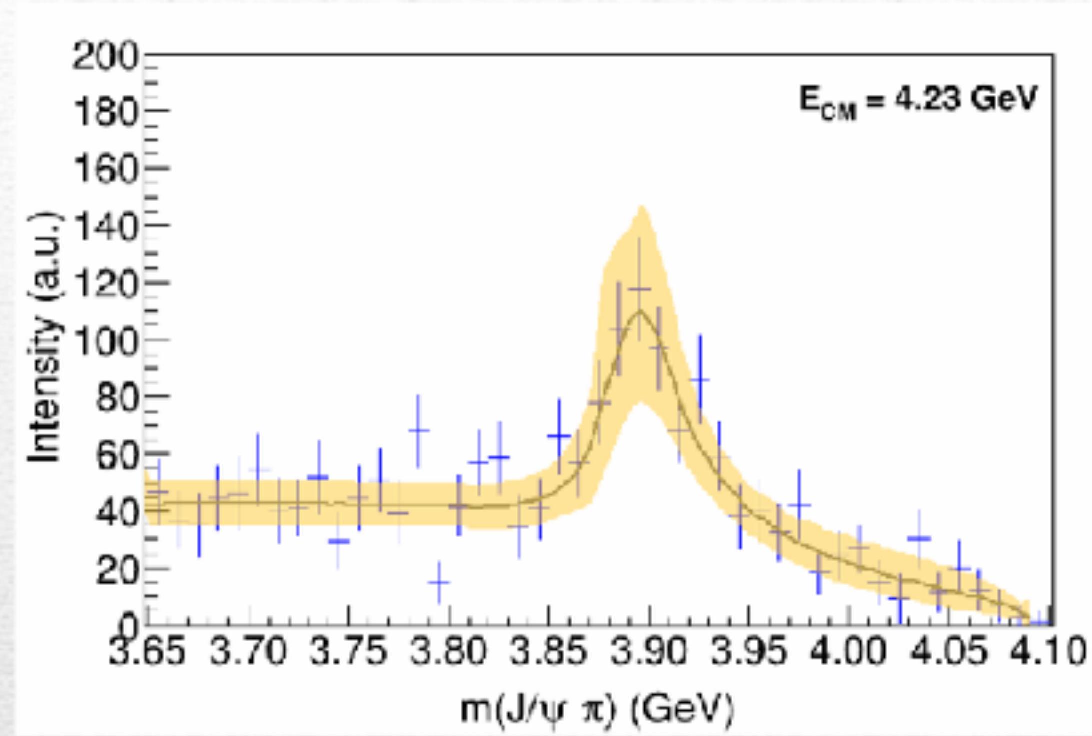
Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

A.Pilloni

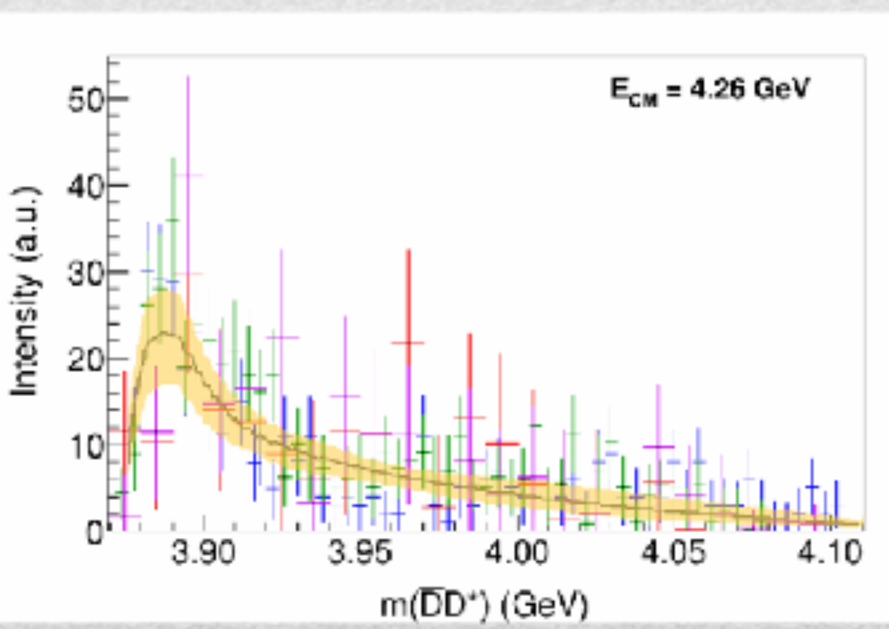
Fit: III



E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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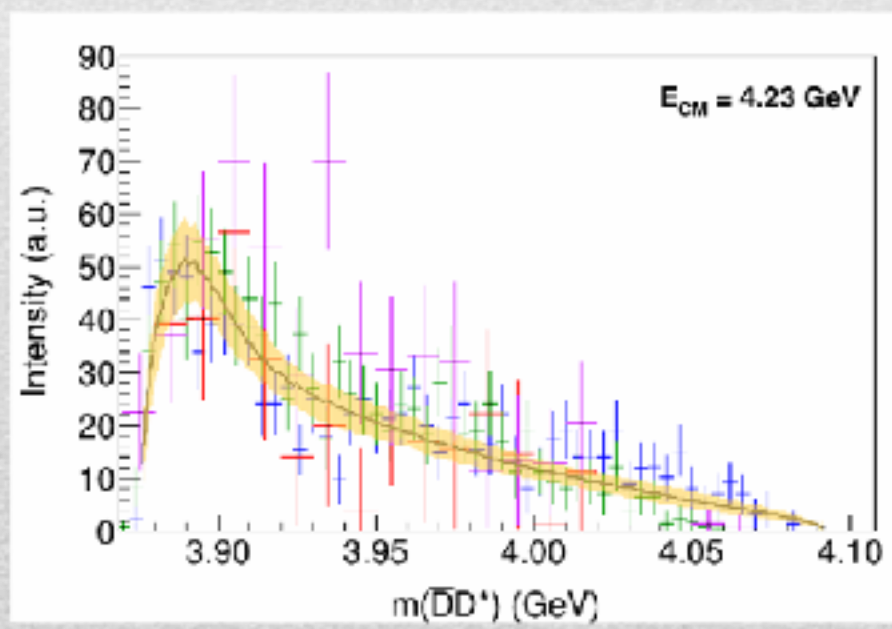

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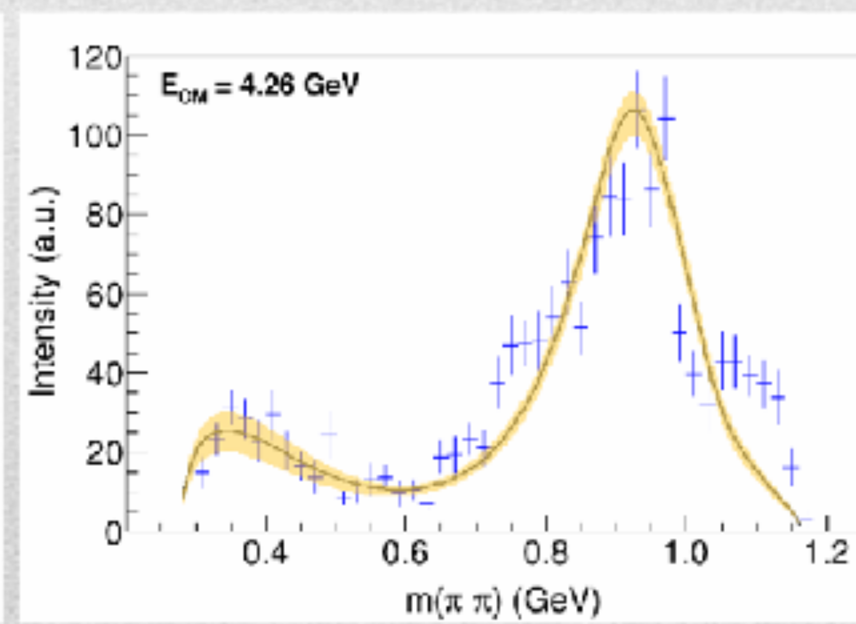

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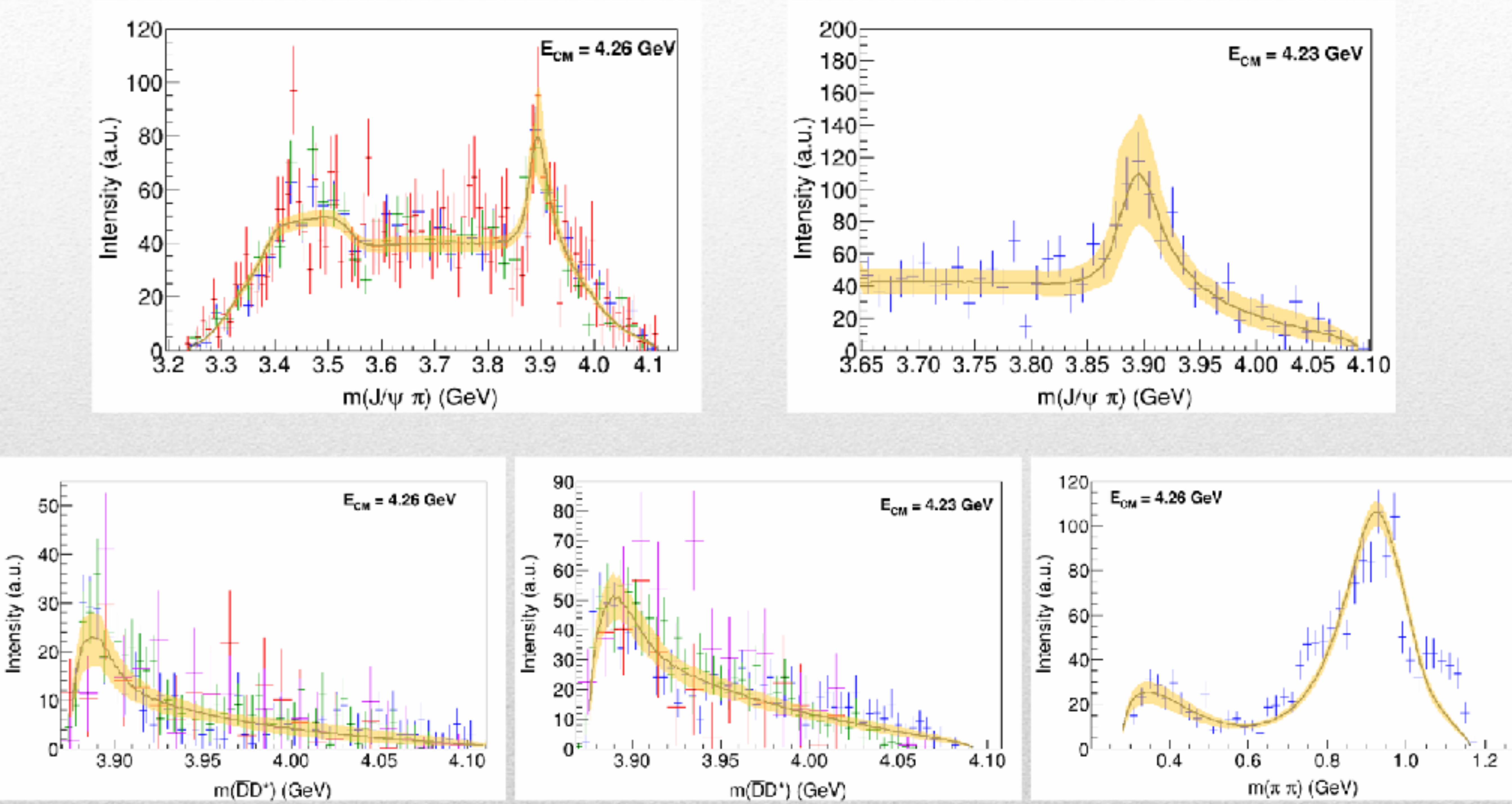
Fit: III



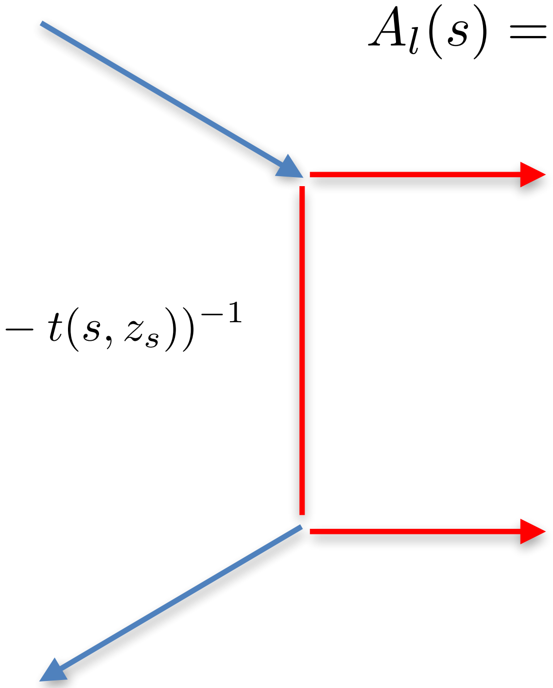
E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Two models for cusps

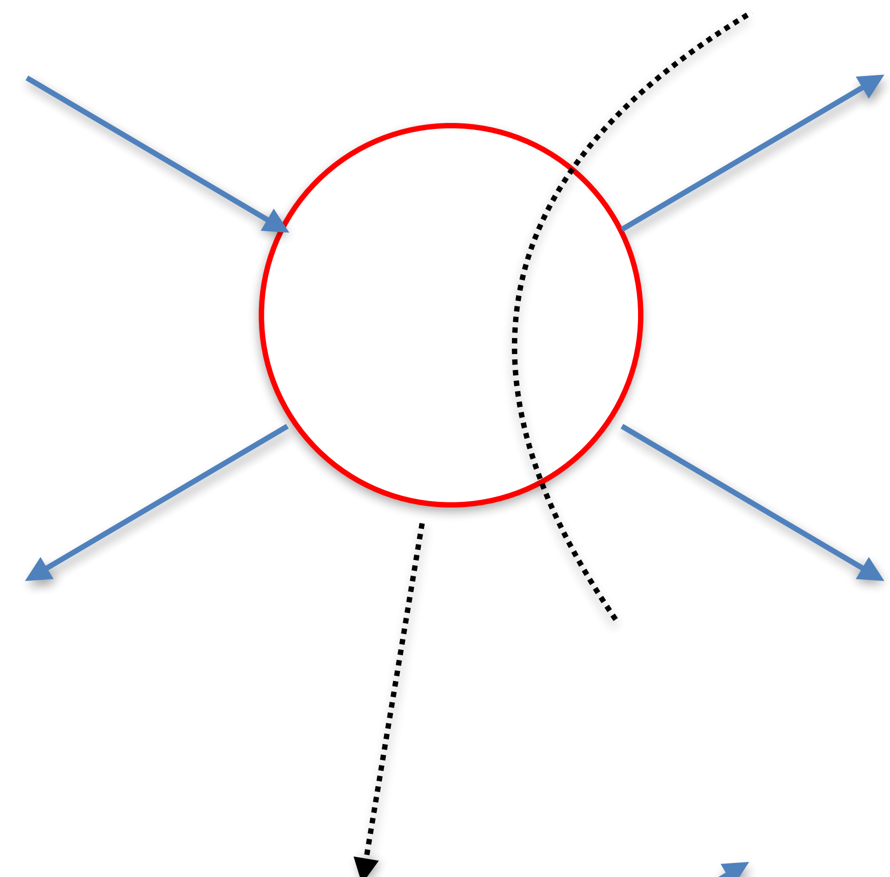
(2nd sheet singularities near physical region)

$$A_l(s) = \int dz_s A(s, t(s, z_s), u(s, z_s)) P_l(\cos\theta)$$

$$t = -\frac{(s-4m^2)}{2}(1 - z_s)$$

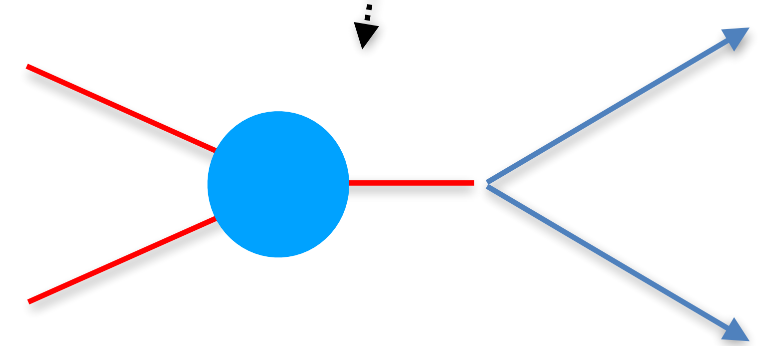


$\propto (m_e^2 - t(s, z_s))^{-1}$



$$A_0(s) \sim \int_{-1}^{1} dz_s \frac{1}{m_e^2 + \frac{(s-4m^2)}{2}(1 - z_s)}$$

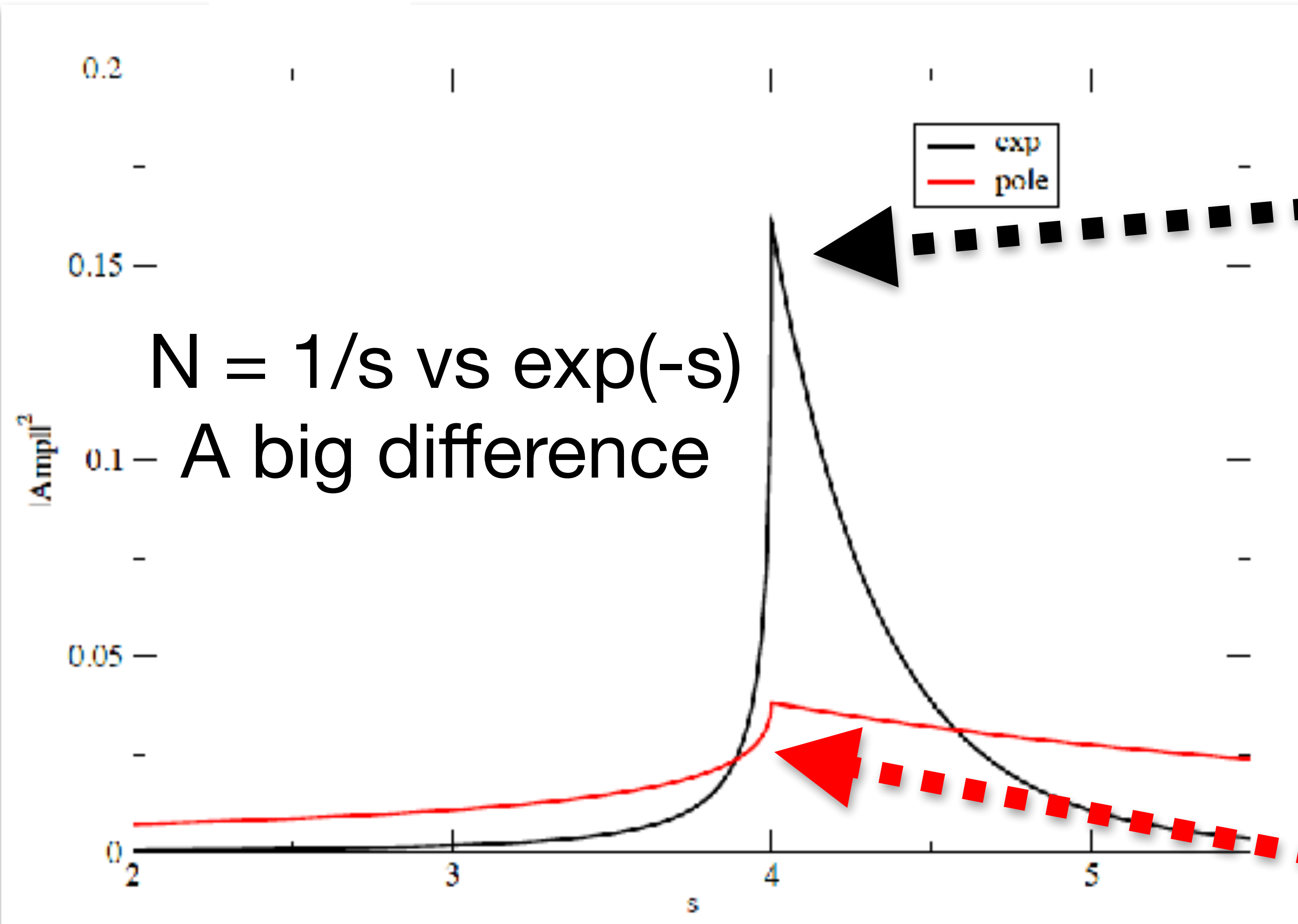
t/u channel exchange close to
the s-channel physical region
(Leads to a cusp in the s-channel)

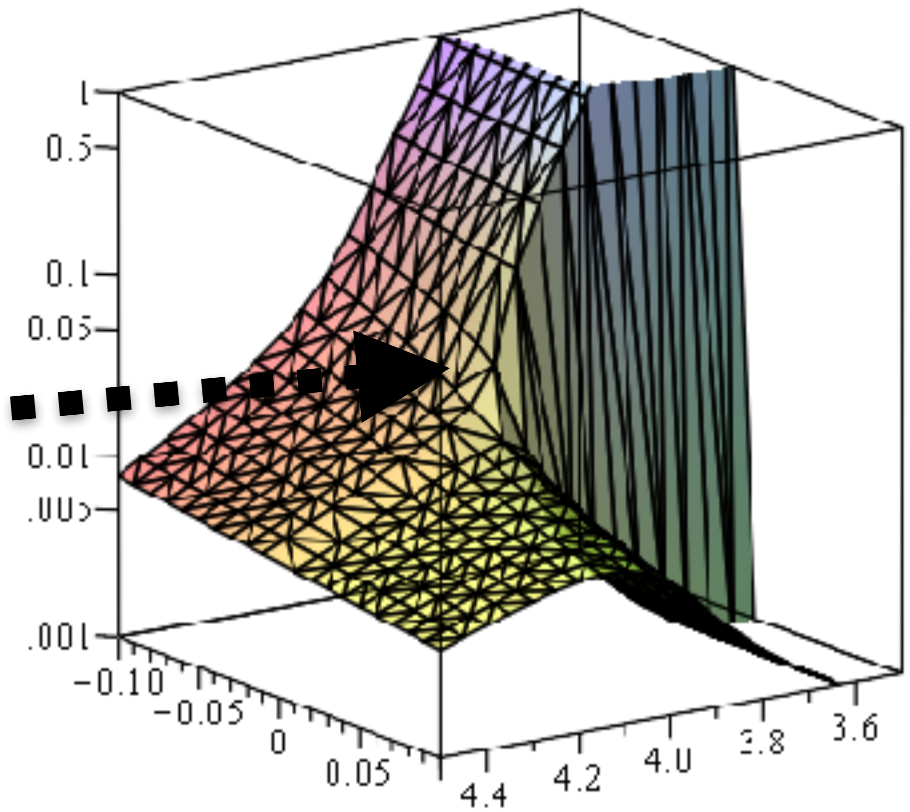


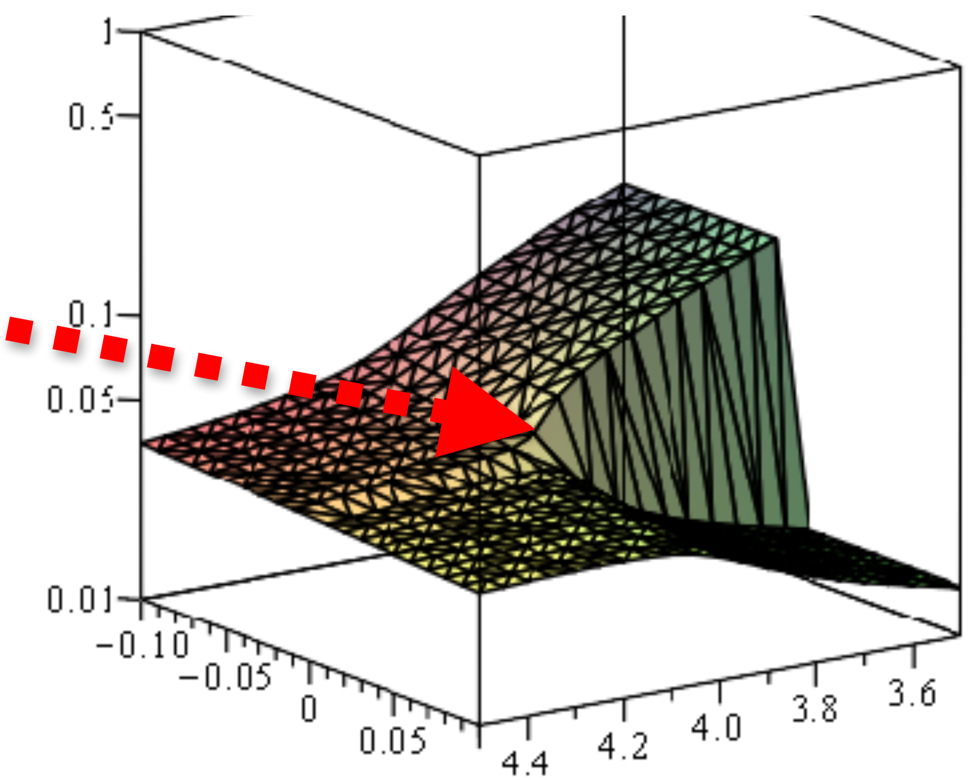
"Form factor"

A cusp from a vertex

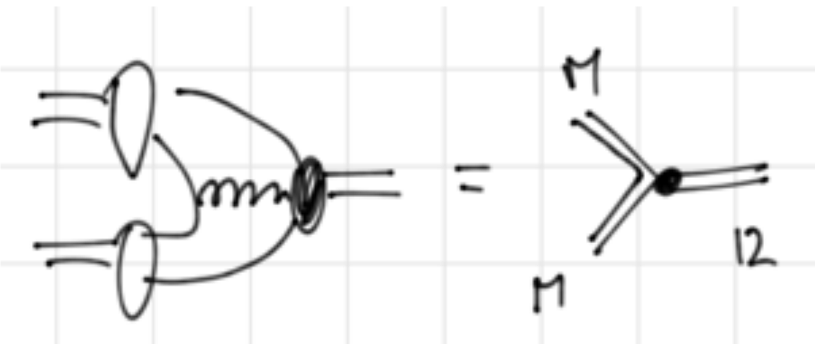
$$\int_4 ds' \sqrt{1 - \frac{4}{s'}} \frac{N(s')}{s' - s}$$

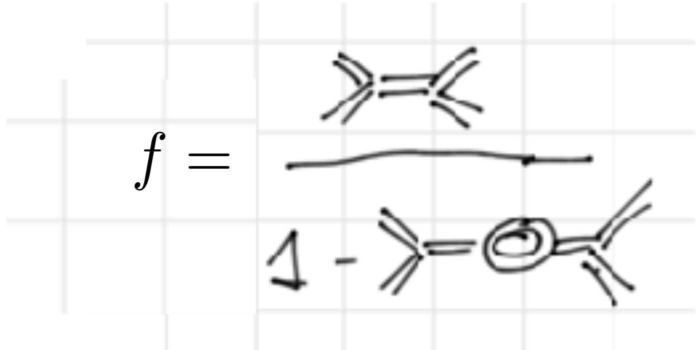


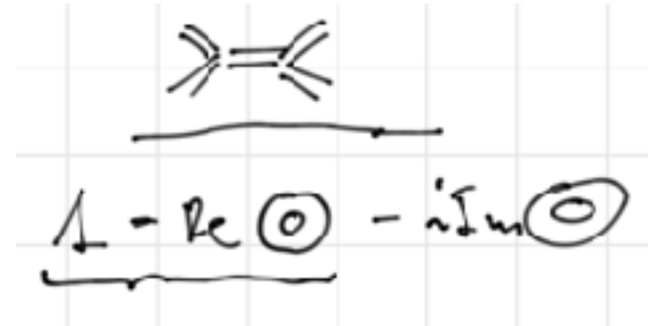




Scattering through resonances







$$f(s) = \frac{1}{K^{-1}(s) - i\Gamma(s)}$$

∞ number of poles = confinement

$K^{-1}(s)$ needs to have ∞ number of poles (and zeros)

Quadratically spaced radial trajectories

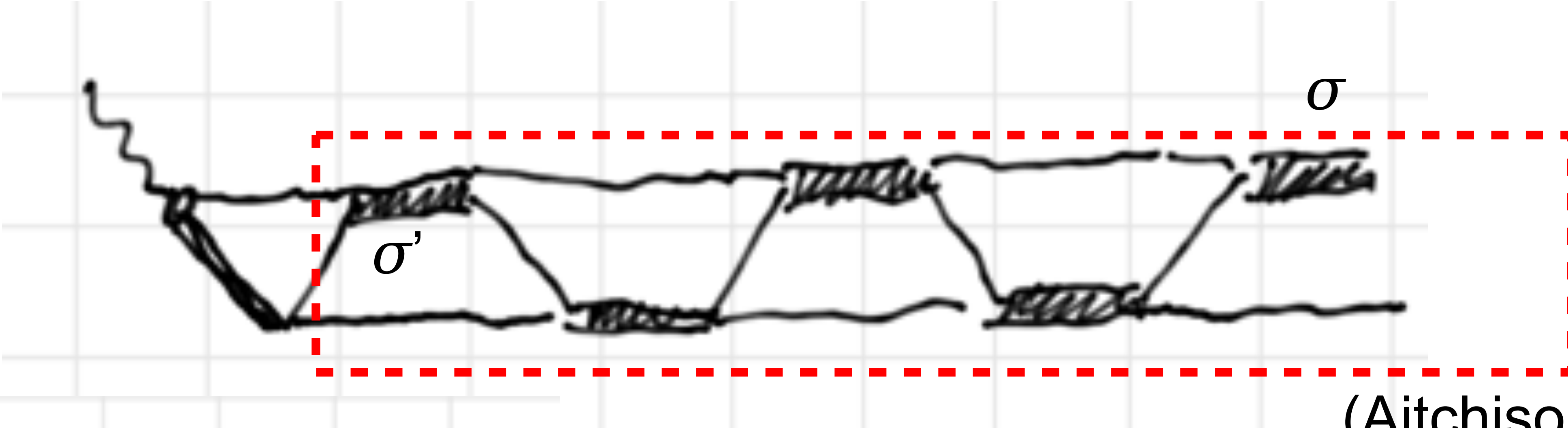
$$K(s) = \sum_{r=1}^{\infty} \frac{g_r^2}{m_r^2 - s} \to \sum_r \frac{1}{r^2 - s} \sim \frac{\cos(\pi\sqrt{s})}{\sin(\pi\sqrt{s})}$$

Linearly spaced radial trajectories (Veneziano)

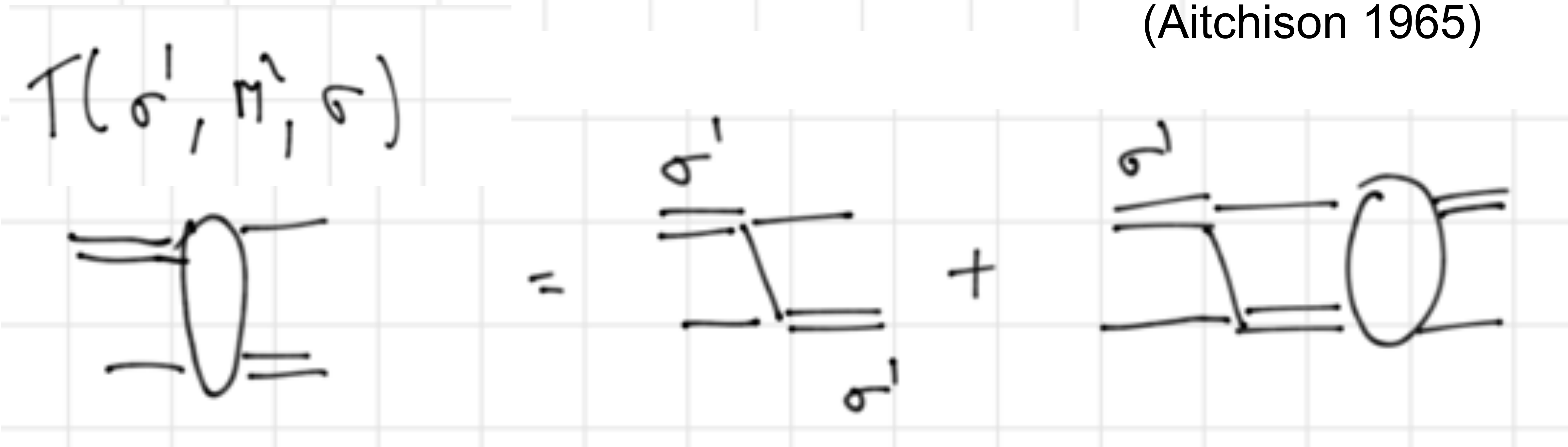
$$K(s) \sim \frac{\Gamma(a-s)}{\Gamma(b-s)}$$

Exponential form factors related to infinite number of particles (confinement) !

3-to-3 amplitude from a 2-to-2 (1-to-3) KT model



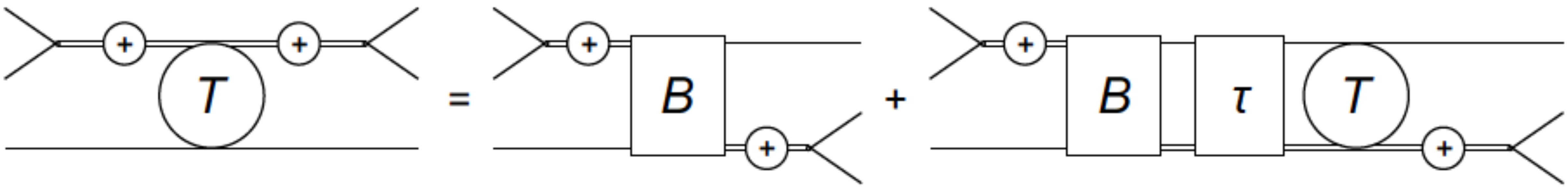
(Aitchison 1965)



$$T(\sigma', M^2, \sigma) = B(\sigma', M^2, \sigma)$$

$$+ \int d\sigma'' B(\sigma', M^2, \sigma'')\tau(\sigma'', M^2)T(\sigma'', M^2, \sigma)$$

Relation to other approaches



$$T(q,p;s) = B(q,p;s) - \int \frac{d^3 l}{(2\pi)^3} B(q,l;s) \frac{\tau(\sigma(l))}{2E(l)} T(l,p;s)$$

$$T_{3\to3}(\sigma', s, \sigma) = \sum_{\sigma''} \left[\frac{1}{1 - \tau(s) B(s)} \right]_{\sigma', \sigma''} [B(s)]_{\sigma'', \sigma}$$

M. Mai et al. (JPAC)
I.Aitchison (Khuri-Treiman)
H.Hammer et al. (EFT)

$\longrightarrow$ "ϱ vs Chew-Madelstam" phase space (properly removes unphysical singularities from ϱ)

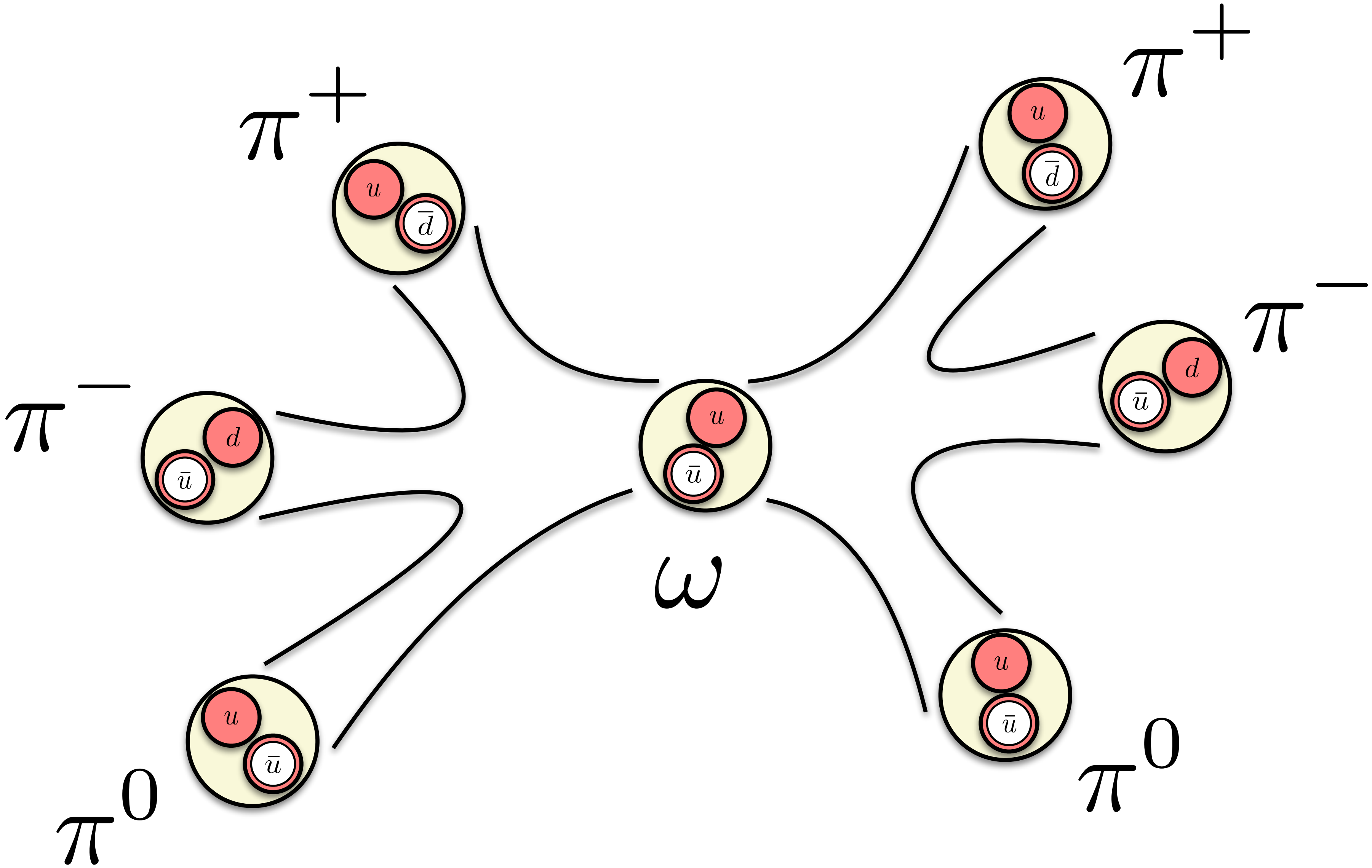
- Reproduces 3-to-3 unitarity on the real axis only
- Analyticity in sub-channel variables ?

$$T = \frac{1}{K^{-1}(s) - i\rho(s)}$$

$$T = \frac{1}{K^{-1}(s) - \frac{1}{\pi} \int_{tr} ds' \frac{\rho(s')}{s'-s}}$$

Next step is to derive the proper dispersive representation

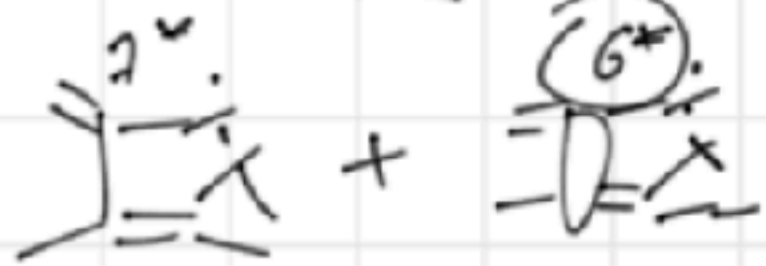
3-to-3 scattering from dispersion relations



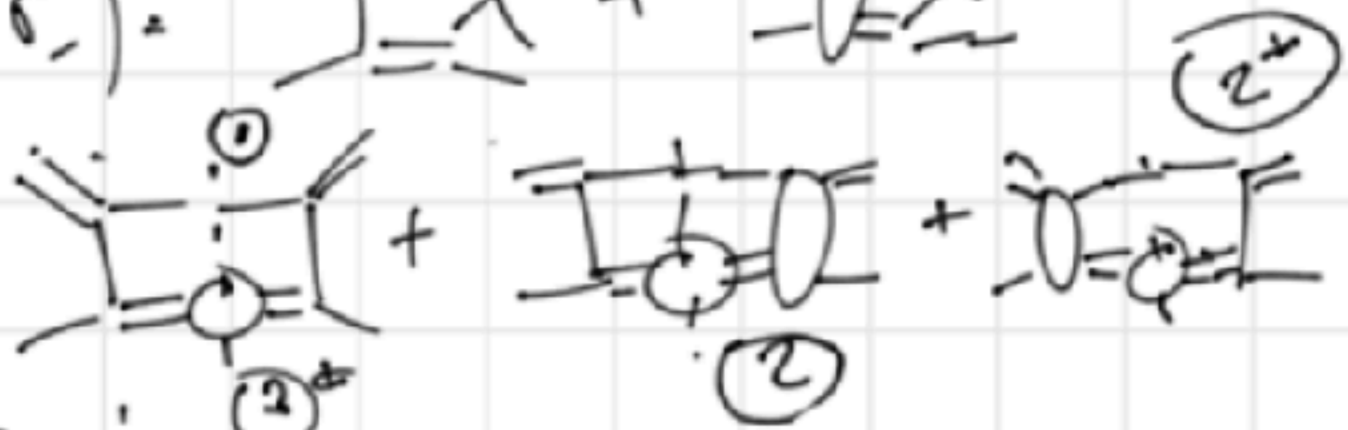
Special thanks to Andrew Jackura

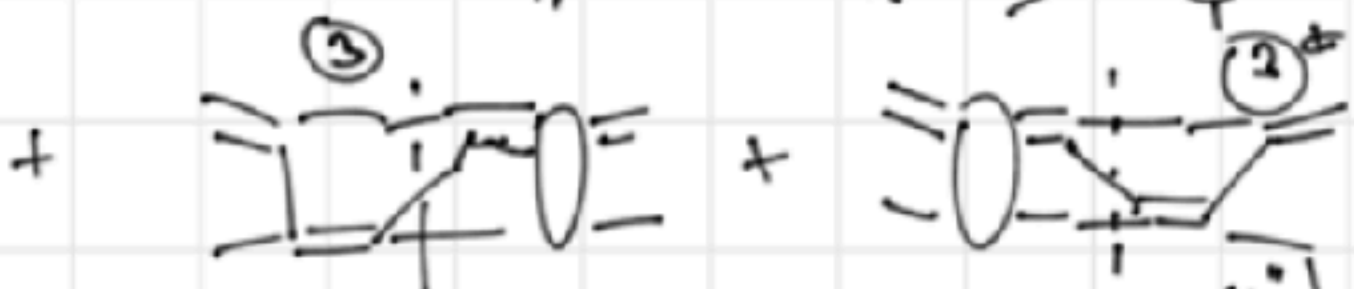
$C_1(\sigma'_+ \, s_+ \, \sigma_+) - C_1(\sigma'_- \, s_+ \, \sigma_+) =$

$C_1(\sigma'_- \, s_- \, \sigma_+) - C_1(\sigma'_- \, s_- \, \sigma_-) =$



$\frac{1}{\varepsilon}$: $C_1(\sigma'_- \, s_+ \, \sigma_+) - C_1(\sigma'_- \, s_- \, \sigma_+) =$





$+$ Boxes

$(m_2+m_3)^2 < \sigma''_{23} < (\sqrt{s}-m_1)^2$

s_- real

$$(2) = \frac{1}{2}\int d\sigma''_{23} \, \frac{\lambda^{1/2}(s,\sigma''_{23},m_1^2)}{4s} \int \frac{dz}{2} \left[\frac{1}{2\pi} \frac{N(\sigma'_{12})N(\sigma''_{23})}{\mu^2 - t(z,s,\sigma'_{12},\sigma''_{23}) + i\varepsilon} \right] \ln\left(\frac{1}{D(\sigma''_{23})}\right) C_{3,1}(\sigma''_{23+}, s_+, \sigma_{12+})$$

no singularities in σ'_{12-}

$$(1) = \frac{1}{2}\int d\sigma''_{23} \, \frac{\lambda^{1/2}(s,\sigma''_{23},m_1^2)}{4s} \int \frac{dz}{2} \left[\frac{1}{2\pi} \frac{N(\sigma'_{12})N(\sigma''_{23})}{\mu^2 - t(z,s,\sigma'_{12},\sigma''_{23}) + i\varepsilon} \right] \ln\left(\frac{1}{D(\sigma''_{23})}\right) \left[\int \frac{dz}{2} \frac{N(\sigma''_{23})N(\sigma_{12+})}{\mu^2 - t(z,s,\sigma''_{23},\sigma_{12}) - i\varepsilon} \frac{1}{2\pi} \right]$$

Summary

- Existing 3 particle scattering amplitudes may have spurious singularities — analyticity not enforced.

- Proposed framework: unitarity + analyticity (only normal thresholds in particle-isobar amplitudes) → dispersion relations + short range inputs → N/D equations → amplitudes !