50th anniversary, Florence, 11

Gabriele Veneziano

The beginning of a 50-years-old adventure*)

*)

and

so many youngsters in those pioneering days

Outline

1. The cheapest bootstrap
2. FESR days @ Rehovot and Cambridge
3. The cheapest solution
4. DRM days, part I
5. Not an accident

1. The cheapest bootstrap

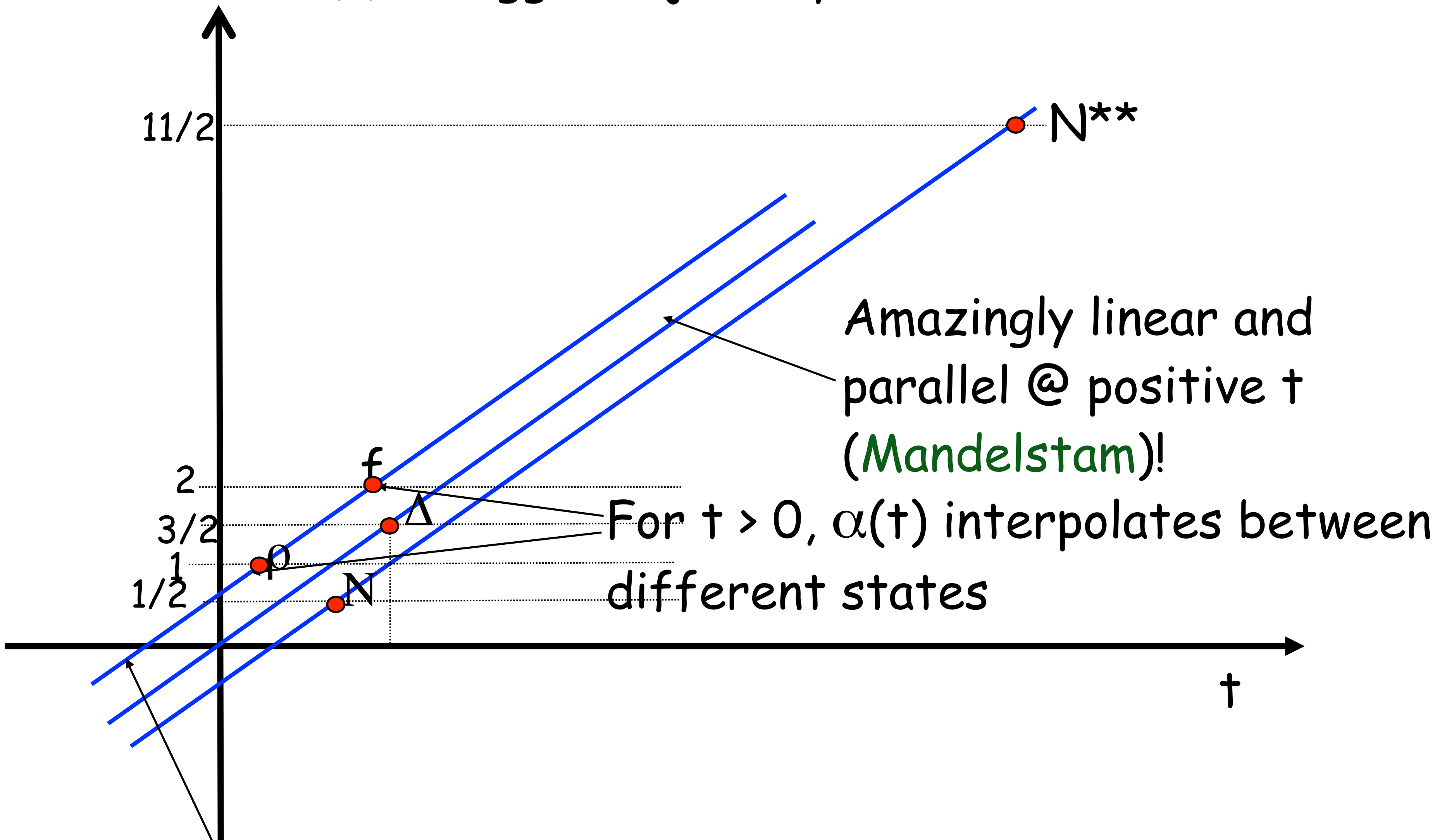
Chew's "expensive" bootstrap…

Add to the general constraints of symmetry, causality, unitarity of the S-matrix that of Nuclear Democracy:

All hadrons lie on Regge trajectories @ $t>0$;

All asymptotics fixed by the same trajectories @$t<0$

J = α(t) = Regge trajectory



For t < 0, α controls high-energy scattering at momentum transfer^2 ~ -t.

Will this give a unique S-matrix?

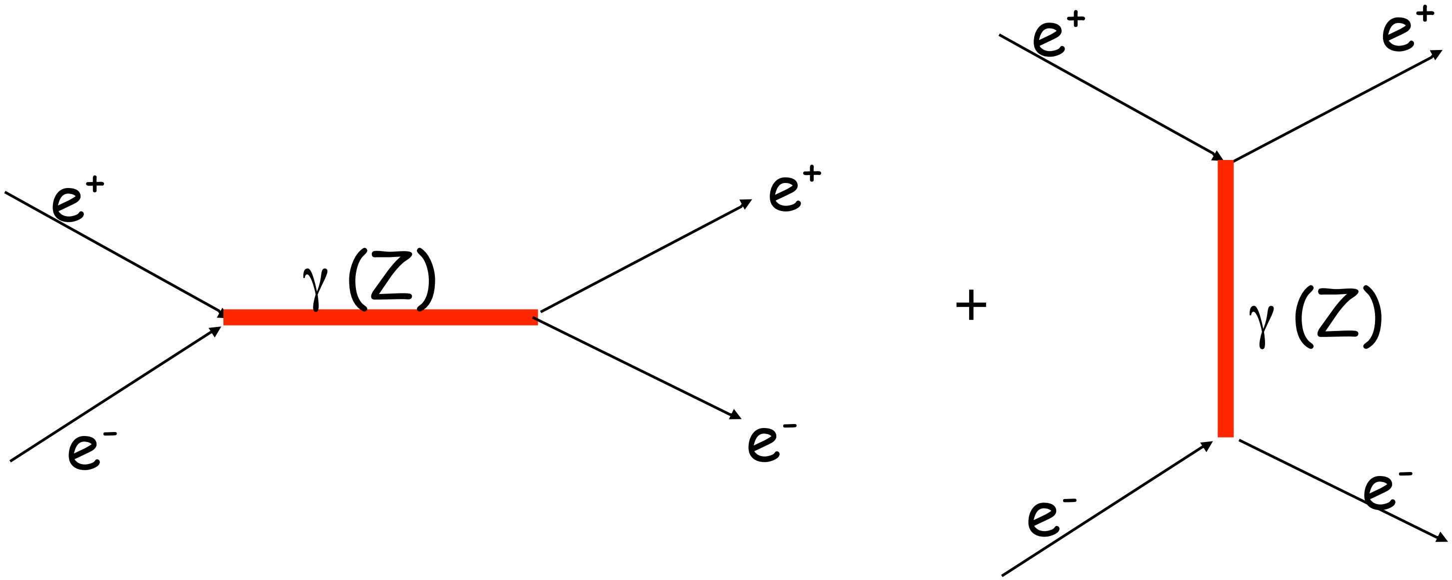
We now believe that the answer is no, but nuclear democracy came out to be true.

The S-matrix knew about both uses of Regge poles:

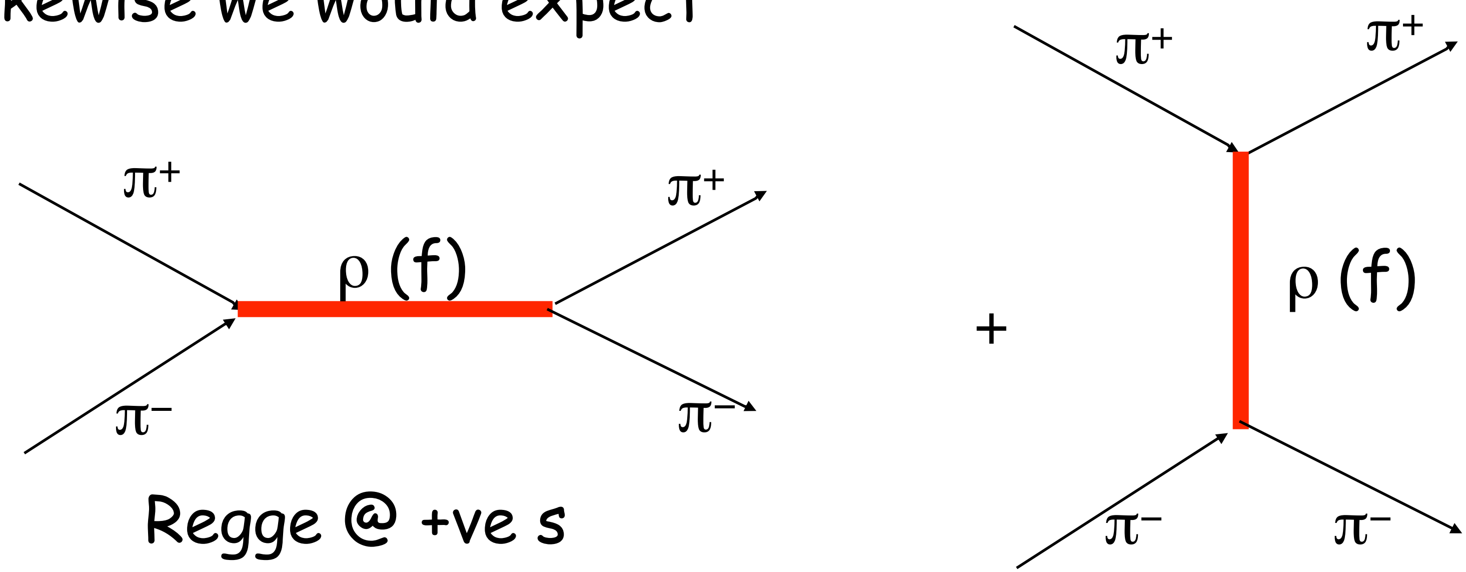
$$S = S_{s\text{-channel}} + S_{t\text{-channel}}$$

In QED $e^+ e^- \to e^+ e^-$ is given (to lowest order) by the coherent sum of 2 (actually 3) Feynman*) diagrams

*) whose 100th birthday happens to be just today!



Likewise we would expect



Regge @ +ve s

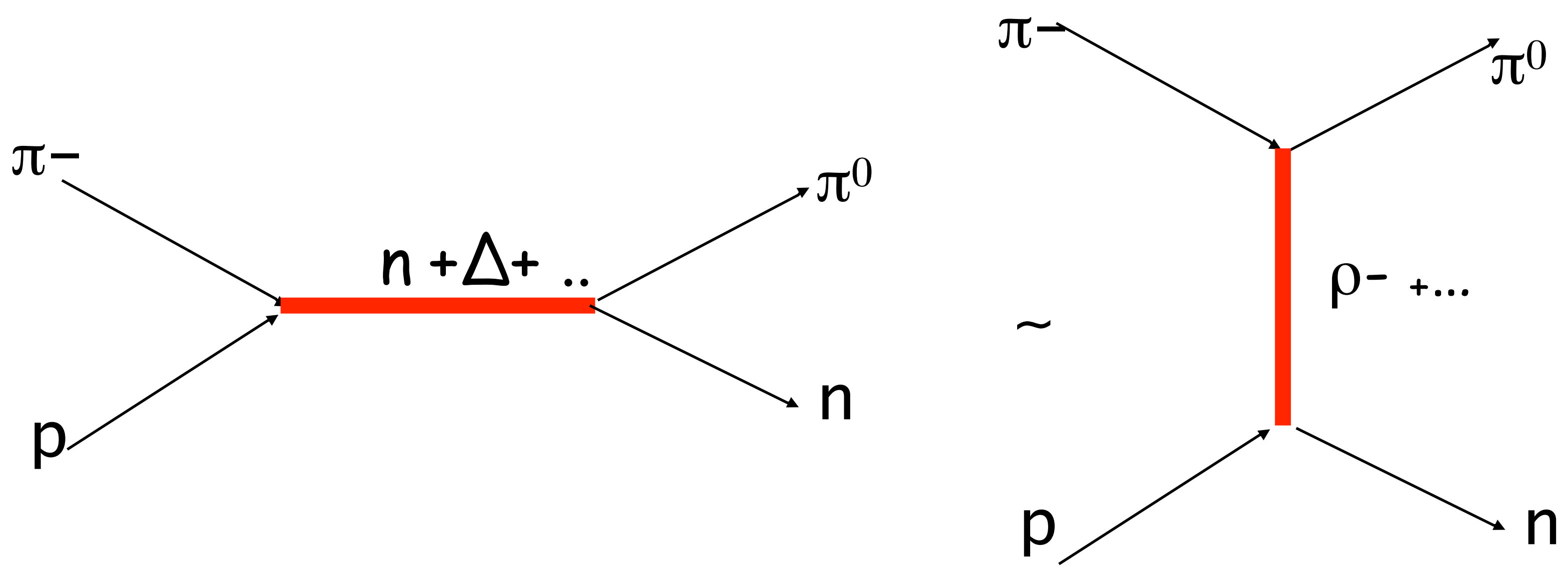
Regge @ -ve t

Erice summer 1967

M. Gell Mann bringing news from Caltech

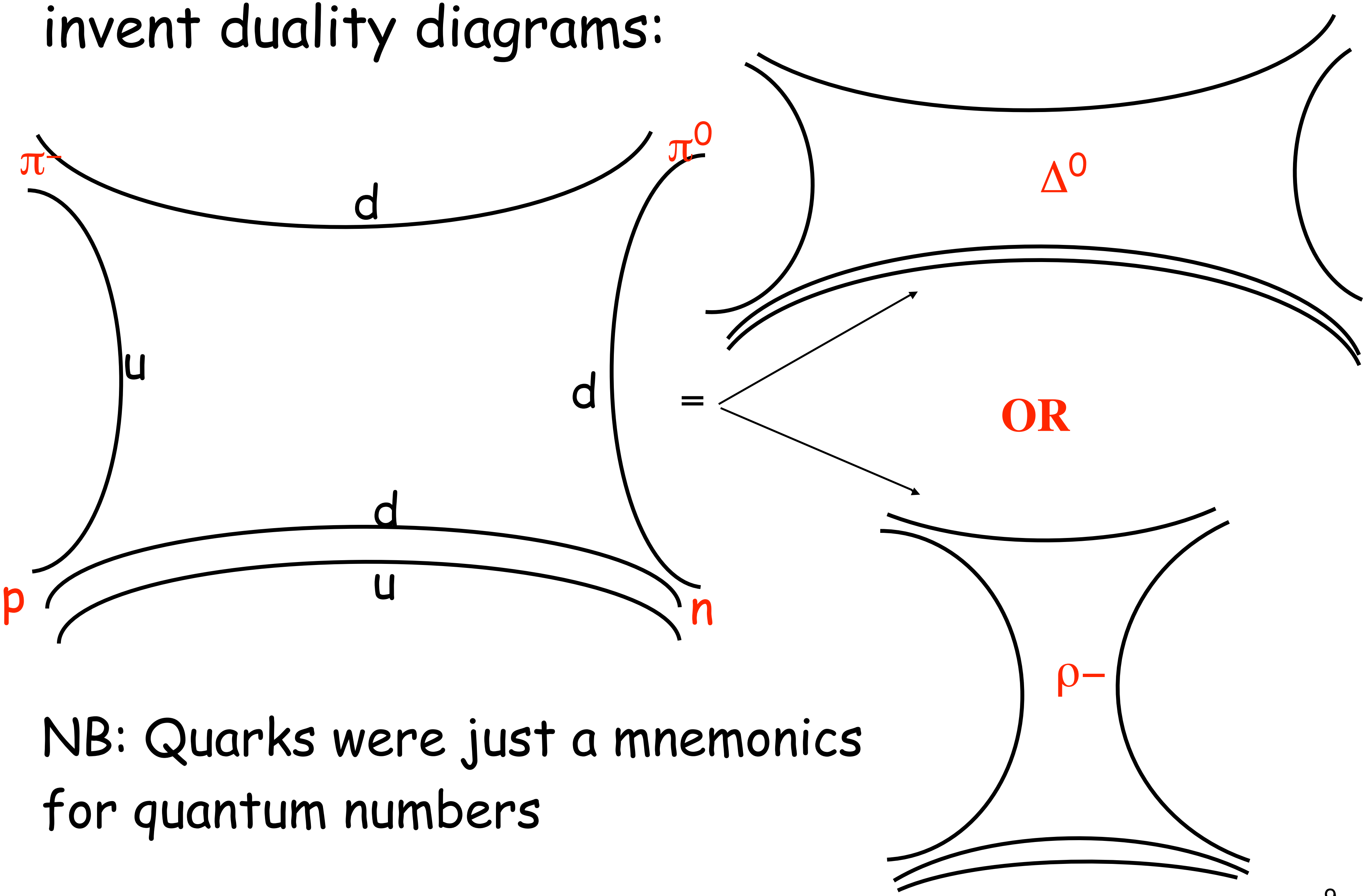
Dolen-Horn-Schmit (DHS) duality:

In $\pi^- p \rightarrow \pi^0 n$ s and t-channel descriptions are roughly (i.e. on average) equivalent, complementary, DUAL



A non-trivial, yet linear ("CHEAP") bootstrap…

DHS duality prompted Harari and Rosner to invent duality diagrams:



NB: Quarks were just a mnemonics for quantum numbers

2. FESR days @ Rehovot & Cambridge

The cheapest bootstrap is working (too?) well

π N scattering didn't look like the best choice
We* decided to consider a better process:

$\pi\,\pi \rightarrow \pi\,\omega$

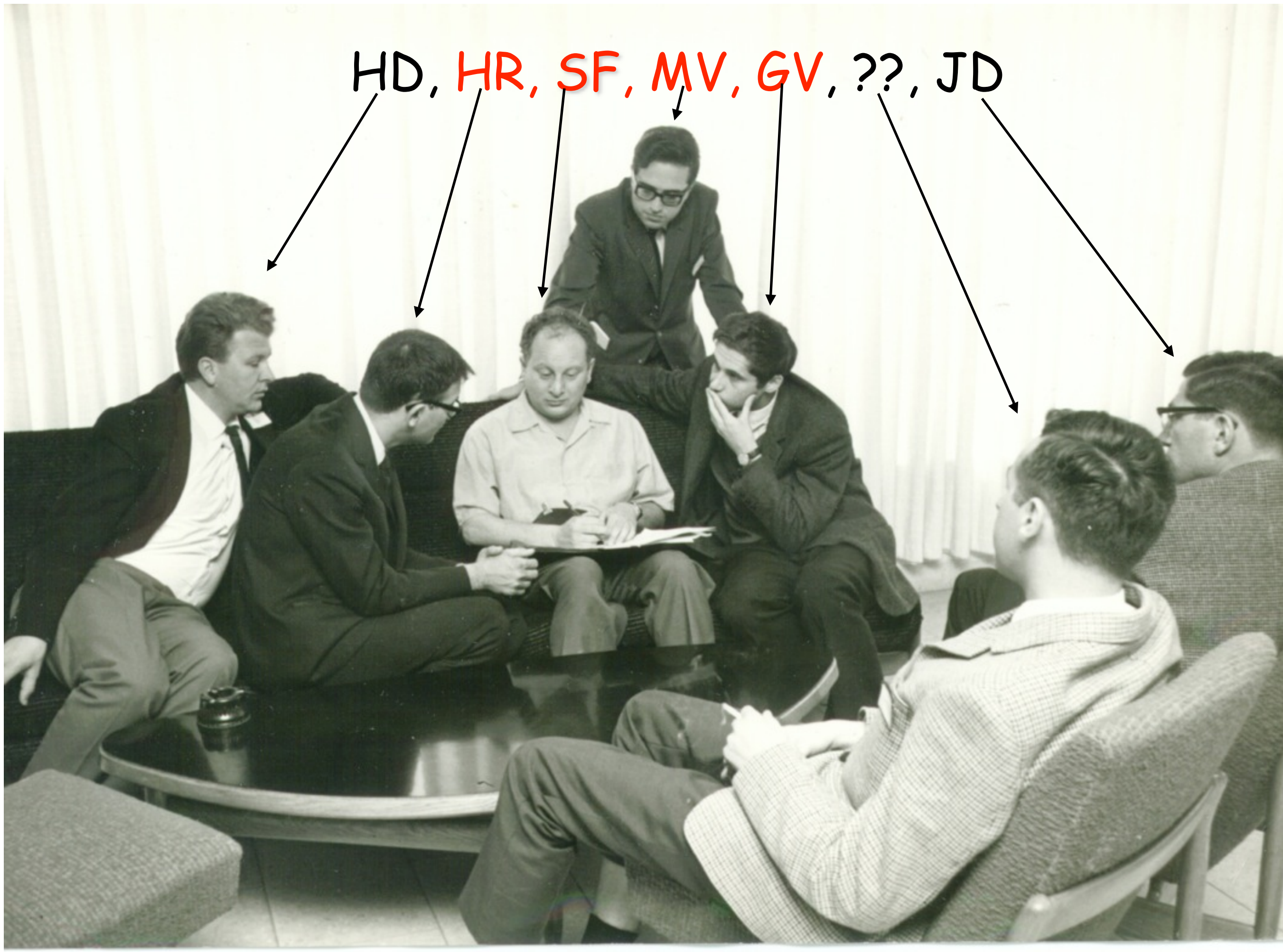
Very symmetric & selective in QN's (ρ, ρ^*..):
a real bootstrap!

Between the fall of 1967 and the summer of 1968 we* made much progress in finding accurate (yet approximate) solutions to this "cheap bootstrap".

Perhaps an exact solution was at hand…

*) Ademollo, Rubinstein, Virasoro, GV (+Bishari & Schwimmer) with much advice and encouragement by Sergio Fubini

Weizmann Institute's common room, winter '68?



3.The cheapest solution of the cheapest bootstrap

The cheap bootstrap was formulated in terms of Im *A*
The ansatz that worked amazingly well in $\pi\,\pi \to \pi\,\omega$ was:

$$Im\, A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))}(\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

with: $\beta(t) \sim const.,\ \alpha(t) = \alpha_0 + \alpha' t$

i.e. a linear leading Regge trajectory accompanied by parallel "daughters". Adding daughters enlarged the t-range of good agreement.

3 steps led from an approximate to an exact solution.

1. Look at A rather than Im A (A = analytic function)
2. Impose exact crossing symmetry: $A(s,t) = A(t,s)$
3. Emphasize resonances over Regge (A = meromorphic fnct.)

We can satisfy both 2. and 3. by simply writing:

$$A(s,t) = \beta \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} = \beta\, B(1-\alpha(s), 1-\alpha(t))$$

Its generalization to more than 4 external legs led to the so-called Dual Resonance Model (DRM).
By no means easy to "sell"!

4. DRM days: part I

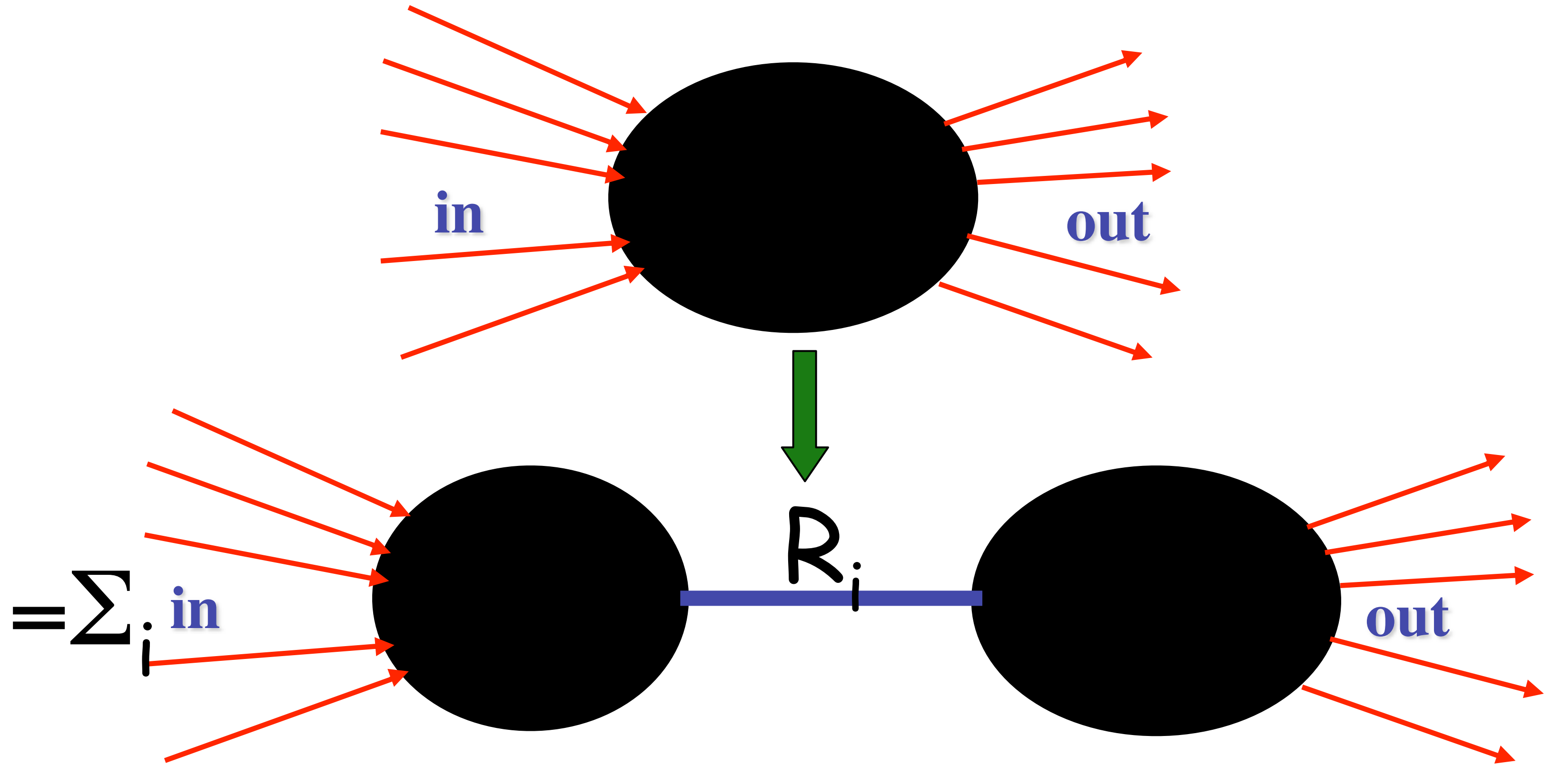
(see also next 3 talks)

Counting states

- There was a big worry based on previous experience (e.g. of SF): possibly, in order to satisfy all the constraints, the model had to contain "ghosts", states produced with negative probability. If so the model would have been inconsistent.
- To answer that question one had to identify first all the states. The way to do so was via a property of S, known as factorization.
- It is what unitarity reduces to in the single-particle-exchange approximation.

Factorization

Q: How many terms are needed (in the sum over i) in order to have, for all **in** and **out** states,



- This could not be done using just the Beta function, but, after a short while, in the fall of 1968, several people (BR, V, GS, CT, CP, KN) had found its (pretty unique) generalization to multi-particle initial and final states.
- The result on the counting of states (FV, BM, 1969) turned out to be very surprising.
- Because of the parallel daughters, we were expecting a mild degeneracy (increasing, say, like a power of M). Instead, the number of states grew much faster, like $\exp(b\, M)$, with b some constant (with dimensions 1/mass and of order $(\alpha')^{1/2}$).

- Although unexpected, this was just the behavior postulated by Hagedorn a few years earlier (~1965) on more phenomenological grounds (e.g. a Boltzmann factor in final particle spectra)
- And, sure enough, there were ghosts!
- The FV-BM factorization procedure was cumbersome. It was soon replaced by a much more handy operator formalism (FGV, Nambu)

$$|N_{n,\mu}\rangle \sim \prod_{n,\mu} \left(a^{\dagger}_{n,\mu}\right)^{N_{n,\mu}} |0\rangle \ , (n = 1, 2, \ldots ; \mu = 0, 1, 2, 3)$$

$$\alpha' M^2 = \sum_{n,\mu} n\, a^{\dagger}_{n,\mu}\, a_{n,\mu} \equiv L_0 - p^2$$

- In that formalism a sufficient set of states consisted of the energy levels of an infinite set of decoupled harmonic oscillators with quantized frequencies:

$$[a_{n,\mu} \ , \ a^{\dagger}_{m,\nu}] = \delta_{n,m}\eta_{\mu\nu} \ , \ \eta_{\mu\nu} = diag(-1, 1, 1, 1)$$

Because of the "wrong" sign of the timelike c.r., states created by an odd number of timelike operators were ghosts. Was the DRM doomed? Well, almost.

One (tiny?) hope remained: all those states were sufficient but perhaps only a (ghost-free) subset was necessary

In FV's original paper the following (so-called "spurious") states were found to be unnecessary

$$L_{-1}|X\rangle \equiv \left(p \cdot a_1^\dagger + \sum_n \sqrt{n(n+1)}\, a_{n+1}^\dagger \cdot a_n \right) |X\rangle$$ (with |X> any state)

This was probably sufficient to eliminate the ghosts created by the time component of a_1. But what about all others?

The situation looked almost desperate...until Virasoro (1969) made a crucial discovery. Iff $\alpha(0) = 1$ one could enlarge enormously the space of "spurious" states to:

$$L_{-m}|X\rangle \equiv \left(p \cdot a_m^\dagger + \sum_n \sqrt{n(n+m)}\, a_{n+m}^\dagger \cdot a_n \right) |X\rangle$$ (with m=1,2,..)

=> for $\alpha(0) = 1$, there was a chance to eliminate all the ghosts!!

$\alpha(0) = 1$ gives a massless J=1 state but people kept hoping...

Formal developments

Between the summer of 1969 and the spring of 1970 several developments took place:

1. Discovery (Gliozzi & Chiu-Matsuda-Rebbi) that $(L_0 , L_{\pm1})$ satisfy an SU(1,1) algebra.
2. Construction (FV and Gervais, 1969) of fields (Q(z)) and «Vertex Operators», V(k); their correlators, SU(1,1) action on them, as a result:
3. Duality, factorization and spurious/physical-state conditions all came out algebraically
4. After Virasoro's work, FV (1970) extended all this to the whole set of L_n and guessed (too) quickly their algebra… missing the crucial «central charge», soon pointed out by Joe Weis (Cf. FV's NAIP => Virasoro algebra)

Virasoro's crucial discovery

Virasoro never wrote the famous VA but
his discovery, a posteriori, was even more
fundamental than the algebra itself: this newly
discovered theory could only be consistent in the
presence of massless J=1,2 states.

It meant already the end of the hadronic string
and laid the basis for its reinterpretation as a
quantum theory of gauge interactions and gravity at a
deeper level.

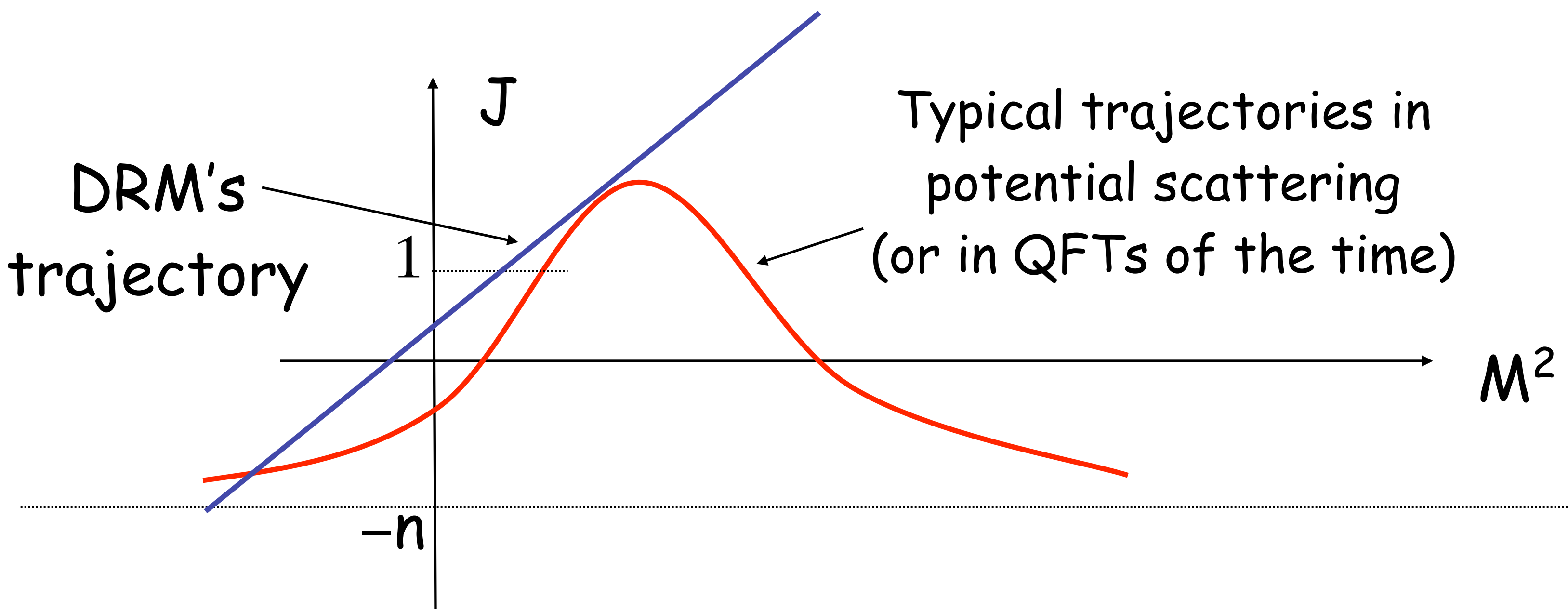
Towards the no-ghost theorem

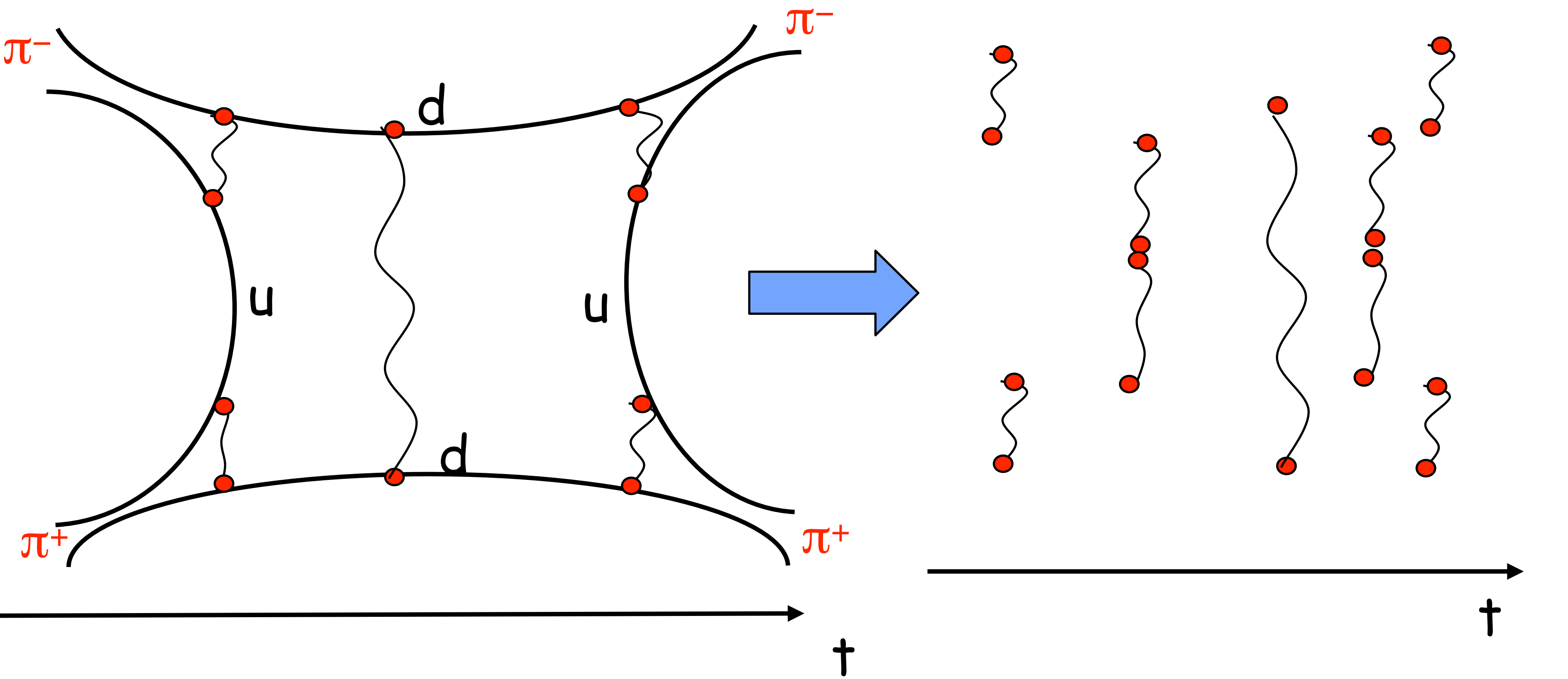
- At this point the machinery was almost ready for a final assault to the ghost-killing program;
- An essential step turned out to be the construction of the DDF (Di Vecchia, Del Giudice, Fubini) positive-norm states. They were in one-to-one correspondence with (D-2) sets of harmonic oscillators;
- Loops, Lovelace, and D= 26;
- A talk to the MIT mathematicians: no proof came out of them, but Kac-Moody algebras etc.
- The no-ghost theorem was proven instead by R. Brower and by P. Goddard & C.Thorn, see following talks.

5. Not an accident!

Hints of an underlying string

1. From linear Regge trajectories ($J \sim M^2 \Rightarrow M \sim L$)
2. From duality and duality diagrams
3. From the harmonic oscillators
4. From an underlying 2-d field theory
5. ……


J
DRM's trajectory
1
−n
M^2
Typical trajectories in potential scattering (or in QFTs of the time)



joining and splitting of strings ?

Strong interactions love strings!

1. From confinement (Cf. the String tension)
2. From large-N book-keeping (Riemann surfaces)
3. Duality should be (almost) automatic in large-N QCD (just need UDR @ some negative t, see AZ's talk?)
4. Even the fixed (unphysical) angle limit should look like the one of the DRM (see ZK's talk?)

Hopefully we will find out, one day, the true string of QCD, at least in the large-N_c limit!

And, possibly, it will look very much like:

$$A(s,t,u) = \frac{\bar{\beta}}{\pi}\left[B(1-\alpha(t), 1-\alpha(s)) + B(1-\alpha(t), 1-\alpha(u)) + B(1-\alpha(s), 1-\alpha(u))\right] \qquad (3)$$

where we have introduced the Euler B-function $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

Scanned from the original manuscript of the paper "Construction of a crossing symmetric, Regge behaved amplitude for linearly rising trajectories" by Gabriele Veneziano, 1968

in some kinematical region (it cannot be at large -ve t or at fixed angle because of asymptotic freedom): perhaps, also the new-bootstrap people should look at this lucky reaction…it worked, at least once!

That would close
a 50-years-old circle!

THANK YOU